Range, Radial Velocity, and Acceleration MLE Using Frequency Modulation Coded LFM Pulse Train

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Abstract—Linearly frequency modulation (LFM) pulse train and linearly stepped frequency (LSF) pulse train are mostly used in radar systems. However, the estimation performance of target motion parameters may be affected by the high recurrent lobe levels and the range-Doppler coupling phenomenon appearing in ambiguity function (AF). In multi-target scenarios, the estimation performance becomes even worse. The Costas frequency-modulation coded (CFMC) LFM pulse train has an ideal thumbtack-shaped AF, thus it can provide motion parameter estimation with high precision. However, the estimation of target motion parameter for the coherent CFMC LFM pulse train has not been investigated in depth. In this paper, we first analyze the properties of the AF of the CFMC LFM pulse train. Based on its convexity and narrow mainlobe width, a fast method to implement the maximum likelihood estimator (MLE) is proposed to estimate the motion parameters. To reduce the computation complexity, the Chirp-Z Transform (CZT) is introduced. Then, the Cramer-Rao lower bounds (CRLBs) on range, velocity and acceleration for frequency-modulation coded (FMC) pulse train are derived. It is shown that the CRLBs are relevant to the frequency coding patterns. Finally, Monte Carlo simulations are performed to verify the performance of the MLE. The results show that the performance of our proposed method can achieve the CRLBs when the signal-noise ratio (SNR) is higher than the threshold SNR.

Keywords—Frequency modulation coded radar, Costas codes, maximum likelihood estimation, Chirp-Z Transform.

I. INTRODUCTION

The fast and accurate estimation of target range, range rate (i.e., radial velocity), and

acceleration from sampled radar return signals is necessary for some radar applications. In radar imaging [1]-[6], the motion estimation with higher precision ensures better performance of the motion compensation to obtain the focused target images in synthetic aperture radar (SAR) and inverse synthetic aperture radar (ISAR) [7]-[11]. In the tracking of high altitude Earth-orbiting satellites [12] [13], it is essential for long-time integration realization to obtain reliable detection and position measurement.

Apparently, the high resolution waveform design is an effective method to improve the precision of motion estimation. As a category of high resolution signal, the frequency-modulation coded (FMC) pulse train is derived from frequency coded signals [14]-[23] and linearly frequency modulation (LFM) signals [24]-[27]. Meanwhile it combines advantages of both kinds of signals mentioned above, e.g., reducing the relatively high recurrent lobe levels in ambiguity functions (AF). AF is a two-dimensional function of time delay and Doppler frequency, and commonly used to analyze such inherent characteristics as the optimum resolution, the measurement precision of range, velocity and the ability of cluster rejection. The linearly stepped frequency-modulation (LSF LFM) pulse train, due to the high recurrent lobe levels and the range-Doppler coupling appearing in its AF, suffers from inaccurate estimation of the range and velocity [1]. In the 1960s, J.P. Costas [2] proposed a Costas array which has an ideal thumbtack-shaped AF and the minimum periodic sidelobes. Since then, varieties of new radar waveforms are widely explored [3]-[5], especially in the characteristics research of Costas coded sinusoidal pulse train and Costas frequency-modulation coded (CFMC) LFM pulse train.

The CFMC LFM pulse train has an ideal thumbtack-shaped AF, thus it can provide motion parameter estimation with high precision. Although many methods of target motion parameter estimations using other FMC pulse trains have been researched and proposed, the target motion parameter estimation for the coherent CFMC LFM pulse train has not been investigated in depth yet. The time-domain/frequency-domain velocity measurement method for the linearly stepped frequency (LSF) pulse train was studied in [28]. Also, a minimum entropy (ME) method was used to estimate the motion parameters by minimizing the entropy of the motion-compensated frequency domain signal after the matching filter [29]. However, a tradeoff between the computation complexity and parameter estimation accuracy needs to be considered. The high-precision

estimation requires large computation load, which makes it unavailable in real-time applications. Meanwhile, many methods have been developed, including the phase-gradient autofocus [30], methods based on range profiles of the prominent points [31] [32], keystone transform and generalized dechirp process (KTGDP) [33], inverse probability [34]. The maximum-likelihood estimator (MLE) method has been proved to be one kind of effective parameter estimation method, i.e., searching for the maximum of the nonlinear 3-dimension (3D) likelihood function [35]. In the 1960s, Bello [34] and Kelly [35] lead on estimating motion parameters using the MLE. Abatzoglou [36] further proposed a method to implement the MLE for joint estimation of range, radial velocity, and acceleration of a target using LFM pulse train. In [37], Deng extended the MLE method from slow-speed echo model to high-speed echo model. Jia Xu et al. [38] proposed Radon-Fourier transform (RFT) to realize the long-time coherent integration of the second-order-motion targets (range, radial velocity and acceleration). Furthermore, the generalized RFT (GRFT) [39] was defined for radar target detection with arbitrarily parameterized motions. However, the AF of LFM pulse train used in [36]-[39] suffers from high recurrent lobe levels, i.e., blind speed side lobe (BSSL), around the main peak, which has been discussed in [40]. Moreover, in [36], when using the Newton's method to implement the MLE, the coarse estimates are required to be located in the convergence region of the Newton's method, hence, it is only valid for moderate SNRs. When using the GRFT method to implement the MLE, the ergodic search for GRFT implementation along the whole parameter space of moving targets leads to a computation burden [39]. Although the improved methods [41] [42] were presented for the fast implementation of RFT, the particle swarm optimization (PSO) [41] is sensitive to the initial parameters set and easily gets trapped in the local optima when solving multi-targets problem. The Chirp-Z transformation (CZT) is used in pulse dimension, i.e., slow time dimension, to accelerate the MLE [42], but it becomes unsuitable when the carrier frequencies are coded by Costas array or other arrays, because the signal in pulse dimension does not represent a specific arcs any more.

On the other hand, there are other researchers who take the waveform design, property research and the motion parameter estimation into account. Along this direction, [43] put forward a novel twice bandwidth resynthesizing processing method to measure the velocity of high speed targets using cross stepped frequency sequence (CSF) pulse train. Inspired by reference [43], this paper

proposed a fast method for implementing the MLE to estimate the motion parameters using the CFMC LFM pulse train. To evaluate the performance of proposed MLE method, we examine it in four aspects, i.e., the hopping frequency patterns, the resolvability of multi-targets, the inherent accuracy limitations, i.e., the Cramer-Rao lower bound (CRLB), and the BSSL suppression phenomenon. Moreover, the CZT is used to accelerate the MLE by searching for motion parameters along the parameter space of interest, and different from the previous work [42], the CZT is used in range frequency domain in this paper, instead of slow-time dimension, which makes the CZT operation feasible when the FMC pulse train is used. Finally, Monte-Carlo simulations are performed to verify the performance of the MLE. It is shown that the proposed MLE can effectively suppress the BSSL and mitigate range-Doppler coupling problems, furthermore, overcome the drawback of the Newton's method and GRFT method.

The remainder of this paper is organized as follows. Section II provides background on the signal model. In Section III, the properties of the CFMC LFM pulse train are discussed and the MLE on range, velocity and acceleration is proposed. In Section IV the CRLB of FMC pulse train is derived. The fast MLE implementation using CZT is discussed in Section V. In Section VI, Monte-Carlo simulations are performed. Finally, Section VII concludes the paper briefly.

II. MATHEMATICAL MODEL

In this section, we modify and extend the signal model of the conventional LFM pulse train in [36] [38] to FMC pulse train, whose intra-pulse frequency is linearly modulated, and the inter-pulse frequency is coded instead of keeping constant. The basic steps are outlined below.

The complex envelope of the transmitted pulse is defined by

$$s(t) = \text{rect}(t/T) \exp(j\pi\gamma t^2) \quad (1)$$

where $\text{rect}(u) = \begin{cases} 1, & |u| < 1/2 \\ 0, & \text{else} \end{cases}$, T is the pulse width, and γ is the frequency sweep rate. The m th

transmitted signal can be expressed as

$$s_i(\hat{t}, t_m) = s(\hat{t}) \exp(j2\pi f_{c,m} \hat{t}) \quad (2)$$

where $\hat{t} = t - t_m$ is the fast time, t is the full time, $m = -M, \dots, 0, \dots, M-1$ is the transmitted

pulse index, $t_m = mT_r$ is the slow time, T_r is the pulse repetition interval, and $f_{c,m}$ is the coded carrier frequency. In this paper, a Costas array is used to code the inter-pulse frequency.

When receiving the radar echo signal, the mixer removes the high frequency signal components and the remaining baseband signal can be written as

$$s_{rb}(\hat{t}, t_m) = \hat{A}s(t - 2r(\hat{t}, t_m)/c) \exp(-j4\pi f_{c,m} r(\hat{t}, t_m)/c) + u(\hat{t}, t_m) \quad (3)$$

where $r(\hat{t}, t_m)$ represents the distance between the radar and target at time t_m , $u(\hat{t}, t_m)$ is the complex additive white Gaussian noise with zero mean and variance σ^2 at baseband, c is the speed of light, $\hat{A}$ is the complex backscattering coefficient of the target.

Sampling the m th baseband signal, we obtain

$$x_m(n) = \hat{A}s(nT_s - \tau_m) \exp(-j2\pi f_{c,m} \tau_m) + u_m(n) \quad (4)$$

where $s(nT_s)$ is the sampled signal of $s(t)$, $T_s = 1/f_s$ is the sampling period, n is the sampling index, $N = f_s T$ is the total sampling number and $\tau_m = 2r(\hat{t}, t_m)/c$ is the round-trip delay of the signal transmitted at t_m .

Then the matched filter is adopted at the output of analog digital conversion (ADC) and work in the range frequency domain, which is similar to references [36] [37]. In our case, the inter-pulse frequency is coded by a Costas array instead of keeping constant. The discrete Fourier transformation (DFT) of $x_m(n)$ is

$$\begin{aligned} X_m(k) &= \sum_{n=0}^{N-1} x_m(n) \exp(-j2\pi kn/N) \\ &= \hat{A} \sum_{n=0}^{N-1} s(nT_s - \tau_m) \exp(-j2\pi kn/N) \exp(-j2\pi f_{c,m} \tau_m) + U_m(k) \\ &= \hat{A} S(k) \exp(-j2\pi k \Delta f \tau_m) \exp(-j2\pi f_{c,m} \tau_m) + U_m(k) \end{aligned} \quad (5)$$

where $S(k) = \sum_{n=0}^{N-1} s(n) \exp(-j2\pi kn/N)$ is the DFT output of $s(n)$, $\Delta f = f_s/N$ is the sampling

frequency interval, $U_m(k) = \sum_{n=0}^{N-1} u_m(n) \exp(-j2\pi kn/N)$ is the DFT output of $u_m(n)$.

The output of matched filter is

$$X_{m,k}(k) = X_m(k) S^*(k) = \tilde{A} \exp(-j2\pi k \Delta f \tau_m) \exp(-j2\pi f_{c,m} \tau_m) + U_{m,k} \quad (6)$$

where $-N_b/2 \leq k \leq N_b/2 - 1$, N_b is the total frequency sampling number within the effective

bandwidth, $U_{m,k} = U_m(k) \exp(-j2\pi k \Delta f T/2) S^*(k)$ is the complex Gaussian random variable at the output of the matched filter, and $\tilde{A} = \hat{A} S(k) S^*(k) = A e^{j\theta}$ is treated as a deterministic unknown variable.

Consider that the round-trip delay of second-order motion of the target can be approximately expressed as

$$\tau_m \approx 2 \frac{r_0}{c} + 2 \frac{v_0}{c} t_m + \frac{a_0}{c} t_m^2 \quad (7)$$

where r_0 , v_0 and a_0 are respectively the initial distance, radial velocity, and acceleration between the radar and target at $t = 0$. It is assumed that the target follows the second-order motion model (range, radial velocity and acceleration), and this assumption can be reasonable by limiting the coherent integration time.

Therefore, formula (6) can be rewritten as

$$X_{m,k} = \tilde{A} \exp \left(-j \frac{4\pi}{c} (k \Delta f + f_{c,m}) \left(r_0 + v_0 t_m + \frac{1}{2} a_0 t_m^2 \right) \right) + U_{m,k} \quad (8)$$

III. MAXIMUM LIKELIHOOD ESTIMATION

Since the DFT operation is a linear operation, the real and imaginary parts of $U_{m,k}$ still follow the Gaussian distribution. Assuming that the complex white Gaussian noise samples are of variance σ'^2 , the joint posterior probability density function of their real and imaginary parts can be expressed as

$$p(U_{m,k} | A, \theta, r, v, a) = \frac{1}{(\pi \sigma'^2)^{N_b \times (2M+1)}} \times \exp \left(- \frac{\sum_{k,m} |X_{m,k} - \tilde{A} \exp(-j2\pi(k\Delta f + f_{c,m})\tau_m(r, v, a))|^2}{\sigma'^2} \right). \quad (9)$$

According to the definition of likelihood function and ignoring the constant term, the corresponding log-likelihood function is

$$L(A, \theta, r, v, a) = - \sum_{k,m} |X_{m,k} - A \exp(-j(\theta + 2\pi(f_c + k\Delta f)\tau_m(r, v, a)))|^2. \quad (10)$$

Maximizing (10) is equivalent to maximizing (11) by finding the proper r , v , a , when the amplitude A and the phase θ are ignored.

$$H(r, v, a) = \left| \sum_{k,m} X_{m,k} \exp(j2\pi(k\Delta f + f_{c,m})\tau_m(r, v, a)) \right|. \quad (11)$$

$$(\hat{r}, \hat{v}, \hat{a}) = \arg(\max_{r,v,a} H(r, v, a)). \quad (12)$$

In order to obtain the motion parameter estimation $\hat{r}$, $\hat{v}$, and $\hat{a}$, it is necessary to perform a search over the whole parameter grids. The ergodic research for second-order GRFT implementation of MLE [38]-[41] can be used here. However, given that the ergodic research brings large computation cost, a CZT implementation of MLE is introduced in section V. In the remainder of this section, we provide detailed comparisons between the MLE used in [36] [38]-[41] and our MLE on four aspects, i.e., the hopping frequency patterns, the resolvability of multi-targets, the inherent accuracy limitations, i.e., the CRLB, and the BSSL phenomenon.

A. Comparison of the hopping frequency patterns

From (11), it is seen that $H(r, v, a)$ is relative to the coding arrays which are used to code the inter-pulse frequency, and the different coding arrays lead to different $H(r, v, a)$ profiles, that is, (11) provides a unified framework of the likelihood function using different coding arrays for the moving target with second-order motion. Note that if $f_{c,m}$ keeps constant during the coherent integration time, the MLE is the same with the one used in [36] [38]-[41], as a special case of second-order GRFT [39], namely the MLE of the LFM pulse train afterwards. In our case, a Costas array is used to code the inter-pulse frequency of the pulse train. More accurately, the likelihood function of a CFMC LFM pulse train can be rewritten as:

$$H(r, v, a) = \left| \sum_{k,m} X_{m,k} \exp(j2\pi(k\Delta f + f_{c,\min} + (C_m - 1)\Delta f_c)\tau_m(r, v, a)) \right|. \quad (13)$$

where C_m is the Costas codes, and Δf_c is the interval of the inter-pulse hopping frequency.

Costas array is one of the widely used sequences to code the inter-pulse frequency of pulse train, and is one special case of random codes. There exist a number of analytical procedures for constructing Costas frequency hopping arrays, for example, the Welch construction and the Lempel-Golomb construction method. Although Costas arrays may exist in principle for any positive integer N_F (in our case, $N_F = 2M + 1$ represents the pulse number), these analytical construction methods are typically limited to values of N_F related to prime numbers [44] [45]. In this paper,

the total pulse number is limited to a positive integer for which a Costas array can be constructed by Welch construction method, including type 1: when $N_F = n - 1$, type 2: when $N_F = n - 2$, type 3: when $N_F = n - 3$, where n is an odd prime.

By the way, if C_m is a continuous positive integer sequence, Eq. (13) becomes the likelihood function of the LSF LFM pulse train. In this way, It can be concluded that the MLEs of LFM pulse train, LSF pulse train and CFMC LFM pulse train are all special cases of the unified MLE (12), if the moving target follows the second-order motion model.

Fig. 1 shows the comparison of the global distribution of likelihood function $H(r, v, 0)$ along range-velocity plane when using the LFM pulse train, the LSF LFM pulse train and the CFMC LFM pulse train, respectively. It is interesting that the LFM effect existing in a single LFM pulse which results in sheared ambiguity function contour [1] disappears in the likelihood function $H(r, v, 0)$ of the LFM pulse train. However, the sheared ambiguity function contour, i.e., the delay-Doppler coupling phenomenon, can be observed in the $H(r, v, 0)$ for the LSF LFM pulse train. For the CFMC LFM pulse train case, its $H(r, v, 0)$ profile is thumbtack-shaped, which consists of a main peak with lower sidelobes. Comparing with the LSF LFM pulse train, the CFMC LFM pulse train avoids the delay-Doppler coupling which is brought in when the inter-pulse frequency linearly varies. In this way, the CFMC LFM pulse train can achieve locally optimal in the region surrounding the main lobe and obtain high accuracy of estimate for range and velocity. And its convexity and narrow mainlobe width near the origin are important for designing an implementation of a MLE including a coarse search and a fine search. The convexity of likelihood function can be used in the fine search via recursive algorithms, e.g., the Newton's method or local spectrum estimation techniques, e.g., CZT.

Fig. 1 The global distribution of range-velocity $H(r, v, 0)$ of (a) the LFM pulse train (upper), (b) the LSF LFM pulse train (middle), and (c) the CFMC LFM pulse train (bottom).

B. Comparison of the resolvability of multi-targets

According to AF theory, the resolvability of multi-targets is reflected by the main peak width of the AF. In order to distinguish targets which are close to each other, the AF is required to be isolate and thumbtack-shaped. Meanwhile, no other peaks are allowed to exist around the main peak, because they may cover other targets nearby. The ability to distinguish multi-targets can be intuitively observed from Fig. 1. The likelihood function of the CFMC LFM pulse train is nearly isolate and thumbtack-shaped, comparing with that of the LFM pulse train.

C. Comparison of CRLBs

It is known that the measurement precisions of the unknown parameters depend on the “sharpness” of the likelihood function, namely the curvature, the second derivative of the likelihood function [46]. Intuitively, from Fig. 1, measurement precisions of motion parameters using the CFMC LFM pulse train is obviously superior to the one using the LFM pulse train. Quantitatively, the difference of the CRLBs of motion parameters will be derived and simulated in section IV.

D. Comparison of BSSL phenomenon

The BSSL phenomenon can be easily observed for the case of LFM and LSF LFM pulse train signals. Although the BSSLs are reduced with the increase of the blind-speed integer, they may still lead to false alarms or loss detections in a real multi-target scenario. In the contrast, the BSSL phenomenon is effectively suppressed for the CFMC LFM pulse train case as shown Fig. 1. Actually, when $f_{c,m}$ keeps constant, labeled as f_{const} , during the integration time [40], the maximum unambiguity speed is calculated by $c / (2f_{const}T_r)$, while when $f_{c,m}$ is coded by random array, the ambiguity speed occurs at $v_{f_{c,m}} = c / (2T_r)$. The randomness of the coding array breaks the periodicity of recurrent lobe levels.

IV. CRLBs OF RANGE, VELOCITY, AND ACCELERATION FOR THE FMC WAVEFORM

The CRLBs of motion parameters estimation for LFM pulse train are derived in [36] [39]. In our work, we extend it to be applied for the FMC pulse train, and compare the difference of CRLBs between the LFM pulse train, the LSF LFM pulse train and the CFMC LFM pulse train using

simulations.

The likelihood function of the FMC pulse train can be defined as

$$F(A, \theta, r, v, a) = -\frac{1}{\sigma^2} \sum_{m,k} |X_{m,k} - D_{m,k}|^2 = -\frac{1}{\sigma^2} \sum_{m,k} (X_{m,k} - D_{m,k})(X_{m,k} - D_{m,k})^* \quad (14)$$

where $D_{m,k} = Ae^{j\theta} \exp(j4\pi(k\Delta f + f_{c,m})(r + vt_m + at_m^2/2)/c)$. The elements of the Fisher matrix are

$$\mathbf{J}_{A,A} = \frac{2N_b \times (2M+1)}{\sigma'^2} \quad (15)$$

$$\mathbf{J}_{A,\theta} = \mathbf{J}_{\theta,A} = -E \left\{ \frac{\partial^2 F}{\partial \alpha \partial \theta} \right\} = 0 \quad (16)$$

$$\mathbf{J}_{A,r} = \mathbf{J}_{r,A} = -E \left\{ \frac{\partial^2 F}{\partial \alpha \partial r} \right\} = 0 \quad (17)$$

$$\mathbf{J}_{A,v} = \mathbf{J}_{v,A} = -E \left\{ \frac{\partial^2 F}{\partial \alpha \partial v} \right\} = 0 \quad (18)$$

$$\mathbf{J}_{A,a} = \mathbf{J}_{a,A} = -E \left\{ \frac{\partial^2 F}{\partial \alpha \partial a} \right\} = 0 \quad (19)$$

$$\mathbf{J}_{\theta,\theta} = -E \left\{ \frac{\partial^2 F}{\partial \theta^2} \right\} = \frac{2A^2}{\sigma'^2} 2N_b (2M+1) \quad (20)$$

$$\mathbf{J}_{\theta,r} = \mathbf{J}_{r,\theta} = -E \left\{ \frac{\partial^2 F}{\partial \theta \partial r} \right\} = \frac{8\pi A^2}{c\sigma'^2} \sum_{m,k} (f_{c,m} + k\Delta f) \quad (21)$$

$$\mathbf{J}_{\theta,v} = \mathbf{J}_{v,\theta} = -E \left\{ \frac{\partial^2 F}{\partial \theta \partial v} \right\} = \frac{8\pi A^2}{c\sigma'^2} \sum_{m,k} (k\Delta f + f_{c,m}) m T_r \quad (22)$$

$$\mathbf{J}_{\theta,a} = \mathbf{J}_{a,\theta} = -E \left\{ \frac{\partial^2 F}{\partial \theta \partial a} \right\} = \frac{8\pi A^2}{c\sigma'^2} \sum_{m,k} (k\Delta f + f_{c,m}) m^2 T_r^2 / 2 \quad (23)$$

$$\mathbf{J}_{r,r} = -E \left\{ \frac{\partial^2 F}{\partial r \partial r} \right\} = \frac{32\pi^2 A^2}{c^2 \sigma'^2} \sum_{m,k} (f_{c,m} + k\Delta f)^2 \quad (24)$$

$$\mathbf{J}_{r,v} = \mathbf{J}_{v,r} = -E \left\{ \frac{\partial^2 F}{\partial r \partial v} \right\} = \frac{32\pi^2 A^2}{c^2 \sigma'^2} \sum_{m,k} (f_{c,m} + k\Delta f)^2 m T_r \quad (25)$$

$$\mathbf{J}_{r,a} = \mathbf{J}_{a,r} = -E \left\{ \frac{\partial^2 F}{\partial r \partial a} \right\} = \frac{32\pi^2 A^2}{c^2 \sigma'^2} \sum_{m,k} (f_{c,m} + k\Delta f)^2 m^2 T_r^2 / 2 \quad (26)$$

$$\mathbf{J}_{r,a} = \mathbf{J}_{a,r} = -E \left\{ \frac{\partial^2 F}{\partial r \partial a} \right\} = \frac{32\pi^2 A^2}{c^2 \sigma'^2} \sum_{m,k} (f_{c,m} + k\Delta f)^2 m^2 T_r^2 / 2 \quad (27)$$

$$\mathbf{J}_{v,a} = \mathbf{J}_{a,v} = -E \left\{ \frac{\partial^2 F}{\partial v \partial a} \right\} = \frac{32\pi^2 A^2}{c^2 \sigma'^2} \sum_{m,k} (k\Delta f + f_{c,m})^2 m^3 T_r^3 / 2 \quad (28)$$

$$\mathbf{J}_{a,a} = -E \left\{ \frac{\partial^2 F}{\partial a^2} \right\} = \frac{32\pi^2 A^2}{c^2 \sigma'^2} \sum_{m,k} (k\Delta f + f_{c,m})^2 m^4 T_r^4 / 4 \quad (29)$$

Taking the inverse of the Fisher matrix, we obtain the CRLB matrix $\mathbf{I}$, and the CRLBs of r , v , a can be obtained as

$$E \{ (r - r_0)^2 \} \geq \frac{\mathbf{I}_{v,v} \mathbf{I}_{a,a} - \mathbf{I}_{v,a} \mathbf{I}_{a,v}}{|\mathbf{I}|} \quad (30)$$

$$E \{ (v - v_0)^2 \} \geq \frac{\mathbf{I}_{r,r} \mathbf{I}_{a,a} - \mathbf{I}_{r,a} \mathbf{I}_{a,r}}{|\mathbf{I}|} \quad (31)$$

$$E \{ (a - a_0)^2 \} \geq \frac{\mathbf{I}_{r,r} \mathbf{I}_{v,v} - \mathbf{I}_{r,v} \mathbf{I}_{v,r}}{|\mathbf{I}|} \quad (32)$$

From (30), (31) and (32), we can see that the coding arrays can affect the computation of CRLBs. When $f_{c,m}$ keeps constant, the CRLBs derivation of motion parameters becomes the same of that proposed in [36] [39]. Fig. 2 intuitively reflects the CRLBs difference caused by different kinds of coding arrays, including the LFM pulse train, the LSF LFM pulse train, and the CFMC LFM pulse train. It is interesting that the theoretical performance of range estimation for the LFM pulse train is similar to that for the LSF waveform, but is poorer than that of the CFMC LFM pulse train, which has intuitively been explained from the likelihood function $H(r, v, 0)$ shown in Fig. 1. While the theoretical performance of velocity estimation for the LFM pulse train is similar to that for the CFMC waveform. Since the LSF LFM pulse train has the delay-Doppler coupling phenomenon, the theoretical estimation performance of the LSF pulse train is poorer than that of the CFMC LFM pulse train both in the range and velocity dimensions. However, in the acceleration dimension, the CRLBs are not sensitive to the different coding arrays, the CRLBs for these three kinds of pulse train tend to be the same.

The length of coding array used in Fig. 2 is 35. The length of Costas array influences the CRLBs. Fig. 3 qualitatively shows the CRLBs of range, velocity and acceleration estimates when the length of Costas codes is respectively 15 and 35. Besides, a quantitative result can be derived from (30), (31) and (32).

Fig. 2 CRLBs of (a) range, (b) velocity and (c) acceleration estimation.

Fig. 3 The influence of the length of Costas codes on CRLBs of (a) range estimation (upper), (b) velocity estimation (middle), and (c) acceleration estimation (bottom)

V. FAST MLE IMPLEMENTATION USING CHIRP-Z TRANSFORM

The RFT implementation of MLE of range, radial velocity, and acceleration [38] [39] needs to search for the maximum in a 3D likelihood function, thus the computational load is very heavy. In order to reduce the computation complexity, the CZT is used to implement the MLE. The CZT method has been used for fast RFT implementation [42], but it is used in slow-time dimension, instead of the range frequency domain, which makes the CZT operation unavailable when using the FMC pulse train. The function of the CZT operation is to explore uniformly-spaced points along a specific arcs in the z plane, however, the signal in slow-time dimension, $\exp(j4\pi f_{c,m}vmPRI/c)$ does not represent a specific arcs any longer, because the inter-frequency is coded.

Assuming the search step of range, velocity and acceleration are separately Δr , Δv and Δa , the notation $[r_{\min}, r_{\max}]$, $[v_{\min}, v_{\max}]$, $[a_{\min}, a_{\max}]$ represent the search grid of r , v , a , and can be discretely expressed as $n_r\Delta r$, $n_v\Delta v$ and $n_a\Delta a$, where n_r , n_v and n_a are respectively the search index along range, velocity and acceleration grid. N_r , N_v and N_a are the total search number over range, velocity and acceleration dimensions. Then likelihood function (11) can be rewritten as

$$H(n_r, n_v, n_a) = \left| \sum_{k,m} X_{m,k} C_{m,k} \exp\left(j \frac{2\pi k n_r}{N_r}\right) \exp\left(j \frac{4\pi}{c} f_{c,m} n_r \Delta r\right) \right| \quad (33)$$

where

$$C_{m,k} = \exp\left(j \frac{4\pi}{c} (k\Delta f + f_{c,m})(n_v\Delta v m T_r + n_a\Delta a m^2 T_r^2)\right) \quad (34)$$

$$N_r = \frac{c}{2\Delta f \Delta r}. \quad (35)$$

It is straight-forward to implement the MLE (33) using DFT. However, comparing with the DFT, the CZT operation is able to explore uniformly-spaced points along a specific arcs in the z plane, instead of sampling the z plane uniformly along the whole unit circle, such that less frequency sampling points are needed to meet the required estimation accuracy.

The CZT of $x(n)$ is defined as

$$X(k) = \text{CZT}_{\theta_0, \varphi_0, M}(x(n)) = \sum_n x(n) z_k^{-n} = \sum_n x(n) A_0^{-n} W^{nk} \quad (36)$$

where $k = 0, \dots, M' - 1$ is the searched frequency index, $A_0 = e^{-j\theta_0}$, $W = e^{-j\varphi_0}$, θ_0 is the phase of the first sampling point z_0 , and φ_0 is the phase angle step between two sampling points in the z plane.

Considering (33) and (36), the CZT can simplify the likelihood function as

$$H(n_r, n_v, n_a) = \left| \sum_m Q_{m,n_r} \exp\left(j \frac{4\pi}{c} f_{c,m} n_r \Delta r\right) \right| \quad (37)$$

where Q_{m,n_r} is the output of Nr' point CZT of $X_{m,k} C_{m,k}$, $A_0 = e^{-j\theta_0} = \exp(j4\pi r_{\max} \Delta f / c)$, and $W = e^{-j\varphi_0} = \exp(j4\pi \Delta r \Delta f / c)$.

Next, we briefly analyze the difference of computation complexity by using DFT and CZT. It is given that the required range accuracy is Δr , the length of signal $X_{m,k} C_{m,k}$ is N_b , the corresponding frequency sampling interval is Δf . When using DFT to implement (33), the search size of range need to be $N_r = \frac{c}{2\Delta f \Delta r}$. N_r point-DFT requires N_r complex multiplications and $N_r - 1$ complex additions. While using CZT, in general, the search size of range N'_r can be significantly smaller than N_r , because CZT only focuses on the range grid of interest. Since CZT can be implemented using three times DFT, hence, three times $L = N'_r + N_b - 1$ point-DFT are required, consisting of $3(N'_r + N_b - 1)$ complex multiplications and $3(N'_r + N_b - 2)$ complex additions. We only analyze the computation complexity when searching along the range dimension above. When the computational costs of the ergodic search along the other two dimensions (radial velocity and acceleration) are added, it takes $N_v \times N_a \times N_r$ complex multiplications and $N_v \times N_a \times (N_r - 1)$ complex additions for the case of using DFT, while $N_v \times N_a \times 3(N'_r + N_b - 1)$ complex multiplications and $N_v \times N_a \times 3(N'_r + N_b - 2)$ complex additions for the case of using CZT. It is shown that using CZT instead of DFT in range frequency dimension greatly reduce the computation burden for implementing MLE.

Table 1 Comparison of algorithm complexity of DFT and CZT

Methods	Computation Costs	
	Complex multiplications	Complex additions
DFT	$N_v \times N_a \times N_r$	$N_v \times N_a \times (N_r - 1)$
CZT	$N_v \times N_a \times 3(N'_r + N_b - 1)$	$N_v \times N_a \times 3(N'_r + N_b - 2)$

Table 2 The radar parameters and target motion parameters

Radar parameters	Bandwidth	5 MHz
	Pulse width	25 μs
	Pulse repetition interval	0.0033s
	Carrier frequency step	5 MHz
	Sampling frequency	10 MHz
	Minimum carrier frequency	5.6 GHz
	Maximum carrier frequency	5.635 GHz
Target motion parameters	The initial range	300 km
	The initial velocity	3000 m / s
	The initial acceleration	10 m / s ²

VI. SIMULATIONS

Simulations are conducted to characterize the performance of the MLE, which uses CZT to accelerate the MLE for LFM pulse train and CFMC LFM pulse train. 1000 Monte Carlo trials per SNR are conducted to compute the root mean squared errors (RMSEs) of the estimates of range, radial velocity and acceleration. Besides, the Newton's method [36] with the LFM pulse train, the CFMC LFM pulse train, the RFT method [38] with the LFM pulse train are implemented as a comparison. The RMSEs of parameter estimation of the RFT method for the CFMC LFM pulse train are not provided here, since the computation load of the RFT for the CFMC LFM pulse train is very heavy. On the other hand, the fast implementation method proposed in [42] is unsuitable for the CFMC LFM pulse train, which has been explained above. Table 2 gives the radar parameters

and target motion parameters used in the simulations.

Fig. 4 shows the simulation results of the RMSEs of the estimators and the CRLBs for range, radial velocity and acceleration at different SNRs, and the length of Costas codes used is 35. When the length of Costas code equals 15, the results are given as a comparison in Fig. 5. The simulation results show that the MLE method used for the CFMC LFM pulse train can achieve the CRLBs with properly set search step Δr , Δv and Δa . Meanwhile, the CZT method is more tolerate for low SNR, comparing with the Newton's method, which requires the accuracy of the initial parameter estimates located at the convergence domain. The CZT method is suitable for FMC pulse train with high efficiency, compared with the RFT method.

The RMSEs of the MLE for range, radial velocity and acceleration on the cases of the LFM pulse train, the LSF LFM pulse train and the CFMC LFM pulse train are shown in Fig. 6, respectively. The carrier frequency of the LFM pulse train is $f_c = 5.6\text{GHz}$ and the length of coding array for the CFMC LFM pulse train is 35. It is shown that the CFMC waveform provides the highest precision for range and velocity estimation, and the RMSEs of motion parameter estimations are matched with the theoretical CRLBs results.

We only discuss the single target scenario above. Without loss of generality, the proposed method can be extended to multi-target detection scenario. Once the coarse parameter estimations of each moving target are given by using the CLEAN technique, the fine estimations can be obtained by respectively using CZT in the each search grid of interest. The difficulty in the multi-target scenario is that the determination of the number of targets and the impact of the spectrum leakage from large targets to the small targets in the field of radar detection. This will be our future work.

Fig. 4 The root mean square error (RMSE) of (a) range estimation (upper), (b) velocity estimation (middle) and (c) acceleration (bottom) estimation when the length of Costas codes used is 35.

Fig. 5 The root mean square error (RMSE) of (a) range estimation (upper), (b) velocity estimation (middle) and (c) acceleration (bottom) estimation when the length of Costas codes used is 15.

Fig. 6 The root mean square error (RMSE) of (a) range estimation (upper), (b) velocity estimation (middle) and (c) acceleration (bottom) estimation in the context of the LFM pulse train, the LSF LFM pulse train and CFMC LFM pulse train.

VII. CONCLUSIONS

The CFMC LFM pulse train with ideal thumbtack-shaped ambiguity function can provide better range resolution and higher motion parameter estimation accuracy than other types of FMC pulse train used in modern radar systems. According to the convexity and narrow mainlobe width properties of its likelihood function, an implementation of the MLE algorithm in terms of a coarse search and a fine search is designed. In the fine search step, the CZT is used to compute the local likelihood function surrounding the main lobe, thus the computational cost is reduced. The CRLBs of motion parameter estimation for FMC pulse trains are derived and it is shown that the CRLBs are relevant to the frequency modulation patterns. The simulation results show that the performance of the proposed method can achieve the CRLB when the signal-to-noise ratio (SNR) is higher than the threshold SNR.

ACKNOWLEDGEMENT

The research was supported by the National High-tech R&D Program (863) of China, the Open-End Fund National Laboratory of Automatic Target Recognition (ATR), the National Science Foundation (No. 61471012) and the Open-End Fund of BITTT Key Laboratory of Space Object Measurement and the Fundamental Research Funds for Xiamen University (Grant No. 20720160105).

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