Fast L_1 Regularized Iterative Forward Backward Splitting with Adaptive Parameter Selection for Image Restoration

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Abstract

We consider the L_1 regularized iterative forward backward splitting (IFBS) algorithm for image restoration. The main aim of the paper is to develop a fast and adaptive method with an automatic selection of regularization parameter. The regularization parameter is updated in each iteration without requiring the initial value of the parameter. We establish analytically monotonicity results of the adaptive parameter. Such an algorithm corresponds to solving an adaptive iterative thresholding process. Experimental results demonstrate that the adaptive parameter method is efficient and provide competitive performance.

Keywords: L_1 Regularization; Regularization Parameter; Splitting methods; Image Restoration.

1. Introduction

Image restoration is a classical linear inverse problem, which plays an important role in the area of image processing such as medical imaging, astronomy, remote sensing, etc, see, for instance, [1, 2, 3, 4, 5, 6, 7, 8]. A

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basic linear inverse problem leads us to study the following linear system

$$f = Au + \eta \quad (1)$$

where A is a linear operator representing, for the deblurring problem, the blur point spread function (PSF). η is a white Gaussian noise with variance σ^2 , $f \in \Omega$ represents the blurred image and $u \in \Omega \subset R^2$ is the unknown true image. Many linear inverse problems are ill-posed, i.e., either the linear operator does not admit inverse or it is near singular, yielding highly noise sensitive solutions. One of the techniques to avoid this problem is the regularization framework which solves the following minimization problem:

$$\min_u \{\lambda J(u) + H(u)\} \quad (2)$$

where $J(u)$ is the regularization term which characterizes some features of the desired solution u , $H(u)$ is the fidelity term given by $H(u) = \frac{1}{2}\|f - Au\|_{L^2}^2$ which measures the difference between f and Au . $\lambda > 0$ is a regularization parameter tuning the weight between the regularization term and the fidelity term. If we consider λ as a variable, then the corresponding constrained problem is the following least square problem with regularization constraint:

$$\begin{aligned} \min_u \quad & \frac{1}{2}\|f - Au\|_{L^2}^2, \\ \text{s.t.} \quad & J(u) \end{aligned} \quad (3)$$

with $J(u)$ the regularization function.

A well-known regularized inverse problem is the Tikhonov regularization model [9], which can be formulated as a one-step filter via Fourier transform for image deconvolution. An alternative regularization [10] uses H^1 semi-norm of u as the regularization term called Wiener filter. This method is linear and the edges of restored image are usually smeared. A successful edge preserving image restoration regularization model is the well-known total variation (TV) regularization model [11], which was proposed by Rudin, Osher, and Fatemi. There are many methods to minimize the TV regularization models, such as FTVd method [12], forward-backward splitting technique [13], iterative Bregman algorithm [14, 15], etc.

Frequently, the dominant information in an image or signal is much "sparser" than the image or signal itself under a proper representation. Com-

pressed sensing is the latest signal acquisition and processing theory and has great application prospects in image and video coding, though the High Efficiency Video Coding (HEVC) is the newest video coding standard for improvement in video/image data compression [16, 17, 18, 19]. The fundamental principle of emerging technology of compressive sensing is that u can be recovered from relatively few incomplete measurements by solving the l_0 -minimization problem

$$\min_u \|u\|_0, \quad s.t. \quad Au = f \quad (4)$$

As this problem is known to be NP-hard, approximation algorithms are often considered. Moreover, Candès, Romberg and Tao [20]-[21], Donoho [22] and their colleagues have shown that, under some reasonable conditions on u and A , the sparsest solution u^* of the above problem can be found by solving the following problem

$$\min_u \|u\|_1, \quad s.t. \quad Au = f \quad (5)$$

For more information see [22], [23]. In this work, we consider this L_1 regularized model with $J(u) = \|u\|_1$ for image restoration.

In Eq.(2), the parameter λ is known as the regularization or the penalization parameter. Many researchers have successfully solved the regularized model by fixing λ first. In the work [24], a technique called the L-curve has been proposed in the case of Tikhonov regularization. A significant amount of research in the literature are developed for the parameter selection for Tikhonov regularization [25]. For the TV regularization, the parameter can be automatically selected by some methods or rules such as generalized cross-validation (GCV)[27], the discrepancy principle (DP), the L-curve and the majorization-minimization (MM) [26] method. The GCV method [27] can be applied to determine regularization parameters when the input error level is unknown. A parameter selection rule called the discrepancy principle is strongly dependent on the knowledge of the noise variance. Lin and Wohlberg in [28] extended the method of unbiased predictive risk estimator (UPRE) to the TV regularization when information about the perturbation level is not available. However, the computational cost is too high. In [29], an adaptive scale parameter has been proposed for the wavelet-based iterative regularization algorithm. A different approach is considered in [30, 31, 32] that one can compute λ as the Lagrange multiplier to solve the Karush Kuhn Tucker (KKT) system of the constrained problem (3), based on the estimate of the

noise level for TV regularization. Applying variable splitting technique, the authors [33] updated the regularization parameter conveniently according to Morozov's discrepancy principle (DP) for TV image restoration when the noise variance is known. In [34], a new adaptive rule for the L_1 regularization image restoration model was proposed without theoretical analysis.

The main aim of this paper is to propose an adaptive automatic technique for the estimation of the regularization parameter, which has been derived in the context of the L_1 regularized iterative forward-backward splitting method (IFBS). This method can be applied to determine the regularization parameter with the unknown noise variance. One can update the regularization parameter in each different iterative procedure, the restored image becomes increasingly close to the original one. Our numerical experiments demonstrate that the proposed algorithm is efficient for image deblurring.

The following is organized as follows. In the next section, we summarize some facts of the iterative forward-backward splitting (IFBS) method. Section 3 is mainly concerned with the derivation of the adaptive regularization parameter and a fast, effective and adaptive IFBS algorithm is proposed. The results of several numerical experiments are presented in section 4 and section 5 concludes the paper.

2. L_1 Regularized Iterative Forward-Backward Splitting

In this section, we consider the unconstrained minimization problem (2). The authors in [13] developed a forward-backward algorithm to minimize the problem. First, a proximal operator is defined as:

$$\text{Prox}_J(f) := \min_u \{\lambda J(u) + H(u)\} \quad (6)$$

For the convexity of the functional $J(\cdot)$ and $H(\cdot)$, the solution of Eq.(2) satisfies the condition:

$$\lambda \partial J(u) + \partial H(u) = 0 \quad (7)$$

For a certain positive number β , one has:

$$(u + \beta \lambda \partial J(u)) - (u - \beta \partial H(u)) = 0 \quad (8)$$

which leads to the forward-backward splitting algorithm:

$$u = \text{Prox}_{\beta \lambda J}(u - \beta \partial H(u)) \quad (9)$$

With the additional hypothesis that $\beta \leq (2/L)$, the solution of the above equation can be evaluated by the following iterative forward–backward splitting (IFBS) method:

$$u_{n+1} = \underbrace{\text{Prox}_{\beta\lambda J}}_{\text{backward step}} \left(\underbrace{u_n - \beta\partial H(u_n)}_{\text{forward step}} \right) \quad (10)$$

where the explicit updating represents the forward step, whereas the evaluation of the proximity operator represents the implicit backward step.

In the problem (2), one has $H(u) = \frac{1}{2}\|f - Au\|_{L^2}^2$, whose Lipschitz constant is given by the maximum modulus eigenvalue of matrix $A^T A$. For a suitable choice of β , the solution can be evaluated with the IFBS method (10). The L_1 based IFBS algorithm can be described as follows.

Algorithm 1: L_1 Regularized IFBS Algorithm

Initialize $\lambda, \beta, A, f, u_0$ and $\{\alpha_n\}$ and set $\tilde{u}_0 = u_0$

For iteration $n = 0, 1, \dots$ do

• **Forward step:**

$$v_{n+1} = u_n + \beta A^T(f - Au_n)$$

• **Backward step:**

$$\begin{aligned} \tilde{u}_{n+1} &= \text{Prox}_{\lambda J}(v_{n+1}) \\ &= \arg \min_u \left\{ \lambda|u|_1 + \frac{1}{2}\|u - v_{n+1}\|_{L^2}^2 \right\} \end{aligned}$$

• **Correction step:**

$$u_{n+1} = \tilde{u}_{n+1} + \alpha_n(\tilde{u}_{n+1} - \tilde{u}_n)$$

In [35] the sequence $\{\alpha_n\}$ in the above algorithm has been chosen in a suitable bounded interval. It is not difficult to see that the backward step has the following solution:

$$\tilde{u}_{n+1} = \max(|v_{n+1}| - \lambda, 0) \cdot \frac{v_{n+1}}{|v_{n+1}|} \quad (11)$$

which corresponds to the soft threshold of v with the threshold λ . We denote

$$\begin{aligned}\tilde{u}_{n+1} &= \mathcal{S}_\lambda(v_{n+1}) \\ &= \begin{cases} v_{n+1} - \lambda \cdot \text{sign}(v_{n+1}), & |v_{n+1}| > \lambda \\ 0, & |v_{n+1}| \leq \lambda \end{cases} \end{aligned} \quad (12)$$

where $\mathcal{S}_\lambda(\cdot)$ denotes the soft thresholding operator.

3. L_1 IFBS with Adaptive Parameter Selection

3.1. Adaptive Parameter Selection

The ability of image reconstruction, in a sense, depends on the parameter λ for the L_1 regularized IFBS method. Fig.1 shows the original clean image and the corrupted one by BSNR=30 and Gauss 2 with size 7 blur. Fig.2 shows the deblurring results with different regularization parameters using the IFBS algorithm described in section 2. Fig.2 illustrates that, the small value of λ corresponds that the fidelity term plays an important role with u closing to f , while large λ yields over-smoothed u .

Figure 1: The original clean image and the corrupted one by BSNR=30 and Gauss 2 with size 7 blur.

Following, an adaptive parameter is obtained from L_1 regularized IFBS method. The parameter λ plays an important role in the backward step. For the backward step, the typical denoising model is as follows:

$$u^* = \arg \min_u \{ \lambda \|u\|_1 + \frac{1}{2} \|u - v\|_{L^2}^2 \} \quad (13)$$

λ can be viewed as a Lagrange multiplier and can be solved by the Euler-Lagrange equation. The Euler-Lagrange equation of the above model has

Figure 2: Deblurred results with different parameters using algorithm 1: the left with $\lambda = 0.5$, right $\lambda = 50$.

the formulation of

$$\lambda \text{sign}(u) + (u - v) = 0 \quad (14)$$

for $u \neq 0$. Then, we have

$$\text{sign}(\lambda) \text{sign}(u) = \text{sign}(v - u) \quad (15)$$

so,

$$\text{sign}(u) = \text{sign}(v - u) \quad (16)$$

Eq.(14) can be rewritten as follows:

$$\lambda \cdot \text{sign}(v - u) = v - u \quad (17)$$

Multiplying Eq.(17) by $(u - v)$ and integrating over the whole image domain Ω

$$\lambda \iint_{\Omega} \text{sign}(v - u)(v - u) dx dy = \iint_{\Omega} (v - u)(v - u) dx dy \quad (18)$$

Then, we have the following equation:

$$\lambda = \frac{\iint_{\Omega} (v - u)(v - u) dx dy}{\iint_{\Omega} |v - u| dx dy} \quad (19)$$

Remark: For image denoising model, $v \triangleq f = u + n$, n is a white Gaussian noise with variance σ^2 . We denote $V(u)$ the variance of an image u : $V(u) \doteq \frac{1}{|\Omega|} \iint (u - \bar{u})^2 dx dy$, $\bar{u}$ is the mean value: $\bar{u} \doteq \frac{1}{|\Omega|} \iint u dx dy$. Therefore, $V(n) = \sigma^2$, and

$$\lambda = \frac{|\Omega|\sigma^2}{\iint_{\Omega} |v - u| dx dy}. \quad (20)$$

However, for the IFBS algorithm, u and v are changed with the number of iterations. In numerical implementations, we use u_n, v_n to approximate u and v respectively, and accordingly λ_n denotes λ in Eq.(19). So, a new iterative regularization parameter is obtained that does not require any information about the perturbation process with the following form:

$$\lambda_n = \frac{\iint_{\Omega} (v_n - u_n)(v_n - u_n) dx dy}{\iint_{\Omega} |v_n - u_n| dx dy} \quad (21)$$

In order to accelerate the convergence of the above algorithm, the fast method of Beck et al.[36] is applied. In this way, u is updated as follows:

$$u_{n+1} = \tilde{u}_{n+1} + \left(\frac{t_n - 1}{t_{n+1}} \right) (\tilde{u}_{n+1} - \tilde{u}_n) \quad (22)$$

where $t_{n+1} = \frac{1 + \sqrt{1 + 4t_n^2}}{2}$ with $t_0 = 1$, and $\frac{t_n - 1}{t_{n+1}} \in [0, 1]$ is the step length.

Applying the proposed adaptive regularization parameter to the fast based algorithm, a detailed description of the proposed algorithm named AFIFBS algorithm is outlined as follows:

Algorithm 2: Adaptive Fast Iterative Forward-Backward Splitting (AFIFBS) Algorithm

Initialize β, A, f, u_0, v_0 and set $\tilde{u}_0 = u_0, t_0 = 1$

For Iteration $n = 0, 1, \dots$ do

• **Updating the parameter step:**

Compute Eq.(21) to update the parameter

• **Forward step:**

$$v_{n+1} = u_n + \beta A^T (f - Au_n)$$

• **Backward step:**

$$\begin{aligned} \tilde{u}_{n+1} &= \text{Prox}_{\beta J}(v_{n+1}) \\ &= \arg \min_u \left\{ \lambda_n |u|_1 + \frac{1}{2} \|u - v_{n+1}\|_{L^2}^2 \right\} \\ &= \mathcal{S}_{\lambda_n}(v_{n+1}) \end{aligned}$$

• **Correction step:**

$$t_{n+1} = \frac{1 + \sqrt{1 + 4t_n^2}}{2}$$

$$u_{n+1} = \tilde{u}_{n+1} + \left(\frac{t_n - 1}{t_{n+1}} \right) (\tilde{u}_{n+1} - \tilde{u}_n)$$

Remarks: (1). The observed image is used as the initial value of u_0 , the value of β is set to 1 and otherwise the condition $\beta < \frac{2}{\lambda_{\max}(A^T A)}$ is necessary for the convergence of the IFBS method. The algorithm requires, for each iteration, the solution of a problem of the form (2) with λ fixed. This algorithm does not need to know the initial value of the regularization parameter. **(2).** In the proposed algorithm, the updating parameter step and the backward step of the two dimensional shrinkage consume the most computation. In each iteration, the computation cost of λ is about $O(N^2 + 1)$ and the backward step can be solved by two dimensional shrinkage in $O(N^2)$ multiplicative operations for an $N \times N$ image. Therefore, the computation cost is about $O(2N^2 + 1)$ for each iteration.

3.2. Well Definedness of The Iteration

In this subsection, we give the convergence analysis of the defined parameter sequence $\{\lambda_n\}$ and of the energy functional sequence $\{F(\lambda_n, u_{\lambda_n})\}$. We consider the functional

$$F(\lambda_k, u_{\lambda_k}) = \lambda_k |u_{\lambda_k}|_1 + \frac{1}{2} \|f - Au_{\lambda_k}\|_2^2 \quad (23)$$

with

$$u_{\lambda_k} = \arg \min_u \{ \lambda_k |u|_1 + \frac{1}{2} \|f - Au\|_2^2 \} \quad (24)$$

The following two propositions illustrate the stability of the AFIFBS algorithm.

Proposition 1. The sequence $\{\lambda_{n+1}\}$, $n \geq 1$, defined in Algorithm 2 is decreasing as $n \rightarrow \infty$.

Proof:

In Algorithm 2, considering the backward step by utilizing Eq.(12), then

$$\tilde{u}_{n+1} = \begin{cases} v_{n+1} - \lambda_n \cdot \text{sign}(v_{n+1}), & |v_{n+1}| > \lambda_n \\ 0, & |v_{n+1}| \leq \lambda_n \end{cases} \quad (25)$$

and

$$\begin{aligned}
\lambda_{n+1} &= \frac{\iint_{\Omega} (v_{n+1} - \tilde{u}_{n+1})(v_{n+1} - \tilde{u}_{n+1}) dx dy}{\iint_{\Omega} |v_{n+1} - \tilde{u}_{n+1}| dx dy} \\
&= \frac{\iint_{\Omega_1} (\lambda_n \text{sign}(v_{n+1})) (\lambda_n \text{sign}(v_{n+1})) dx dy + \iint_{\Omega_2} v_{n+1} v_{n+1} dx dy}{\iint_{\Omega_1} |\lambda_n \text{sign}(v_{n+1})| dx dy + \iint_{\Omega_2} |v_{n+1}| dx dy} \\
&= \frac{\iint_{\Omega_1} \lambda_n^2 dx dy + \iint_{\Omega_2} v_{n+1} v_{n+1} dx dy}{\iint_{\Omega_1} \lambda_n dx dy + \iint_{\Omega_2} |v_{n+1}| dx dy} \\
&= \lambda_n \frac{M + \frac{1}{\lambda_n^2} \iint_{\Omega_2} v_{n+1} v_{n+1} dx dy}{M + \frac{1}{\lambda_n} \iint_{\Omega_2} |v_{n+1}| dx dy}
\end{aligned}$$

where Ω_1 denotes a subset of Ω , that in Ω_1 , $|v_{n+1}| > \lambda_n$. And $\Omega_2 = \Omega/\Omega_1$. Then the second equation holds. M denotes the area of Ω_1 .

For simplicity, in what follows, we denote $U = M + \frac{1}{\lambda_n^2} \iint_{\Omega_2} [v_{n+1} v_{n+1}] dx dy$ and $D = M + \frac{1}{\lambda_n} \iint_{\Omega_2} |v_{n+1}| dx dy$. Then,

$$\begin{aligned}
U - D &= \frac{1}{\lambda_n^2} \iint_{\Omega_2} [v_{n+1} v_{n+1}] dx dy - \frac{1}{\lambda_n} \iint_{\Omega_2} |v_{n+1}| dx dy \\
&= \frac{1}{\lambda_n^2} \iint_{\Omega_2} [|v_{n+1} v_{n+1}| - \lambda_n |v_{n+1}|] dx dy \\
&\leq 0
\end{aligned}$$

For $\Omega_2 = \Omega/\Omega_1$, that is, in Ω_2 , $|v_{n+1}| \leq \lambda_n$, the last inequality is hold. So $\frac{U}{D} \leq 1$, and $\lambda_{n+1} \leq \lambda_n$. $\{\lambda_{n+1}\}$ is decreasing as $n \rightarrow \infty$. $\sharp$

The following proposition indicates that with the decreased parameter sequence the energy functional sequence is indeed decreasing.

Proposition 2. Let u_{λ_k} and $u_{\lambda_{k+1}}$ be the corresponding minimizers of (24), then the corresponding functional $F(\lambda, u_{\lambda})$ satisfies

$$F(\lambda_k, u_{\lambda_k}) > F(\lambda_{k+1}, u_{\lambda_{k+1}}) \quad (26)$$

Proof:

From (23) and (24), we have

$$\begin{aligned}
F(\lambda_k, u_{\lambda_k}) &= \lambda_k |u_{\lambda_k}|_1 + \frac{1}{2} \|f - Au_{\lambda_k}\|_2^2 \\
&> \lambda_{k+1} |u_{\lambda_k}|_1 + \frac{1}{2} \|f - Au_{\lambda_k}\|_2^2 \\
&\geq \lambda_{k+1} |u_{\lambda_{k+1}}|_1 + \frac{1}{2} \|f - Au_{\lambda_{k+1}}\|_2^2 \\
&= F(\lambda_{k+1}, u_{\lambda_{k+1}})
\end{aligned}$$

This ends the proof. $\sharp$

4. Numerical examples

In this section, we present some numerical examples of image restoration to illustrate the effectiveness of our proposed fast IFBS algorithm. The peak-signal-to-noise ratio (PSNR), and the blurred signal-to-noise ratio (BSNR) are used to compare the quality of the restoration results. They are defined as follows:

$$\begin{aligned}
PSNR(u) &= 20 \cdot \log_{10} \left(\frac{N \times 255}{\|\bar{u} - u\|^2} \right) \\
BSNR(u) &= 20 \cdot \log_{10} \left(\frac{\|f - \bar{f}\|_2}{\sigma \cdot \sqrt{N}} \right)
\end{aligned}$$

where f , u and $\bar{u}$ denote the observed image, the original clear image and the restored image respectively, $\bar{f}$ denotes the mean of f , and σ the standard deviation of Gauss white noise. N is the image size.

Figure 3: Test Images: peppers, satellite and finger, of size(256×256).

To test the proposed algorithm we have used three different types of images shown in figure 3, named peppers(256×256), satellite(256×256) and finger(256×256). The results presented in the next subsections are obtained by running a Matlab implementation on the PC with Intel Core(TM) Duo CPU (2.40GHz) and 2GB of RAM under windows 7.

4.1. Experiment 1.

In this experiment, we will demonstrate the convergence property of the adaptive parameter and of the energy functional. We consider the Pepper image blurred by Gaussian (2) kernels of size 9, motion kernels of length 9 and uniform 3×3 . A Gauss white noise is added to the blurred image with $\text{BSNR} = 40\text{dB}$. The left column of Figure 4 shows the evolution of λ with the iteration number, and the right column displays the evolution of the energy functional with the iteration number. From the experiment we validate the correctness of our theory.

4.2. Experiment 2.

To evaluate the performance of the L_1 regularized AFIFBS algorithm, we compare the results with the IFBS algorithm 1 and the AFIFBS algorithm 2.

First, we consider the 'pepper', 'satellite' and 'finger' images perturbed by different Gauss white noise $\text{BSNR} = 20, 30, 40\text{dB}$. The three images are blurred by Gauss (2) kernel of 7 size. For non-adaptive algorithms, one can manually choose $\lambda = 15, 10, 5$ corresponding to the BSNRs. The results of the PSNRs and the CPU times obtained by using the IFBS and the proposed AFIFBS algorithm are shown in Table 1. We see from the table that, the AFIFBS algorithm is competitive in the PSNRs and the CPU times.

In the following, we experiment the effect of varying the size of the convolution kernel on the adaptive algorithm. To this aim, we consider these three images blurred with motion kernel of lengths 9, 15 and 21 and blurred with uniform size 5×5 , 9×9 and 13×13 . For all the experiments, the images are perturbed by adding Gaussian white noise with $\text{BSNR} = 40\text{dB}$. One can see from Table 2 - Table 3 that the effectiveness of the Algorithm 2 is maintained for severely blurred images.

For easy observation, we take the 'satellite' image, which is corrupted with motion blur with kernel size 15 and added Gaussian white noise with $\text{BSNR}=40$, as an example and magnify a small region of the restoration results with these two algorithms. The restoration results are shown in Fig.5.

Figure 4: Pepper, different types blur and BSNR=40. Left column shows λ evolves with the iteration number, and right one shows $F(\lambda, u_\lambda)$ evolves with the iteration number; first row: Gauss(2)kernel of 7 size ; second row: Motion kernel of length 9; third row: Uniform size of 3×3 .

Table 1: PSNRs and CPU times for the 'pepper', 'satellite', 'finger' images blurred by Gaussian (2) kernels of 7 size and different BSNRs.

BSNR	Images	IFBS		AFIFBS	
		PSNR	Times	PSNR	Times
20	pepper	22.9482	0.7443	22.9522	0.9979
	satellite	23.7160	1.0166	24.4548	0.5215
	finger	18.3352	3.5482	19.4398	0.8379
30	pepper	25.5792	3.4427	26.7426	1.4619
	satellite	24.8280	3.6125	25.3292	1.2849
	finger	20.3630	14.7500	21.0928	1.9072
40	pepper	28.9337	18.0667	29.5035	2.1492
	satellite	26.6628	40.6345	26.6359	3.1818
	finger	23.0556	81.9949	23.3046	4.6435

Table 2: PSNRs and CPU times for the 'pepper', 'satellite', 'finger' images blurred by motion kernel of lengths 9, 15 and 21.

motion length	Images	IFBS		AFIFBS	
		PSNR	Times	PSNR	Times
9	pepper	30.2103	9.4706	30.5769	1.2513
	satellite	30.9630	22.8941	31.4514	2.1621
	finger	25.4105	49.1089	25.8930	3.0001
15	pepper	29.0858	14.5627	28.0038	2.7632
	satellite	30.5463	26.5447	31.4743	2.3887
	finger	24.6785	105.6828	24.8467	5.0592
21	pepper	27.8292	27.8220	25.6040	4.0089
	satellite	28.4981	37.3602	29.5865	3.2897
	finger	23.3128	90.9896	23.4512	4.8725

Table 3: PSNRs and CPU times for the 'pepper', 'satellite', 'finger' images blurred with uniform kernel of size 5×5 , 9×9 and 13×13 .

uniform size	Images	IFBS		AFIFBS	
		PSNR	Times	PSNR	Times
5×5	pepper	30.5909	6.3503	31.1255	1.0838
	satellite	30.3480	26.4261	30.2065	2.3941
	finger	25.4846	31.5136	25.7203	2.6104
9×9	pepper	28.3435	22.0048	26.9973	3.0220
	satellite	26.7202	32.5523	27.1266	3.5809
	finger	21.1621	98.4876	21.3054	4.8787
13×13	pepper	26.9446	48.1087	24.0781	4.9262
	satellite	24.8636	27.9102	25.7259	4.7103
	finger	19.7371	170.4528	19.8775	6.8549

We can easily see visual improvement in the image by using our algorithm. In Fig.6, we depict the objective function value and the PSNR value with respect to the iteration number for 'satellite' image. Fig.7 includes the visual results of 'finger' image blurred with uniform size 9×9 and added Gaussian white noise with BSNR=40. Iterations 20 and 80 are describe in Fig.7. The relative error at iteration n is denoted as R_n . It is clear to see that the reconstructed image with the proposed AFIFBS algorithm have more detailed information and are much closer to the original clear image. The AFIFBS algorithm is much faster than IFBS algorithm with a higher PSNR.

Figure 5: Experimental results from different restoration algorithms for satellite image. (a) Original clear image; (b) IFBS restoration; (c) AFIFBS restoration.

Figure 6: Satellite, Motion blur with blur size 15, BSNR=40dB, restored with IFBS and AFIFBS algorithms. (a) PSNRs, as a function of iteration number ; (b) $F(\lambda, u_\lambda)$, as a function of iteration number.

Figure 7: Iterations of IFBS and AFIFBS algorithms for 'finger' image deblurring. R_n denotes the relative error between the restored image u^* and the clear image u .

4.3. Experiment 3.

In this experiment, we further study the behavior of the proposed adaptive regularization parameter and compare the AFIFBS algorithm with the recently proposed adaptive parameter estimation under L_1 based image restoration [34]. Both the two algorithms are based on the IFBS method. In [34], the authors proposed an iterative L_1 -based image restoration algorithm with the following adaptive parameter estimation:

$$\begin{aligned}\lambda_1 &= r \cdot \frac{\|Au_0 - f\|_2^2}{J(u_0)}, \quad \lambda_2 = \lambda_1 \cdot \frac{F(\lambda_1, u_1)}{F(\lambda_1, u_0)} \\ \lambda_{n+1} &= \lambda_n \cdot \frac{F(\lambda_n, u_n)}{F(\lambda_{n-1}, u_{n-1})}\end{aligned}\tag{27}$$

with $r \in (0, 1)$ and the first two values of λ are fixed when u_1 and v_1 are computed in the initialization step.

The proposed parameter selection rule can be applied to determine the parameter without knowing the noise level. The crucial point for the performance of the algorithm is the choice of the value of λ . It is common to make use of the empirical reduction rule

$$\lambda_n = w \cdot \frac{\iint_{\Omega} (v_n - \tilde{u}_n)(v_n - \tilde{u}_n) dx dy}{\iint_{\Omega} |v_n - \tilde{u}_n| dx dy}\tag{28}$$

where $w \in (0, 1)$ is the reduction factor. We consider the example 'pepper' blurred by Gaussian (2) kernel of size 9, BSNR=40dB and Fig.8 displays the results obtained by the AFIFBS algorithm using the update λ Eq.(28), with $w = 0.2$, $w = 0.6$ and $w = 1$. Fig.8 shows the behavior of λ and of the values of PSNRs as the function of the iteration number. We note that the AFIFBS algorithm with the update Eq.(28) maintains the monotonic decrease of the regularization parameter, so the selection of λ with Eq.(28) seems reasonable. One can see from this example that, (1)when $0 < w < 1$, this choice ensures faster decrease of the parameter λ than the choice $w = 1$; (2)when $0 < w < 1$, the behavior of λ reveals the same performance of the proposed adaptive algorithm. Table 4 displays the PSNRs and CPU times obtained by IFBS, Algorithm [34], proposed AFIFBS algorithm with $w = 0.2$ and $w = 1$. One can see the effectiveness of the AFIFBS algorithm with $w = 0.2$. Furthermore, without knowing the initial value of the parameter, our proposed algorithm can guarantee the monotonicity of parameter

theoretically. However, there is no theoretical analysis for the automatically estimated regularization parameter [34] which requires a prior estimation of the initial λ .

Figure 8: Pepper Gaussian (2) kernel of size 9, BSNR=40dB; (a) Values of λ evaluated by Eq.(28) evolves with the iteration number with $w = 0.2$, $w = 0.6$ and $w = 1$; (b) PSNRs as a function of the iteration number.

Table 4: Comparison of PSNRs with different algorithm. The 'pepper' blurred with Gauss (2) kernel of size 7, motion blur of length 9, uniform kernel of size 5×5 and BSNR=40dB.

	Gauss		Motion		Uniform	
	PSNR	Times	PSNR	Times	PSNR	Times
IFBS	28.9337	18.0667	30.2103	9.4706	30.5909	6.3503
Algorithm[34]	30.3710	2.1059	32.1740	1.2020	32.9040	1.0383
AFIFBS($w = 1$)	29.5035	2.1492	30.5769	1.2513	31.1255	1.0838
AFIFBS($w = 0.2$)	30.3711	1.8094	32.2927	0.8226	32.9471	0.6808

5. Conclusion

In this paper, a fast, effective and adaptive algorithm, referred to as AFIFBS, has been proposed for L_1 regularized image restoration. We studied the selection of regularization parameter in L_1 regularized image restoration. By updating the adaptive parameter in each iteration, the restored

image can be obtained. The proposed AFIFBS algorithm does not require any information about the initial value of the parameter. The proposed algorithm leads to adaptive iterative soft threshold process. Numerical examples demonstrate the effectiveness of the AFIFBS algorithm.

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