Robust Multi-View Stereo synthesized by Various Parameters Model [☆]

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Abstract

In this paper, we have developed a novel and robust framework of combining a matrix splitting with multi-view stereo reconstructions to separate reconstruction inaccuracies from a various parameters model for high-accuracy multi-view stereo reconstruction. Instead of performing the fixed parameters reconstruction procedure, we apply the variational based 3D reconstruction algorithm multi-times with various parameters to derive a set of hypothetic 3D models, and then synthesized the final result by formulating the problem as a low-rank matrix splitting problem. Benefited from the matrix splitting formulation, the outliers and bad matches, which are treated as the noise in the synthesized model, are effectively removed and thus lead to a 3D reconstruction with higher accuracy than the existing fixed parameters reconstructions. Constrained convex optimization is introduced for matrix splitting with an accelerated proximal gradient (APG) algorithm integrated for fast convergence. Both the experiments on the Middlebury and real-world data sets have demonstrated the effectiveness of the proposed method.

Keywords: Multi-view Stereo, Variational Method, Matrix Splitting, Low-rank Matrix, Accelerated Proximal Gradient.

1. Introduction

Multi-View Stereo (MVS) reconstruction algorithm [1] [2] [3] [4][5] [6] [7] [8] [9] [10] [11], is used to construct 3D model from multiple calibrated images of

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4 a realistic object. It has drawn intensive interests in practical applications, such as
5 industrial design, characters modelling for films, electronic games, cultural relics
6 preservation, commercial advertisement, etc.

7 Generally, 3D object model reconstruction can be divided into two steps. First
8 of all, all point clouds are computed from the calibrated image pairs of adjacent
9 viewpoints. Many techniques have been proposed to generate the point clouds
10 from the image pairs. Compbell *et al.* [12] proposed to compute the point clouds
11 using discrete Markov Random Field (MRF). Gargallo *et al.* [13] developed a
12 Bayesian model for the point clouds generation. Liu *et al.* [14] applied the varia-
13 tional approach to estimate continuous depth map, which is verified to be more ac-
14 curate than other solutions [15] [16] [17] [18]. Secondly, point clouds are merged
15 together properly to produce the 3D model. Therefor, there are many works about
16 point cloud fusion strategies concentrated on how to reduce the redundancy and
17 improve the accuracy of the fused point cloud. Merrel *et al.* proposed a fast
18 algorithm for depth map fusion [19]. They considered selecting the most repre-
19 sentative depth information by projecting it to the original calibrated images with
20 the least distance. Then, the redundancy among the rough point cloud is reduced
21 by clustering. Liu *et al.* [14] and Bradley *et al.* [10] took a similar techniques to
22 reduce the redundancy point cloud using normalized cross correlation(NCC).

23 However, both point clouds estimation strategy and merging step have two
24 prominent drawbacks:

- 25 1. The point clouds estimation is usually nonadaptive, in other words, the qual-
26 ity of the estimated point clouds highly depend on the parameter tuning. An-
27 alytical 3D object models generally include a data and smooth term when
28 we apply the variational method to derive the point cloud, and the recon-
29 structed 3D object would vary in details (i.e., texture, edges, etc.) when
30 adapting different value of the terms' weights. Typically, reconstructed ob-
31 jects will have smoother surface if we use larger weighting smooth term;
32 oppositely, it will have richer texture details if we use smaller weighting
33 smooth term.
- 34 2. The 3D locations of a single scene point computed from different image
35 pairs are often misaligned due to the point clouds corruption, resulting in
36 severe accuracy damage.

37 To address these issues, a various parameters models synthesized 3D recon-
38 struction framework is proposed in this paper. To preserve both smooth surface
39 and texture details, we have experimented various parameter sets to construct mul-
40 tiple point clouds (for the same object). Then we stack all these point clouds to

Figure 1: Reconstruction pipeline of our proposed method. Firstly, roughed point cloud were estimated using variational method with different parameters. Then, refined point cloud were computed by matrix splitting method. Finally, accurate 3D model was meshed using refined point cloud.

41 derive the low-rank matrix for further splitting so as to product the optimized
 42 model for more accurate 3D object reconstruction. As shown in Fig. 1, the ini-
 43 tial point clouds are computed by the variational depth map estimation method
 44 with various parameters of a large range. The synthesized models of the same
 45 object (i.e., the initial point clouds) with different details and corruption errors
 46 are formulated into a series of column vectors, and then a matrix splitting model
 47 is employed to extract the final 3D model from the error corrupted hypotheses
 48 matrix. The *Contributions* of our work can be summarized as follows:

- 49 1. The idea of combining a matrix splitting(robust PCA) with multi-view stereo
 50 reconstructions to separate reconstruction inaccuracies from a various pa-
 51 rameters model for high-accuracy multi-view stereo reconstruction is pro-
 52 posed firstly.
- 53 2. For extracting the final model from the hypotheses, a low-rank matrix split-
 54 ting problem, which is more suitable for the sparse error distribution is ap-
 55 plied.
- 56 3. By using a convex optimization of the nuclear and l_1 -norm combined ob-
 57 jective, the proposed algorithm can be computed efficiently and accurately.

58 Benefiting from the above three main contributions, the proposed algorithm
 59 improves the robustness and the quality of results greatly than the existing frame-
 60 work straightly combining the variational based points clouds estimation and naive
 61 clouds merging. It is worth noting that the initial point clouds estimation in the
 62 proposed method can be easily replaced by any other binocular stereo algorithms,
 63 so that our method is of much flexibility with the development in the 3D recon-
 64 struction research realm.

65 The reminder of this paper is organized as follows: the related work is given in
 66 Section 2. The various parameters 3D reconstruction framework is illustrated in
 67 Section 3 and the implementation details are presented in Section 4. Experiments
 68 is performed to demonstrate the effectiveness of our scheme in Section 5 using
 69 the Middlebury and real-world data sets. We conclude this paper in Section 6.

70 2. Related Work

71 In this section, we review the previous works about multi-view stereo recon-
 72 struction, especially on depth maps fusion techniques.

73 Kolmogorov and Zabih [3] computed a set of depth maps using multi-baseline
 74 stereo with graph cuts, then merge the point clouds into a voxel volume by com-
 75 puting the intersections of occluded volumes form each view-point. Vogiatzis [4]

76 proposed a correlation cost volume in the neighborhood of the visual hull. A min-
77 imum cost surface is then computed using volumetric min-cut. Furukawa *et al.*
78 [20] used wide-baseline stereo matching to recover the 3D coordinates of salient
79 feature points, then shrink a visual hull model so that the recovered points lie on its
80 surface, and refine the result using energy minimization. Goesele *et al.* [21] com-
81 puted a depth map from each viewpoint and merged the results using volumetric
82 integration.

83 Noise reduction (i.e., outlier and bad matches removal) is an important re-
84 search topic for in 3D object model reconstruction. In the literature, a large num-
85 ber of 3D point cloud fusion approaches has been proposed which can be roughly
86 grouped into two main classes: filtering and optimization methods. The former
87 using the physical properties (the distance of each point) of point clouds to reduce
88 the noise, while the latter builds an appearance model of the object and formu-
89 lates the fusion problem as a mathematical optimization problem. Liu [14] and
90 Bradley [10] took advantages of the NCC metric to refine the depth maps. The
91 method reviewed above can be categorized as filtering techniques since they take
92 advantages of the physical properties of point clouds to remove the noise in the
93 generated depth maps. But, the 3D models of [10] [14] have shown filtering only
94 is often not effective. This is because, as detailed in in [14], the NCC metric is
95 limited to remove those isolated patches from the surface and refined point cloud
96 still include a large quantity of errors. Campbell *et al.* [12] enhanced the quality
97 of depth maps by extracting multiple depth candidates for each pixel, and then im-
98 posed global optimization algorithm to remove outliers and achieve point cloud
99 smoothness simultaneously. Zach *et al.* [22] used total variation with l_1 norm
100 regularizer to compute the point clouds, making sure that outliers and errors could
101 be reduced. Deng *et al.* [23] have further introduced noisy depth maps fusion for
102 multi-view stereo via matrix completion. The aforementioned methods [12] [22]
103 [23] can be categorized as optimization since they using mathematical formulate
104 to optimize the object. They could further reduce the redundancy and improve the
105 accuracy of the estimated point clouds, but the models' complexity makes the op-
106 timization procedure difficult to find the optimal solutions(hard to overcome the
107 local minimum). So, these optimization methods not enough robust for multi-view
108 stereo reconstruction problem.

109 **3. Synthesized 3D Reconstruction**

110 In this section, we will propose our robust multi-view stereo method by using
111 the various parameters models synthesized 3D reconstruction framework. As is

Figure 2: Rough point clouds of DinoSparseRing data sets and TempleSparseRing data sets, the red rectangles and circles mark the outliers and the bad matched respectively

Figure 3: Rough point clouds of DinoSparseRing datasets and TempleSparseRing datasets are split into two parts, refined points and outliers respectively

112 shown in Fig. 4, the initial rough point clouds are computed from the input image
 113 pairs with different parameters. Depending on the parameter settings, a series
 114 of rough point clouds with different levels of outlier errors and smoothness are
 115 derived. By arranging these initial hypotheses into a sequence of column vectors,
 116 a matrix splitting model is used in this paper to handle the outlier errors extraction.
 117 In the following we will give the details of the proposed framework.

118 3.1. various parameters Point Clouds Estimation

119 Among all the depth map estimate algorithms, the variational-based method
 120 has demonstrated its strength by presenting high accuracy in developing contin-
 121 uous correspondence between motion images [12] [24] [25] [26]. Thus, we use
 122 the variational based method [14] to estimate the hypothetic point clouds in this
 123 paper. Considering the self-containing and convenience to expression, we briefly
 124 introduce the method here. (We refer the readers to [14] for details.)

According to [14], the depth map $d(w)$ can be computed by minimizing the target energy function, which includes both the data terms and the smoothness

terms. w denotes the shift from the pixel position on the target image to the optimal correspondence on the epipolar line in the reference image, named disparity. The projection of w on the epipolar line is inversely proportional to the depth $d(w)$. It is formulated as:

$$E(w) = E_D(w) + \alpha E_S(w) \quad (1)$$

where

$$E_D(w) = \int_{\Omega} \psi_D(|I_r(p + d(w)) - I_t(p)|^2 + \gamma(|\nabla I_r(p + d(w)) - \nabla I_t(p)|^2) dx dy$$

and

$$E_S(w) = \int_{\Omega} \psi_S(|\nabla w|^2) dx dy$$

125 Here, I_r and I_t denote the reference image and the target image respectively.
 126 $\psi(s^2) = \sqrt{s^2 + \epsilon^2}$ is a robust operator, where ϵ is a small positive constant. ψ
 127 can be applied separately to each term. $\nabla I(x, y)$ represents the gradient of the
 128 $I(x, y)$ and $\nabla = (\partial_x, \partial_y)^\top$, where x and y represents image spatial direction. α is
 129 a positive weighting parameter that trades off the data terms versus the smoothness
 130 terms.

For simplicity, we use the following abbreviations for derivatives and differences: $I_x =: \partial_x I_r(p + d(w))$; $I_{xy} =: \partial_{xy} I_r(p + d(w))$; $I_y =: \partial_y I_r(p + d(w))$; $I_{yy} =: \partial_{yy} I_r(p + d(w))$; $I_z =: I_r(p + d(w)) - I_t(p)$; $I_{xz} =: \partial_x I_r(p + d(w)) - \partial_x I_t(p)$; $I_{xx} =: \partial_{xx} I_r(p + d(w))$; $I_{yz} =: \partial_y I_r(p + d(w)) - \partial_y I_t(p)$. The problem of finding w that minimize the target variational energy function can be translated to the minimization of Euler-Lagrange equation [24]. Using the aforementioned abbreviations, Euler-Lagrange equation is shown as:

$$\begin{aligned} & \psi'_D(I_z^2 + \gamma(I_{xz}^2 + I_{yz}^2)) \times (I_z I_x i + \gamma I_{xz} I_{xx} i + \gamma I_{yz} I_{xy} i) \\ & - \alpha \text{div}(\psi'_S(|\nabla w|^2) \nabla w) = 0 \\ & \psi'_D(I_z^2 + \gamma(I_{xz}^2 + I_{yz}^2)) \times (I_z I_y j + \gamma I_{xz} I_{xy} i + \gamma I_{yz} I_{yy} j) \\ & - \alpha \text{div}(\psi'_S(|\nabla w|^2) \nabla w) = 0 \end{aligned} \quad (2)$$

131 Here, $(i, j)^T$ is the unit normal vector of direction of epipolar line corresponding
 132 to pixel p .

133 As seen, both data and smoothness terms in Eq. (2) are non-linear functions.
 134 In order to remove the non-linearity in the Eq. (2), two fixed point iterations are

135 applied. In the first step, the Eq. (2) is converted to a linear system by a multi-
 136 resolution strategy with two nested fixed point iterations. In order to implement a
 137 multi-scale approach, a coarse-to-fine strategy with down-sampling factor η which
 138 can be more close to the global optimum of the energy function is used. It is pro-
 139 posed here to use an arbitrary $\eta \in (0, 1)$. We assume that w_0 is the initialization
 140 of w for each pixel in the target image, and w_0 was pre-computed from the vi-
 141 sual hull (file generated by the *EPVH* tool [27]). Finally, the linear system of
 142 equations about w is solved using multi-grid method [28]. In contrast to other
 143 numerical methods, multi-grid algorithm is able to damp both high frequency and
 144 low frequency errors effectively.

145 As soon as a fixed point in w^k is reached, we can calculate the point clouds at
 146 each pixel according to the known camera parameters. With the help of epipolar
 147 geometry, optical flow map of the adjacent views can be derived from the function
 148 above. Then the partial point clouds can be computed from optical-flow maps and
 149 calibration parameters. Finally, 3D model point clouds are generated by combin-
 150 ing the partial point clouds of each view point.

151 Fig. 2 illustrates the point clouds of both Middlebury Data sets *Dino* and *Tem-*
 152 *ple*, from which we could see outliers (from the corrupted estimation in optical
 153 flow) and bad matches (from conflicting estimation of the points captured by dif-
 154 ferent cameras). As a result, the key problem is how to remove these noise for
 155 point clouds refinement so as to form an accurate and complete 3D object mode
 156 reconstruction. We formulate the problem as an optimization based low-rank ma-
 157 trix splitting shown in the following sections.

158 3.2. Outlier Extraction Using Matrix Splitting

A hypothetical point clouds D can be modeled as the combination of the ground
 truth data A and the outliers E . It is formulated as:

$$D = A + E \quad (3)$$

159 where D is the matrix representation of the rough point clouds which are estimated
 160 using variational method, A is the matrix denoted accurate point clouds and E is
 161 the matrix representation of the noise data. We demonstrate the key module of
 162 removing outliers and bad matches and how we cast the problem of 3D model
 163 reconstruction as a problem of splitting a low-rank matrix.

The matrix $D \in R^{m \times n}$ is constructed by stacking all the vectorized rough
 point clouds $vec(d_i)$ as:

$$D = [vec(d_1), \dots, vec(d_i)]$$

164 where $vec(d_i)$ ($i = 1, \dots, n$) is vectorized rough point as illustrated in Fig. 4.
 165 d_i represents different point clouds generated using different parameters listed in
 166 Table 1 variational method of the same object. Especially, n indicates the number
 167 of generated rough point clouds using variational method. We set $n = 16$ in our
 168 simulations. In other words, ten similar 3D models that contains a few outliers
 169 are generated by variational method. To clearly readers how to determine which
 170 points from different views should be put in the same column. We denote the
 171 estimated 3-D point corresponding to pixel $I_x^{(k)}$ by d_x^k . Projecting d_x^k to another
 172 image $I^{(t)}$, its 2-D correspondence in that image can be derived. Considering
 173 depth estimation error and calibration inaccuracy, this corresponding point may
 174 deviate from the real one. Generally, this corresponding point in the neighboring
 175 region of $I_p^{(t)}$. If the distance between d_x^k and $d_x^{k,t}$ is less than a specific thresh-
 176 old(determined by experiments), these two points are regarded as describing the
 177 same surface point and will be placed in the same column of the reconstructed ma-
 178 trix. In practice, the nearest point is stacked in the reconstructed matrix column if
 179 there are multiple points.

Table 1: Parameters used in the experiments

Parameter	Value
Smooth parameter α	2 : 2 : 8
The weight factor γ	85 : 5 : 100
Multi-gride iteration	5
Outer fixed point iteration	5
Inner fixed point iteration	10
Reduction factor η	0.7

D is stacked using the same model point clouds which are generated by dif-
 ferent parameters in variational method. The data matrix D obeys the low-rank
 model and the rank of D is one. In practice, there are only few points in each row
 were corrupted and majority of the points are correct, hence most entries in E will
 be zero, i.e., E is a sparse matrix. [29] was introduced to recover a low rank ma-
 trix with sparse noise on the correct points. Thus, the problem can be expressed
 as a constrained optimization problem:

$$\begin{aligned}
 \min_{A, E} \quad & rank(A) + \lambda \|E\|_0 \\
 s.t. \quad & D = A + E
 \end{aligned} \tag{4}$$

180 where $\|\cdot\|_0$ denotes the ℓ_0 -norm(number of non-zero entries in the matrix) and
 181 λ is a positive weighting parameter that trades off the rank of the A versus the
 182 sparsity of the error E .

183 While Eq. 4 follows from our formulation, it is not tractable since both $rank(\cdot)$
 184 and ℓ_0 -norm are non-convex and discontinuous functions. Solving this optimization problem efficiently will be the topic of discussion in the Section 4.

Figure 4: The formation of low-rank matrix D is stacked using the same model point cloud which is generated by different parameters in variational method

185

186 4. Implementation Details

187 As discussed above, the outlier extraction is formulated as a matrix splitting
 188 optimization to remove outliers and bad matches, as shown in Fig. 3. As seen, the
 189 optimization problem given in Eq.(4) is NP-hard [11]. In this section, we propose
 190 to solve it efficiently based on convex relaxation of matrix rank minimization [29]
 191 [30] [31] [32].

192 4.1. Convex Relaxation

In the paper [33], Wright *et al* have proposed that the problem in Eq.(4) can be solved by replacing the cost function with its convex relaxation, provided that the rank of the matrix A is not too high and the number of non-zero entries in the matrix E is sparse. Candès *et al*[29] have proposed that the cost function was replaced by $\min_{A,E} \|A\|_* + \lambda \|E\|_1$. This convex relaxation, named Robust

Principal Component Analysis(RPCA) in [29], replaces $rank(\cdot)$ with the nuclear norm (sum of the singular values of the matrix) and the l_0 -norm with the l_1 -norm (sum of the absolute values of all entries of the matrix). Therefore, the optimization problem given in Eq.(4) can be transformed as:

$$\begin{aligned} \min_{A,E} & \|A\|_* + \lambda \|E\|_1 \\ \text{s.t.} & D = A + E \end{aligned} \quad (5)$$

where $\|\cdot\|_*$ ($\|A\|_* = \sum_{i=1}^r \sigma_i(A)$) represents the nuclear norm and $\|\cdot\|_1$ is ℓ_1 -norm, λ is a trade-off parameter, and $\sigma_i(X)$ is the i -th singular value of the matrix X . $\|\cdot\|_1$ is defined as $\|E\|_1 = \sum_{i,j=1}^n |E_{i,j}|$, which is the sum of the absolute value of matrix entries. It has been proved in [29]. Eq. (4) has the same optimal solution as Eq.(5) and $\lambda = C/\sqrt{\max(m,n)}$, where C is a constant.

4.2. Fast Algorithm Using Accelerated Proximal Gradient

There are many works on developing for nuclear-norm and l_1 norm minimization [32] [33]. In the next, We show how the Accelerated Proximal Gradient(APG) can be adapted to efficiently solve Eq. (5). Relaxation of the optimization problem in Eq. (5) can be formulated as

$$\min_{A,E} F(A, E) = \mu \|A\|_* + \mu \lambda \|E\|_1 + \frac{1}{2} \|D - A - E\|_F^2 \quad (6)$$

where μ is a small positive scalar and $\|\cdot\|_F$ is the Frobenius norm. Accelerated proximal gradient (APG) algorithm [34] is utilized to reach the solution of A in an iterative fashion, where the basic APG iteration is given by

$$\begin{aligned} A_{k+1} &= \mathcal{S}_{\mu/L_f}(Y_k^A - \frac{1}{L_f}(Y_k^A + Y_k^E - D)) \\ E_{k+1} &= \mathcal{S}_{\mu/L_f}(Y_k^E - \frac{1}{L_f}(Y_k^E + Y_k^A - D)) \end{aligned} \quad (7)$$

where Y_k is updated using the following rule for fast convergence:

$$\begin{aligned} Y_k^A &= A_k - \frac{t_{k-1} - 1}{t_k}(A_k - A_{k-1}) \\ Y_k^E &= E_k - \frac{t_{k-1} - 1}{t_k}(E_k - E_{k-1}) \\ t_{k+1}^2 - t_{k+1} &\leq t_k^2 \end{aligned} \quad (8)$$

203 Meanwhile, A_k and E_k are updated following

$$\begin{aligned} A_{k+1} &= U \mathcal{S}_{\frac{\mu_k}{2}}[S] V^T \\ E_{k+1} &= \mathcal{S}_{\frac{\lambda \mu_k}{2}}[G_k^E] \end{aligned} \quad (9)$$

204 where USV^T represents the singular decomposition of G_k^E , as defined in
 205 Algorithm 1. $\mathcal{S}_\epsilon[x]$ is a shrinkage operator shown below:

$$\mathcal{S}_\epsilon[x] =: \begin{cases} x - \epsilon & \text{if } x > \epsilon \\ x + \epsilon & \text{if } x < -\epsilon \\ 0 & \text{otherwise} \end{cases}$$

206 where $x \in R$ and $\epsilon > 0$.

207 μ is a small positive scalar. A classical technique [30], which varies μ , starting
 208 from a large initial value μ_0 and decreasing it geometrically for each iteration
 209 until it reaches the floor $\bar{\mu}$, can speed up the convergence dramatically. μ_k is a
 210 monotonically increasing positive sequence. The entire algorithm to solve Eq. (6)
 211 has been summarized as Algorithm 1.

Algorithm 1: Data Matrix Splitting via APG

Input : Data matrix $D \in R^{m \times n}$.
Initialization: $A_0, A_{-1} \leftarrow 0; E_0, E_{-1} \leftarrow 0; t_0, t_{-1} \leftarrow 1; \hbar \mu \leftarrow \delta \mu_0$.
1 repeat
 2 $Y_k^A \leftarrow A_k + \frac{t_{k-1}-1}{t_k}(A_k - A_{k-1}), Y_k^E \leftarrow E_k + \frac{t_{k-1}-1}{t_k}(E_k - E_{k-1});$
 3 $G_k^A \leftarrow Y_k^A - \frac{1}{2}(Y_k^A - Y_k^E - D);$
 4 $(U, S, V) \leftarrow \text{svd}(G_k^A), A_{k+1} = U \mathcal{S}_{\frac{\mu_k}{2}}[S] V^T;$
 5 $G_k^E \leftarrow Y_k^E - \frac{1}{2}(Y_k^A + Y_k^E - D);$
 6 $E_{k+1} = \mathcal{S}_{\frac{\lambda \mu_k}{2}}[G_k^E];$
 7 $t_{k+1} \leftarrow \frac{1 + \sqrt{4t_k^2 + 1}}{2};$
 8 $\mu_{k+1} \leftarrow \max(\eta \mu_k, \hbar \mu);$
 9 $k \leftarrow k + 1$.
10 until convergence;
Output : $(A, E) \leftarrow (A_k, E_k)$.

Figure 5: Comparison results of DinoSparseRing data sets between the coarse models using NCC method and the refined models using our method, which demonstrate the strength of our refine model

Figure 6: Comparison results of TempleSparseRing data sets between the coarse models using NCC method and the refined models using our method, which demonstrate the strength of our refine model

5. Experiments

To evaluate the performance of the matrix splitting algorithm, we tested this method using data sets from the Middlebury Vision Evaluation Website [35] and Real-World data sets [36]. We implemented our algorithm using C++ with OpenCV library. Firstly the rough point clouds are estimated using variational method, then the matrix splitting is applied to refine the point clouds. 3D models are generated from the rough point clouds and the refined point clouds using Poisson surface reconstruction [37] separately. We compare the two 3D models to demonstrate that our method removes the noise and errors effectively.

In these experiments, we verify the effectiveness of our method quantitatively. We compare our results with other state-of-the-art reconstruction algorithms. The comparison is reported in Table 2, where we use A to represent the reconstruction accuracy, C to indicate the completeness and T is the processing time at minute scale.

5.1. Hypothetic Point Clouds Estimation Using Variational Method

Firstly, *DinoSparseRing* and *TempleSparseRing* datasets are used to generate the rough point clouds using variational method. The data sets can be obtained from the website [35], which contain 16 images and a calibration parameters file. Left parts of Fig. 5 shows the rough 3D model of *Dino* using filter in PMVS [7]. In Fig. 5 and Fig. 6, the colored rectangles and circles mark the regions that are corrupted severely. The rough point clouds estimation module is implemented based on Brox’s optical flow method [24].

5.2. Outlier Extraction Using Matrix Splitting

It can be seen from Fig. 6 that the reconstruction accuracy of variational method vibrates over different parts of the model. By contrast, a matrix splitting method as Algorithm 1 can refine the point clouds robustly in this experiments. In the above experiments, λ is a important weighting parameter. We have fixed the value of the parameter $\lambda = 1/\sqrt{n}$, as suggested by [29]. n is the number of all rough points, i. e. , dimensions of the vector $vec(d_i)$ ($i = 1, \dots, n$). From Fig. 6 we can see that the proposed refined method could dramatically remove outliers and bad matches.

The quantitative results of our framework are shown in Table 2. At the moment when the results were submitted to the Middlebury Vision Evaluation Website, the accuracy and completeness measurements of the results about two data sets both rank top, especially, the 3D model of *Dino* was in top three. Fig. 7 show refined 3D

models, which are more accurate and complete than existing state-of-art methods [7] [10] [14] [23].

5.3. Middlebury Datasets

Table 2: Results for Middlebury datasets

Methods	Dino Sparse Ring			Temple Sparse Ring		
	A. [mm]	C. [%]	T. [min]	A. [mm]	C. [%]	T.[min]
Deng [23]	0.49	97.8	13	0.74	96.1	12
Liu2 [14]	0.51	98.7	20	0.65	96.9	16
Bradley [10]	0.38	94.7	7	0.48	93.7	4
Furukawa 3 [7]	0.37	99.2	152	0.63	99.3	129
Ours	0.37	99.2	21	0.62	95.2	18

According to the results, we can conclude that the algorithm achieves the best accuracy among all the reconstruction algorithm applied in DinoSparseRing data sets. In the condition of the same accuracy and completeness, Furukawa’s [7] reconstruction algorithm spent 10 more times of computational cost than our method. Bradley’s method [10] achieves higher accuracy about TempleSparse data sets, but results of them have low scores in completeness since they can’t generate closed models. Overall, our method is among the best reconstruction accuracy on both evaluation data sets, as well as a attractive balance on both reconstruction completeness and computational efficiency.

5.4. Real-World Data sets

To further evaluate the proposed method, we applied our algorithm to several real-world data sets: *beethoven* datasets[36], *bunny* datasets[36], *pig* datasets[36]. Ground truth for these data sets were not available, and thus qualitative evaluation of appearance of the reconstruction results was adopted in the experiments.

The *beethoven*, *bunny* and *pig* data sets include 33, 36, 26 images respectively. They were captured by a set of synchronized multiple cameras. The *beethoven* data sets present textureless surface, and the *pig* data sets present smooth surface. Our experiment results of these data sets are illustrated in Fig. 8. The proposed method is able to handle noise and outliers effectively, even for the textureless situation. The *bunny* data sets include only 36 images of highly textured images. Unlikely, some details of wrings of the bunny were not kept. The reason may be that these small details be affect by the sooth data factor when rough points evaluation using variational method.

Figure 7: The comparison of reconstruction results on both DinoSparseRing and TempleSparseRing data sets

273 6. Conclusions

274 In this paper, we propose a novel technique for accurate reconstruction of 3D
275 geometrical shape from multiple calibrated images of the same object. The rough
276 point clouds are generated by integrating the silhouette information and epipolar
277 constraint into the variational method. To remove outliers and bad matches effec-
278 tively, We first introduce a low-rank matrix splitting on 3D model reconstruction
279 which can be solved efficiently using accelerated proximal gradient algorithm.
280 Compared with the existing state-of- art methods, we have shown the top perfor-
281 mance in both accuracy and completeness measurements using Middlebury and
282 real-world data sets.

283 Exploring the advantage of the low-rank minimization, we consider that the
284 proposed technique can be extended to handle similar problems in other low-level
285 vision tasks in the future.

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Figure 8: Images and 3D reconstruction results of several real datasets: (a)bunny (b)beethoven (c)pig. The red box represent of the results of [24], green box represent of our results.

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