Digital image analysis to quantify carbide networks in ultrahigh carbon steels

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Abstract

A method has been developed and demonstrated to quantify the degree of carbide network connectivity in ultrahigh carbon steels through digital image processing and analysis of experimental micrographs. It was shown that the network connectivity and carbon content can be correlated to toughness for various ultrahigh carbon steel specimens. The image analysis approach first involved segmenting the carbide network and pearlite matrix into binary contrast representations via a grayscale intensity thresholding operation. Next, the carbide network pixels were skeletonized and parceled into braches and nodes, allowing the determination of a connectivity index for the carbide network. Intermediate image processing steps to remove noise and fill voids in the network are also detailed. The connectivity indexes of scanning electron micrographs were consistent in both secondary and backscattered electron imaging modes, as well as across two different (50x and 100x) magnifications. Results from ultrahigh carbon steels reported here along with other results from the literature generally showed lower connectivity indexes correlated with higher Charpy impact energy (toughness). A deviation from this trend

was observed at higher connectivity indexes, consistent with a percolation threshold for crack propagation across the carbide network.

Keywords

Ultrahigh carbon steel; Carbide network; Network analysis; Image segmentation; Fracture toughness; Percolation theory

1. Introduction

Ultrahigh carbon steels (UHCS) have been used for many years in various applications demanding high strength and wear resistance. UHC steels have carbon contents from 1-2.1 wt%, in excess of the 0.76 wt% Fe-C eutectoid composition (Figure 1), and as a result there is precipitation of proeutectoid cementite (Fe_3C) during cooling from casting and/or heat treatment processes. The hard and brittle cementite contributes to the high hardness and wear resistance of UHCS, which is desirable for applications involving the cutting or shaping of other metals. UHCS have been used in rolling mills as far back as 1913(1) and are also commonly used in some tool steels. There has been interest over the last few decades in using UHCS for other applications where traditionally lower carbon content steels have been utilized, such as sheet and automotive steels(2, 3). Recent UHCS research has focused on improving ductility/toughness by modifying existing manufacturing processes to improve the resulting microstructure.

One major goal in UHCS processing is modification of the network of brittle carbides (primarily cementite) distributed throughout the microstructure. Networks of proeutectoid cementite form in UHCS alloy during cooling, primarily due to precipitation in the $\gamma+\text{Fe}_3\text{C}$ phase field (depending on the steel composition and cooling rate, a small amount of eutectic

solidification may also occur). The carbide network provides crack initiation sites and pathways for crack propagation that reduce UHCS toughness/ductility (4, 5) by allowing cracks to circumvent the ductile matrix. Figure 2 shows an example of a crack propagating preferentially within the carbide network. Modification of the network, namely breaking it up, has the potential to reduce pathways for crack propagation and force cracks to travel through the matrix, thereby increase toughness.

It is possible to process UHCS so as to completely eliminate the network or significantly reduce its connectivity. In this case, toughness may depend on microstructural parameters such as the type of eutectoid transformation products and the prior austenite spacing (6). Successful approaches have included thermomechanical processing (3, 7) and chemistry modification by small additions of rare earths or niobium (8–10); both approaches have resulted in greatly improved mechanical properties measured at room temperature. These relatively new manufacturing methods have not yet achieved widespread adoption, possibly because of additional processing difficulty and cost. Another possible processing alternative is using heat treatments alone (11–13). In all cases, these studies utilize optical microscopy (OM) or scanning electron microscopy (SEM) to qualitatively evaluate the carbide network connectivity in relation to processing conditions.

While there is interest in breaking up the network in UHCS, there currently exists no metric for quantifying the degree of network break-up beyond calculating the network volume fraction. Establishing a standard method for quantifying connectivity of carbide networks would improve evaluation and comparison of proposed network modification processes.

The concept of network connectivity analysis is already well established in various fields. Percolation theory (14–16) provides an essential framework for describing networks in terms of

connections between points/nodes on a grid. Transportation science (17, 18) defines a “connectivity index” as a ratio of streets to intersections and cul-de-sacs. Tortuosity in porous substances (19), is found by quantifying the ratio of distance traveled along a convoluted path to the direct point to point distance. Medical science describes a connectivity index in cancellous human bone (20, 21) in terms of the maximal number of connections needed to be broken to separate one network into two. For the case of UHCS steels, carbide networks provide a “pathway” for crack propagation. Thus in this work, the transportation science metric for network quantification is leveraged to quantify carbide networks in UHCS.

The purpose of this paper is to propose, demonstrate and standardize a potential method of analyzing OM/SEM images of UHCS microstructures and quantifying the network connectivity. In this study, commercial UHCS roll mill specimens were imaged and analyzed to produce quantified metrics for carbide network connectivity. These measurements were compared to experimental toughness measurements performed in this study as well as literature values. Details of the digital image analysis approach are described.

2. Digital Image Analysis Methods

This section describes our approach for quantifying network connectivity as applied for as-cast and heat treated UHCS specimens. ImageJ (22), an image processing software program in the public domain (<http://rsb.info.nih.gov/ij/index.html>), was utilized for network connectivity quantification. Network analysis requires deconstructing contrast features in micrographs into a network composed of links and nodes in space, illustrated in Figure 3. Nodes may be endpoints, linked to exactly one other node, or internal junctions connected through links to two or more other nodes. The ratio of links to nodes, known as the connectivity index (18), is a useful

parameter in evaluating such a simplified network. Information such as the probability of each node being an endpoint or junction, and the average link length, is also of interest.

The critical image processing step allowing these parameters to be extracted is skeletonization, an operation that thins the network to a single-pixel width so that nodes and links (Figure 3) can be identified. The following sections describe the image processing steps developed to extract connectivity index from digital micrographs of UHCS materials: thresholding for phase segmentation, particle size filtering for artifact removal, outlier removal for network void removal, and skeletonization for network quantification.

2.1. Thresholding for phase segmentation

The initial objective was to digitally select and isolate the carbide network from other phases present in a typical UHCS micrograph. The first step was to ensure clear image contrast between constituent phases in the steel microstructure under OM/SEM imaging. For UHCS containing cementite and ferrite, the common etchant, Nital (5% nitric acid and 95% alcohol such as ethanol or methanol) was useful towards this aim. Nital preferentially etched the ferrite, revealing proeutectoid cementite as well as cementite lamellae in the pearlite. The resulting rougher eutectoid pearlite morphology produced a higher local secondary electron (SE) yield than the smoother morphology network proeutectoid cementite. Thus the proeutectoid cementite network exhibited lower grayscale intensity in the SE image. Figure 4(a) shows the typical appearance of a Nital-etched pearlitic UHCS microstructure as imaged by SEM. User-controlled gain settings ensured the proeutectoid cementite was dark and distinct from the lighter pearlitic matrix. There was some grayscale variation in the pearlite matrix due to spatial lath orientation variations. Ultimately, any suitable etchant process for the analyzed steel should yield such

micrographs of distinct constituent phase contrast, whether by SEM using SE or backscattered electron (BSE) detection or by OM as will be demonstrated later in this article. It should be noted that the etching process will reveal additional cementite beneath the original plane of polish. Given that etching typically does not penetrate more than a micrometer into the steel, and our microstructural feature of interest (the carbide network) has a length scale of tens to hundreds of microns, we do not anticipate a significant deviation in measured network connectivity due to the effects of etching, except possibly in cases of severe over-etching.

The next step was to segment phases within the micrograph. Segmentation began by establishing a grayscale intensity threshold in order to distinguish between constituent phases in the microstructure. The pixel grayscale histogram for Figure 4(a) is shown in Figure 4(b) and exhibits two peaks in grayscale intensity, each peak distinguishing the carbide network (darker grayscale) from the pearlite matrix (brighter grayscale). Intensity thresholding was used to retain only pixels below a threshold value of grayscale intensity. In **Error! Reference source not found.**(c) the highlighted red pixels denote those pixels below the intensity threshold. These pixels identify the carbide network for subsequent analysis. The threshold intensity was chosen by fitting a Gaussian profile to the range of pixels representing the network peak and setting the threshold at either two or three standard deviations from the peak center (2σ or 3σ from the maximum intensity). Taking three standard deviations ensured that most pixels in the network were included within the threshold, but substantial image noise might necessitate a 2σ selection. Simply setting the threshold at the intersection of the matrix and network peaks is not always sufficient to guarantee retention of sufficient network pixels after thresholding. The vertical black line in Figure 4(b) denotes the threshold grayscale value at a distance of 3σ from the network peak center, and the intensity region to be retained is highlighted in red. After

thresholding, a binary operation was performed to generate an 8-bit binary image where all retained pixels have one grayscale value of 0 (black) and the rest have values of 255 (white).

2.2. Particle size filtering for artifact removal

The next step in the image processing procedure was removal of noise artifacts from the binary image. Artifact pixels result from the overlap in grayscales between network and matrix phases in the histogram, as well as random fluctuations in SE signal during SEM image acquisition due to noise. Phase signal overlap was due to appreciable grayscale variation in the matrix, where the darkest regions in the pearlite matrix were as low in grayscale value as the brightest regions of the carbide network and thus were not segmented out during thresholding. Random SE signal fluctuations from noise depend on beam dwell time on each pixel, and thus introduced artifacts on the order of a few pixels. Artifacts from phase signal overlap depend on the microstructure itself as well as the quality of sample preparation, and thus can be of any size. It is critical to remove as many artifacts from the image as possible while retaining the actual carbide network signal.

Artifact removal was accomplished through employment of a particle size filter. Particles (collections of adjacent pixels) must exceed a critical user-defined area to be retained. The average carbide network width was measured and the size filter was set to exclude areas less than the square of the network width. Network width was determined by measuring line profiles of grayscale intensity across the binary image as shown in Figure 6(a) and plotted as a function of distance as shown in Figure 6(b). The peak widths for each line were averaged and the mean of the peak widths along the six lines was used as an estimation of the average carbide network widths. Since most particles on the network were elongated and most artifact particles tend to be

more equiaxed in shape, filtering out particles with areas less than the square of the average network width removed few network particles (as long as there is not too much variation in the distribution of network branch widths) and targeted the maximum number of artifact pixels. Figure 7(a) shows a thresholded binary image with particles to be removed highlighted in red, and Figure 7(b) shows the results after the filtering operation.

2.3.2. Outlier removal operation for network void removal

In addition to its utility in removal of noise artifacts by size filtering, the measured network width was also employed to remove voids from the carbide network. The term “voids” refers to small pockets of matrix that existed within the branches of the network, mostly due to eutectic solidification. If these voids were not removed, the skeletonization step will create many artificial network pathways that were not necessarily representative of crack propagation paths through the network. Figure 8 provides an example micrograph illustrating this point, where a crack has taken a straight path through a network carbide containing many voids. Given that voids exist entirely within the network branches, their average widths should generally be smaller than the previously measured average carbide network branch width.

To remove the voids, a “remove outliers” operation was utilized in ImageJ and demonstrated in Figure 9. This operation allowed assessment of individual white or black pixels and their neighbors. For the case of void removal, each white pixel was assessed and if the surrounding pixels, up to a user-defined radius, were more black than white, that pixel was considered within a void and converted to a carbide pixel, thus rendered black. The radius required to remove voids was equal to or somewhat greater than the diameter of the observed voids themselves, so the average network branch width was a reasonable value to use as the user-

defined radius. Comparison of Figure 9(a) to Figure 9(c) shows that the operation removed many voids throughout a given micrograph. Failure to remove voids from the network led to a skeleton that included many extra small loops, as shown in the unprocessed Figure 9(b) as compared to Figure 9(d). Inclusion of these loops in the analysis created many non-endpoint nodes, and thus contributed to an artificially larger connectivity index.

2.2. Skeletonization for network quantification

The final image processing was skeletonization. ImageJ uses a thinning algorithm for skeletonization, which iteratively removed pixels from all the edges of particles in the image until the thickness was reduced to one pixel. Figure 10(a) shows a micrograph after all the steps described above: phase segmentation by thresholding, artifact removal by particle size filter, and void filling by the remove outliers operation. Skeletonization of this image resulted in the simplified form seen in Figure 10(b). The longest shortest paths within each network tree, i.e. the longest distance that could be traveled along the tree without doubling back, are shown in Figure 10(c). Figure 9(c) is included to help visualize the most likely crack paths through this microstructure along the plane of polish.

Following skeletonization, quantitative information about the skeletonized network was extracted using the “AnalyzeSkeleton” plugin for ImageJ (23, 24). AnalyzeSkeleton labels each pixel: end-point pixels are those with exactly one neighbor, slab pixels have two neighbors, and junction pixels have more than two neighbors. The plugin condensed connected slab pixels into branches (B) and connected junction pixels into junctions (J). Endpoints (E) and junctions are the nodes of the network, and branches are links between them as previously illustrated in Figure 3. Relevant input and output from the Plugin is summarized in

Tables

The numerical output was useful for calculating connectivity index, average branch length, and probability of any node to be an endpoint. As mentioned earlier in Section 2, the connectivity index is the ratio of links to nodes. Therefore, the connectivity index can be defined as $CI = (B/(E+J))$ and will relate the probability of any two nodes being connected within the network. Higher values of connectivity index will mean higher probability of nodes being connected, thus a more interconnected network. The probability of any node to be an endpoint is

found by dividing the number of endpoints by the total number of nodes ($E/(E+J)$). The next sections show the application of this method to actual UHCS samples.

3. Materials and Experimental Methods

The image analysis process described in the previous sections has been applied to various UHCS specimens. In the present study, UHCS specimens consisted of 0.5 in. x 0.5 in. x 0.5 in. samples cut from commercial roll mill castings. Specimens from two different 36 inch diameter roll mills were studied, one in the as-cast condition and one after a heat treatment that involved an austenitizing hold above 1000°C. The composition of the material is shown in Table 2.

The microstructure was characterized using a scanning electron microscope (Phillips XL-30 SEM, operating at 20kV accelerating voltage). For SEM analysis, specimens were cut using an abrasive alumina wheel, ground with SiC paper, polished in alumina slurry and etched for 30 seconds in 4% Nital (nitric acid/methanol) solution. Connectivity index and network area fraction were quantified as described previously in Section 2 for numerous SEM micrographs of these UHCS specimens.

Two instrumented Charpy impact tests were performed commercially by Westmoreland Mechanical Testing and Research for each of the as-cast and heat treated UHCS conditions. Standard size Charpy V-notch (CVN) specimens were machined per ASTM E-23c (25) and, due to low toughness of the samples, instrumented Charpy tests were conducted per ASTM E2298-15 (26) for the as-cast and heat treated steels to examine potential correlation between measured impact toughness and microstructure.

4. Results

4.1. Effects of magnification and imaging conditions

The previously described image analysis approach was applied to SEM micrographs of the UHCS specimens, examples presented in Figure 10. Quantitative analysis results were found to be consistent across different magnifications and using different imaging conditions as shown in Table 3.

4.2 Comparison of as-cast and heat treated UHCS

The next objective of our study was to compare two different UHCS microstructures in terms of measured connectivity indexes. Figure 13 shows example SEM micrographs of heat treated and as-cast UHCS samples. **Error! Reference source not found.** reports image analysis and Charpy impact testing results. Qualitatively, there was little difference in the apparent connectivity of the two UHCS conditions. Nevertheless, digital analysis of the micrographs indicates that the heat treated UHCS had a lower connectivity index than the as-cast sample. The carbon contents in the samples are different, 2.1 wt% in the as-cast roll and 1.7% in the heat treated roll, so no claim is made that the heat treatment is solely responsible for the decrease in connectivity index. Charpy impact energy absorption did not differ significantly between the heat treated and as-cast conditions. For our purposes, a significant difference would be on the order of one to several Joules. We do not report a standard deviation for the Charpy impact energy values because only two tests were conducted for each condition. Additional measurements from UHCS microstructures with significant differences in impact energy were desired to investigate a possible link between index and Charpy impact energy, so we sought to further utilize our image processing approach for published micrographs in the literature.

4.2 Quantifying network index in previously published research.

The image analysis methods described above were also applied to previously published studies of UHCS. Hamidzadeh (8, 27) and Liu (28) explored the effect of chemical additives on carbide network breakup and resulting toughness enhancements, measured using Charpy impact tests. Chemistries of these UHCS are shown in Table 6.

Figure 15(a-c) reproduces, with permission, OM micrographs of UHCS specimens previously published by Hamidzadeh *et al.*(8). The OM micrographs qualitatively reflect a breakup of the carbide network with increased Nb addition. The current image analysis approach was applied to quantify carbide network connectivity index for each OM micrograph. Segmented binary images, shown in Figure 15(d-f), captured the carbide network for each corresponding OM micrograph (Figure 15 (a-c)). The measured connectivity index for each micrograph showed a clear decrease with additional Nb content. Area fraction also decreased with Nb content. The images of Liu showed little change in connectivity with the addition of rare earth modifiers, but network area fraction was increased due to the modification.

5. Discussion

5.1 Relevance of connectivity index

The primary objective of this study was to develop a metric to connect UHCS microstructure and performance. Since network break up is often associated with enhanced toughness, results from this study and published literature (8, 9, 27) were analyzed to explore possible trends in material performance (quantified by Charpy impact testing) and microstructural metrics. Comparisons for Charpy impact energy versus connectivity index and network area fraction are shown in Figure 13. We do not show error bars in Charpy impact energy in Figure 13 for the data of Liu *et al* (9) because no standard deviation in Charpy impact energy data was reported in their study. Our own Charpy data, which consists of two

measurements for each of the heat treated and as-cast conditions, appears as a single point for each condition because the data points completely overlap when both measurements are plotted at the scale of the figure.

Two trends were identified in Figure 13(a) based on the grouping of UHCS samples by similar steel chemistries. Both trends show that impact energy decreased as connectivity index decreased in the lower connectivity index range (below 0.7). We noted a possible inflection point at about 0.70-0.75, above which connectivity index ceased to have an effect on impact energy (plateau region). The two different energy trends and the existence of two plateau energies in Figure 13(a) are attributed to differences in alloy chemistry and carbide type. Hamidzadeh *et al.* (8, 27) reported primarily M_7C_3 carbides in their steel (upper trend), as compared to the Fe_3C carbides reported by Liu *et al.* (9) and this study (lower trend). The upper trend had a plateau energy close to 8.5 J/cm^2 , while the lower trend plateau energy was close to 2.5 J/cm^2 . The difference in plateau energies might reflect the energy dissipation capacities of the different types of carbides. However, we have not been able to find any literature values of impact energy for pure Fe_3C or M_7C_3 .

Figure 13(b) shows little correlation between impact energy and network area fraction across all steels, though a decreasing trend was observed in the work of Hamidzadeh. This trend was attributed to the relationship between connectivity and network area fraction as shown in Figure 14. The carbide network formed between austenite grains, so it was expected that higher amounts of carbide would correlate with a more connected network, and thus a clearer connection between network area fraction and the connectivity index.

5.2 Crack Propagation behavior and Percolation Threshold

The existence of the plateau and inflection point in Figure 16 (a) is attributed to a possible transition in crack propagation behavior through the UHCS microstructure. The plateau region, where network connectivity does not influence toughness, most likely corresponds to crack propagation primarily along the brittle carbide network. In this high connectivity index region, the network has sufficient connectivity to span the entire microstructure without appreciable discontinuities, so further increases in connectivity of the network have but minor influence on toughness. We estimate that this plateau region begins when connectivity exceeds about 0.7, as seen in Figure 16 (a) from interpolation of trendlines for both Hamidzadeh and Liu/*this study*. For connectivity indexes lower than 0.7, absorbed impact energy increases as index decreases, strongly indicating that cracks must traverse greater proportions of ductile matrix to propagate across the UHCS microstructure. UHCS in which the carbide network has been eliminated should have toughness controlled by matrix microstructural parameters.

This concept of a critical connectivity level required for a network to effectively span a given length-scale is in congruence with well-established percolation theory, specifically the concept of “percolation threshold” (14). Percolation theory addresses connected paths of specified length within a system. Systems can be modeled as a lattice of nodes in space, with each node having a probability of being connected to its nearest neighbor(s). The percolation threshold refers to the critical connection probability required to have a greater than 50% chance of having a connected path span across the entire lattice. In systems of infinite size, the probability of spanning the lattice is a stepwise function; above the percolation threshold there will always exist a spanning path, and below it such a path will never exist. The more nodes there are in the lattice, the closer the probability of spanning approaches a stepwise function. If we envision an UHCS microstructure as a lattice of matrix and carbide nodes, the connectivity of

the network of carbide nodes as a whole is related to the probability of connection between individual carbide nodes. Thus, there should be a critical network connectivity enabling the network to span the entire UHCS microstructure. Connectivity at or above this threshold should lead to cracks being able to propagate entirely within the carbide network without passing through the matrix, resulting in the observed plateaus in impact energy in Figure 16 (a). The 2D network sections analyzed in this work could accurately reflect the true 3D connectivity, given a sufficiently isotropic network. Analysis of different regions spanning millimeters in the UHCS specimens of this study yielded similar connectivity indices but also varying directionality in the local network structure due to dendritic solidification. With sufficient statistical analysis of 2D images over many regions of a sample, the resulting microstructural properties should be adequately representative for the 3D case as well. Thus, it should be possible to estimate the true 3D connectivity of the network from 2D analysis. Furthermore, serial sectioning over large length scales (hundreds of microns) to produce 3D microstructural data could yield 3D connectivity indices to compare with 2D analysis, a process we are pursuing using new plasma focused ion beam instruments now commercially available (29).

Future work will include analytically calculating the percolation threshold for a 3D UHCS microstructure and thereby predicting the critical 2D connectivity index needed to exit the plateau phase. Critical metrics like average network branch length (length-scale) and probability of each node to be an endpoint/internal junction should be sufficient to perform this calculation. Both of these parameters are extracted from 2D micrographs using the AnalyzeSkeleton plugin as described in Section 2.2. Heat treatments would need to break up the carbide network and surpass a threshold connectivity index in order to yield measurable improvements in UHCS toughness.

5.2 Network length-scale

For crack propagation along a network, a higher number of deflections correlates to higher toughness (30). The connectivity index defined above is dimensionless, but a measure of network length scale can be separately quantified for the above-mentioned images by using the average distance between branches in the network along a line profile of grayscale intensities. A similar method using line profiles was demonstrated for finding the network branch width in Section 2.2, except now the average distance between carbide branches was quantified. This distance would depend on cooling rate, depend on chemistry (8, 31) and would correspond to the secondary dendrite arm spacing of the primary austenite. For the UHCS tested in this report, average size was approximately 56 μm while the average size from the work of Hamidzadeh and Liu was approximately 35 μm . Therefore toughness differences could also be attributed to higher amount of crack path deflections occurring in the more closely spaced networks. The connection between network connectivity and length-scale will be further examined in future studies.

Conclusions

This study presents a method for quantitatively assessing carbide network connectivity in UHCS. The connectivity index used in transportation science is analogous to crack propagation along a “pathway” of carbide networks in UHCS. Therefore, we demonstrated a quantitative image analysis approach to determine this connectivity index values from SEM and OM micrographs in order to gain insight on crack propagation and toughness for UHCS microstructures. The process required a series of standard image processing steps that common

commercial software programs, such as ImageJ, can perform. Experimental micrographs were segmented by phase using grayscale intensity thresholding. Artifacts in the segmented images were removed by filtering out particles with area less than the square of the average network branch width. Voids inside the carbide branches were filled and then the networks were skeletonized to convert them into systems of links and nodes for determining connectivity index.

The developed image processing procedure demonstrated consistent connectivity index values with respect to different magnifications and imaging conditions for the same UHCS samples. Toughness was correlated with connectivity index below a critical value of around 0.7 and plateaued above that value. This observation was attributed to connectivity index exceeding a percolation threshold. In general, the connectivity index shows promise as a useful metric to predict comparative toughness for UHCS specimens modified by various processing strategies.

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Figure and Table Captions

Figure 1. Fe-C phase diagram indicating temperature/composition regions of interest in UHCS.

[This phase diagram was generated using the Factsage® thermochemical software(32)]

Figure 2. SEM micrograph of a crack propagating through the cementite network in an UHCS (BSE imaging mode).

Figure 3. Schematic illustration of (a) a simple network and (b) the same network indicating branches and nodes.

Figure 4. (a) An experimental SEM micrograph of a UHCS microstructure containing a proeutectoid cementite network with (b) grayscale intensities plotted in a histogram; the highlighted red portion indicates the retained grayscale intensities that are (c) overlaid in red on the original micrograph and denote the selected carbide network phase.

Figure 6. Approximate width of branches in the network is determined by plotting grayscale intensity across the micrograph, demonstrated by (a) selecting various lines across the binary micrograph, indicated by dotted black lines. A plot of grayscale intensity along the solid red line is shown in (b).

Figure 7. Thresholded binary micrograph (a) showing particles with area less than the square of the average network branch width highlighted in red and (b) the same binary micrograph after particles below the size threshold have been removed.

Figure 8. Secondary electron micrograph of a carbide network (dark gray) in an UHCS with a crack propagating across a number of voids.

Figure 9. Binary micrograph of an UHCS carbide network with a region of voids in the network highlighted in the lower right corner. (a) Network segment before void removal via Remove Outliers operation in ImageJ showed (b) many small loops upon skeletonization, while (c) the same network segment after Remove Outliers operation showed (d) no small loops upon skeletonization.

Figure 10. Image processing output of an SEM image (a) as a binary network representation, (b) after skeletonization and (c) after terminating branches are removed to show longest connected paths (likely in-plane crack propagation pathways).

Figure 10. Examples of binary and skeletonization image analysis steps on SEM micrographs of as-cast UHCS (a) at 50x magnification using a SE detector, (b) at 100x magnification using a SE detector, and (c) at 100x magnification using a BSE detector. (Note that the index as shown in each figure is not averaged. For mean and standard deviation of five such indexes, see Table 3.)

Figure 13. SEM micrographs of different Nital-etched UHCS (a) in the as-cast condition and (b) in the heat treated condition.

Figure 15. OM micrographs reproduced from Hamidzadeh et al. (8) show a 1.4 wt% C UHCS with chemistry modified by (a) no Nb addition, (b) 0.5 wt% Nb addition, and (c) 1 wt% Nb addition. Segmented binary images show a network connectivity change from (d) 0.94 to (e) 0.76 to (f) 0.63 upon increasing additions of niobium. [Fig 12(a,b,c) adapted from (8), with permission from Elsevier]

Figure 16. Plots of (a) connectivity index versus Charpy impact energy and (b) network area fraction versus Charpy impact energy, incorporating data from three different research studies (including the present). Lines in (a) drawn in manually to help guide the eye to the plateau and inflection in Charpy impact energy as a function of connectivity index. Impact energy is plotted in units of J/cm^2 to be consistent with data from published literature(8, 9, 27).

Figure 14. Plot of network area fraction versus connectivity index.

Table 1. Inputs and outputs from AnalyzeSkeleton plugin for ImageJ.

Table 2. Nominal composition for UHCS in wt. %

Table 3. Quantitative analysis results on images of as-cast UHCS networks

Table 4. Image analysis data and toughness results

Table 6. Chemistries of steels in some other UHCS studies. (RE – rare earth elements added to break up carbide network)

Figures

Tables

Input Pixel	Example	Condensed To:	Numerical Output
Type Labels:	(Red Pixel)		
Endpoint Pixel		Endpoint (E)	Number of Endpoints
Slab Pixel		Branch (B)	Number of Branches, Length of Each Branch, Longest Shortest Path
Junction Pixel		Junction (J)	Number of Junctions.

Sample	C	Si	Mn	Cr	Ni	Mo
As Cast	2.02	0.65	0.72	3.86	1.45	.33
Heat Treated	1.71	.55-.90*	.55-.90*	3.7-4.2*	1.00-1.50*	.15-.45*
Carbon content measured by LECO, other elements by ICP. (Except as noted by *)						
*Nominal composition range for this class of steel.						

Condition	Image	Connectivity Index ^[1]
Etched, as-cast, 50x magnification, SE image	Figure 10(a)	0.91±0.02
Etched, as-cast, 100x magnification, SE image	Figure 10(b)	0.92±0.04
Unetched, as-cast, 100x magnification, BSE image ^{[2],[3]}	Figure 10(c)	0.92±0.03

[1] - Value and standard deviation from five different secondary electron SEM images of UHCS microstructure.

[2] Unsharp mask filter has been applied to the image for better contrast.

[3] The diamond-shaped region in the center of Figure 10(c) is an artifact from Vickers indentation, which was manually removed for connectivity index determination by selecting its pixels using the “magic wand” tool in ImageJ.

Condition	As-Cast	Heat Treated
Carbon Content (Wt. %)	2.1	1.7
Network Area Fraction (%)	14 ±3	10 ±1
Connectivity Index	0.92±0.01	0.84±0.02
CVN Impact Energy	1.82 J and 1.83 J	1.98 J and 2.06 J

Sample	C	Si	Mn	Cr	Nb	Mo	V	RE	Ref. No.
This Study	2.02	0.65	0.72	3.86	0.02	.33		-	-
Hamidzadeh A	1.45	0.18	0.37	11.45	-	0.58	0.88	-	(8)
Hamidzadeh B	1.44	0.18	0.39	11.30	0.48	0.61	0.89	-	(8)
Hamidzadeh C	1.38	0.18	0.34	11.22	1.06	0.61	0.86	-	(8)
Hamidzadeh D	1.35	0.18	0.33	10.95	1.54	0.60	0.80	-	(8)
Hamidzadeh RE	1.45	0.18	0.37	11.45	-	0.58	0.88	0.03	(27)
Liu Unmodified	1.94	0.53	0.47	1.43	-	-	-	-	(9)
Liu RE	1.93	0.58	0.49	1.46	-	-	-	>0	(9)