Lattice Discrete Particle Modeling of Fiber Reinforced Concrete: Experiments and Simulations

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Abstract

Naturally accounting for material heterogeneity, the Lattice Discrete Particle Model (LDPM) is a meso-scale model developed recently to simulate the meso-structure of quasi-brittle materials by a three-dimensional (3D) assemblage of polyhedral particles. A meso-scale constitutive law governs the interaction between adjacent particles and simulates various features of the meso-scale response, including cohesive fracturing, strain softening in tension, strain hardening in compression and material compaction due to pore collapse. LDPM has been extensively calibrated/validated, showing superior capabilities in predicting qualitative and quantitative behavior of concrete. As a natural extension for this discrete model to include the effect of dispersed fibers as discrete entities within the meso-structure, LDPM-F incorporates this effect by modeling individual fibers, randomly placed within the volume according to a given fiber volume fraction. In this investigation, the theoretical basis for LDPM-F is reviewed, and to calibrate/validate the numerical model, an extensive experimental study has been conducted to investigate the mechanical properties of various prismatic specimens containing different types (steel and synthetic) and dosages of fibers. Excellent predictive capability of LDPM-F is demonstrated through a rigorous calibration/validation procedure.

Keywords: Fiber Reinforced Concrete, Experiments, Discrete Models

1. Introduction

Concrete is by nature a brittle material that performs well in compression, but is considerably less effective when in tension, and thus reinforcement is often used to absorb these tensile forces so that cracking is either eliminated, e.g., in the case of prestressed members, or, at least, reduced. For many years, steel in the form of bars or mesh has been used as reinforcement for a large variety of structural elements. In recent years, new forms of reinforcement to improve concrete toughness have been developed, such as short randomly distributed fibers of small diameters, made out of steel, plastic, natural materials, recycled reinforcements, etc. [1]. Fiber reinforced concrete, characterized by a significant residual tensile strength in post-cracking regime and enhanced capacity to absorb strain energy due to fiber bridging mechanisms across the crack surfaces, shows a dramatic improvement in service properties over plain concrete. ACI Committee 544 reported that the addition of fibers in a concrete matrix improves some mechanical properties of concrete, including impact strength and flexural toughness [2]. The most beneficial aspects of the use of fibers in concrete structures include durability improvements through reduction in crack width and permeability [3, 4]; potential cost savings where fibers are substituted for labor-intensive steel-reinforcing bars [5]; and the mitigation of deterioration owing to shrinkage and the associated early age cracking. Fiber reinforced concrete has found many applications in tunnel linings, bridge decks, airport pavements, slabs on grounds, industrial floors, dams, pipes, fire protection coatings and marine structures. In addition, it can be also used for repair, rehabilitation, strengthening and retrofitting of existing concrete structures [2].

Many numerical models have been developed to simulate the behavior of fiber reinforced concrete at different scales. Most approaches simulate the mechanical response of these composites by means of phenomenological macroscopic models combined with fracture mechanics techniques, such as damage, crack and microplane models [6, 9, 8, 7, 10, 11]. Fanella¹ and Krajcinovic developed a nonlinear analytical model based on the principles of the continuum damage mechanics for the stress-strain behavior of fiber reinforced concrete subjected to monotonic compressive and tensile loading [6]. This model was later modified by Li and Li to characterize the tensile stress-strain response of the fiber reinforced concrete [9]. Peng and Meyer proposed a simplified continuum damage mechanics model to describe the inelastic behavior of concrete reinforced with randomly distributed short fibers,

including anisotropic damage, unilateral effect, coupled volumetric and deviatoric inelastic deformation and complicated failure mechanisms [8]. Caner et al. proposed a M7f model for fiber reinforced concretes under static and dynamic loads, which features many improvement over the earlier versions of microplane model, including a more realistic description of the fiber pullout and breakage [7]. Ferreira addresses the use of R-curves to study the fracture behavior of high-strength concrete and steel-fiber-reinforced concrete subjected to cracking in a three-point bending configuration, where the R-curves are modeled through an effective approach based on the equations of linear-elastic fracture mechanics [10]. Han et al. proposed a constitutive model based on total strain and applied it to simulate structural component tests, which captures fiber reinforced concrete's unique reversed cyclic loading behavior [11].

However, experimental studies on fiber reinforced concrete have confirmed that the mechanisms responsible for the macroscopic mechanical responses mainly involve phenomena that occur at the meso-structural level. The cement fracture is the mechanism that triggers the failure of fiber reinforced concrete, but the subsequent mechanisms leading to complete failure is completely modified by the relative contents of fibers in the composite, and more importantly, by the bond characteristic at the fiber-matrix interface and all the phenomena associated with this effect [12]. Consequently, many discrete frameworks, such as rigid particle and lattice models, have been developed, taking explicitly into account the meso-scale phenomena [13, 14, 15, 16, 17]. A sequential multi-scale homogenization approach has been proposed by Kabele [18], whose framework links analytical and computational models covering scales from micro- to macro-level. Recently, Oliver et al. [19] proposed a two-scale approach in which the macroscopic model, at the structural level, takes into account the meso-structural phenomenon associated with the fiber-matrix interface bond/slip process.

In the present work, as first proposed by Cusatis et al. [20, 21, 22], fiber reinforced concrete is analyzed by a multi-scale model, called LDPM-F, in which the fine-scale fiber-matrix interaction is solved independently and the overall response is analyzed in a 3D meso-scale framework based on the recently formulated lattice discrete particle model (LDPM). LDPM is a realistic 3D model of concrete meso-structure developed by Cusatis et al. [23], which has been extensively calibrated/validated under a wide range of quasi-static and dynamic loading conditions, showing superior capabilities in predicting qualitative and quantitative behavior of concrete [25]. LDPM is an improve-

ment over the confinement-shear lattice model [26, 27], which, along with LDPM, has constitutive laws analogous to those of the microplane model [28, 29]. As a natural extension for this discrete model to include the effect of dispersed fibers as discrete entities within the meso-structure, LDPM-F incorporates this effect by modeling individual fibers, randomly placed within the framework according to a given fiber volume fraction [30].

LDPM-F has many salient and unique features: (1) a realistic, three-dimensional modeling of concrete mesostructure, including a discrete representation of individual fibers randomly distributed therein; (2) a multiscale approach in which the effect of embedded fibers on the structural response is based directly on the micromechanics of the fiber-matrix interaction; (3) the ability to simulate not only tensile fracturing but also multiaxial compressive loading; and (4) true predictive capability demonstrated through a rigorous calibration/validation procedure. In this investigation, the theoretical basis for LDPM-F is reviewed, to calibrate/validate the numerical model, an extensive experimental study has been conducted to investigate the mechanical properties of various prismatic specimens containing different types (steel and synthetic) and dosages of fibers. The predictive capability of LDPM-F is demonstrated through a rigorous calibration/validation procedure.

2. Review of LDPM-F

In this section, the theoretical basis for LDPM-F provided by Schaufert and Cusatis [21] is reviewed, and details of the previously established [31] micromechanical fiber pullout model adopted for this study are presented.

2.1. *Extension of LDPM to Include Fiber-Reinforcing Capability*

LDPM simulates concrete mesostructure by considering only coarse aggregate. Particles with assumed spherical shape and in accordance with typical mix designs and granulometric distributions, are introduced randomly into the volume by a procedure which avoids particle overlapping and ensures that all particles are contained within the volume of interest. A Delaunay tetrahedralization of the particle centers, along with the nodes used to describe the external surface of the volume, is used to define the lattice system which represents the mesostructure topology. A 3D domain tessellation, anchored to the Delaunay tetrahedralization, creates a system of polyhedral cells. Each cell contains one aggregate particle (or a surface node), and adjacent cells interact through the triangular facets where they are in contact.

Fig. 1(a) shows two adjacent particles along with their polyhedral cells and the associated tetrahedron edge. In the LDPM formulation, the interface facets are interpreted as potential crack surfaces. A detailed description of the tessellation procedure and LDPM idealization of concrete mesostructure can be found in Cusatis et al. [23, 25].

The extension to LDPM-F to include fiber-reinforcing capability is performed by inserting individual fibers with randomly generated positions and orientations [21, 22]. The geometry of an individual fiber is characterized by diameter d_f , length L_f , and curvature. In this study, fibers are assumed to be straight, and non-circular cross-sections are simulated through an equivalent diameter calculated as $d_f = 2(A_f/\pi)^{1/2}$, where A_f is the fiber cross-sectional area. For a given fiber volume fraction V_f , the number of fibers N_f contained in a concrete volume V is determined by $N_f = \lceil 4V_f V / (\pi d_f^2 A_f) \rceil$, where $\lceil x \rceil$ indicates the ceiling function. The fiber system is then overlapped with the cell system and all the facets intersected by fibers are detected. For each intersection, the shorter and longer fiber lengths, denoted as L_s and L_l , respectively, on each side of the facet, along with the orientations of the fiber and the facet, characterized by unit vectors $\mathbf{n}_f$ and $\mathbf{n}$, respectively, are computed. In accordance with the current LDPM formulation, the facet depicted in Fig. 1(b) is the projection of the original tessellation facet onto a plane orthogonal to the associated tetrahedron edge. The projected facet is used in order to avoid non-symmetric behavior for the case of purely tangential movement between two particles and the consequent stress-locking effect which can occur [23].

LDPM describes meso-structure deformation through the adoption of rigid body kinematics. For given displacements and rotations of the particles associated with a given facet, the relative displacement $\llbracket \mathbf{u}_C \rrbracket$ at the centroid is used to define measures of strain as $\varepsilon_N = \mathbf{n}^T \llbracket \mathbf{u}_C \rrbracket / \ell$, $\varepsilon_M = \mathbf{m}^T \llbracket \mathbf{u}_C \rrbracket / \ell$, and $\varepsilon_L = \mathbf{l}^T \llbracket \mathbf{u}_C \rrbracket / \ell$, where ℓ is the tetrahedron edge length associated with the facet, and $\mathbf{m}$ and $\mathbf{l}$ are two mutually orthogonal unit vectors tangential to the facet. It can be demonstrated analytically that this measure of strains correspond to the projections onto the facets of the classical strain tensor of continuum mechanics [24]. The strains are then used to compute facet stresses as $\boldsymbol{\sigma}_c = \sigma_N \mathbf{n} + \sigma_M \mathbf{m} + \sigma_L \mathbf{l}$ according to the constitutive equation reviewed in the next section. When a facet strains beyond the tensile elastic limit, the meso-scale crack opening can be calculated as $\mathbf{w} = w_N \mathbf{n} + w_M \mathbf{m} + w_L \mathbf{l}$, where $w_N = \ell(\varepsilon_N - \sigma_N/E_N)$, $w_M = \ell(\varepsilon_M - \sigma_M/E_T)$, and $w_L = \ell(\varepsilon_L - \sigma_L/E_T)$ with E_N , E_T being the elastic normal and tangential LDPM stiffnesses. At

this point fibers are engaged and equilibrium considerations at the facet level allow for the reasonable assumption of a parallel coupling between the fibers and the surrounding concrete matrix. Total stresses on each LDPM facet are then computed as $\boldsymbol{\sigma} = \boldsymbol{\sigma}_c + (\sum_{f \in A_c} \mathbf{P}_f)/A_c$, where A_c is the facet area, $\mathbf{P}_f$ is the crack-bridging force for each fiber crossing the given LDPM facet. The following assumptions are made: (1) $\mathbf{P}_f \approx \mathbf{0}$ for both $\varepsilon_N < 0$ and $\mathbf{w} = \mathbf{0}$; and (2) the bridging force depends on the micromechanical crack-bridging mechanisms associated with an increasing meso-scale crack opening, $\mathbf{P}_f = \mathbf{P}_f(\mathbf{w})$. The crack-bridging force $\mathbf{P}_f$ is calculated by analyzing separately the crack-bridging effect of one individual fiber on one single meso-scale crack.

2.2. LDPM Constitutive Law

A vectorial constitutive law governing the behavior of the material is imposed at the centroid of each facet. The elastic behavior is described by assuming that the normal and shear stresses are proportional to the corresponding strains: $\sigma_N = E_N \varepsilon_N$; $\sigma_M = E_T \varepsilon_M$; $\sigma_L = E_T \varepsilon_L$; where $E_N = E_0$, $E_T = \alpha E_0$, E_0 = effective normal modulus, and α = shear-normal coupling parameter. E_0 and α are assumed to be material properties that can be identified from results of experimental tests in the elastic regime. At the macroscopic level, concrete elastic behavior is statistically homogeneous and isotropic. As such, it can be modeled effectively through the classical theory of elasticity characterized by Young's modulus, E , and Poisson's ratio, ν . The relationship between the meso-scale LDPM parameters, α and E_0 , and the macroscopic elastic parameters, E and ν , can be obtained by considering a limiting case in which an infinite number of facets surrounds one aggregate piece. In this case, the LDPM formulation corresponds to the kinematically constrained formulation of the microplane model without deviatoric/volumetric split of the normal strain component, and it can be written as:

$$E_0 = \frac{E}{1 - 2\nu} \iff E = \frac{2 + 3\alpha}{4 + \alpha} E_0 \quad (1)$$

and

$$\alpha = \frac{1 - 4\nu}{1 + \nu} \iff \nu = \frac{1 - \alpha}{4 + \alpha} \quad (2)$$

For stresses and strains beyond the elastic limit, LDPM mesoscale failure is characterized by three mechanisms as described below.

The first one is fracture and cohesion due to tension and tension-shear. For tensile loading ($\varepsilon_N > 0$), the fracturing behavior is formulated through

an effective strain and stress:

$$\varepsilon = \sqrt{\varepsilon_N^2 + \alpha(\varepsilon_M^2 + \varepsilon_L^2)}, \quad \sigma = \sqrt{\sigma_N^2 + (\sigma_M + \sigma_L)^2/\alpha} \quad (3)$$

which define the normal and shear stresses as

$$\sigma_N = \varepsilon_N(\sigma/\varepsilon), \quad \sigma_M = \alpha\varepsilon_M(\sigma/\varepsilon), \quad \sigma_L = \alpha\varepsilon_L(\sigma/\varepsilon) \quad (4)$$

An internal variable ω characterizes the coupling between normal strain ε_N and total shear strain $\varepsilon_T = \sqrt{\varepsilon_M^2 + \varepsilon_L^2}$, or the normal stress σ_N and total shear stress $\sigma_T = \sqrt{\sigma_M^2 + \sigma_L^2}$, as:

$$\tan(\omega) = \varepsilon_N/\sqrt{\alpha}\varepsilon_T = \sigma_N\sqrt{\alpha}/\sigma_T \quad (5)$$

The effective stress σ is incrementally elastic ($\dot{\sigma} = E_0\dot{\varepsilon}$), and must be less than or equal to the strain-dependent boundary:

$$\sigma_{bt} = \sigma_0(\omega) \exp[-H_0(\omega)\langle\varepsilon - \varepsilon_0(\omega)\rangle/\sigma_0(\omega)] \quad (6)$$

where $\langle x \rangle = \max\{x, 0\}$. For pure tensile stress ($\omega = \pi/2$), the tensile boundary represents strain-softening with exponential decay, characterized by: (1) the mesoscale tensile strength σ_t ; and (2) an initial slope of the exponential decay, or softening modulus H_t . In order to preserve the correct energy dissipation during mesoscale damage localization, the softening modulus in pure tension is expressed as:

$$H_t = 2E_0/(l_t/l_e - 1) \quad (7)$$

where $l_t = 2E_0G_t/\sigma_t^2$; l_e is the length of the tetrahedron edge under consideration; and G_t is the mesoscale fracture energy. For pure shear stress ($\omega = 0$), the effective stress boundary represents perfectly plastic behavior characterized by the mesoscale shear strength σ_s . LDPM provides a smooth transition between pure tension and pure shear. With the ratio of shear strength to tensile strength given as $r_{st} = \sigma_s/\sigma_t$, the parabolic variation for the strength is expressed as:

$$\sigma_0(\omega) = \sigma_t r_{st}^2 \left(-\sin(\omega) + \sqrt{\sin^2(\omega) + 4\alpha \cos^2(\omega)/r_{st}^2} \right) / [2\alpha \cos^2(\omega)] \quad (8)$$

The second physical phenomenon simulated is compaction and pore collapse from compression. For compressive loading ($\varepsilon_N < 0$), the normal stress

is computed by imposing the inequality $-\sigma_{bc}(\varepsilon_D, \varepsilon_V) \leq \sigma_N \leq 0$, where σ_{bc} is a strain-dependent boundary depending on the volumetric strain ε_V and the deviatoric strain ε_D . The volumetric strain is computed at the tetrahedron level as $\varepsilon_V = (V - V_0)/3V_0$, where V and V_0 are the current and initial volumes of a tetrahedron, respectively. The upper limit of elastic behavior is defined by a volumetric strain value given as: $\varepsilon_{c0} = \sigma_{c0}/E_0$, where σ_{c0} is a material parameter termed the mesoscale compressive yield stress. Beyond this limit, the compressive boundary is assumed to have an initial linear evolution, modeling pore collapse, up to a volumetric strain value of $\varepsilon_{c1} = \kappa_{c0}\varepsilon_{c0}$. Disregarding the deviatoric-to-volumetric strain ratio, $r_{DV} = \varepsilon_D/\varepsilon_V$, the stiffness of this linear segment is some fraction of the elastic stiffness and is expressed as H_{c0} . For axial compression with lateral confinement, r_{DV} can significantly affect the response. To simulate the observed horizontal plateau featured by typical experimental data, the stiffness of the pore collapse segment must tend to zero for increasing values of r_{DV} . This can be achieved by setting:

$$H_c(r_{DV}) = H_{c0}/(1 + \kappa_{c2} \langle r_{DV} - \kappa_{c1} \rangle) \quad (9)$$

where H_{c0} , κ_{c1} , and κ_{c2} are material parameters. The boundary for the pore collapse phase ($\varepsilon_{c0} \leq -\varepsilon_V \leq \varepsilon_{c1}$) can now be expressed as:

$$\sigma_{bc} = \sigma_{c0} + \langle -\varepsilon_V - \varepsilon_{c0} \rangle H_c(r_{DV}) \quad (10)$$

Compaction and rehardening occur beyond pore collapse ($-\varepsilon_V \geq \varepsilon_{c1}$). In this case one has the following:

$$\sigma_{bc} = \sigma_{c1}(r_{DV}) \exp [(-\varepsilon_V - \varepsilon_{c1}) H_c(r_{DV}) / \sigma_{c1}(r_{DV})] \quad (11)$$

$$\sigma_{c1}(r_{DV}) = \sigma_{c0} + (\varepsilon_{c1} - \varepsilon_{c0}) H_c(r_{DV}) \quad (12)$$

Finally, the LDPM constitutive equation simulates friction due to compression-shear. The incremental shear stresses are computed as:

$$\dot{\sigma}_M = E_T(\dot{\varepsilon}_M - \dot{\varepsilon}_M^p), \quad \dot{\sigma}_L = E_T(\dot{\varepsilon}_L - \dot{\varepsilon}_L^p) \quad (13)$$

where $\dot{\varepsilon}_M^p = \dot{\lambda} \partial \varphi / \partial \sigma_M$, $\dot{\varepsilon}_L^p = \dot{\lambda} \partial \varphi / \partial \sigma_L$, and $\dot{\lambda}$ is the plastic multiplier with loading-unloading conditions $\varphi \dot{\lambda} \leq 0$ and $\dot{\lambda} \geq 0$. The plastic potential is defined as:

$$\varphi = \sqrt{\sigma_M^2 + \sigma_L^2} - \sigma_{bs}(\sigma_N) \quad (14)$$

where the nonlinear frictional law for the shear strength is assumed to be as follows:

$$\sigma_{bs} = \sigma_s + (\mu_0 - \mu_\infty)\sigma_{N0}[1 - \exp(\sigma_N/\sigma_{N0})] - \mu_\infty\sigma_N \quad (15)$$

where σ_{N0} is the transitional normal stress, and μ_0 and μ_∞ are the initial and final internal friction coefficients.

2.3. Constitutive Law for Fiber-Matrix Interaction

In the present study, a semi-empirical formulation based on a number of assumptions, including that the fiber is initially straight, elastic, and has negligible bending stiffness, is adopted [31]. Prior to a purely frictional pull-out stage, an embedded fiber segment must completely debond from the surrounding matrix. The debonding stage described as a “tunnel-type” cracking process is characterized by two parameters: the bond fracture energy G_d , and a constant value of frictional stress τ_0 . The slippage has a critical value v_d representing full debonding. For a generic embedment length L_e , it can be expressed as [32]:

$$v_d = \frac{2\tau_0 L_e^2}{E_f d_f} + \left(\frac{8G_d L_e^2}{E_f d_f} \right)^{1/2} \quad (16)$$

where E_f is the modulus of elasticity of the fiber. For the debonding stage ($v < v_d$), fiber load is given as [32]:

$$P(v) = \left[\frac{\pi^2 E_f d_f^3}{2} (\tau_0 v + G_d) \right]^{1/2} \quad (17)$$

For the pull-out phase ($v > v_d$), the resistance is entirely frictional and one has [32]:

$$P(v) = P_0 \left(1 - \frac{v - v_d}{L_e} \right) \left[1 + \frac{\beta(v - v_d)}{d_f} \right] \quad (18)$$

where $P_0 = \pi L_e d_f \tau_0$, and β is a dimensionless coefficient.

In a general situation, the orientation of the embedded segment and the free segment, with associated crack-bridging force P_f , are different. The original deflection angle between the bridging segment and the embedded end is denoted as φ_f . At the point where the fiber exits the matrix and changes orientation, bearing stresses are created in the underlying matrix. When this localized stress field reaches a sufficient intensity, spalling occurs, and the embedment length of the fiber is reduced by a corresponding length s_f . The force in the bridging segment experiences a sudden drop, and the

deflection angle between the two fiber segments is reduced to φ'_f . In the current study, the spalling length is obtained by [31]:

$$s_f = \frac{P_{fN} \sin(\theta/2)}{k_{sp} \sigma_t d_f \cos^2(\theta/2)} \quad (19)$$

where P_{fN} is the normal component of the total force P_f , σ_t is the matrix tensile strength, k_{sp} is a material parameter, and $\theta = \arccos(\mathbf{n}_f^T \mathbf{n})$.

The snubbing effect is taken into account by assuming that at the point where it exits the tunnel crack, the fiber wraps around the intact matrix in a perfectly flexible manner. The summation of all slip-friction and debonding forces acting parallel to the embedded length is denoted as P . The additional friction and bearing forces at the exit point imply that $P_f > P$. The following analytical formulation is adopted:

$$P_f = \exp(k_{sn} \varphi'_f) P(v) \quad (20)$$

where P has now been properly expressed as a function of relative fiber slippage v , and k_{sn} is a dimensionless material parameter.

The formulations above describe the fiber-matrix interaction only when the load does not cause the rupture of the fiber, it is then necessary to check for each fiber that:

$$\sigma_f = 4P_f / \pi d_f^2 \leq \sigma_{uf} e^{-k_{rup} \varphi'_f} \quad (21)$$

where k_{rup} is a dimensionless material parameter, and σ_{uf} is the ultimate tensile strength of the fiber.

For steel fibers, the experimental studies usually provide σ_{uf} , and sometimes the elastic modulus, usually close to $E_f = 200$ GPa.

Consider a fiber, with initial orientation $\mathbf{n}_f$, subject to a crack opening of vector $\mathbf{w}$. The tangential component of $\mathbf{w}$ is defined as $w_T = \sqrt{w_M^2 + w_L^2}$. Assuming that the spalling length s_f is the same on both sides, the vector for the crack-bridging segment can be computed as $\mathbf{w}' = \mathbf{w} + 2s_f \mathbf{n}_f$. The bridging segment force vector is assumed to be coaxial with the fiber $\mathbf{P}_f = P_f \mathbf{n}'_f$, where $\mathbf{n}'_f = \mathbf{w}' / \|\mathbf{w}'\|$. The frictional pulley model applies equally to both the shorter and longer ends, with initial lengths L_s and L_l , relative slippages v_s and v_l , and pullout resisting forces P_s and P_l , respectively. Equilibrium of the fiber bridging segment leads to $P_f = \exp(k_{sn} \varphi'_f) P_s(v_s) = \exp(k_{sn} \varphi'_f) P_l(v_l)$, where $\varphi'_f = \arccos(\mathbf{n}_f^T \mathbf{n}'_f)$, which implies that $P_s(v_s) = P_l(v_l)$, and compatibility has to be enforced, which is $\|\mathbf{w}'\| = 2s_f + v_s + v_l$.

As discussed in the sections above, the LDPM-F response depends on two sets of parameters: (a) the LDPM material parameters governing the behavior of plain concrete; and (b) the parameters governing the fiber-matrix interaction model, which needs to be identified by fitting experimental data.

3. Experimental Data and Test Setup

The bending test of prismatic specimens is the most popular and widely used experimental technique for assessing the post-cracking behavior of fiber reinforced concrete. It is relatively simple to perform and can be representative of many practical situations. To calibrate/validate the proposed numerical model, an extensive experimental study was conducted to investigate the three-point bending behavior of various prismatic specimens containing different types (steel and synthetic) and dosages of fibers.

The fracture properties of the fiber reinforced concrete were determined according to the European Standard EN14651 [34] and EN14845 [35] which require bending test (3PBT) on small beam specimens ($150 \times 150 \times 550$ mm). The width and depth of test specimens are greater than three times the length of the fibers used in this study, so the preferential fiber alignment effect is minimized [36]. Fracture tests were carried out with a closed-loop hydraulic testing machine by using displacement as control parameter. The experimental results were reported in the form of load versus Crack Mouth Opening Displacement (CMOD) curves, and the latter was measured by means of a resistive displacement transducer positioned astride a notch (having a depth of 25 mm and a width of 4 mm) at mid-span.

In the present work, only a subset of the obtained experimental data was used. In particular, the experimental data on the behavior of the specimens made of plain concrete and concrete containing the fibers were used, as shown in Fig. 2. Fibers SF are hooked steel fibers, and Fibers MS2 are polystyrene fibers. The most important properties of these fibers are listed in Table 1. For each type of fiber, two different dosages were tested, corresponding to the minimum and the maximum dosages suggested by the fiber manufacturers, as shown in Table 2. For each dosage, 7 specimens were tested. Although the experimental setup and test procedure have been described in Ref. [33], the experimental data used in the current study have not been published before.

Existing literature has shown that to improve the compressive concrete strength, flexural strength, and shrinkage, deformed steel fibers are more effective than straight fibers due to mechanical anchorage created by the

deformed shape [37, 38, 39], and thus it is not surprising that almost all commercially available steel fibers at present are mechanically deformed, such as hemi-circular, hooked, and corrugated fibers [37, 40, 41]. Steel fibers with hooked ends were used in this research. These fibers represent the most widely used steel fibers' geometries used in concrete reinforcement [42].

4. Model Calibration and Validation

The proposed numerical model is calibrated by simulating three point bending tests on prismatic notched specimens and by comparing the numerical results with the experimental data. The LDPM concrete parameters are determined by fitting a simulation to the plain concrete response. The fiber model calibration is then performed by fitting a simulation to one of the non-zero V_f response curves. For validation purposes, the numerical responses for other fiber contents, determined by altering only the fiber volume fraction V_f , are then generated, and the results are compared with the experimental curves.

4.1. Plain Concrete

In three-point bending experiments, damage typically occurs only in a small region of the beam centered about the mid-span notch. Therefore, in order to reduce the computational cost of the simulations, the mid-span portion was modeled with LDPM-F, and the ends of the beams were modeled with standard elastic finite elements connected to the LDPM-F region through a master-slave algorithm, as shown in Fig. 3(a). The simulation results obtained from the discrete model with LDPM-F defined over the whole specimen are exactly the same as those obtained from the model mixed with LDPM-F in the middle and finite elements at two ends. In Fig. 3(c) the load vs Crack Mouth Opening Displacement (CMOD) curves are shown for plain concrete. In this case the experimental curve (red line) represents the average of two experimental tests, and the numerical curve (green line) represents the average response from two distinct meso-structural models (The figures appear in color in the electronic version of this article). The LDPM concrete parameter values determined by matching the numerical results with experimental data are shown in Table 3.

4.2. Steel Fiber Reinforced Concrete

For steel fiber reinforced concrete, LDPM-F was calibrated by fitting numerical simulation results to the SF-20 ($V_f = 0.26\%$) curve, and the fiber

parameters determined from that fit are summarized in Table 4. Fig. 4 shows the specimen geometry and discretization in the numerical simulations, the load vs CMOD curve, and the meso-scale crack patterns at CMOD = 0.8 mm. In this case the experimental curve (red line) represents the average of seven experimental tests, and the numerical curve (green line) represents the average response from seven distinct meso-structural models.

Numerical prediction of the higher V_f response was obtained by changing only V_f 's value to 0.45%. The numerical load vs CMOD curves were compared with the experimental curves, as shown in Fig. 5. In this case the numerical results show higher values than the experimental data. The difference between the experimental average and the numerical average, calculated at CMOD = 0.8 mm, is about 45%. As shown in Fig. 3(d), Fig. 4(c), and Fig. 5(c), for each set of specimen analyzed, the maximum crack opening is about 80% of the CMOD value.

In order to clarify the discrepancy in the simulation with $V_f = 0.45\%$, the fiber arrangement within each specimen was analyzed. It was hoped that such an approach would develop a better understanding of the observed discrepancy, as shown in Fig. 5(b). In the experiments, each specimen was analyzed at the end of the test, and the number of fibers bridging the crack in the three divisions of the cross section, as shown in Fig. 3(b), was counted and the relevant values are reported in Table 5. To improve reliability of the comparison, seven more numerical simulations were conducted for each case, and the number of fibers bridging the cracks was extracted from the output files of the numerical simulation, as shown in Table 6.

In Fig. 6(a), at CMOD = 0.8 mm, the load increment due to the effect of fibers, calculated as $100(P_*/P_0 - 1)$, where P_* is the load for the fiber reinforced concrete specimens and P_0 is the average load for the plain concrete specimens, is plotted against the number of active fibers in the lower two-thirds of the ligament length, i.e., *Bottom + Middle*. It can be observed that there is a clear linear relationship between the load increment and the number of active fibers, and that both the experimental data and the numerical results demonstrate a similar trend. However, for the numerical specimens, the range of active fibers increase from [22-39] to [47-70] as the fiber volume fraction nearly doubles from $V_f = 0.26\%$ to $V_f = 0.45\%$, while for the experimental specimens, the increase of the number of active fibers, from [17-39] to [17-50], is not consistent with the doubling of the fiber volume fraction. It can be concluded that, for the higher value of V_f , the experimental specimens contain less active fibers than expected, probably due to the inherent

difficulty of obtaining a uniform fiber distribution during casting. This could be one of the major reasons for the discrepancy observed in the curves shown in Fig. 5(b), and at the same time it highlights the fact that the actual fiber distribution within the specimens is a crucial information needed for the numerical simulation to accurately capture the mechanical behavior of fiber reinforced concrete.

It is worth observing here that the reason that the number of active fibers in the *Bottom + Middle* part of the specimen was plotted was motivated by analyzing the crack pattern in the numerical models. Fig. 7 shows the color-scale isosurfaces from two typical meso-scale crack openings. Each isosurface corresponds to a different crack opening value. The minimum value that was plotted corresponds to a crack opening magnitude of $2v_d$. This is the overall crack opening associated with a full debonded straight fiber, orthogonal to the crack surface. At this level of crack opening, fibers can be considered fully active. As depicted in Fig. 7, the zone characterized by fully active fibers extends to approximately two-thirds of the ligament length above the notch. The choice of plotting isosurfaces at $\text{CMOD} = 0.8$ mm for the comparison is due to the fact that at this value the macroscopic load becomes almost constant.

4.3. Synthetic Fiber Reinforced Concrete

For synthetic fiber reinforced concrete, LDPM-F was calibrated by fitting numerical simulation results to the MS2-5 ($V_f = 0.37\%$) curve, and the fiber parameters determined from that fit are summarized in Table 4. Fig. 8 shows the specimen geometry and discretization in numerical simulation, the load vs CMOD curve, and the meso-scale crack patterns at $\text{CMOD} = 0.8$ mm. Table 4 shows that, besides E_f and σ_{uf} , only k_{rup} and τ_0 are modified compared to the previous case of steel fibers.

Fig. 9 shows the comparison between the synthetic fiber behavior and the steel fiber behavior during a two way pull-out test. Fig. 9(a) shows that the synthetic fiber undergoes a greater deformation before the complete debonding of the short embedment end since its elastic modulus is much lower than that of the steel fiber. It can be seen that, for the synthetic fiber, the complete debonding of the short embedment end is achieved at $v = 0.268$ mm, while for the steel fiber it is achieved at $v = 0.086$ mm. Fig. 9(b) shows that, due to the differences in elastic modulus and tensile strength, the synthetic fiber undergoes a much lower load level (0.101 kN), before the load drop, than the steel fiber does (0.474 kN). Besides, the synthetic fiber

load drop is about 0.014 kN, whereas the steel fibers load drop is close to 0.071 kN.

Numerical prediction of the higher V_f response was obtained by changing only the value of V_f to 0.74%. The numerical load vs CMOD curves were compared with the experimental curves, as shown in Fig. 10. In this case, LDPM-F is able to predict very accurately the response for the higher fiber content. The error, compared to the experimental results, is only a small percentage at the peak and initial post-peak, and remains less than 20% even in the far post-peak.

The numerical simulations are further investigated in Fig. 3(d), Fig. 8(c), and Fig. 10(c), where contours of the meso-scale crack openings at CMOD = 0.8 mm, are reported for plain concrete and synthetic fiber reinforced concrete. Due to the presence of the notch and the associated stress concentration, one main macroscopic crack develops in all three cases. This crack, however, is the result of a band of meso-scale cracking characterized by a certain width. This result is in general agreement with the theoretical interpretation provided by Bažant and Oh [43]. For plain concrete, the crack is very localized, and in the vicinity of the notch tip, the crack band width is about 20 mm. This is commensurate with the maximum aggregate size, suggesting very little crack-bridging effect, and is in accordance with the nearly complete loss of load-carrying capacity indicated in Fig. 3(b) for CMOD = 0.8 mm. For synthetic fiber reinforced concrete, the width of the crack band increases significantly with increasing fiber volume fraction. For the case with $V_f = 0.74\%$, the crack band width is approximately 75 mm, nearly 3.5 times the maximum aggregate size and nearly twice the fiber length.

Notwithstanding the good agreement between the experimental data and the numerical results for the model validation, the same analysis on the fiber arrangement within each specimen was conducted. Experimental data for the number of fibers bridging the crack in the three divisions of the cross section have been shown in Table 5. Seven numerical models were analyzed for each case, and the number of fibers bridging the cracks was extracted and shown in Table 6.

The crack pattern in the numerical models was analyzed in Fig. 11, which shows the color-scale isosurfaces for two typical meso-scale crack openings. The minimum crack opening ($2v_d$) is 0.300 mm in this case. As depicted in Fig. 11, the zone characterized by fully active fibers extends to approximately one half of the ligament length above the notch. For this reason the comparison between the experimental data and the numerical results of the load

increment versus the number of active fibers was now performed by considering two cases: (a) only the numbers of active fibers in the *Bottom*, as shown in Fig. 6(b); and (b) the numbers of active fibers in the *Bottom + Middle*, as shown in Fig. 6(c). Again, it can be observed that there is a linear relationship between the load increment and the number of active fibers, and that both the experimental data and the numerical results demonstrate a similar trend.

4.4. Comparison and Discussion

The LDPM-F numerical simulations capture many salient features of the experimental results remarkably well: (a) by comparing the load vs CMOD curves for plain concrete and fiber reinforced concrete, the increased load-carrying capacity of fiber reinforced concrete proportional to fiber volume fraction can be observed during the post-peak period; (b) due to the difference in fibers' mechanical properties, despite the increased fiber volume fraction used in the case of synthetic fibers, the load vs CMOD curves of the synthetic fiber reinforced concrete show much lower load levels during the post-peak compared with steel fiber reinforced concrete; (c) at the peak load, it can be observed that there are no correlations between the load level and the fiber volume fraction since, at that time, the crack opening is so small that the fibers are not active yet and the material response is mainly driven by the concrete matrix, whose maximum strength depends on its random meso-scale structure; and (d) for fiber reinforced concrete, because the fibers' bridging effect leads to the load transmission to the cracked concrete matrix, the size of the crack propagation zone is much greater than that in the case of plain concrete.

However, it can be seen that the numbers of fibers in the numerical models tend to be less scattered than those obtained from the experimental results. This incongruence is caused by the difference in the approaches by which the fibers are arranged within the specimen. During the experiments, the fibers are introduced into the cement matrix non yet aged and then distributed by mixing, whereas during the numerical simulation, LDPM-F uniformly distributed the fibers by a random iterative procedure. To make an efficient and reliable comparison between the experimental and the numerical data, it must be highlighted the importance of recording the exact number of fibers arranged in the volume fraction in which the cracking leads to the specimen failure during the test. It is interesting to note that to predict the evolution of distribution and orientation of fibers in the fiber reinforced self-compacting

concrete during the casting process, a two-way coupled model for flow of suspension of rigid solid particles in a non-Newtonian fluid with a free surface has been developed recently [44]. This model could be adopted by LDPM-F for the extension to include non-uniform fiber distribution.

5. Analysis of the Fracture Process Zone

For cementitious materials, the Fracture Process Zone (FPZ) is defined as the inelastic zone around the crack tip in which energy is dissipated at a certain stage of analysis. The behavior of the FPZ is dominated by complicated mechanisms, such as micro-cracking, crack deflection, bridging, crack face friction, crack tip blunting by voids, and crack branching. Because its length is related to the characteristic length of the cementitious materials, the size, extent, and location of the FPZ have been the subject of countless research efforts for several decades [45, 46, 47, 48]. The understanding of FPZ has been significantly hindered by the fact that, in most experiments, only global quantities, such as the overall strength of the structure, are determined, and local quantities, such as the spatial distribution of dissipated energy and fracture patterns, could not be obtained. Experimentally, local quantities so far have been evaluated by acoustic emission measurements [49, 50, 51, 52], X-ray tomography [53], moiré interferometry [54], and optical microscopy [55]. Digital image correlation techniques have also been applied to study fracture processes in concrete [56, 57, 58] and other materials [59, 60]. However, more detailed results are required to fully understand the mechanics of the evolution of fracture processes of concrete, especially fiber reinforced concrete.

The main purpose of this section is to create a 3D numerical methodology to identify the change in fracture characteristics of concrete reinforced with different types of fibers. A somewhat similar analysis, but in 2D, was proposed recently by Grassl et al. [61]. For this purpose, the numerical models analyzed in previous sections for plain concrete and fiber-reinforced concrete were post-processed to understand how the fibers affect the shape and size of the FPZ, as shown in Figs. 12–14, where contours of the meso-scale combined stresses are reported. The combined stress of each facet is defined as the square root of the sum of the squares of the normal stress σ_N , and the two shear stresses σ_M and σ_L . It should be noted that only the facets that undergo tensile stress, i.e., $\sigma_N > 0$, are considered, since their progressive failure is responsible for the crack propagation.

It is well-known that the fracture process zone is subdivided into an inner softening zone and an outer micro-fracture zone. To study the inner zone where the energy dissipation is concentrated, the facets located around it, weakly contributing to the dissipation, are not plotted. Figs. 12–14 are obtained by setting a minimum dissipated power value of 0.001 J/s and a minimum combined stress value of 0.2 MPa, i.e, the facets with power or stress lower than these values are not shown. During all the tests, the minimum value set for power is less than 1.5% of the maximum, and the minimum value set for combined stress is less than 5.5% of the maximum.

5.1. Plain Concrete

The evolution of the FPZ for plain concrete is shown in Fig. 12. The minimum power value of 0.001 J/s equals to 1.24% of the maximum at the peak load, which is 0.0803 J/s, and 0.33% of the maximum value at CMOD = 0.8 mm (when the load becomes almost constant), which is 0.3023 J/s. The minimum stress of 0.2 MPa is 5.21% of the maximum value at the peak load, which is 3.84 MPa, and 4.07% of the maximum value at CMOD = 0.8 mm, which is 4.92 MPa. In the beginning, localized presence of high energy dissipation can be observed just above the notch where the stress is concentrated. With the load increasing, the FPZ size becomes larger and it reaches its maximum width immediately after the peak load. The FPZ width starts to decrease after that because the specimen fails by cracking, and correspondingly, the high stress values move from the notch to the top surface where the concrete matrix has not been damaged yet. In the meantime, the facets localized just above the notch undergo a significant stress drop. At the end of the test, only the facets near to the top surface still contribute to the energy dissipation by enduring combined stress values greater than zero. However, it can be observed that some facets close to the notch, where the concrete matrix has been fully damaged, still have non-zero combined stress values. This apparent inconsistency is caused by the frictional effect between two opposite crack faces while they undergo relative displacement due to not perfectly symmetric specimen deflection.

5.2. Steel Fiber Reinforced Concrete

The evolution of the FPZ for steel fiber reinforced concrete with $V_f = 0.26\%$ is shown in Fig. 13. The minimum power value of 0.001 J/s equals to 1.27% of the maximum at the peak load, which is 0.0789 J/s, and 0.13% of the maximum value at CMOD = 0.8 mm, which is 0.7672 J/s. The minimum

stress of 0.2 MPa is 4.74% of the maximum value at the peak load, which is 4.22 MPa, and 3.74% of the maximum value at $\text{CMOD} = 0.8$ mm, which is 5.35 MPa. Fig. 13 shows that the FPZ size keeps increasing until the peak load is reached, but different from the previous case of plain concrete, due to the bridging effect between fibers and matrix by which the applied load can be transmitted to the facets even with large crack opening displacements, the facets close to the notch still undergo a certain amount of stress at the end of the test.

5.3. Synthetic Fiber Reinforced Concrete

The evolution of the FPZ for synthetic fiber reinforced concrete with $V_f = 0.74\%$ is shown in Fig. 14. The minimum power value equals to 1.28% of the maximum at the peak load, which is 0.0781 J/s, and 0.23% of the maximum value at $\text{CMOD} = 0.8$ mm, which is 0.4376 J/s. The minimum stress is 3.87% of the maximum value at the peak load, which is 5.15 MPa, and 3.85% of the maximum value at $\text{CMOD} = 0.8$ mm, which is 5.20 MPa. Similar to the previous cases of plain concrete and steel fiber reinforced concrete, Fig. 14 shows that the FPZ size keeps increasing until the peak load is reached, and then the high stress values move from the notch to the top surface where the concrete matrix has not been damaged yet. Comparison with the cases of steel fiber reinforced concrete shows that the FPZ is much smaller at the end of the test in synthetic fiber reinforced concrete and much less facets experience non-negligible stress levels although higher fiber volume fractions are used. This has been explained by the significantly lower capacity of the synthetic fibers in supporting high stress concentrations, as shown in Fig. 9. For this reason, after an initial phase during which the fibers bridge the cracks close to the notch and transmit the load to the concrete matrix, the fibers' pull-out occurs, leading to a significant drop in the facets' stress levels.

5.4. Comparison and Discussion

To compare the FPZ shape and size during the tests for the above discussed cases, Table 7 reported the FPZ width, i.e., the width of the area composed of facets that undergo non-negligible power and stress values, the number of facets in the FPZ, and the value of the total power, both immediately after the peak load at $\text{CMOD} = 0.1$ mm when, for each analyzed specimen, the FPZ reaches its maximum width, and at the end of the tests when $\text{CMOD} = 1.0$ mm and fibers are fully active. Fig. 15 shows the FPZ

widths at $\text{CMOD} = 0.1 \text{ mm}$ and $\text{CMOD} = 1.0 \text{ mm}$ for the five different specimens. Fig. 16 shows the trend of the total power during the tests.

For plain concrete, Table 7 and Fig. 15 show that the FPZ width at $\text{CMOD} = 1.0 \text{ mm}$ is much smaller than that at $\text{CMOD} = 0.1 \text{ mm}$. As shown in Fig. 16, the total power at the end of the test is about 60% lower than that at $\text{CMOD} = 0.1 \text{ mm}$. This is caused by the concrete matrix failure, which leads to a significant decrease of the number of facets (68% less) in which energy is dissipated. The described behavior is representative of the inability of cracked concrete to transfer tensile stress across the crack. The LDPM-F simulation captures this feature very well.

By contrast, for the steel fiber reinforced concrete with $V_f = 0.26\%$, the FPZ width at the end of the test is almost the same as immediately after the peak load. The number of facets that make up the FPZ at the end of the test is much larger than that in the case of plain concrete and only 12% lower than the value at $\text{CMOD} = 0.1 \text{ mm}$. As shown in Fig. 16, the trend of the total power of steel fiber reinforced concrete appears very different from that of the plain concrete. The energy dissipation for plain concrete at the end of the test is about one third of the value immediately after the peak load, while for steel fiber reinforced concrete with $V_f = 0.26\%$, the total power at the end is still 6% greater than the value at $\text{CMOD} = 0.1 \text{ mm}$. The difference is caused by the fact that in steel fiber reinforced concrete, facets with large crack openings can still support tensile loading due to the fibers' bridging effect.

A similar trend has been observed in steel fiber reinforced concrete with $V_f = 0.45\%$. But in this case, because of the higher fiber volume fraction, a larger number of facets in the FPZ and a higher value of the total power are obtained at the end of the test. The number of facets in the FPZ at $\text{CMOD} = 1.0 \text{ mm}$ is 10% lower than the value at $\text{CMOD} = 0.1 \text{ mm}$ while the total power is about 60% higher.

For synthetic fiber reinforced concrete with $V_f = 0.45\%$, the FPZ width at the end of the test (38.4 mm) is much smaller than that at $\text{CMOD} = 0.1 \text{ mm}$ (84.3 mm). Correspondingly, the number of facets that make up the FPZ and the value of the total power is 62% smaller and 40% lower than the values immediately after the peak load, respectively. The specimen with synthetic fiber volume fraction of 0.74% shows a better behavior due to the higher fiber content, but still, the number of facets at the end of the test is 53% smaller than that immediately after the peak load, and the value of the total power is 52% lower. The poor fibers' contribution has been explained

by the significantly lower capacity of the synthetic fibers in supporting high levels of stress.

From Table 7, it can be seen that the reported values for the five different specimens at $\text{CMOD} = 0.1$ mm do not differ significantly from each other. This is because at $\text{CMOD} = 0.1$ mm, the maximum crack openings for fiber reinforced concrete specimens are: 0.079 mm for SF-20, 0.064 mm for SF-35, 0.082 mm for MS2-5, and 0.078 mm for MS2-10, respectively, and so the fibers are not active yet. Recall that the crack openings at complete debonding are 0.094 mm and 0.300 mm for steel fibers and synthetic fibers, respectively.

From Fig. 16, it is interesting to observe that the total power of MS2-10 becomes lower than that of MS2-5 after $\text{CMOD} = 0.7$ mm, although MS2-10 has a much higher fiber volume fraction. This could possibly be explained by the randomly distributed fibers and cracks, resulting in fewer active fibers in MS2-10 than in MS2-5.

6. Characterization Through Round Panel Test

As reported in previous sections, characteristic values determined from three-point bending tests have a large scatter on the results. It has been shown that this large scatter is due to a nonuniform fiber distribution. Such a scatter becomes particularly large when low fiber volume fractions are used [62]. One of the possible solutions to avoid the large scatter is to increase the cracked area. For this reason, a round panel test, first developed by Bernard [63, 64], was proposed by ASTM [65] in 2004. This type of test has a statically determinate configuration (a round slab having a diameter of 800 mm and a thickness of 75 mm with three supports at 120 degrees) where the crack pattern is predictable and the post-cracking material properties can be adequately determined. It appears to be particularly suitable for evaluating the performance of fiber reinforced concrete because the test method involves relatively large deflections. However, handling and placing such a specimen is quite complicated due to the large size and, consequently, high weight (91 kg). In addition, standard servo-controlled loading machines may not fit with the geometry of the panel, which is too large for most of them. To meet the need of having a specimen easier to handle, Minelli and Plizzari [66, 67] have proposed the use of a smaller round panel having a diameter of 600 mm and a depth of 60 mm, with a weight of only 40 kg. Recently, Ciancio et al. [68] have proposed to use even smaller panels having a diameter of 500 mm. It

has been shown that results from smaller round panels are consistent, reliable and provide a repeatable and predictable crack pattern with a consistently lower dispersion than classical beam tests.

In this section, LDPM-F is calibrated/validated by simulating round panel tests on both large and small specimens and by comparing the numerical results with the experimental data provided by Minelli and Plizzari [66, 67].

6.1. *Small Round Panel Tests*

The numerical simulation for small round panel tests was performed on round panels with a diameter of 600 mm (effective diameter of 550 mm) and a thickness of 60 mm, with the load applied to the panels using a rigid cylinder (50 mm of diameter) above the center of the top surface. The panels were supported by three pivots (40 mm of diameter), 120 degrees apart radially and positioned 25 mm from the edge of the panel. The geometry of the specimen is shown in Fig. 17. The translation of the nodes on the bottom surface of the panel in contact with the three pivots is set to be fixed in z direction only, and their rotation is fixed in both x and y directions. The test is displacement controlled by means of the central displacement. The nodes on the top surface of the panel in contact with the loading cylinder are selected, and the loading history is assigned to these nodes.

Since only part of the material parameters are provided in [66, 67] and they are the same as those used in previous sections, it is assumed here that the material parameters for concrete and steel fiber used in the experiments [66, 67] are the same as those used in previous sections, and so the parameters shown in Table 3 are used in this section for plain concrete, and the parameters for SF shown in Table 1 and Table 4 are used for steel fibers.

The relationship between load and deflection (obtained from three numerical specimens in each case) for different kinds of fiber reinforcement, together with the curves for plain concrete, are shown in Fig. 18. The simulation results have shown that as the panel was loaded, cracks first appeared at the bottom of the panel at its central point, radiated to the edges, and then opened upwards. The typical crack pattern of each round panel was three cracks at angles about 120 degrees, with the panel breaking into 3 pie-shaped wedges at large deflections. Fig. 19(a) shows the crack propagation through the round panel obtained from one of the numerical models on plain concrete at $\delta = 0.31$ mm, $\delta = 0.41$ mm, $\delta = 0.48$ mm, and $\delta = 0.60$ mm. Fig. 19(b) shows the crack propagation obtained from one of the numerical

models on steel fiber reinforced concrete with $V_f = 0.39$ at $\delta = 0.32$ mm, $\delta = 0.45$ mm, $\delta = 0.54$ mm, and $\delta = 0.63$ mm.

Typically, the P - δ curves of the panels are almost linear up to the peak load. The central deflections at first crack load for both steel fiber reinforced concrete panels are similar, much larger than for the plain concrete. The increase in peak load due to steel fiber reinforcement was not significant. However, it can be seen that the toughness of the concrete panel has been increased dramatically due to steel fiber reinforcement. Fig. 18 shows the comparison between experimental data and numerical results. Note that the use of experimental data is complicated by the fact that the displacement measurement is highly susceptible to inaccuracies caused by several factors, such as the deformation of the supports and the loading platen. To make it simple, the inaccurate part of the displacement is assumed to be linear with the loading, and thus all the experimental data of displacement, δ , are modified as $\delta^* = (\delta - P/k) + P/k^*$ with $k = 50$ kN/mm and $k^* = 90$ kN/mm, intentionally adjusting the slope of the elastic part of the curves to create a match with that of the numerical results. With this adjustment, while not an exact match, the agreement between experimental and numerical curves is quite reasonable.

For plain concrete, Fig. 18 shows three distinct load drops after the peak load, associated with the subsequent formation of the three main cracks, and then a softening behavior by which the load tends to become zero. For steel fiber reinforced concrete, load drops can be observed during the test for all the cases, but the trend is much smoother than in the case of plain concrete, due to the fact that the fibers bridging effect leads to smaller load jumps. In fact, it is difficult to distinguish the three distinct load drops, because when a crack starts to propagate from the panel center, and the load starts to drop, fibers can be intercepted, and the bridging effect leads to load transmission. Nevertheless, in all the cases of steel fiber reinforced concrete, notwithstanding this difference from plain concrete, at the end of the test, three macro-cracks can again be distinguished, but now around them a larger crack propagation zone caused by the fibers bridging effect can be observed.

6.2. Large Round Panel Tests

The simulation process and material parameters for large round panel test are exactly the same as those used in small round panel test except for the geometry of the specimen. The numerical simulation for large round panel

tests was performed on round panels with a diameter of 800 mm (effective diameter of 750 mm) and a thickness of 75 mm, using the same setup as the small round panel test. The panels were supported by three pivots (40 mm of diameter), 120 degrees apart radially and positioned 25 mm from the edge of the panel.

The load–deflection curves for different fiber reinforcements, together with the curves for plain concrete, are shown in Fig. 18. Once again, the load–deflection curves computed by LDPM-F are considered to exhibit a reasonable agreement with those determined experimentally. Fig. 20(a) shows the crack propagation through the round panel bottom surface obtained from one of the numerical models on plain concrete at $\delta = 0.31$ mm, $\delta = 0.41$ mm, $\delta = 0.48$ mm, and $\delta = 0.60$ mm. Fig. 20(b) shows the crack propagation obtained from one of the numerical models on steel fiber reinforced concrete with $V_f = 0.39$ at $\delta = 0.48$ mm, $\delta = 0.57$ mm, $\delta = 1.43$ mm, and $\delta = 1.90$ mm.

Since for each numerical specimen, three cracks occur, the overall number of fibers involved in the load bearing is more similar among the panels than in case of three point bending test, in which only a single crack occurs in the middle where the notch is located. That is the reason why the load–deflection curve for each round panel shows approximately the same load values in the post-peak softening regime where all the three main cracks have been fully established.

7. Conclusions

In this investigation, the fundamental concepts governing the formulation of LDPM-F were reviewed and the mathematical formulation was highlighted. Fiber-matrix interaction is modeled by adapting an established micro-mechanical theory providing the fiber crack-bridging force as a function of the crack opening. For all fibers that intersect an LDPM facet, the crack-bridging forces are added together and assumed to act in parallel with the LDPM constitutive behavior. For calibration/validation purpose, an extensive experimental study was conducted on various prismatic specimens containing different types (steel and synthetic) and dosages of fibers. The numerical simulations conducted on three-point bending tests and round panel tests demonstrated that LDPM-F is able to accurately reproduce the fiber toughening mechanisms, and the associated increases in macroscopic strength and ductility that can be seen in the results of experimental testing while

keeping the computational cost manageable. In conclusion, the work presented in this paper demonstrates that LDPM-F is an accurate and efficient simulation tool for studying the mechanics of the evolution of fracture processes of fiber reinforced concrete.

8. Acknowledgment

This material is partly based upon work supported by the National Science Foundation under grant no. CMMI-1201087. M. Savoia and G. Cusatis conceived and initiated the project. N. Buratti and M. Savoia designed and performed the experimental work. M. Stacchini performed the computation in Section 4 and Section 5. C. Jin performed the computation in Section 6, analyzed data and wrote the manuscript. G. Cusatis supervised all the computational tasks.

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Table 1: Mechanical Properties of Fibers Used in the Experiments

Type	Fiber Code	L_f [mm]	d_f [mm]	L_f/d_f [-]	E_f [GPa]	σ_{uf} [MPa]	Shape
Steel	SF	50	1	50	210	1100	Hooked
Polystyrene	MS2	40	0.83	48	11.3	600	Hooked

Table 2: Fiber Dosage Used in the Experiments

Mix Code	Number of Specimens	Dosage [kg/m ³]	Volume Fraction [%]
MS2-5	7	5.0	0.37
MS2-10	7	10.0	0.74
SF-20	7	20.0	0.26
SF-35	7	35.0	0.45

Table 3: LDPM Concrete Parameter Values

E_0 [GPa]	α [-]	σ_t [MPa]	l_t [mm]	G_t [N/m]	σ_s/σ_t [-]	n_t [-]
51	0.25	2.21	900	43	2.5	0.2
σ_{c0} [MPa]	H_{c0}/E_0 [-]	κ_{c0} [-]	κ_{c1} [-]	κ_{c2} [-]	μ_0 [-]	μ_∞ [-]
195	0.2	2.7	1	5	0.25	0.0
σ_{N0} [MPa]	E_d/E_0 [-]	d_0 [mm]	c [kg/m ³]	w/c [-]	n_F [-]	ρ [kg/m ³]
600	1	10	357	0.49	0.5	2400
a/c [-]	d_a [mm]					
5.02	22					

Table 4: LDPM-F Parameter Values Used in Numerical Simulations

	SF-20	SF-35	MS2-5	MS2-10
G_d [N/m]	5.0	5.0	5.0	5.0
τ_0 [MPa]	6.0	6.0	1.5	1.5
β [-]	0.0	0.0	0.0	0.0
k_{sp} [-]*	6.2	6.2	6.2	6.2
k_{sn} [-]	0.5	0.5	0.5	0.5
σ_{uf} [MPa]	1100	1100	600	600
k_{rup} [-]	0.0	0.0	0.3	0.3
E_f [GPa]	210	210	11.3	11.3

*In numerical simulation, a simplified procedure has been adopted. Most studies provide the macroscopic compressive strength σ'_c of the concrete or mortar being investigated. Using a mid-range estimate, it is assumed that $\sigma'_t = \sigma'_c/10$, which allows for the approximate calculation of $k_{sp}\sigma_t$ as $6.2\sigma'_c/10 \approx 150$ Mpa.

Table 5: Numbers of the Fibers Bridging the Cracks in Three Divisions of the Cross Section

Experiments on SF-20							
Specimen	#1	#2	#3	#4	#5	#6	#7
Bottom	8	10	22	18	12	15	18
Middle	9	15	17	9	11	13	14
Top	15	16	23	4	13	24	25
Experiments on SF-35							
Specimen	#1	#2	#3	#4	#5	#6	#7
Bottom	12	17	15	15	19	15	28
Middle	5	28	12	15	15	12	22
Top	19	12	21	22	14	27	25
Experiments on MS2-5							
Specimen	#1	#2	#3	#4	#5	#6	#7
Bottom	24	18	28	29	32	18	25
Middle	34	13	20	21	34	15	23
Top	21	18	25	24	30	26	24
Experiments on MS2-10							
Specimen	#1	#2	#3	#4	#5	#6	#7
Bottom	67	31	21	25	37	38	37
Middle	47	39	27	38	40	30	37
Top	37	34	34	40	49	46	40

Table 6: Numbers of the Fibers Bridging the Cracks in Three Divisions of the Cross Section

Numerical Simulation on SF-20							
Specimen	#1	#2	#3	#4	#5	#6	#7
Bottom	17	12	10	16	15	8	17
Middle	17	11	21	14	15	14	16
Top	11	13	12	9	5	13	10
Specimen	#8	#9	#10	#11	#12	#13	#14
Bottom	10	11	28	21	21	19	12
Middle	18	15	9	18	15	20	16
Top	9	12	14	9	4	12	19
Numerical Simulation on SF-35							
Specimen	#1	#2	#3	#4	#5	#6	#7
Bottom	25	31	20	27	29	19	33
Middle	22	23	31	26	32	31	37
Top	22	16	23	27	19	10	16
Specimen	#8	#9	#10	#11	#12	#13	#14
Bottom	27	28	24	35	25	21	16
Middle	26	25	22	27	28	29	29
Top	15	17	19	19	21	19	20
Numerical Simulation on MS2-5							
Specimen	#1	#2	#3	#4	#5	#6	#7
Bottom	18	25	24	27	26	27	23
Middle	26	28	23	29	24	20	22
Top	12	24	13	22	17	11	20
Numerical Simulation on MS2-10							
Specimen	#1	#2	#3	#4	#5	#6	#7
Bottom	37	43	37	44	44	43	50
Middle	36	33	43	38	40	42	57
Top	38	40	39	40	33	32	38

Table 7: Fracture Process Zone Features at $\text{CMOD} = 0.1 \text{ mm}$ and $\text{CMOD} = 1.0 \text{ mm}$

	CMOD [mm]	FPZ Width [mm]	Number of Facets	Total Power [J/s]
Plain Concrete	0.1	75.5	6050	93.3
Plain Concrete	1.0	36.1	1934	36.9
SF-20	0.1	75.6	5566	85.1
SF-20	1.0	76.2	4876	90.1
SF-35	0.1	80.8	6866	79.7
SF-35	1.0	73.8	6170	126.4
MS2-5	0.1	84.3	6664	99.4
MS2-5	1.0	38.4	2648	56.7
MS2-10	0.1	82.2	6180	89.7
MS2-10	1.0	65.8	3108	43.4

Figure 1: LDPM simulation for concrete mesostructure: (a) Two adjacent particles; (b) Facet with intersecting fiber.

Figure 2: Fibers used in the experiments. MS2 represents synthetic fibers, and SF represents steel fibers. Scales are in cm.

Figure 3: LDPM simulation for plain concrete: (a) Specimen geometry and discretization in numerical simulation; (b) Three divisions of the cross section; (c) Load vs CMOD curve; and (d) Meso-scale crack patterns at CMOD = 0.8 mm. The experimental curve (red line) represents the average of two experimental tests, and the numerical curve (green line) represents the average response from two distinct meso-structural models.

Figure 4: LDPM simulation for steel fiber reinforced concrete SF-20 with $V_f = 0.26\%$: (a) Specimen geometry and discretization in numerical simulation; (b) Load vs CMOD curve; and (c) Meso-scale crack patterns at CMOD = 0.8 mm. The experimental curve (red line) represents the average of seven experimental tests, and the numerical curve (green line) represents the average response from seven distinct meso-structural models.

Figure 5: LDPM simulation for steel fiber reinforced concrete SF-35 with $V_f = 0.45\%$: (a) Specimen geometry and discretization in numerical simulation; (b) Load vs CMOD curve; and (c) Meso-scale crack patterns at CMOD = 0.8 mm. The experimental curve (red line) represents the average of seven experimental tests, and the numerical curve (green line) represents the average response from seven distinct meso-structural models.

Figure 6: (a) Load increment versus number of active fibers in *Bottom+Middle* of the steel fiber reinforced concrete; (b) Load increment versus number of active fibers in *Bottom* of the synthetic fiber reinforced concrete; (c) Load increment versus number of active fibers in *Bottom + Middle* of the synthetic fiber reinforced concrete.

Figure 7: Crack opening isosurfaces at CMOD = 0.8 mm for steel fiber reinforced concrete: (a) $V_f = 0.26\%$; (b) $V_f = 0.45\%$.

Figure 8: LDPM simulation for synthetic fiber reinforced concrete MS2-5 with $V_f = 0.37\%$: (a) Specimen geometry and discretization in numerical simulation; (b) Load vs CMOD curve; and (c) Meso-scale crack patterns at CMOD = 0.8 mm. The experimental curve (red line) represents the average of seven experimental tests, and the numerical curve (green line) represents the average response from seven distinct meso-structural models.

Figure 9: Single fiber two way pull-out test: (a) Relationship between fiber slippage and crack opening for steel and synthetic fibers; (b) Relationship between load and crack opening for steel and synthetic fibers.

Figure 10: LDPM simulation for synthetic fiber reinforced concrete MS2-10 with $V_f = 0.74\%$: (a) Specimen geometry and discretization in numerical simulation; (b) Load vs CMOD curve; and (c) Meso-scale crack patterns at CMOD = 0.8 mm. The experimental curve (red line) represents the average of seven experimental tests, and the numerical curve (green line) represents the average response from seven distinct meso-structural models.

Figure 11: Crack opening isosurfaces at CMOD = 0.8 mm for synthetic fiber reinforced concrete: (a) $V_f = 0.37\%$; (b) $V_f = 0.74\%$.

Figure 12: Fracture process zone for plain concrete.

Figure 13: Fracture process zone for steel fiber reinforced concrete with $V_f = 0.26\%$.

Figure 14: Fracture process zone for synthetic fiber reinforced concrete with $V_f = 0.74\%$.

Figure 15: Fracture process zone for (a) plain concrete at CMOD = 0.1 mm, and (b) at CMOD = 1.0 mm; (c) steel fiber reinforced concrete with $V_f = 0.26\%$ at CMOD = 0.1 mm, and (d) at CMOD = 1.0 mm; (e) steel fiber reinforced concrete with $V_f = 0.45\%$ at CMOD = 0.1 mm, and (f) at CMOD = 1.0 mm; (g) synthetic fiber reinforced concrete with $V_f = 0.37\%$ at CMOD = 0.1 mm, and (h) at CMOD = 1.0 mm; (i) synthetic fiber reinforced concrete with $V_f = 0.74\%$ at CMOD = 0.1 mm, and (j) at CMOD = 1.0 mm.

Figure 16: The trend of the total power during the test for the five different analyzed specimens.

Figure 17: LDPM simulation for round panel test on small specimens. This figures shows the specimen geometry and discretization for plain concrete and steel fiber reinforced concrete with $V_f = 0.26\%$ and $V_f = 0.39\%$.

Figure 18: Experimental data and numerical simulation results for the relationship between load and deflection (obtained from three numerical specimens in each case) for different kinds of fiber reinforcement, together with the curves for plain concrete: (a) Small round panel test; (b) Large round panel test.

Figure 19: LDPM simulation for round panel test on small specimens. This figures shows the crack propagation through the panel: (a) Plain concrete at $\delta = 0.30$ mm, $\delta = 0.38$ mm, $\delta = 0.45$ mm, and $\delta = 0.53$ mm; (b) Steel fiber reinforced concrete with $V_f = 0.39\%$ at $\delta = 0.32$ mm, $\delta = 0.45$ mm, $\delta = 0.54$ mm, and $\delta = 0.63$ mm.

Figure 20: LDPM simulation for round panel test on large specimens. This figures shows the crack propagation through the bottom surface: (a) Plain concrete at $\delta = 0.33$ mm, $\delta = 0.49$ mm, $\delta = 0.50$ mm, and $\delta = 1.10$ mm; (b) Steel fiber reinforced concrete with $V_f = 0.39\%$ at $\delta = 0.48$ mm, $\delta = 0.57$ mm, $\delta = 1.43$ mm, and $\delta = 1.90$ mm.