An Adaptive Low-Dimensional Control to Compensate for Actuator Redundancy and FES-Induced Muscle Fatigue in a Hybrid Neuroprosthesis

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Abstract

To restore walking and standing function in persons with paraplegia, a hybrid walking neuroprosthesis that combines a powered exoskeleton and functional electrical stimulation (FES) can be more advantageous than sole FES or powered exoskeleton technologies. However, the hybrid actuation structure introduces certain control challenges: actuator redundancy, cascaded muscle activation dynamics, FES-induced muscle fatigue, and unmeasurable states. In this paper, a human motor control inspired control scheme is combined with a dynamic surface control method to overcome these challenges. The new controller has an adaptive muscle synergy-based feedforward component which requires a fewer number of control signals to actuate multiple effectors in a hybrid neuroprosthesis. In addition, the feedforward component has an inverse fatigue signal to counteract the effects of the muscle fatigue. A dynamic surface control (DSC) method is used to deal with the cascaded actuation dynamics without the need for acceleration signals. The DSC structure was modified with a delay compensation term to deal with the electromechanical delays due to FES. A model based estimator is used to estimate the unmeasurable fatigue and actuator activation signals. The development of the controller and a Lyapunov stability analysis, which yielded semi-global uniformly ultimately boundedness, are presented in the paper. Computer simulations were performed to test the new controller on a 2 degree of freedom fixed hip model after which preliminary experiments were conducted on one able-bodied male subject in the fixed hip configuration.

Keywords: Functional Electrical Stimulation, Muscle Fatigue, Dynamic Surface Control, Synergies, Human Inspired Control, Adaptive Control, State Estimator, Input Delays, Electromechanical Delays.

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1. Introduction

Paraplegia due to a spinal cord injury (SCI) impairs walking function and hinders ability to perform activities of daily living. Functional electrical stimulation (FES) and powered orthoses are two viable technologies that may help restore walking function in persons with SCI. FES artificially revitalizes users' muscles by applying external electric potentials to produce muscle contractions [1], and when specific muscles are stimulated in an appropriate sequence and supplemented with a walker or crutches, walking movements can be reproduced. The first attempt to produce gait patterns using transcutaneous electrical stimulation of the quadriceps and glutei was presented in [2] and the first FES system to improve walking was developed in 1961 by Liberson et al. [3] to correct drop foot. Since then many FES based gait restoration systems have been developed [4–9]. In [6] a combination of stimulation the peroneal nerve, to produce withdraw reflex, and quadriceps to produce a gait motion. However, FES-based walking restoration systems have achieved limited acceptability among persons with mobility disorders due to the rapid onset of muscle fatigue and other challenges such as coordinating multiple muscles with FES, electromechanical delays (EMD), poor and unreliable muscle force generation by electrical stimulation, and upper-body effort required to maintain balance while walking. Moreover, controlling FES can be cumbersome due to the highly nonlinear muscle dynamics.

To reduce the muscle fatigue effects, FES can be used in conjunction with a passive orthosis [10–13]. The addition of an orthosis lowers the metabolic cost of walking by providing a relatively natural and stable gait, reducing the load on an user's arms, and decreasing the stimulation duty cycle of FES. However, this type of a walking system can be disadvantageous because the power is only provided by FES, which deteriorates with FES-induced muscle fatigue. Recently, powered exoskeletons that use electric motors, or some other forms of external actuation, to assist or elicit lower-limb movements have been developed [14–16]. Unlike FES, powered exoskeletons reliably generate joint torque and can coordinate joint movements with high accuracy. However, the powered exoskeleton requires larger heavier batteries to sustain walking for long periods of time, which adds weight and bulk to the device.

A more viable technology for gait restoration is a hybrid device that combines an FES system with a powered exoskeleton [17–21]. With such a system, the limitations of FES such as limited and unreliable muscle torque generation, can be overcome by using an electric motor. Also, intermittent use of FES would provide therapeutic benefits to a user. This would also help conserve the battery charge required for an electric motor because a portion of the joint torque can be generated by FES; thus, resulting in smaller and lighter batteries and motors. However, the use of an electric motor and FES at each joint creates an actuator redundancy problem, which is a control challenge. For example, in a hybrid walking system that controls 4 lower limb joints, 12 actuators need to be controlled if applying FES via surface electrodes. These actuators include FES of flexors and extensors and an electric motor drive at each joint.

Thus a challenge for a controller for the hybrid system is to be able to allocate the control efforts between the redundant actuators to produce a coordinated movement. Recent control methods developed for hybrid walking exoskeletons include open-loop FES control with a feedback controller for controlled brakes [12, 22–24], iterative learning control [21, 25], adaptive gain control [26, 27], a finite state machine (FSM) was used to control an orthosis which combined FES with hydraulic actuators and wrap-spring clutches [10]. Although these pioneering control approaches are interesting, the methods did not consider control of multiple muscle stimulation and multiple electric drives applied to multiple DOFs. Further, these control methods do not follow a formal control development and do not provide stability guarantees. Thus, research on the systematic design of automatic control algorithms for a hybrid walking system and their stability analysis that addresses actuator redundancy due to multiple muscle stimulation and electric motors has been lacking.

To overcome this challenge, an adaptive synergy-based feedforward component for muscle stimulation and a combination of feedforward and feedback control for the electric motors is proposed. The adaptive feedforward component for both FES and electric motor uses the muscle synergy principle; i.e., fewer control signals are used to control multiple actuators in a hybrid neuroprosthesis. Muscle synergies are described as motor primitives that are employed by the human central nervous system (CNS) to efficiently perform complex movements such as posture control and hand manipulations [28–32]. A small number of muscle synergies has been shown to capture various muscle patterns across different behavioral and task conditions. This is typically done by collecting electromyography data from a large number of muscles during tasks and performing statistical analyses. Some analysis techniques used are non-negative matrix factorization (NNMF), principal component analysis (PCA), partial least squares regression (PLSR), or singular value decomposition (SVD) to transform the system into a lower dimensional system while keeping as much of the variance of the data as possible. The underlining concept is the transformation of a high dimensional, complex system to a lower dimensional, simpler system that is easier to control [33]. Muscle synergies have also been studied for human locomotion and the extracted synergies have been successfully applied to complex human walking models to reproduce realistic gait motions [34]. For a more thorough background on the literature pertaining to synergies, the reader is referred to [31, 33]. Recently a low-dimensional adaptive controller, which uses synergies extracted through PCA, was developed for a hybrid neuroprosthesis [35, 36]. However, the control development and stability analysis in this paper did not consider the muscle activation, electromechanical delay (EMD), and fatigue dynamics associated with FES.

Electromechanical delays are classified as a time lag between the application of electrical stimulation to the motor-neurons and the buildup of tension in the muscles as they contract. The causes of EMD are accredited to various phenomena such as finite propagation time of the chemical ions in the muscle, cross-bridge formation between actin-myosin filaments, stretching of the series elastic components in response to the external electrical input, synaptic trans-

mission delays, and others [37, 38]. Typically, in FES-based models, the EMD is modeled as an input delay that occurs in the 1st order activation dynamics. Input delays have been shown to cause performance degradation and can potentially cause instability if not dealt with. Recently, much research has gone into developing automatic closed-loop controllers which are augmented with delay compensation terms to deal with input delays [39–43]. However, the aforementioned works ignore the activation dynamics.

The neuromuscular activation dynamics describe Ca^{2+} ion activation and deactivation dynamics that primarily facilitate the muscle force generation [44] and can be modeled as a first-order ordinary differential equation [45] or a Hammerstein structure [46–48]. The activation dynamics couple neural excitations to the muscle contraction dynamics (muscle force-length and force-velocity relationships) that drive the second order musculoskeletal dynamics. Although the activation dynamics change the behavior of the system, typically controllers designed for NMES ignore them [42, 49, 50] or use ad-hoc methods without considering their effects on the stability [51, 52]. An integrator backstepping-based approach was used to design a neural network-based NMES controller for a musculoskeletal system with fatigue and calcium dynamics [53]. However, the control design required an acceleration signal during the control implementation. This requirement of the acceleration signal is due to the use of the integrator backstepping approach, which is associated with the “explosion of terms” for systems of higher order. Recently, a controller was designed for an NMES-driven musculoskeletal system with activation dynamics that used dynamic surface control (DSC) to avoid the “explosion of terms” problem [54]. DSC is a modified version of multiple sliding surface (MSS) control, which is very similar to integrator backstepping. A DSC approach, first developed by Swaroop et al. [55, 56], is a robust nonlinear control technique that uses the dynamics of first order low-pass filters to avoid the “explosion of terms”.

The main contribution of this paper is the development of an adaptive synergy-based dynamic surface tracking controller for a hybrid neuroprosthesis with actuator redundancy, EMD, fatigue, and actuation dynamics. This control scheme solves the actuator redundancy problem by using an adaptive synergy based feedforward component to provide the rough input profile for all the actuators. Then feedback control of the motors is used to increase the robustness of the controller and to provide better tracking control. The synergies are extracted using principle component analysis on optimal activations, which are pre-computed through dynamic optimizations of a hybrid lower-limb and exoskeleton model [57]. The synergies with the least reconstruction variance were truncated to reduce the amount of signals needed during online implementation. The synergy activation profiles were modified online using an adaptive update law to compensate for any unmodeled phenomena or parametric changes in the system. Because the activation dynamics were considered in the control development, a DSC framework was used to avoid acceleration signals. Also the feedforward and feedback terms are scaled up by the inverse of the fatigue estimate and a control gain to maintain the effectiveness of the feedforward component as the muscles fatigues and in case there is mismatch in model strength.

The fatigue and activation states are not measurable, so model based estimators were designed to estimate the signals for real-time implementation of the controller. A simulation study was performed on a 2 degree of freedom fixed hip model to show the efficacy of the newly developed controller. The controller was then tested in preliminary experiments on an able-bodied male subject.

2. Dynamic Model

Consider a musculoskeletal model of a swinging leg that is given by:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + f(q, \dot{q}) + d = \Gamma, \quad (1)$$

where q , $\dot{q}$, $\ddot{q} \in \mathbb{R}^n$ are the angular position, velocity, and acceleration of the limb segments, respectively. To simplify the presentation of the model the following subscripts are introduced: 1) i denotes the element for the i^{th} limb, 2) j denotes the type of actuator, i.e., extension muscle ($j = ex$), flexion muscle ($j = fx$), and electric motor ($j = m$), and 3) l denotes the l^{th} parameter for the sets of parameters in the subsequently defined passive dynamics.

In (1), $M(q) \in \mathbb{R}^{n \times n}$ is the combined inertia of the semi-powered orthosis and human limbs in the swing phase, $C(q, \dot{q}) \in \mathbb{R}^{n \times n}$ is the centripetal/Coriolis matrix, $G(q) \in \mathbb{R}^n$ is the gravity vector, and $d(t) \in \mathbb{R}^n$ is any unmodeled effects or disturbances in the system such as spasticity. In (1), $f(q, \dot{q}) \in \mathbb{R}^n$ denotes the nonlinear viscoelastic passive dynamics of the muscles and can be defined as $f(q, \dot{q}) = M_e + M_v$, where $M_e \in \mathbb{R}^n$ are the elastic muscle effects that account for muscle elasticity under normal functional usage, hyperextension, and hyperflexion and $M_v \in \mathbb{R}^n$ are the viscous effects in the musculoskeletal system. These nonlinear functions were modeled as

$$\begin{aligned} M_{e_i} &= k_{1_i}(q_i - k_{0_i}) + k_{2_i}e^{k_{3_i}q_i} - k_{4_i}e^{k_{5_i}q_i} \\ M_{v_i} &= B_{1_i}\tanh(-B_{2_i}\dot{q}_i) - B_{3_i}\dot{q}_i \end{aligned}$$

where k_{l_i} ($l = 0$ to 5), B_{l_i} ($l = 1$ to 3) $\in \mathbb{R}$ are subject specific model parameters for an i^{th} joint/DOF. The active torques at the joints are generated by including the muscle dynamics due to FES [58] and the current dynamics due to an electric motor attached at each joint. The torque term is defined as

$$\Gamma \triangleq b(q, \dot{q})\phi(t)\mu, \quad (2)$$

where $\mu(t) \in \mathbb{R}^{3n}$ is the intermediate normalized activation vector containing activation states for the actuators, muscle activation for FES and normalized current for motors, and is defined as

$$\mu \triangleq \begin{bmatrix} \mu_{1_{fx}} & \mu_{1_{ex}} & \cdots & \mu_{n_{fx}} & \mu_{n_{ex}} & \mu_{1_m} & \cdots & \mu_{n_m} \end{bmatrix}^T,$$

$\phi(t) \in \mathbb{R}^{3n \times 3n}$ is the fatigue matrix that contains the fatigue factor corresponding to each stimulated muscle and is defined as

$$\phi \triangleq \text{diag} \left(\begin{bmatrix} \phi_{1_{fx}} & \phi_{1_{ex}} & \cdots & \phi_{n_{fx}} & \phi_{n_{ex}} & 1 & \cdots & 1 \end{bmatrix} \right), \quad (3)$$

and $b(q, \dot{q}) \in \mathbb{R}^{n \times 3n}$ is the control gain matrix containing

$$b = \begin{bmatrix} \psi_{1_{fx}} & 0 & \cdots & 0 \\ -\psi_{1_{ex}} & 0 & \cdots & 0 \\ & \ddots & & \\ 0 & \cdots & 0 & \psi_{n_{fx}} \\ 0 & \cdots & 0 & -\psi_{n_{ex}} \\ \kappa_1 & & 0 & \cdots & 0 \\ & \ddots & & & \\ 0 & \cdots & 0 & & \kappa_n \end{bmatrix}^T, \quad (4)$$

where $\psi_{i_j}(q_i, \dot{q}_i)$ is the nonlinear function for the torque-length and torque-velocity relationships for the muscles and κ_i is a conversion constant (current to torque) of an electric motor at the i^{th} joint. These torque-length and torque-velocity relationships are defined as

$$\psi_{i_j} \triangleq (c_{0_{i_j}} + c_{1_{i_j}} q_i + c_{2_{i_j}} q_i^2)(1 + c_{3_{i_j}} \dot{q}_i),$$

where $c_{l_{i_j}} (l = 1 \text{ to } 3) \in \mathbb{R}$ are a set of constants for the muscles ($j = fx$ and ex) at the i^{th} joint/DOF.

The active torque, Γ , is coupled to the normalized control input, $u \in \mathbb{R}^{3n}$, through the intermediate normalized activation vector, $\mu \in \mathbb{R}^{3n}$. The general actuator dynamics for the electric motor and FES are governed by the following differential equation [45]

$$\dot{\mu}_{i_j} = -w_{i_j} \mu_{i_j} + w_{i_j} u_{i_j} (t - \tau_{i_j}), \quad (5)$$

where $\tau_{i_j} \in \mathbb{R}^+$ is a known constant EMD and $w_{i_j} \in \mathbb{R}$ is the time constant for the activation variables, μ_{i_j} . The input delay is included out of necessity for FES but is generalized for the electric motors as well, and can be assumed to be zero for the motors. The normalized input $u(t) \in \mathbb{R}^{3n}$ is defined as the saturation function vector that contains μ_{i_j} in (5) and is denoted by a piecewise linear function as

$$u = \text{sat}[v] \triangleq \begin{cases} u_{min} & v < v_{min} \\ \frac{u_{max} - u_{min}}{v_{max} - v_{min}} (v - v_{min}) + u_{min} & v_{min} \leq v \leq v_{max} \\ u_{max} & v > v_{max}, \end{cases} \quad (6)$$

where $v_{min}, v_{max} \in \mathbb{R}^{3n}$ are the minimum/maximum input magnitudes for each actuator (stimulation or motor). Based on (5) and (6), a linear differential inequality can be developed to show that $\mu \in [u_{min}, u_{max}]$. The u_{min}, u_{max} values are $[0, 1]$ for muscles because they are unidirectional and $[-1, 1]$ for electric motors because they are bidirectional actuators.

The fatigue dynamics of the muscles, $\phi_{i_j} \in \mathbb{R}$ is generated from the first order differential equation [59]

$$\dot{\phi}_{i_j} = \frac{1}{T_{f_{i_j}}}(\phi_{\min_{i_j}} - \phi_{i_j})\mu_{i_j} + \frac{1}{T_{r_{i_j}}}(1 - \phi_{i_j})(1 - \mu_{i_j}), \quad (7)$$

where $\phi_{\min} \in (0, 1)$ is the unknown minimum fatigue constant of a muscle, and $T_f, T_r \in \mathbb{R}^+$ are unknown time constants for fatigue and recovery in the muscle, respectively. Because $\mu \in [u_{\min}, u_{\max}]$ for muscles, it can be shown that $\phi \in [\phi_{\min}, 1]$, where $\phi = 1$ when the muscle is fully rested, and $\phi = \phi_{\min}$ when the muscle is fully fatigued.

The estimates of the activation and fatigue states in (5) and (7) are generated through the following dynamics

$$\dot{\hat{\mu}}_{i_j} = -\hat{w}_{i_j}\hat{\mu}_{i_j} + \hat{w}_{i_j}u_{i_j}(t - \tau_{i_j}), \quad (8)$$

$$\dot{\hat{\phi}}_{i_j} = \frac{1}{\hat{T}_{f_{i_j}}}(\hat{\phi}_{\min_{i_j}} - \hat{\phi}_{i_j})\hat{\mu}_{i_j} + \frac{1}{\hat{T}_{r_{i_j}}}(1 - \hat{\phi}_{i_j})(1 - \hat{\mu}_{i_j}), \quad (9)$$

where $\hat{w}_{i_j}, \hat{w}_i, \hat{T}_{f_{i_j}}, \hat{T}_{r_{i_j}}$, and $\hat{\phi}_{\min_{i_j}}$ are bounded estimates of the real parameters that can be determined through system identification. Note that these estimators are governed by first-order differential equations, thus the estimates are bounded as $\hat{\mu} \in [u_{\min}, u_{\max}]$ and $\hat{\phi} \in [\hat{\phi}_{\min}, 1]$.

3. Desired Muscle and Electric Motor Response

Let $\mu_d(t) \in \mathbb{R}^{3n}$ be the desired intermediate normalized activation variable vector containing desired muscle activation and motor current levels. This activation variable vector can be computed using dynamic optimizations [57]. Below, the dynamics, excluding the fatigue factor ϕ , are written in terms of the desired control and kinematic trajectories as

$$\begin{aligned} M(q_d)\ddot{q}_d + C(q_d, \dot{q}_d)\dot{q}_d + G(q_d) + f(q_d, \dot{q}_d) \\ \equiv b(q_d, \dot{q}_d)\mu_d(t). \end{aligned} \quad (10)$$

Further, by using PCA, the possibly correlated optimal activations, μ_d , can be transformed into a set of linearly correlated synergy activations, c_d , using the synergies W such as

$$\mu_d = Wc_d(t) + \mu_{loss}, \quad (11)$$

where the rows in $W \in \mathbb{R}^{3n \times p}$ are the synergies and $c_d(t) \in \mathbb{R}^p$ are the first p synergy activation coefficients. PCA is a statistical procedure that uses an orthogonal transformation, i.e., multiplying by W , to transform an ensemble of signals into linearly correlated signals also known as principal components

(synergy activation coefficients). The principal components are ordered such that the first synergy and its activation coefficient accounts for most of the variance of the original signal, the second pair accounts for the second most, and so on. This allows the use of p synergies, where $p < 3n$, to reconstruct μ_d to over a 90% match. After disregarding the $3n - p$ synergies, there will be a mismatch between the reconstructed activation profiles, Wc_d , and the optimal activation profiles μ_d . This reconstruction error is quantified as μ_{loss} , as given in (11).

To simplify the derivations, the following notations are used: (1) the time dependence of a function is dropped (e.g., $e(t) \rightarrow e$) and (2) a signal delayed by τ is notated as a subscript (e.g., $e(t - \tau) \rightarrow e_\tau$). In addition, to facilitate the control development and stability analysis, the following assumptions were made.

ASSUMPTION 1: Only motion in the sagittal plane is considered and the ankle joint is locked by an orthosis.

ASSUMPTION 2: The unmodeled effects or disturbances, τ_d , are bounded as $|\tau_d| \leq \epsilon_1$ where $\epsilon_1 \in \mathbb{R}^+$ is a constant.

ASSUMPTION 3: The transformation coefficients in the matrix, W , are bounded constants and the principal components, c_d , are bounded vectors. The reconstruction error μ_{loss} which accounts for the loss of variability in the activation profile data is bounded.

ASSUMPTION 4: The desired trajectory, $q_d \in \mathbb{R}^n$, and its derivatives, $\dot{q}_d$, $\ddot{q}_d \in \mathbb{R}^n$, are bounded.

4. Control Development

The control objective is to track a continuously differentiable desired trajectory $q_d \in \mathbb{R}^n$. The tracking error, $e \in \mathbb{R}^n$, is defined as

$$e \triangleq q_d - q. \quad (12)$$

To facilitate the control design and stability analysis, the auxiliary error signal $r \in \mathbb{R}^n$ is defined as

$$r \triangleq \dot{e} + \alpha e, \quad (13)$$

where $\alpha \in \mathbb{R}^+$ is a control gain. The open-loop error is derived by multiplying the time derivative of (13) with $M(q)$ and substituting the dynamics in (1) and (2) to obtain

$$M\dot{r} = M\ddot{q}_d + C\dot{q} + G + f + d - b\phi\mu + M\alpha\dot{e}. \quad (14)$$

This expression can be written in the form

$$M\dot{r} = -Cr + \tilde{N} + N_d + d - b\phi\mu - e - b_d\phi e_I, \quad (15)$$

where $e_I \in \mathbb{R}^{3n}$, is defined as $e_{I_{i_j}} \triangleq \int_{t-\tau_{i_j}}^t u_{i_j}(\theta) d\theta$ for each actuator and $\tilde{N} \in \mathbb{R}^n$, is defined as $\tilde{N} \triangleq N - N_d$ and the auxiliary signals $N(q, \dot{q}, e, \dot{e}, e_I, t)$ and $N_d(t)$ are defined as

$$N \triangleq M\ddot{q}_d + C(\dot{q}_d + \alpha e) + G + f + M\alpha\dot{e} + e + b_d\phi e_I,$$

$$N_d \triangleq M(q_d)\ddot{q}_d + C(q_d, \dot{q}_d)\dot{q}_d + G(q_d) + f(q_d, \dot{q}_d).$$

The term $\tilde{N}$ in (15) can be upper bounded by using the Mean Value Theorem as

$$\|\tilde{N}\| \leq \rho_1(\|z\|) \|z\|, \quad (16)$$

where $\rho_1(\|z\|) \in \mathbb{R}$ is a positive monotonic bounded function and $z \in \mathbb{R}^{5n}$ is defined as

$$z = [e^T \quad r^T \quad e_I^T]^T.$$

Note that the auxiliary signal N_d is equal to the left hand side of the desired muscle dynamics in (10). Therefore, (15) can be rewritten as

$$M\dot{r} = -Cr + \tilde{N} + d + b_d\mu_d - b\phi\mu - e - b_d\phi e_I. \quad (17)$$

After adding and subtracting the terms $b_d\hat{\phi}\bar{\mu}$, $b_d\phi\bar{\mu}$, $b_d\phi\hat{\mu}$, $b_d\phi\mu$, and $b_d\phi\mu_f$ where $b_d = b(q_d, \dot{q}_d)$, $\hat{\mu} \in \mathbb{R}^{3n}$ and $\hat{\phi} \in \mathbb{R}^{3n \times 3n}$ are estimates of the activation state and the fatigue state generated using model based estimators in (8) and (9), $\bar{\mu} \in \mathbb{R}^{3n}$ is the desired activation to be later defined, and $\mu_f \in \mathbb{R}^{3n}$ is a filtered desired activation, and rearranging the terms, (17) becomes

$$M\dot{r} = -Cr + b_d\phi S + b_d\phi y + \tilde{N} + d + b_d\mu_d + \tilde{b}\phi\mu + b_d\phi\tilde{\mu} + b_d\tilde{\phi}\bar{\mu} - b_d\hat{\phi}\bar{\mu} - e, \quad (18)$$

where $\tilde{b} \in \mathbb{R}^{n \times 3n}$ is defined as $\tilde{b} \triangleq b_d - b$, $\tilde{\phi} \in \mathbb{R}^{3n \times 3n}$ is defined as $\tilde{\phi} \triangleq \hat{\phi} - \phi$, and $\tilde{\mu} \in \mathbb{R}^{3n}$ is defined as $\tilde{\mu} \triangleq \hat{\mu} - \mu$.

In (18), the surface error, $S \in \mathbb{R}^{3n}$, is defined as

$$S \triangleq \mu_f - \hat{\mu} - e_I, \quad (19)$$

and the boundary layer error, $y \in \mathbb{R}^{3n}$, for μ is defined as

$$y \triangleq \bar{\mu} - \mu_f. \quad (20)$$

The filtered desired activation μ_f is obtained by passing $\bar{\mu}$ through a low-pass filter such as

$$\zeta_f \dot{\mu}_f + \mu_f = \bar{\mu}; \quad \mu_f(0) = \bar{\mu}(0), \quad (21)$$

where $\zeta_f \in \mathbb{R}^+$ is the low-pass filter time constant. The delay compensation term, e_I , is added to the surface error, S , to deal with the input delay in the actuator dynamics.

After designing the desired activation, $\bar{\mu}$, as

$$\bar{\mu} = \hat{\phi}^{-1} [\zeta_{sf} W \hat{c} + kr], \quad (22)$$

where $\hat{c} \in \mathbb{R}^p$ is the estimate of c_d , $\zeta_{sf} \in \mathbb{R}^{3n \times 3n}$ is a control gain and $k \in \mathbb{R}^{3n \times n}$ is the feedback gain that is chosen to only influence the electric motors. In $\bar{\mu}$, the feedforward component, $\zeta_{sf} W \hat{c}$, and the feedback component, kr , are scaled by the inverse of the fatigue estimate. This feature is included in the controller so that as a muscle fatigues, the stimulation input to that muscle increases gradually to counteract the effects of the fatigue. The estimate of the principal components updates according to the following update law with the projection algorithm

$$\dot{\hat{c}} = \text{proj} (\dot{c}_d + F W^T \zeta_{sf}^T b_d^T r), \quad (23)$$

where $F \in \mathbb{R}^{p \times p}$ is a symmetric positive definite gain matrix. After using (11) and (22), (18) becomes

$$\begin{aligned} M\dot{r} = & -Cr + b_d \phi S + b_d \phi y + \tilde{N} + d + b_d \zeta_{sf} W \tilde{c} + b_d (I - \zeta_{sf}) W c_d \\ & + b_d \mu_{loss} + \tilde{b} \phi \mu + b_d \phi \tilde{\mu} + b_d \tilde{\phi} \hat{\phi}^{-1} \zeta_{sf} W \hat{c} + b_d \tilde{\phi} \hat{\phi}^{-1} kr - b_d kr - e, \end{aligned} \quad (24)$$

where $\tilde{c} \in \mathbb{R}^p$ is defined as

$$\tilde{c} = c_d - \hat{c}.$$

Using the Mean Value Theorem, Assumption 4, and the property of projection algorithm the following terms can be bounded as

$$\begin{aligned} \|\tilde{b} \phi \mu\| & \leq \rho_2(\|z\|) \|z\|, \quad \|b_d\| \leq \zeta, \\ \|b_d \mu_{loss} + b_d (I - \zeta_{sf}) W c_d\| & \leq \epsilon_2, \quad \|b_d \tilde{\phi} \hat{\phi}^{-1} \zeta_{sf} W \hat{c}\| \leq \epsilon_3 \end{aligned} \quad (25)$$

where $\rho_2(\|z\|) \in \mathbb{R}$ is a positive monotonically increasing bounded function and $\epsilon_2, \epsilon_3, \zeta \in \mathbb{R}^+$ are constants.

The surface error dynamics are derived by taking the time derivative of (19) and using (8), resulting in

$$\dot{S} = \dot{\mu}_f + \hat{w} \hat{\mu} - \hat{w} u_\tau - (u - u_\tau). \quad (26)$$

Based on the subsequent stability analysis, the normalized input u is designed as

$$u = \beta S + \dot{\mu}_f, \quad (27)$$

where $\beta \in \mathbb{R}^+$ is a control gain.

Remark: Note that in the overall controller v , the DSC component which is tuned through ζ_3 could potentially amplify the feedforward component, $W \hat{c}$. To avoid this, the feedforward component, $W \hat{c}$, in the definition of $\bar{\mu}$ was scaled by ζ_{sf} in anticipation of this amplifying effect. In addition, this gain could be used

Figure 1: The control schematic for the implementation of the newly developed controller.

to scale down or up the feedforward signals if there is a mismatch between the strength of a subject and the model used to compute the feedforward signals. This scaling factor gain will also give the user control over the relative amount of stimulation and motor effort used in the overall controller.

Therefore, the closed loop surface error dynamics can be written as

$$\dot{S} = -\beta S + \hat{w}\hat{\mu} + (1 - \hat{w})u_\tau. \quad (28)$$

The boundary layer error dynamics are found by taking the time derivative of (20) and using (21), which results in

$$\dot{y} = \eta - \frac{y}{\zeta_f}, \quad (29)$$

where $\eta(e, r, S, y, t)$ is a continuous nonlinear function defined as $\eta = \frac{d}{dt} [\bar{\mu}]$. Based on the definition of u in (27), the control law v is designed as

$$v = \left[\beta S + \frac{\hat{\phi}^{-1} [\zeta_{sf} W \hat{c} + kr] - \mu_f}{\zeta_f} - u_{min} \right] \frac{v_{max} - v_{min}}{u_{max} - u_{min}} + v_{min}. \quad (30)$$

5. Stability Analysis

Theorem 1: The controller designed in (22), (23), and (30) ensures semi-global uniformly ultimately bounded tracking provided that the following gain

conditions are met:

$$K_{min} > \frac{(\rho_1(\|z\|) + \rho_2(\|z\|))^2}{2\zeta}, \quad \gamma_{min}\{b_d k - \Lambda\} > 0, \quad \beta > \vartheta + \frac{\bar{\tau}\zeta_f}{\beta}$$

where $\gamma_{min}\{\cdot\}$ denotes the minimum eigenvalue of a square matrix and K_{min} , $\vartheta, \bar{\tau} \in \mathbb{R}^+$ are subsequently defined constants.

Proof: To prove stability, choose a continuously differentiable Lyapunov candidate $V(x, t) \in \mathbb{R}$, defined as

$$V = \frac{1}{2}e^T e + \frac{1}{2}r^T M r + \frac{1}{2}S^T S + \frac{1}{2}y^T y + \frac{1}{2}\tilde{c}^T F^{-1}\tilde{c} + P. \quad (31)$$

The Lyapunov candidate V can be lower and upper bounded as

$$\lambda_1 \|x\|^2 \leq V \leq \lambda_2 \|x\|^2 + \Upsilon, \quad (32)$$

where Υ , λ_1 , $\lambda_2 \in \mathbb{R}^+$ are constants and $x = [e^T \ r^T \ S^T \ y^T \ \sqrt{P}]^T$. The Lyapunov-Krasovskii functional $P \in \mathbb{R}^+$ is defined as

$$P = \frac{\zeta_f}{\beta^2} \int_{t-\bar{\tau}}^t \left(\int_s^t u(\theta)^T u(\theta) d\theta \right) ds,$$

where $\bar{\tau} = \max(\tau_{i_j})$.

Taking the time derivative of $V(x, t)$ and using (13), (24), (28), (29), the update law in (23), the skew-symmetry property [60] and canceling out the like terms results in

$$\begin{aligned} \dot{V} = & -\alpha e^T e - r^T b_d k r - S^T \beta S - \frac{1}{\zeta_f} y^T y + S^T (\hat{w}\hat{\mu} + (1 - \hat{w})u_\tau) \\ & + y^T \eta + r^T \left(\tilde{N} + d + b_d \mu_{loss} + b_d(I - \zeta_{sf})Wc_d + b_d \phi S + b_d \phi y + \tilde{b}\phi\mu \right) \\ & + r^T \left(b_d \tilde{\phi}\tilde{\mu} + b_d \phi \tilde{\mu} \right) + \frac{\zeta_f}{\beta^2} \bar{\tau} \|u\|^2 - \frac{\zeta_f}{\beta^2} \int_{t-\bar{\tau}}^t \|u(\theta)\|^2 d\theta. \end{aligned}$$

The previous equation can be bounded using (16), (25), the definitions of u , and Assumptions 2 and 3 to get

$$\begin{aligned} \dot{V} \leq & -\alpha e^T e - r^T \left(b_d k - b_d \tilde{\phi} \hat{\phi}^{-1} k \right) r - S^T \beta S - \frac{1}{\zeta_f} y^T y + \|S\| \|\hat{w}\hat{\mu} + (1 - \hat{w})u_\tau\| \\ & + \|r\| [(\rho_1(\|z\|) + \rho_2(\|z\|)) \|z\| + \epsilon_1 + \epsilon_2] + \|y\| \|\eta\| \\ & + \|b_d \phi\| (\|r\| \|S\| + \|r\| \|y\|) + \|r\| \left\| b_d \tilde{\phi} \hat{\phi}^{-1} \zeta_{sf} W \hat{c} \right\| + \|r\| \|b_d \phi \tilde{\mu}\| \\ & + \frac{\zeta_f \bar{\tau}}{\beta^2} \left(\beta^2 \|S\|^2 + 2\beta \|S\| \|\dot{\mu}_f\| + \|\dot{\mu}_f\|^2 \right) - \frac{\zeta_f}{\beta^2} \int_{t-\bar{\tau}}^t \|u(\theta)\|^2 d\theta. \end{aligned}$$

Using the fact that $u \in [u_{min}, u_{max}]$, $\hat{\mu} \in [u_{min}, u_{max}]$, and $\hat{\phi} \in [\hat{\phi}_{min}, 1]$, Young's inequality, and (25) to bound the following terms

$$\begin{aligned}
\|b_d \phi\| \|r\| \|S\| &\leq \frac{\zeta}{2} \|r\|^2 + \frac{\zeta}{2} \|S\|^2, \\
\|b_d \phi\| \|r\| \|y\| &\leq \frac{\zeta}{2} \|r\|^2 + \frac{\zeta}{2} \|y\|^2, \\
\|y\| \|\eta\| &\leq \frac{1}{2\varepsilon} \|y\|^2 \|\eta\|^2 + \frac{\varepsilon}{2}, \\
\|S\| \|\hat{w}\hat{\mu} + (1 - \hat{w})u_\tau\| &\leq \Psi \|S\| \leq \frac{1}{2\varepsilon} \Psi^2 \|S\|^2 + \frac{\varepsilon}{2}, \\
\|r\| \left\| b_d \tilde{\phi} \hat{\phi}^{-1} \zeta_{sf} W \hat{c} \right\| &\leq \frac{\epsilon_3^2}{2\varepsilon} \|r\|^2 + \frac{\varepsilon}{2} \\
\|b_d \phi \tilde{\mu}\| \|r\| &\leq \frac{\zeta^2}{2\varepsilon} \|r\|^2 + \frac{\varepsilon}{2} \\
\frac{2\bar{\tau}}{\beta} \|S\| \|y\| &\leq \frac{\bar{\tau}\zeta_f}{\beta} \|S\|^2 + \frac{\bar{\tau}}{\zeta_f \beta} \|y\|^2
\end{aligned}$$

where $\varepsilon \in \mathbb{R}^+$ is a arbitrary constant, $\Psi = \hat{w} + (1 + \hat{w}) u_{max}$, and ζ is defined in (25). After rearranging the terms this expression becomes

$$\begin{aligned}
\dot{V} &\leq -\alpha e^T e - r^T (b_d k + \zeta - \Lambda) r \\
&\quad - S^T \left(\beta - \vartheta - \frac{\tau \zeta_f}{\beta} \right) S - \left(\frac{\kappa}{\zeta_f} - \frac{\zeta}{2} \right) y^T y \\
&\quad + \|r\| [(\rho_1(\|z\|) + \rho_2(\|z\|)) \|z\| + \epsilon_1 + \epsilon_2] \\
&\quad + \frac{1}{2\varepsilon} \|y\|^2 \|\eta\|^2 + 2\varepsilon - \frac{\zeta_f}{\beta^2} \int_{t-\bar{\tau}}^t \|u(\theta)\|^2 d\theta.
\end{aligned}$$

where $\Lambda = b_d \tilde{\phi} \hat{\phi}^{-1} k + \left(2\zeta + \frac{\zeta^2}{2\varepsilon} + \frac{\epsilon_3^2}{2\varepsilon} \right) I$, $\vartheta = \frac{\zeta}{2} + \frac{\Psi^2}{2\varepsilon} + \zeta_f \bar{\tau}$, and $\kappa = 1 - \bar{\tau} \beta^{-2} - \bar{\tau} \beta^{-1}$. After defining ζ_f such that

$$\frac{1}{\zeta_f} \geq \frac{1}{\kappa} \left[\frac{\zeta}{2} + \frac{\Omega^2}{2\varepsilon} + k_o \right], \tag{33}$$

where $k_o \in \mathbb{R}^+$ is a known constant and $\Omega > 0$ is the maximum of η in the defined compact set $\Xi = \{h \in \mathbb{R}^{8n} | \|h\| < 2\sigma, h = [e, r, S, y]^T\}$ where $\sigma \in \mathbb{R}^+$ is a known constant, the previous equation becomes

$$\begin{aligned}
\dot{V} &\leq -\alpha e^T e - r^T (b_d k + \zeta - \Lambda) r - S^T \left(\beta - \vartheta - \frac{\bar{\tau} \zeta_f}{\beta} \right) S - k_o y^T y \\
&\quad + \|r\| [(\rho_1(\|z\|) + \rho_2(\|z\|)) \|z\| + \epsilon_1 + \epsilon_2] \\
&\quad - \left(1 - \frac{\|\eta\|^2}{\Omega^2} \right) \frac{\|y\|^2 \Omega^2}{2\varepsilon} + 2\varepsilon - \frac{\zeta_f}{\beta^2} \int_{t-\bar{\tau}}^t \|u(\theta)\|^2 d\theta.
\end{aligned}$$

Using nonlinear damping to separate the terms and further bounding results in

$$\begin{aligned}\dot{V} \leq & -\alpha e^T e - r^T (b_d k - \Lambda) r - S^T \left(\beta - \vartheta - \frac{\bar{\tau} \zeta_f}{\beta} \right) S - k_o y^T y \\ & + \frac{(\rho_1(\|z\|) + \rho_2(\|z\|))^2 \|z\|^2}{2\zeta} + \frac{(\epsilon_1 + \epsilon_2)^2}{2\zeta} + 2\varepsilon - \frac{\zeta_f}{\beta^2} \int_{t-\bar{\tau}}^t \|u(\theta)\|^2 d\theta.\end{aligned}$$

Further, by splitting the integral term and using the Cauchy Schwarz inequality

$$\|e_I\|^2 \leq \bar{\tau} \int_{t-\bar{\tau}}^t \|u(\theta)\|^2 d\theta$$

the following term can be bounded as

$$-\frac{\zeta_f}{2\bar{\tau}\beta^2} \left(\bar{\tau} \int_{t-\bar{\tau}}^t \|u(\theta)\|^2 d\theta \right) \leq -\frac{\zeta_f}{2\bar{\tau}\beta^2} \|e_I\|^2.$$

This expression can be bounded as

$$\begin{aligned}\dot{V} \leq & - \left(K_{min} - \frac{(\rho_1(\|z\|) + \rho_2(\|z\|))^2}{2\zeta} \right) \|z\|^2 - S^T \left(\beta - \vartheta - \frac{\bar{\tau} \zeta_f}{\beta} \right) S \\ & - k_o y^T y + \frac{(\epsilon_1 + \epsilon_2)^2}{2\zeta} + 2\varepsilon - \frac{\zeta_f}{2\beta^2} \int_{t-\bar{\tau}}^t \|u(\theta)\|^2 d\theta,\end{aligned}\tag{34}$$

where K_{min} is defined as $K_{min} = \min \left\{ \alpha, \gamma_{min} \{b_d k - \Lambda\}, \frac{\zeta_f}{2\bar{\tau}\beta^2} \right\}$. Because

$$\begin{aligned}\int_{t-\bar{\tau}}^t \left(\int_s^t \|u(\theta)\|^2 d\theta \right) ds & \leq \bar{\tau} \sup_{s \in [t, t-\bar{\tau}]} \left[\int_s^t \|u(\theta)\|^2 d\theta \right] \\ & = \bar{\tau} \int_{t-\bar{\tau}}^t \|u(\theta)\|^2 d\theta,\end{aligned}$$

(34) can be rewritten as

$$\begin{aligned}\dot{V} \leq & - \left(K_{min} - \frac{(\rho_1(\|z\|) + \rho_2(\|z\|))^2}{2\zeta} \right) \|z\|^2 - S^T \left(\beta - \vartheta - \frac{\bar{\tau} \zeta_f}{\beta} \right) S \\ & - k_o y^T y + \frac{(\epsilon_1 + \epsilon_2)^2}{2\zeta} + 2\varepsilon - \frac{1}{2\bar{\tau}} P,\end{aligned}\tag{35}$$

Using the definitions of $x(t)$ and $z(t)$ this expression can be upper bounded as

$$\dot{V} \leq -\bar{K}_{min} \|x\|^2 - \left(K_{min} - \frac{(\rho_1(\|z\|) + \rho_2(\|z\|))^2}{2\zeta} \right) \|e_I\|^2 + \frac{(\epsilon_1 + \epsilon_2)^2}{2\gamma} + 2\varepsilon, \quad (36)$$

where $\bar{K}_{min}$ is defined as

$$\bar{K}_{min} = \min \left\{ K_{min} - \frac{(\rho_1(\|z\|) + \rho_2(\|z\|))^2}{2\zeta}, \beta - \vartheta - \frac{\bar{\tau}\zeta_f}{\beta}, k_o, \frac{1}{2\bar{\tau}} \right\}.$$

This expression can be further bounded if $K_{min} - \frac{(\rho_1(\|z\|) + \rho_2(\|z\|))^2}{2\zeta} > 0$, which is true if the condition $\|z\|^2 < \bar{\rho}^{-2}(\sqrt{2K_{min}\zeta})$ is satisfied, where $\bar{\rho}$ is a positive monotonically increasing bounded function defined as $\bar{\rho} = \rho_1(\|z\|) + \rho_2(\|z\|)$. Considering (32), the definitions of x and z , and $P > \frac{\zeta_f}{\bar{\tau}\beta^2}e_I^2$, a set $\mathbb{S}$ defined as

$$\mathbb{S} \triangleq \left\{ \begin{aligned} &x(t) \in \mathbb{R}^{8n+1} \mid \|x(0)\| \\ &< \sqrt{\frac{\lambda_1}{\lambda_2} \left(\min \left\{ 1, \frac{\zeta_f}{\bar{\tau}\beta^2} \right\} \bar{\rho}^{-2} \left(\sqrt{2\zeta K_{min}} \right) - \frac{\Upsilon}{\lambda_1} \right) - \frac{B\lambda_2}{\delta\lambda_1}} \end{aligned} \right\},$$

where $B \in \mathbb{R}^+$ is a subsequently defined constant. In $\mathbb{S}$, $\bar{K}_{min}(\|z\|)$ is bounded by a constant $\delta \in \mathbb{R}^+$ as

$$\bar{K}_{min}(\|z\|) \geq \delta.$$

Adding and subtracting $\frac{\delta}{\lambda_2}\Upsilon$ to (36) and using (32), (36) becomes

$$\dot{V} \leq -\frac{\delta}{\lambda_2}V + B, \quad (37)$$

where $B = \frac{\delta}{\lambda_2}\Upsilon + \frac{(\epsilon_1 + \epsilon_2)^2}{2\zeta} + 2\varepsilon$. (37) can be integrated with respect to time to obtain

$$V(x, t) \leq V(0)e^{-\frac{\delta}{\lambda_2}t} + \frac{B\lambda_2}{\delta} \left(1 - e^{-\frac{\delta}{\lambda_2}t} \right). \quad (38)$$

From (38) it is evident that $V(x, t)$ decays exponentially to a bound $\frac{B\lambda_2}{\delta}$ which can be minimized using the control gains. Further analysis can be done to show that the $\|x\|$ decays to the ball of radius $\sqrt{\frac{B\lambda_2}{\delta\lambda_1}}$. Therefore, it can be concluded that $V \in \mathcal{L}_\infty$ and the states $e, r \in \mathcal{L}_\infty$. By Theorem 4.18 in [61], it can be concluded that the origin of x is semi-global uniformly ultimately bounded (SGUUB). $\blacksquare$

Figure 2: A schematic of the fixed hip model with no ground model. The pink muscle bellies labeled u_{he} , u_{hf} , u_{ke} , and u_{kf} indicate the stimulated muscles that produce hip/knee flexion and extension and the torques produced by the motors at both joints are labeled T_{hm} and T_{km} .

6. Simulations

The efficacy of the newly developed controller was tested in a simulation study on a 2-DOF fixed hip model of a leg. The fixed hip model represents the movement of the lower extremities while the hip is fixed. In this model, only the hip and knee joints are actuated (i.e., the ankle joint is fixed) and only motion in the sagittal plane is considered. This model considers actuation of the antagonistic muscle pairs via FES and an electric motor at each joint, i.e., 6 overall inputs for a 2-DOF system. A schematic of the fixed hip model can be seen in Fig. 2.

The musculoskeletal model and parameters used in the simulations were taken from [58] for a person with SCI. Optimizations were conducted to compute the optimal inputs that track normal gait data taken from [62] for one complete gait cycle. For the optimizations, the convex cost function's objective was to minimize the position and velocity error, and minimize the activations of the system and is defined as

$$\min_u \Pi = \int_{t_0}^{t_f} (e(t)^T Q_1 e(t) + \dot{e}(t)^T Q_2 \dot{e}(t) + \mu(t)^T R \mu(t)) dt \quad (39)$$

$$\begin{aligned} \text{subject to: } & M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + G(q) + f(q, \dot{q}) = b(q, \dot{q}) \mu \\ & \mu \in [\mu_l, \mu_u] \end{aligned}$$

Figure 3: Optimal activations generated from dynamic optimizations. The peak in the hip flexion and knee extension are intentionally computed by the dynamic optimizations to counteract the passive torques near hyper flexion and hyper extension.

where $Q_1 \in \mathbb{R}^{2 \times 2}$ is a weight on the position tracking error, $Q_2 \in \mathbb{R}^{2 \times 2}$ is the weight on the velocity tracking error, the matrix $R \in \mathbb{R}^{6 \times 6}$ is a positive-definite matrix of weights on the activation vector, and the lower and upper bound on the activations are defined as μ_l and $\mu_u \in \mathbb{R}^6$. The optimal activations computed through the optimization to reproduce the gait data can be seen in Fig. 3. For more details on the dynamic optimization used to compute optimal activations, see [57].

The synergies, W , and their activation, c_d , were then extracted from the optimal activations using PCA. The synergies that account for little variance in the reconstructed activation data were truncated to decrease the dimensionality of the feedforward component. For this simulation study, the dimensions were reduced from 6 to 3 variables while still accounting for about 94% of the variance as can be seen in Fig. 4. The reduced 3 synergies and their activations are shown in Fig. 5.

Figure 4: Number of synergies vs. Reconstruction variance

Figure 5: (a) Three synergies: w_1 , w_2 , and w_3 after dimension reduction. (b) The corresponding time-varying synergy activation c_1 , c_2 , c_3 , of the three synergies.

After the synergies were extracted from the optimal activations and reduced dimensionally, the controller in (30) was simulated for 5 minutes to demonstrate the effect of the fatigue estimate on the controller. The fatigue and activation dynamics are modeled as a first order differential equations [49, 59]. To illustrate the effect of the fatigue, the plots in Fig. 6-9 depict the simulation results for the first 5 seconds, where there is little fatigue, and for the last five seconds, where the muscles are more fatigued. The tracking performance of the controller can

Figure 6: The control performance of the joint angles resulting from simulating the newly developed controller. Top plots the desired and actual hip joint angle. Bottom plots the desired and actual knee joint angle, each for the first and last five seconds of the 5 min. simulation.

be seen in Fig. 6. The root mean squared errors for the hip and knee joints were found to be 2.91° and 2.82° , respectively. There are multiple components in this controller whose roles can be seen in the control results. The effects of the adaptive synergy-based feedforward component, $\hat{\phi}^{-1}\zeta_{sf}W\hat{c}$, in $\bar{\mu}$ can be seen in Fig. 7. The effect of the fatigue estimate becomes apparent when comparing the normalized muscle activations from the first and last five seconds of the results. By design, the fatigue estimate does not affect the normalized motor currents as motors do not fatigue. However, from Fig. 7, a slight increase in amplitude (motor) is still apparent. This is due to the adaptation of the synergy activation coefficients. The feedback component, kr , in $\bar{\mu}$ was only applied to the motors and is shown in Fig. 8. After applying the DSC structure, the actual input signals can be seen in Fig. 9. The estimate of the fatigue can be seen in Fig. 10 where it can be seen that the knee extensors and hip flexors fatigued the most, which means that their input is scaled the most due to the $\hat{\phi}^{-1}$ term; this is evident in Fig. 7 & 9.

7. Experimental Results

Experiments were conducted on a single leg of an able bodied subject (Male; Age: 24 years), on two testbeds which can be seen in Fig. 11. Prior to any experimentation, an approval from the Institutional Review Board at the University of Pittsburgh was obtained. The participant was instructed to relax and avoid any voluntary interference during the electrical stimulation. The real-time implementation of the controller requires subject specific parameters for the state estimators. These parameters and the minimum/maximum stimulation levels

Figure 7: Feedforward component, $\hat{\phi}^{-1}\zeta_{sf}W\hat{c}$ in $\bar{\mu}$, reconstructed through the reduced synergies after adaptation and with the scaling up from the fatigue estimates. The first and last 5 seconds of the signals are shown to illustrate the increases in the signal amplitude as the muscles fatigue.

Figure 8: The first and last 5 seconds of the feedback component, kr in $\bar{\mu}$, which was only applied to the two motors.

Figure 9: The normalized inputs to the system, u . The first and last 5 seconds are shown for the stimulation inputs to show the effect of the scaling due to the fatigue estimate.

Figure 10: The normalized fatigue variable for the four muscles, hip and knee flexors/extensors, over the 5 minute trial. The fatigue variable ranges from zero to one, which correspond to full fatigue and no fatigue, respectively.

Figure 11: The testbeds used to conduct the experiments: (a) a hybrid exoskeleton with electric motors at the hip and knee joint and stimulation of quadriceps and hamstrings was used to implement the new controller and (b) a modified leg extension machine with a load cell to measure force was used to perform system identification experiments.

for each muscle were empirically identified through system identification experiments. These experiments were conducted on a leg extension machine (LEM) modified to hold the subjects leg in an isometric configuration and instrumented with a load cell (Omega, USA) to measure the torque generated through FES elicited contractions.

The system identification process consisted of three sets of experiments for each muscle. During these experiments the subject is seated in the LEM with their shank strapped to the load cell in an isometric configuration (position is constant). In the first experiment, the muscle is stimulated with a stimulation ramp while the torque produced is measured. The purpose of this experiment was to determine the $v_{\max}$ and $v_{\min}$ parameters in (30). In the second set, the muscles are stimulated at $v_{\max}$ for a few seconds to elicit a step response. From the normalized torque data, the EMD and time constant for the first order activation dynamics were determined. This same procedure was used to determine the activation time constant for the motor which was mounted on the LEM. In the final set of experiments, the subject was stimulated at $v_{\max}$ for two minutes to fatigue the muscles followed by one second pulses every ten seconds to capture the recovery rate of the muscles. From the normalized torque measurements, the fatigue and recovery rates for each muscle can be determined from the two data sets. These parameters are then used in the estimators in (8) and (9) during real-time implementation of the controller. These identified parameters can be seen in Table 1.

Before the controller can be implemented, the optimal activations and synergies must be computed offline. For the dynamics optimizations, parameters for the passive dynamics and torque-length/torque-velocity relationships were taken from data reported for healthy subjects in [58]. Any mismatch or model error this would cause is considered as additional disturbances in the system

Parameter	Quadriceps	Hamstring	Motor
$v_{\min}$	15 mA	20 mA	-10 volts
$v_{\max}$	45 mA	50 mA	10 volts
$\hat{w}$	.085	.096	.062
τ	111 ms	130 ms	-
$\hat{T}_r$	29.02 s	33.24 s	-
$\hat{T}_f$	33.01 s	45.62 s	-
$\hat{\phi}_{\min}$	0.051	0.404	-

Table 1: The subject specific parameters and motor parameters extracted in the system identification experiments. Notice that the motors do not have any fatigue parameters or input delays.

and in u_{loss} . The cost function expressed in (39) was modified for a 2-DOF system with 4 inputs, thus the parameters are redefined as $R \in \mathbb{R}^{4 \times 4}$ and μ_l , and $\mu_u \in \mathbb{R}^4$. The swing phase gait trajectories from [62] were used, except that the trajectories were slowed down to 4 sec. step cycle instead of a 1 sec. step cycle. This was done because the electric motors of the exoskeleton are not able to generate the joint torque required at faster speeds. The amount of signals needed in the feedforward component were reduced by 50% by using 2 synergies. These 2 synergies and their desired activation profiles can be seen in Fig. 12.

Figure 12: The synergies used in the preliminary experiments: (a) Two synergies: w_1 and w_2 after dimension reduction. (b) The corresponding time-varying synergy activation profiles c_1 and c_2 .

The newly developed controller was then tested on a hybrid exoskeleton consisting of electric motors and FES of multiple muscles. The electric motors (Harmonic Drive LLC, MA, USA) were located at the hip joint and knee joint and can produce a maximum torque of 40 Nm. Although the simulations consider

stimulation of the flexors and extensors at both joints, the use of transcutaneous electrical stimulation to produce hip movements is not feasible as the muscles are located deep and can be difficult to stimulate through surface electrodes. Therefore only stimulation of the knee joint flexors and extensors was used in the experimental trials. A RehaStim 8-channel stimulator [Hasomed Inc., DE] was used to generate the current modulated biphasic pulse trains transmitted to the transcutaneous electrodes at 35 Hz. The QPIDE [Quanser Inc, Ontario Canada] DAQ board was used to interface with the motor drivers and run the controller in real-time at 1 kHz. The system identification and control algorithms were coded in Simulink [MathWorks Inc, USA] and implemented using the Quarc real-time software [Quanser Inc, Ontario Canada]. The full controller was tested in a 20 sec. long trial. These experiments were conducted on another day after the first set of experiments to allow the subjects muscle to fully recover from the fatigue trials in the system identification experiments.

The tracking performance from the experimental trial can be seen in Fig. 13. The root mean squared errors for the hip and knee joints were found to be 0.31° and 0.18° , respectively. The feedforward component, $\hat{\phi}^{-1}\zeta_{sf}W\hat{c}$, and feedback component, kr , in $\bar{\mu}$ can be seen in Figs. 14 & 15. As seen in Fig. 18 the effect of the fatigue estimate is not as apparent in the experiments. This is because the fatigue parameters identified for the subject were very slow resulting in the muscles barely fatiguing. However, for SCI subjects muscle fatigue occurs more rapidly, hence this is still a practical feature in the controller. Another important aspect of the controller is the βS term in (27), which includes the delay compensation term and serves to minimize the error between the desired activation and the activation estimate. Therefore, the DSC structure determines the control input which minimizes this error, as seen in Fig. 16. After applying the DSC structure, the actual input signals can be seen in Fig. 17.

8. Discussion

Hybrid exoskeletons that combine electric motors and FES may have a potential to induce greater walking durations than the systems that strictly use electric motors or FES; in addition to providing a light weight walking restoration technology. However, the cooperative use of electric motor and FES in such hybrid devices results in an actuator redundancy. In this paper an adaptive synergy-based dynamic surface controller that can compensate for actuator dynamics, muscle fatigue dynamics, and input delays was developed and tested. The controller solves the control allocation problem by using optimized feedforward components which are dimensionally reduced through synergies. The optimized feedforward signals were computed from control signals resulting from a dynamic optimization that determined how to allocate control between the electric motors and FES. Feedback control to only the motors was included to improve robustness and performance. The newly developed controller was validated through simulations on a fixed hip model of the leg, and through experiments on an able-bodied person in a hybrid exoskeleton device.

Figure 13: The joint angles resulting from the newly developed controller in experiments. Top plots the desired and actual hip joint angle. Bottom plots the desired and actual knee joint angle, each for five cycles.

Figure 14: The feedforward component of $\bar{\mu}$ which is reconstructed through the synergies after adaptation and with the scaling up from the fatigue estimate.

Figure 15: The feedback component of $\bar{\mu}$, which were only applied to the motors.

Figure 16: A comparison between the desired activation $\bar{\mu}$ and the estimated activation $\hat{\mu}$.

Figure 17: The inputs for the motors and stimulation during the experiments. Note that the stimulation levels start at the minimum value which differs for each muscle.

Figure 18: The normalized fatigue variable estimate for the knee flexors and extensors. The fatigue variable ranges from zero to one, which correspond to full fatigue and no fatigue, respectively.

Hybrid exoskeletons come in many configurations including any combination of FES, active actuators such as electric motors, or passive actuators such as controlled brakes and wrap spring clutches [10, 11, 63, 64]. This newly developed controller employs a control structure in which simultaneous control of electric motors as well as FES is used instead of fine control of FES using controlled brakes. The latter approach, which is used in [22], uses open-loop control of stimulation to provide the active torque and controlled brakes are used like a dynamic stopper to restrict a single joint to a desired joint angle. In [12], this method was extended to a full orthosis with FES and controlled brakes at both the hip and knee joints of each leg. The method was tested on four paraplegic subjects. The disadvantage of this method is that the antagonistic muscles are stimulated at the maximum stimulation to generate a raw joint torque, which is fine tuned through the controlled brakes. This results in massive over stimulation of the muscles leading to the rapid onset of muscle fatigue. Later in [10], a hybrid neuroprosthesis with 16 intramuscular stimulation channels combined with a locking orthosis was tested on able-bodied and SCI subjects. The hybrid neuroprosthesis was controlled with an FSM with predetermined open-loop control signals for FES for multiple tasks such as sitting, standing, walking, and ascending and descending stairs. Unlike the aforementioned papers, in [27] a combination of FES and active actuators (electric motors) were used to control knee extensions using an adaptive gain-based controller to allocate the control effort. In [21] and [26] a cooperative controller for hybrid knee-ankle-foot exoskeletons were tested on human subjects. The controllers used an FSM that coordinated the PD controller for the electric motors and an iterative learning controller (ILC) for FES. In [21], the FSM also uses a fatigue estimator to detect the onset of fatigue and modify the stimulation parameters. Although the aforementioned control approaches are interesting, but the methods did not consider control of multiple muscle stimulation and multiple electric drives applied to multiple DOFs. Further, these control methods do not follow a formal control development and do not provide stability guarantees.

The control approach proposed in our paper follows a systematic control design approach for hybrid walking exoskeletons. The approach is inspired from the muscle synergy principle. The studies proposing muscle synergy as a basis of human motor control employ an approach of recording EMG signals (collected from multiple muscles) and then decompose these signals to muscle synergies (lower dimensions). Unlike these studies, our focus is on the design of synergies for stimulating multiple muscles and actuating electric motors in a hybrid exoskeleton. For the latter design approach, using optimized control signals offers multiple advantages and convenience. The optimization framework allows to incorporate external inputs to the exoskeleton and artificial stimulation inputs for FES at the same time. The traditional synergy studies investigate natural modes of muscle recruitment, not electric motor inputs as with a powered exoskeleton. Therefore, the muscle synergies extracted from EMG recordings cannot be mapped easily, and also optimally, to design artificial inputs for FES and electrical motors (extra redundancy). In addition, the optimization framework can be used to design or plan a new joint angle trajectory that can be

specific to a subject with a neurological impairment. The traditional synergy will not allow this as some of these subjects may not have voluntary movements, especially for persons with complete SCI (i.e., no EMG recordings). In our previous work [57], we have pointed out that the dynamic optimizations can be used to design subject-specific optimal control inputs for a hybrid orthosis. These optimal control inputs, which are redundant, can be then transformed to a lower dimensional control by using methodology inspired from the traditional muscle synergy extraction approach.

Optimizations are used to compute the optimal activations to ensure that the redundant actuators are not counteracting each other, i.e. the motors and FES should not be working against each other. The relative amount of stimulation and motor effort can be adjusted through two aspects of the control scheme: 1) via the cost function of the optimization or 2) via the scaling gain in the feedforward loop in (22). The optimal stimulation activation profiles, used in the feedforward component of the controller, are computed based on the model and are limited by modeling errors and parameter mismatch. We used majority of the model parameters from [58] to compute the synergy-based feedforward component during the experimental validation on the able-bodied subject. This parameter mismatch may have decreased the effectiveness or contribution of the feedforward component. However, it did not affect the control performance, which depicts the robustness of the overall control scheme. We also suspect that the feedforward stimulation inputs from the controller may not generate enough torque to overcome the impedance due to electric motors. The current work ignored the impedance in the motors during the optimizations. We will address these limitations and the following limitations in our future work.

The new controller can be implemented on any type of trajectory, therefore simpler trajectories can be used in accordance with this controller. We opted to use trajectories from normal gait studies [62]. One can use the feedforward controller for a simple set of trajectories and the feedback controller to reject any disturbances that may result during gait; e.g., upper body forces during walking. However, the current study was focused on the control design for only the swing phase. In our future work, numerous improvements will be made to extend the controller to full gait cycle. These modifications include adding impedance control and accounting for upper-body effort. The preliminary experimental results that validated the new controller on an able-bodied subject are promising; however, the clinical efficacy of the control design remains to be seen. Our future efforts includes verifying the controller on a person with complete SCI and developing control methods that incorporate voluntary effort; e.g. extending the control designs to persons with stroke and incomplete SCI.

Precise measurement or accurate estimate of fatigue and activation dynamics is still an open research problem. EMG based fatigue estimators are not feasible, especially when the EMG signals are masked by artifacts due to surface stimulation. The use of force sensors to estimate fatigue would also prove challenging as there are multiple interaction forces which would alter the signals and have to be used in an isometric configuration to measure fatigue. As an alternative, the control development and stability analysis uses model based

estimates of the activation and fatigue as opposed to an assumption that they are measurable. These estimates are computed on-line using a model based estimators that use the equations presented in the dynamic model section. In addition, the controller has been implemented experimentally in real-time on an able bodied subject. System identification experiments were conducted to determine the parameters needed for the estimators. Nonetheless, the control development provides a systematic way to integrate fatigue measurements or fatigue estimates in the control design. The controller, or the control development can be easily replaced with an accurate model of fatigue or a measurable fatigue signal, if real-time measurements of fatigue or improved mathematical descriptions of FES-induced muscle fatigue become feasible in the future.

9. Conclusion

In this paper, an adaptive synergy-based low dimensional controller is presented for a lower-limb hybrid neuroprosthesis comprising of an FES system and a powered exoskeleton. The new controller, which was inspired from the muscle synergy principle, was designed to account for redundant actuation structure. Further, the control design also accounted for activation and fatigue dynamics and EMD due to FES. Model based estimators were used to estimate the actuator activation and the fatigue signals. A Lyapunov-based stability analysis of the new controller yielded SGUUB tracking, despite unmodeled disturbances. The efficacy of the controller was tested in simulations and an experiment. The experiment was conducted on an able-bodied subject to track a swing-phase gait trajectory. The controller showed ability to coordinate FES of the quadriceps and hamstrings muscles and electric motors at the hip joint and knee joint of the exoskeleton. The controller also showed robustness to modeling errors. Future research efforts will be to extend the developed controller to a full gait cycle control and verifying the control design on a person with neurological impairment.

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