An improved collaborative optimization for multidisciplinary problems with coupled design variables

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Abstract: In this paper, the collaborative optimization is improved to tackle multidisciplinary problems with coupling design variables within disciplines. The design variables are first distinctly classified into private and public ones according to disciplines. A criterion is then proposed to judge the coupling of private variables with the public variables. For the coupling case, a strategy is suggested to decouple the variables before performing the collaborative optimization. To demonstrate the effectiveness of the improved collaborative optimization, two practical optimization problems are comparatively studied for uncoupled design variables as well as coupled ones. It is shown that the present framework can achieve the better optimal results as compared with other methods. Furthermore, the judge criterion and the decoupling strategy are also validated.

Keywords: Collaborative optimization; Public variables; Private variables; Coupling criterion; Decoupling strategy

1. Introduction

In engineering, a practical optimization problem often involves various disciplines. For instance, the design of aircraft involves disciplines such as aerodynamics, structure analysis, propulsion, and control ^[1-5], and hence the issue of multidisciplinary design optimization (MDO) was put forward. In the early design stage, the optimization is individually carried out for each discipline. However, in the modification process, an optimal decision can only be made by coordinating all the disciplines.

To tackle a MDO problem, two kinds of methods have been proposed, i.e. the monolithic one and the distributed one. For the monolithic kind, a MDO problem is recast as a single optimization problem. Simultaneous Analysis and Design ^[6,7], Individual Discipline Feasible^[8] and Multidisciplinary Feasible^[8,9] are attributed to this kind. In the distributed kind, a MDO problem is however decomposed into a set of smaller disciplinary optimization problems. Collaborative

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Optimization (CO) ^[1,4,10], Concurrent Subspace Optimization^[11,12], Bi-level Integrated System Synthesis^[13], and Analytical Target Cascading^[14,15] are attributed to this kind.

As the method of distributed kind, the idea of CO is to mirror the natural divisions with different disciplines in design companies. Disciplinary analysis tools tend to be complex in nature, and it is often impractical to integrate multiple analysis codes for the purpose of MDO. Instead, the CO offers a means of coordinating separate analyses. As a result, relative to other methods of distributed kind, the CO provides the disciplinary subspaces with an unusually high level of autonomy, which enhances the ability to independently make design decisions pertinent to the corresponding discipline^[16]. On this basis, we focus on the CO in this paper.

Among the disciplines to be optimized, coupling often appears for design variables. In most of cases in the CO, coupling variables will be recognized as the public variables on the system level and hence is eventually tackled by limiting the respective design domains. An exceptional situation is that coupling variables are recognized as private ones as well for certain disciplines, leading to coupling between public variables and private ones. Nevertheless, the CO works still well for pure parametric optimizations (e.g. see [17, 18]) because the coupling variables can be easily decoupled by limiting their design domains.

However, for the combination of parametric and parameter-free design variables^[19], coupling of variables within disciplines is no more evident and difficult to decompose, the CO is not applicable in a direct manner for this type of MDO problems. In this case, the sequential optimization (SO) is often used, which first solves parameter-free optimization problems and then solves parametric optimization problems, so that the disciplines with parameter-free variables and the ones with parametric variables are decoupled by separate optimization. But the results are not optimal because the design space is apparently reduced in this one-by-one optimization process. Though this problem can be decoupled by an empirical decouple approach in mechanical science and engineering^[22], it is necessary to study the coupling criterion and the decoupling strategy for general MDO problems.

This paper attempts to solve MDO problems with variables uncoupled or coupled within disciplines by improving the CO. To this end, the paper is outlined as follows. In Section 2, the conventional CO (CCO) is re-formulated to be a two-level framework by classifying design variables into public variable and private variable. In Section 3, a criterion is presented to judge the coupling of variables in disciplines, and then a decoupling strategy is suggested. Two practical optimization problems are conducted to illustrate the present approach in Section 4. The concluding remarks are finally made in Section 5 to close this paper.

2. Re-formulation of the conventional collaborative optimization

Mathematically, an optimization problem involving several disciplines is expressed as

$$\begin{cases} \text{Find } \mathbf{v}, \text{ such that} \\ \min F(\mathbf{v}) \\ \text{s.t. } c_n(\mathbf{v}) \leq 0 \end{cases} \quad (1)$$

where F is the overall objective and c_n denotes the n constraints. $\mathbf{v}$ is the design variable and assumed to be vectorial in this context.

In Problem (1), without loss of generality, the objective F is assumed to be the weighted sum of f_i for discipline i . In the light of disciplines, design variables $\mathbf{v}$ is split to two kinds, i.e. private variables $\mathbf{x}_i$ which belong to the individual discipline i , and public variables $\mathbf{z}$ which belong to one more disciplines^[23,24]. Accordingly, constraints $c_n(\mathbf{v})$ are also classified into two kinds, i.e. private constraints which are relevant to private variables $\mathbf{x}_i$ for discipline i as well as the public variables $\mathbf{z}$. The public constraints only relevant to public variables $\mathbf{z}$. Thus, the MDO problem (1) is re-expressed as

$$\begin{cases} \text{Find } \mathbf{z} \text{ and } \mathbf{x}_i, \text{ such that} \\ \min F(\mathbf{z}, \mathbf{x}_i) = \sum_j f_j(\mathbf{z}, \mathbf{x}_j) \\ \text{s.t. } c_j(\mathbf{z}, \mathbf{x}_j) \leq 0 \end{cases} \quad (2)$$

Fig. 1 The framework of the CCO

Based on the CO in [1,4,10] and the enhanced collaborative optimization in [16,25], which are generally referred to as the conventional collaborative optimization (CCO) in this context, Problem (2) is decomposed into two levels, i.e. the system level and the discipline level as shown in Fig. 1, in which, for convenience of explanation, only two disciplines are considered. The framework in Fig. 1 is summarized as Procedure (3).

- Step-0: Initialize public variables $\mathbf{z}$;

Step-1: Find private variables $\mathbf{x}_i$ and temporary public variables $\mathbf{y}_i$ in discipline i by conducting each i -th disciplinary optimization ($i = 1 \sim 2$ in case of Fig. 1) on the discipline level;

Step-2: Find the public variables $\mathbf{z}$ by conducting the whole optimization on the system level;

Step-3: Judge if the optimization converges via the system objective F . If yes, the optimization stops; if not, go to step-0 after setting $\mathbf{z}$ obtained in Step-2 to be the initial value of public variables.
- (3)

It should be noted that the two variables in round brackets are all design variables if they are separated by a comma, while the former are design variables and the latter are parameters if they are separated by a semicolon.

It is seen that a collaborative optimization will be achieved in Step-2 by considering the interaction of the temporary public variables with the public variables. It is not difficult to find that with Procedure (3), optimal problems containing several disciplines such as those in Refs [1-4,26,27] can be readily tackled.

3. The collaborative optimization with coupling design variables

In the CCO shown in Fig. 1, coupling among private variables does not occur because they belong to different disciplines. However, from the mathematical point of view, private variables may couple with public variables within disciplines. As will be shown in 4.2.2, the CCO gives a wrong optimal result for this case. To ensure the CCO to work well, it is necessary to first judge coupling between variables and then decouple the variables before the CCO is performed. To this end, the framework of the CCO is improved.

3.1 The criterion to judge a coupling

Let X be a private variable and Y a public variable within a discipline. Assume Ω_X and Ω_Y be respectively the normed spaces of same kind determined by variable X or Y with the distance of same kind defined as

$$d(a, b) = |a - b| \quad (4)$$

for any two points a and b in Ω_X or Ω_Y respectively, in which $|\cdot|$ is the norm operator. Accordingly, we define the distance of two spaces Ω_X and Ω_Y as

$$d_{X-Y} = \min_{\substack{a_X \in \Omega_X \\ b_Y \in \Omega_Y}} d(a_X, b_Y) \quad (5)$$

With Equations (4) and (5), the criterion is suggested that if the condition

$$d_{X-Y} \leq \varepsilon \quad (6)$$

is satisfied for a given small positive number ε , two variables X or Y are referred to as coupling.

To help understand the criterion, an example shown in Fig. 2 is examined with the two design variables, i.e. private variable X , a parameter-free variable for a topology optimization, and public variable Y , a parametric variable denoting the inner radius of a hole. The two normed spaces Ω_X and Ω_Y are determined respectively by the design variables X and Y are shown in Fig. 2 in grey. It is clear that, the domains of Ω_X and Ω_Y are the same kind of spaces over which the distance can be defined.

Since the shaded region in Fig. 2 is the common region of Ω_X and Ω_Y , we have $d_{X-Y} = 0$ from Equation (5) for this case, and thus Inequality (6) is satisfied for any positive number ε . So, Variables X and Y are coupling.

It should be noted that, if the private variable X is parametric, Ω_X will be also regular like Ω_Y in Fig. 2, and hence the criterion is readily applied.

Fig. 2 An example of two possible coupled variables

3.2 Decoupling of variables

It is seen from Inequality (6) that the key parameter in judging the coupling of two variables X and Y is the distance of two spaces (i.e. Ω_X and Ω_Y). In other words, X and Y will be decoupled if Inequality (6) is no more obeyed by changing Ω_X and/or Ω_Y to some extent.

Noticing the fact that X is private variable while Y is public variable, for the purpose to influence the design space in the system level as little as possible, we stipulate the strategy that Ω_X is only shrunk by inserting an additional constraint so that Inequality (6) violates for a given ε as shown in Fig. 3 as compared with that in Fig. 2.

Fig. 3 Illustration of the decoupling strategy

3.3 An improved collaborative optimization with coupling design variables

In this section, an improved collaborative optimization (ICO) will be established for a general MDO problem. In contrast to the CCO shown in Fig. 1, in the ICO the private variables are first judged if they are coupled with the public variables within disciplines in Step-1 in Procedure (3). For uncoupled disciplines (e.g. D1 in Fig. 4), the disciplinary optimization is performed as in Fig. 1. For coupled disciplines (e.g. D2 in Fig. 4), the disciplinary optimization is performed by imposing the additional constraint (e.g. $d_{y_2-x_2} \geq \varepsilon$ for D2 in Fig. 4) for all or part of coupled variables against Inequality (6). So, the framework of the CCO is changed to be the one of the ICO shown in Fig. 4.

Fig. 4 The framework of the ICO

4. Illustrative examples

To validate the ICO, two practical examples are examined. Actually, Example 1 is to optimize

an elastic support system in which both the two disciplines are uncoupled while Example 2 is to optimize an anti-roll torsion bar system in which one of the three disciplines is coupled.

4.1 Optimization of an elastic support system

Fig. 5 Sketch of the elastic support system

The elastic support system in an engine serves as an isolator between the engine and the chassis to reduce the influence of the engine on the chassis by suppressing the unbalanced inertial force. The geometry of the elastic support system is shown in Fig. 5, which consists of two components, i.e. the carrier and the cushion block. Under the working condition, the carrier is subjected to axial and radial excitations, leading to a complex stress state for the cushion block.

For the carrier, the Young's modulus and Poisson's ratio are 210GPa and 0.3 respectively. For the cushion block, the material is assumed to be rubber-like described by the Mooney-Rivlin hyper-elastic model ^[28, 29]. Under 1750 rpm for the engine, the excitation frequency is 29.16 Hz.

4.1.1 Statement of the optimization problem

In close accordance with the framework of the ICO in Fig. 4, the physical significances corresponding to the concepts and notations for this problem are summarized in Table 1. We can see that the involved two disciplines are Carrier (D1) and Cushion block (D2), respectively. The objective for D1 is to minimize the maximum shear stress while the objective for D2 is to minimize the vibration isolation coefficient.

In this problem, the average width is $\bar{W} = (W_s + W_c)/2$. The constraints of D1 include that within [204, 276] (in KN/m) for the axial stiffness and that within [204, 276] (in KN/m) for the radial stiffness. The constraint of D2 is within [700, 1000] (in Hz) for the fundamental frequency. On the discipline level, the sequential quadratic programming is used as the optimization algorithm while the simulated anneal algorithm is used on the system level. w_1 (w_2) and s_1 (s_2) are respectively the weighting factor for the relative importance of discipline and the scaling factor

for dimensional homogeneity for D1 (D2). $w_1 = w_2 = 0.5$, $s_1 = 100$ and $s_2 = 1$ are used in this problem.

Table 1 Corresponding notations in the elastic support system (Example 1)

Notations in the ICO	Notations for the elastic support system	Physical significance
System	Carrier and cushion block	Minimize the complex objective $f = \frac{w_1}{s_1}f_1 + \frac{w_2}{s_2}f_2$
D1	Carrier	Structural analysis discipline
D2	Cushion block	Vibration analysis discipline
f_1	$\tau_{\max}$	Maximum shear stress of the carrier in MPa
f_2	η	Vibration isolation coefficient of the cushion block
Public variable: z_1	$\bar{W}$	Average width of the Carrier and Cushion block
Private variables for D1: x_1	θ	Support angle
Private variables for D2: x_2	Q , design domain (see Fig. 5)	Shape of the cushion block

4.1.2 Results and discussion

Since the minimum distance between the related spaces is as large as 5mm for the two disciplines D1 and D2, the variables are uncoupled after the judge by using Inequality (6) as long as $\varepsilon < 5\text{mm}$, and therefore the ICO is performed without any additional constraint. The ICO converges after 127 iterations. The public variable $\bar{W}$ and the private variable θ for D1 are obtained as listed in Table 2 as compared with those from the SO, while the designed domain Q for D2 is obtained as shown in Fig. 6. The optimal objectives are listed in Table 3.

Table 2 The design variables after optimization with different methods for Example 1

Variable type	Design variable	Range	SO*	Present ICO
Public	$\bar{W}$ (mm)	[15, 20]	20	19.02
Private for D1	θ (°)	[40, 60]	43.17	49.12

*The results are calculated by authors according to the SO.

Table 2 and Fig. 6 show that the optimal design variables from the ICO are quite different from those from the SO. From Table 3, we see that, with the optimal design variables, the vibration isolation coefficient η from the ICO is much better than that from the SO, leading to a reduction of the total objective f by 40.2% as compared with the initial value.

Fig. 6 Shape of the elastic support system (i.e. Q)

(a) Initial shape; (b) Optimal shape from the SO; (c) Optimal shape from the ICO

Table 3 Optimal objectives after optimization with different methods for Example 1

Level	Objectives	Initial value	Optimal value	
			SO	ICO
D1 on the discipline level	Minimize f_1	33.70	23.72	20.32
D2 on the discipline level	Minimize f_2	0.325	0.231	0.193
The system level	Minimize f	0.662	0.468 (-29.3%)	0.396 (-40.2%)

4.2 Anti-roll torsion bar problem

The second example is a MDO problem for a torsion bar system.

4.2.1 Statement of the problem

Fig. 7 Sketch of the torsion bar system

As shown in Fig. 7, the anti-roll torsion bar system for a high speed rail consists of torsion bar, reverse arm, gasket and other components. With rolling of the vehicle body, the torsion bar bears a torque, and meanwhile provides a reverse torque to resist the body to roll. The Young's modulus and Poisson's ratio are, respectively, 210GPa and 0.3 for both the bar and arm materials. The gasket material is assumed to be rubber described by the Mooney-Rivlin hyper-elastic material model, the material parameters of C_{10} , C_{01} and D_1 are 0.95, 0.23 and 0.0001

respectively. The three disciplines are the torsion bar (D1) for topology optimization, the gasket (D2) and the reverse arm (D3) for size optimization, and the optimal objective is the total weight since this topic is invariable for the whole vehicle design. Thus, this is a system optimization with three different disciplines.

Table 4 Corresponding notations in the torsion bar system (Example 2)

Notations in the ICO	Notations in Example 2	Physical significance
System	Bar, arm and gasket	Minimize the total weight $f = \frac{w_1}{s_1} f_1 + \frac{w_2}{s_2} f_2 + \frac{w_3}{s_3} f_3$
D1	Bar	Fatigue analysis
D2	Gasket	Vibration analysis
D3	Arm	Structural analysis
f_1	W_1	Weight of the bar
f_2	W_2	Weight of the gasket
f_3	W_3	Weight of the arm
Public variable z_1	R	Outer radius of the bar and inner radius of the arm
Private variable for D1: x_1	r	Inner radius of bar
Private variable for D2: x_{21}	R_g	Outer radius of gasket
Private variable for D2: x_{22}	W_g	Width of gasket
Private variable for D3: x_3	Q , design domain (see Fig. 7)	For the topology of the arm

In accordance with the ICO shown in Fig. 4, the corresponding notations to this problem are summarized in Table 4.

In this problem, the constraints of D1 include that the resulting maximum Mises stress cannot exceed the material yield stress of 650 MPa and that the fatigue life is more than 50000 times. The constraint of D2 is that the vibration isolation coefficient is less than 0.2 while the constraint of D3 is that the resulting maximum Mises stress cannot exceed the material yield stress of 650 MPa.

Same as in the problem in Section 4.1, the sequential quadratic programming and the simulated anneal algorithm are respectively used for the optimization on the discipline level and the system level. For the same purpose as in Section 4.1, $s_1 = s_2 = s_3 = 1$, $s_3 = 1$ and $w_1 = w_2 = w_3 = 1/3$ are used in this problem.

4.2.2 Results and discussion

Direct use of the CCO shown in Fig. 1 will lead to an abnormal stop during the optimization because the resulting stress is nearly twice the yield stress, and hence gives a divergent design

result as shown in Fig. 8 which cannot smoothly connect the arm and the bar via the public variable z_1 (i.e. R).

Fig. 8 The design result if the coupling judge and decoupling are ignored

Fig. 9 Illustration of coupled design variables

Fig. 10 The decoupling strategy for Example 2 ($\alpha = 0$)

The fact is that, as shown in Fig. 9, the private variable Q (i.e. x_3) is coupled with the public variable R (i.e. z_1) in D3 after judge with Inequality (6) for any positive ϵ , and hence the decoupling strategy must be introduced by prescribing an additional constraint on the coupling variables in D3. As already discussed in Section 3.2, the strategy is used to decouple the variables Q and R by shrinking the domain Q when $\epsilon = 12\text{mm}$ is taken, as shown in Fig. 10 as compared with that in Fig. 9.

Table 5 The design variables after optimization with different methods for Example 2

Variable type	Design variable	Range	Initial value	Optimal value	
				SO ^[30]	Present ICO
Public	R (mm)	[28,36]	32	30	31.2
Private for D1	r (mm)	[0, 21]	0	11	16
Private for D2	R_g (mm)	[28, 36]	45	45	43
Private for D2	W_g (mm)	[75,100]	90	90	85

With the additional constraint for D3, the optimization procedure via the ICO is conducted. The ICO result converges after 237 iterations. The design variable values of public variable R and private variables r , R_g and W_g in D1 and D2 are listed in Table 5, the design domain Q and the Mises stress in D3 are shown in Fig. 11, and the gasket size and the shear stress are shown in Fig. 12. From Table 5, Fig. 11 and Fig. 12, it is seen that the optimal design variables from the present ICO are different from those from the SO.

Fig. 11 The size for bar (or the topology for arm) and the Mises stresses:
(a) Initial; (b) The optimal results by the present ICO

Fig. 12 The size and the shear stress of gasket:
(a) Initial; (b) The optimal results by the present ICO

The optimal objectives are summarized in Table 6. It is seen that, the overall weight of torsion bar system is reduced by 49.6%, better than 36.1% from the SO^[30].

Table 6 The optimal objectives after optimization with different methods for Example 2

Level	Objectives	Initial value	Optimal value	
			SO	Present ICO
D1	Minimize f_1	30614	16163 ^[30]	11900
D2	Minimize f_2	318	318	256
D3	Minimize f_3	16120	13590 ^[30]	11581
The system level	Minimize f	15684	10023 (-36.1%)	7912 (-49.6%)

4.2.3 Validity check of the decoupling strategy

It can be well understood in physics that the decoupling strategy in Section 3.2 is reasonable. However, it is of significance to check the validity of this strategy.

As a matter of fact, variables X and Y will be uncoupled as long as Inequality (6) violates. For this purpose, there are many other decoupling strategies. Without loss of generality, parameter α is introduced to distinguish different strategies so that the present strategy for only the private variable is responsible for $\alpha = 0$ while the strategy for only the public variable is for $\alpha = 1$. For the case of $0 < \alpha < 1$, the strategy is for a combination of two variables, as shown in Fig. 13.

(a) $\alpha = 0.5$ (change both the design spaces) (b) $\alpha = 1$ (only change the design space of R)

Fig. 13 The sketch of decoupling strategies

The optimal results from different decoupling strategies are compared in Fig. 14. It is seen that the present strategy is the best one, and hence the validity is confirmed.

Fig. 14 Comparison of optimal results from different decoupling strategies

5. Conclusions

The ICO was proposed in this paper for generic MDO problems, especially with coupled design variables. In the present ICO, the MDO was decomposed into two levels and the possible coupling was judged and tackled before the discipline optimization was conducted. As a complete

framework, the criterion to judge the coupling in disciplines and an effective strategy to decouple the variables was suggested. Two practical MDOs were carried out to validate the present ICO, as compared with the SO.

In this paper, two examples are all optimization problems with both parametric and parameter-free variables. Application of the present ICO is also straightforward for the optimization problems either with pure parametric variables or with pure parameter-free variables.

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