Hydrokinetic Energy Conversion by two Rough Tandem-Cylinders in Flow Induced Motions: Effect of Spacing and Stiffness

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ABSTRACT

Flow Induced Motions (FIMs) of rigid circular cylinders, and particularly VIV (Vortex Induced Vibrations) and galloping, are induced by alternating lift. The VIVACE (VIV for Aquatic Clean Energy) Converter uses single or multiple cylinders, in tandem, on elastic end-supports, in synergistic FIM, to convert MHK energy to electricity. Selectively distributed surface roughness is applied to enhance FIM and increase efficiency. In this paper, two cylinders are used in tandem with center-to-center spacing of 1.57, 2.0 and 2.57 diameters, harnessing damping ratio $0.00 < \zeta < 0.24$, for Reynolds number $30,000 \leq Re \leq 120,000$. The virtual spring-damping system V_{ck} in the Marine Renewable Energy Laboratory (MRELab) enables embedded computer-controlled change of viscous-damping and spring-stiffness for fast and mathematically correct oscillator realization, without including the hydrodynamic force in the closed control loop. Experimental results for oscillatory response, energy harvesting, and efficiency are presented and the envelope of optimal power is derived. All the experiments were conducted in the Low Turbulence Free Surface Water (LTFSW) Channel of the MRELab of the University of Michigan. The main conclusions are: (1) For the tested cylinder spacing, two cylinders harness power is between 2.56 and 13.49 times the power of a single cylinder, the efficiency of two cylinders is between 2.0 and 6.68 of a single cylinder. (2) The MHK power harnessed by the upstream cylinder is increased by up to 100%, affected by the downstream cylinder. (3) The MHK power harnessed by the downstream cylinder and its FIM are affected to a lesser extent by the interaction. (4) VIVACE can harness energy from flows as slow as 0.4m/s with no upper limit in flow velocity. (5) Close spacing and high spring stiffness yield highest harnessed power. (6) The optimal harnessed power shifts to softer springs as spacing increases.

KEY WORDS: hydrokinetic energy harnessing; virtual spring-damping; flow induced motions; vortex induced vibrations; galloping; distributed roughness

1. INTRODUCTION

In transverse flow, cylinders respond in FIM (Flow Induced Motions); particularly VIV (Vortex Induced Vibrations) and galloping. Both are driven by alternating lift and result in oscillatory motion. Alternating Lift Technologies (ALT) (Bernitsas 2016) provide an environmentally compatible way of harnessing hydrokinetic energy even from slow flows, starting at 0.4 m/s. (Park *et al.* 2012, 2013a, 2013b) . The VIVACE (VIV for Aquatic Clean Energy) Converter uses multiple cylinders, in tandem, with distributed surface roughness, on elastic supports. In this paper, experiments are conducted with two cylinders in tandem resulting in synergistic FIM.

Rigid cylinders on elastic supports in transverse flows can be readily excited in FIM. The latter can be controlled converting Marine Hydrokinetic (MHK) energy to mechanical and electrical energy. VIV and galloping are the most commonly occurring FIM and are implemented in the VIVACE Converter mechanics along with interaction phenomena between shear layers and wakes of upstream cylinders (Bernitsas *et al.* 2011, 2016). An important question in the Fluid-Structure Interaction (FSI) mechanics of this ALT converter is how close cylinders can be placed to achieve highest hydrokinetic energy conversion. This metric affects directly the power-to-volume density. Fish in schools move effortlessly by proper placement in the wake of fish ahead of them (Liao 2007). VIVACE is being developed to achieve optimal FSI synergy among cylinders organized in a school to capture the energy within a volume of flow like a true 3-D converter.

Multi-cylinder tests have been conducted in the Low Turbulence Free Surface Water (LTFSW) Channel in the Marine Renewable Energy Laboratory (MRELab) at the University of Michigan (Kim, *et al.* 2011, 2015) for Reynolds numbers in the range $30,000 \leq Re \leq 120,000$, which falls in the high-lift TrSL3 flow regime (Zdravkovich, 1997). Results show that synergy of FSI for four cylinders can produce power as much as 80% greater than four cylinders in isolation (Ding, *et al.* 2015). In this paper, a more systematic study is initiated to establish this discovery starting with two cylinders in tandem. Damping and stiffness are used as parameters to optimize the power output. Distributed roughness in the form of roughness strips is applied to induce galloping and start it at lower flow speeds, thus, increasing the harnessed MHK power across the range of flow velocities over which a converter is designed to operate (Bernitsas 2016, Bokaian 1984,). The recently developed second generation of virtual spring-damping system, V_{ck} , in the MRELab enables embedded computer-controlled change of viscous-damping and spring-stiffness for precise oscillator modeling and fast parameter changes for systematic testing (Sun *et al.* 2015). In both the first (Lee 2011, 2011) and second generations (Sun H. *et al.* 2015, 2016) of V_{ck} , the hydrodynamic force is not included in the closed control loop; thus, the measured output is not biased. This process makes the V_{ck} system fundamentally different from control systems used in VIV experiments (Hover *et al.* 1998, Smogeli *et al.* 2003). Experimental results for harvested power and the supporting data on amplitude response are presented for two cylinders in tandem and compared to the corresponding information for a single-cylinder (that is, single oscillator) Converter (Sun H. 2016).

Flow Induced Motions (FIMs) present a source of challenge for diverse structures where cylinders are placed perpendicularly to a steady flow. Heat exchangers, bridges, buildings, offshore structures, power-transmission cables, and fish-nets are some examples. The most

common studied FIM phenomenon is Vortex-Induced Vibration (VIV), which was first observed by Leonardo da Vinci in 1504. The first mathematical formulation was done by Strouhal in 1878. Comprehensive reviews of VIV have been published by Bearman (1984, 2011), Sarpkaya (2004), and Williamson and Govardhan (2004). Galloping is another form of flow induced motion. It is an aero/hydro-elastic instability characterized by lower frequencies and larger amplitudes than VIV and is perpendicular to the flow (Blevins 1990, Blevins and Coughran 2009). It is more vigorous and destructive than VIV. Galloping is an instability phenomenon, does not depend on vortex formation, occurs above a critical flow speed, and results in bigger amplitude of oscillation than VIV. Galloping amplitude is limited only by the structure configuration, while VIV is hydrodynamically self-limiting in amplitude and velocity range.

Typically, as flow velocity increases, VIV appears first, followed by galloping. Figure 1 shows the various branches of FIMs for a single cylinder with distributed surface roughness to induce galloping. The cylinder is rigid on elastic supports in a cross-flow. As velocity increases, FIM is initiated as VIV due to vortex shedding, alternating from the two sides of the cylinder, resulting in transverse movement of the cylinder. Vortex shedding locks onto the cylinder oscillation over a broad range of synchronization starting with the initial branch as shown in Figures 1 and 2 (Sun *et al.* 2016). This response amplitude increases with the velocity of the flow in the upper branch. It is followed by the lower branch with smaller response, then the desynchronization zone, and then the end of the VIV synchronization range (Park *et al.* 2012). The various branch characteristics may differ significantly depending on whether the flow in the Transition in Shear Layer (TrSL) is flow regime TrSL2 or TrSL3 (Zdravkovitz, 1997). Williamson and Govardhan (2004) conducted experiments for Re about 3,800 in TrSL2 ($1,000 < Re < 20,000$) and response amplitude reached one cylinder diameter. Bernitsas *et al.* (2009) and Raghavan and

Bernitsas (2010) conducted experiments for $16,000 < Re < 140,000$ in the TrSL3 ($20,000 < Re < 300,000$) flow regime. They showed that: (a) the dependence of VIV on Reynolds number is strong; (b) in the upper branch in TrSL3, the amplitude increases reaching 1.8 diameters; and (c) the upper branch overtakes almost completely the lower branch.

Figure 1. FIM regions with small overlap between VIV and galloping. (Sun *et al.* 2016)

Figure 2. FIM regions with transition between VIV and galloping. (Sun *et al.* 2016)

Multiple cylinder interference is critical when designing complicated structures with interacting cylinders, especially under harsh operating environments. The combination of two cylinders was most frequently used as a representative arrangement of multiple cylinders. Zdravkovich (1985, 1997, 2002) classified the interactions of two cylinders in tandem and stationary into three kinds:

- Proximity interference which occurs when the cylinders are in close proximity to each other.
- Wake interference when one cylinder is in the wake of the other.
- No-interference when the cylinders are sufficiently apart so that they are almost fluid dynamically independent of each other.

Most former investigations have been performed on two rigid circular cylinders in tandem in the spacing range L/D (cylinder center-to-center distance over cylinder diameter) < 3.0

(Zdravkovich, 1997; Sumner et al., 2010, Chen 1986). This region is traditionally regarded as proximity interference. Specifically, for $L/D \leq 1.1$, two cylinders behave like a single bluff body and the vortex shedding can only be found after the rear cylinder. For $1.1 \leq L/D \leq 1.6$, the shear layer of the upstream cylinder alternately reattaches to the downstream cylinder and vortex shedding occurs on the downstream one; for $1.6 \leq L/D \leq 2.5$, the shear layer of first cylinder attach to the second cylinder and again the vortex shedding only happens after the second one. For $2.5 \leq L/D \leq 3.2$, vortex shedding can be observed intermittently in the gap between two cylinders. For $3.2 \leq L/D \leq 8.0-10.0$, this is normally considered as wake interference region. The critical separation of proximity interference and wake interference regions is typically considered $L/D=3.0-4.0$. At larger spacing ratio than this critical value, a developed wake was observed to be present in the gap for all flow speeds.

From previous experimental studies of two oscillating cylinders, conclusions are as follows:

- a) Response of both cylinders is largely determined by the in-line distance, Reynold number, Reduced Velocity, and free stream turbulence intensity.
- b) With close proximity of two cylinders, the amplitudes are mutually affected.
- c) Wake interference still exists even for $L/D=25.0$.
- d) In the dynamic response of the downstream cylinder, two or three bifurcations are observed due to the change of wake structure if the in-line distance of the two cylinders is varied.

King and Jones (1980, 1992) examined wake interaction effects between two identical flexible circular cylinders with $1.25 \leq L/D \leq 5$ in flowing water by conducting experiments in a wind tunnel. Ruscheweyh (1983) and Bokaian (1984) referred to the downstream cylinder vibration as wake-galloping. Cooper (2007) observed the wake-galloping oscillation occurred until $L/D=20$. Laneville and Brika (1999) conducted experiments in wind tunnel and revealed that

when the cylinders are coupled only by fluid flow, the upstream cylinder behaves like an isolated one, while the downstream cylinder response depends on the separation and presents a hysteresis and three bifurcations. Two corresponding to the two identified in the first cylinder dynamics and the third discontinuity coinciding with the end of the synchronization of the windward cylinder. Huera-Huarte and Gharib (2011) showed that in far-wake interference (4 and up to 8 diameters) of two flexible cylinders in tandem, with both cylinders being free to move in all directions, the main excitation mechanism is wake-induced-vibration (WIV). The trailing cylinder shows large amplitudes of oscillation at reduced velocities over the typical lock-in of a single-isolated cylinder undergoing VIV.

The objective of this paper is to study systematically the effects of spacing, linear stiffness, and linear viscous damping on a Converter with two tandem-cylinders both oscillating, to attain higher power generation with higher efficiency for hydrokinetic energy harnessing. For rigid cylinders, most prior studies of two circular cylinders in tandem had the upstream cylinder stationary and the downstream cylinder free to oscillate transversely to observe the wake interference effects on the trailing cylinder. With the introduction of additional linear damping to the oscillators, the FIM of cylinder/s change drastically. Previous work at the MRELab has established harnessing of MHK energy by a single PTC-cylinder and a single smooth cylinder using linear viscous damping as parameter. This was done experimentally by Lee et al. (2011) and Sun et al. (2016), and numerically by Ding et al. (2013). Two tandem cylinders in FIM using physical springs and a fixed damping ratio was studied experimentally by Kim et al. (2013) and numerically by Ding et al. (2015). Prior studies on two tandem cylinders in FIM, with physical springs have used fixed system parameters. Damping has been proven to be an important factor; specifically, changing the damping ratios of the two cylinders results in changing their oscillatory

response and the harnessed power. In addition, hydrodynamic interactions, such as shielding effects, oscillation patterns, also depend on the amplitude and damping ratios of the cylinders. The present study focuses on two oscillating rigid circular cylinders in tandem arrangement, in steady uniform, two-dimensional flow, with $L/D=1.57, 2.0$ and 2.57 , where each value results in different kind of interference. Specifically, interference may result in FIM response of the two cylinders as: (1) A single bluff body with no gap flow in between even though there are vortices between the cylinders. (2) Intermittent gap flow between the two cylinders resulting in in-phase motion with some lag. (3) Intermittent gap flow between the two cylinders resulting in out-of-phase motion. (4) Interference similar to that between two stationary cylinders. To the authors' knowledge, this is the first time that harnessing power using two tandem cylinders undergoing flow-induced motions is conducted. Finally, systematic change of the value of the linear viscous damping parameter is enabled by the unique virtual (controlled) oscillators developed in the MRELab (Sun et al. 2015).

2. PHYSICAL MODEL

The experimental facility, the oscillators and their distributed surface roughness, and the virtual system enabling systematic, correct modeling of the oscillators, and the limitations of the test device are described next.

2.1. Experimental Facility LTFSW Channel

All tests were conducted in the Low Turbulence Free Surface Water (LTFSW) Channel, in the MRELab of the University of Michigan (Fig. 3). The LTFSW Channel recirculates 10,000 gallons (37,854 liters) of mixed cold and hot fresh water (to achieve desirable temperature) at speed up to 1.4m/s using an impeller powered by a 20hp induction motor. The test-section is 2.44m long, 1.0m wide, and 1.52m deep. It is made of transparent plexi-glass, enabling visualization of

the flow past cylinders using two 5W argon lasers and aluminum oxide particles of $100\mu\text{m}$. For this study, the water depth at the test-section was set at 1.17m and the maximum flow speed was limited to 1.35m/s for safety against the galloping instability.

Figure 3. Schematic of the LTFSW channel and VIVACE system

A total of up to four single-cylinder VIVACE oscillators have been placed in the LTFSW Channel for FIM testing.

2.2. Cylinders and Distributed Roughness

In order to enhance the FIM of the cylinder, which results in harnessing more horizontal hydrokinetic energy, distributed roughness in the form of roughness strips was introduced and extensively studied in the MRELab experimentally (Bernitsas and Raghavan 2011, Chang et al. 2010, Park et al. 2012, Kim et al. 2013) and numerically (Ding et al. 2013, 2015) as well. The distributed roughness was termed Passive Turbulence Control (PTC). This study resulted in a most valuable tool, the *PTC-to-FIM Map* (Park et al. 2012, 2013a, 2013b). As shown in Fig. 4, PTC

consists of a pair of straight roughness strips attached symmetrically to the surface of the cylinder. According to the *PTC-to-FIM Map*, the location of the roughness strips determines the response FIM of the cylinder. The placement angle $\alpha_{PTC}=20^\circ$, is measured in degrees with respect to the forward stagnation point to the upstream edge of the roughness strip. Based on the previous studies (Chang et al. 2011, Park et al. 2013) on PTC, the strips with commercial roughness designation P60 and width of 12.7mm, which covers 16 degrees on each side of the 88.9mm diameter cylinder have been used in this study. Table 1 shows the details of the roughness strips used in the present study.

Table 1. PTC parameters (P60)

Strip placement angle	α_{PTC} [degree]	20
Angular coverage of strip	θ [degree]	16
Sand paper plus tape thickness	P [mm]	0.587
Average grit height	k [mm]	0.26
Total thickness of strip	$T=P+k$ [mm]	0.847

The total thickness of the PTC is on the order of the boundary layer thickness. The robustness of the *PTC-to-FIM Map* was studied by Park et al. (2013) defining not only the PTC coverage

requirement in the direction of the flow but also the hierarchy of dominance of the zones where PTC is applied.

2.3. Oscillator Simulated by the Vck System

To conduct systematic tests in FIM with variable spring-stiffness K and damping values c , an efficient method is needed to change K and c when conducting experiments. Physical change of K and c requires extensive time for calibration and validation of the oscillator prior to conducting experiments. Further, as time progresses, fatigue results in change of physical parameters. In the MRELab, a system was developed to replace just the springs and dampers with a controller. The critical issues in development of the Vck system in the MRELab are the following:

- (a) The hydrodynamic lift force is not included in the closed control loop. Including it would bias the FIM phenomenon being measured experimentally and will produce incorrect results for converted power due to the phase-shift introduced between lift-force and cylinder velocity. That is, load cells are not used in the MRElab Vck system to measure the lift force, calculate the displacement from the differential equation, and force the displacement at the next time step. The latter approach has been used in VIV experiments as early as 1998 (Hover *et al.* 1998), later by (Hover *et al.* 2003, Muse *et al.* 2009), and recently by (Mackowski and Williamson, 2011).
- (b) The MERLab developed the 1st generation Vck in 2008-2010 (Lee *et al.* 2011). The 2nd generation was developed by Sun et al. (2015) In this second generation of Vck, the controller force in the Vck model can be expressed as:

$$F_{controller} = F_{nonlinear_damping} - c_{virtual}\dot{y} - K_{virtual}y \quad (1)$$

where $F_{controller}$ is the force applied by the virtual system (controller), $F_{nonlinear_damping}$ is the total nonlinear damping force, $K_{virtual}$ is the virtual spring constant, $c_{virtual}$ is the virtual linear viscous damping.

(c) The process is lengthy and has to be repeated for each oscillator separately. The stiffness part of Vck is simple involving determination of the torque-rotation relation to establish the $K_{virtual}$ relation. That of course can be defined in any non linear function not necessarily as a linear classical force-spring relation (Ma *et al.*, 2016). The damping system identification, though, is lengthy as structures seldom have damping in the classical textbook relation of linear viscous damping (Lee *et al.* 2011, Sun *et al.* 2015).

(d) The resulting expression for damping is subtracted from the system and replaced by a mathematically accurate linear (or nonlinear) viscous damping expression.

The features of the Converter model are summarized in Table 2. In the current design, the linear motion mechanism was brought under water, which resulted in most of the Converter components being under water. The whole linear motion system, which consists of the end sliding-blocks, one timing-belt and pulleys, is fitted inside of a 38mm wide slender tube. Fig. 5 shows the side view of two VIVACE with Vck system mounted in the LTFSW channel.

Table 2. Specifications of the experimental apparatus (model Converter)

Mass Ratio	Aspect Ratio	Maximum Oscillatory Amplitude	Longitudinal Distance
$0.634 \leq m^* \leq 2.00$	$L/D = 10.07$	$A_{max}/D_{3.5''} = 5.5$	$1.429 \leq d/D_{3.5''} \leq 6.0$

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268 **2.4. Physical Model Remarks**

269 Digital Position Signals

270 In the 1st generation of Virtual system (Lee et al. 2011), all the position and the torque
 271 information were processed through National Instruments (NI-7340) using analog signals. The
 272 quality of signals is highly depended on the power supply, and the noise was clocked around 200
 273 Hz. Several improvements were achieved in the second generation Vck compared to the first (Sun
 274 et al. 2015). First, the second generation Vck system was developed as a stand-alone application
 275 via an embedded controller rather than using LabView, which added lags that had to be
 276 compensated. Second, digital signals were used instead of analog signals. The sample time was
 277 reduced by three orders of magnitude, thus, eliminating the need to include dynamic terms with
 278 memory in the damping model. Both systems are capable of compensating for the system nonlinear
 279 damping. In both Vck generations in the MRELab, the total nonlinear expression for the viscous

damping of the system was modeled and its coefficients were determined after extensive system identification. The identified system damping was then subtracted from the system through the controller to nullify the system damping. Subsequently, a mathematically accurate linear viscous damping was introduced. Linear viscous damping is hardly achievable in engineering applications; and yet it is used in modeling oscillators in textbooks. With the Vck controller, a perfectly linear viscous damping is achieved and implemented. A resistor bank is used to dissipate the generated power.

Tip-flow effect

In our studies, due to the structural design (Fig.5), both the upstream and downstream oscillators are subject to tip-flow effect. Tip-flow limits the cylinder length over which lift (positive) is exerted in the transverse direction when the cylinder is in FIM. Negative lift is typically exerted on the rest of the length, which is not affected by the tip-flow. In this study, PTC is proven to be an effective way to overcome tip-flow adverse effects in hydrokinetic energy conversion. The estimated loss in lift force when comparing an oscillator model to a nearly ideal VIV oscillator (Kinaci, et al. 2016) is limited to less than 10% of the cylinder length.

Free-surface and the bottom boundary effects

Bernitsas et al. (2007) conducted tests to study the free-surface effect on the FIM of an elastically supported cylinder in water flow in the range of $8 \times 10^3 < Re < 1.5 \times 10^5$ (TrSL2 and TrSL3). The study showed that proximity to the free-surface reduces the range of synchronization and the oscillatory amplitude of the cylinder in VIV. Raghavan et al. (2007) reported that the amplitude response of VIV depends on proximity to the bottom boundary layer. For $0.65 < G/D < 3.0$, where G/D is the ratio of the gap between the bottom wall and bottom of the cylinder to the diameter of the cylinder, the oscillatory amplitude decreased due to the bottom

boundary layer, but the range of synchronization was not affected by it. Table 2 shows that the experimental apparatus allows for a maximum amplitude ratio of the cylinders of $5.5D$. In our experiments, the maximum A/D reached in FIM was $3.3D$. Moreover, the oscillator is specifically designed for mitigating the free-surface and the end-effects. A gap of $1.5D$ to the bottom and $1.7D$ to the free-surface is allowed in addition to the maximum designed amplitude of $5.5D$.

For $G/D > 3.0$, the effect of the bottom boundary layer on FIM is very small. Therefore, the center of the cylinder should be placed higher than $3.5D$ from the bottom of the water channel and as far away as possible from the free-surface to be free from those effects and to increase the oscillatory amplitude and range of synchronization of FIM. The response of multiple cylinders in the TrSL3 regime has not been investigated as yet due to the limitation of the previous water channel of the MRELab. Therefore, the oscillators should have as large allowable amplitude as possible to further investigate galloping response with no blockage effects.

3. MATHEMATICAL MODEL

The oscillator model, the power conversion and the efficiency models are presented next.

3.1. Harnessed and Dissipated Power

The flow is in the x -direction. The motion of the cylinder in the y -direction and is modeled by the second order linear differential equation of an oscillator as

$$m_{osc}\ddot{y} + c_{total}\dot{y} + Ky = F_{Fluid} \quad (2)$$

where y is the direction perpendicular to the flow and the cylinder axis, m_{osc} is the total oscillating system mass, which includes one third of the spring mass, K is the spring stiffness, c_{total} is the total damping coefficient, and F_{Fluid} is the force exerted by the fluid on the body in the y -direction.

In order to convert hydrokinetic energy to mechanical energy in the oscillator and subsequently electrical energy, additional damping is introduced into the system. The total damping (c_{total}) is defined as

$$c_{total} = c_{structure} + c_{harness} \quad (3)$$

where $c_{structure}$ is the existing damping due to losses in the transmission system, in this research, all the $c_{structure}$ are adjusted by Vck system according to m^* and K to achieve same artificial $\zeta_{structure}$, $c_{harness}$ is the damping added through V_{ck} that converts the mechanical energy in the oscillating cylinder to electrical energy simulating the real world.

From equation (3), we know that $c_{structure}$ and $c_{harness}$ can be expressed using damping ratio $\zeta_{harness}$ and $\zeta_{structure}$.

$$\zeta_{structure} = \frac{c_{structure}}{2\sqrt{m_{osc}K}} \quad (4)$$

$$\zeta_{harness} = \frac{c_{harness}}{2\sqrt{m_{osc}K}} \quad (5)$$

Then mechanical power of VIVACE can be expressed as:

$$P_{VIVACE-Mech} = \frac{1}{T_{osc}} \int_0^{T_{osc}} 4\pi(m_{osc} + m_a)\zeta_{total}y^2 f_{n,water} dt = 8\pi^3(m_{osc} + m_a)\zeta_{total}(Af_{osc})^2 f_{n,water} \quad (6)$$

Using equation (6), we obtain the harnessed power and the dissipated power as

$$P_{VIVACE-structure} = 8\pi^3(m_{osc} + m_a)\zeta_{structure}(Af_{osc})^2 f_{n,water} \quad (7)$$

$$P_{VIVACE-harness} = 8\pi^3(m_{osc} + m_a)\zeta_{harness}(Af_{osc})^2 f_{n,water} \quad (8)$$

Since equations (7) and (8) appear to depend explicitly on the added mass m_a , it is worth noting that they can be recast using as follows, respectively.

$$P_{VIVACE-dissipated} = \frac{1}{2}c_{structure}A^2\omega_{osc}^2 \quad (9)$$

and

$$P_{VIVACE-harness} = \frac{1}{2}c_{harness}A^2\omega_{osc}^2 \quad (10)$$

where, A is the amplitude of the cylinder, ω_{osc} is the angular frequency, which can be measure using V_{ck} system. These two equations do not depend on the specific way the added mass is calculated or even the value of the added mass explicitly. Detailed derivation of the above equations, are provided by Sun, et.al. (2016).

As a final note on this short mathematical model, it is important to mention that a controversy exists regarding the value of m_a used in equations (4) and (5) and also in

$$f_{n,water} = \frac{1}{2\pi}\sqrt{\frac{K}{m_{osc} + m_a}} \quad (11)$$

There are two approaches. In the first, the added mass is included as a constant added mass from potential theory. In the second approach the calculated added mass from the lift force in-phase with the acceleration in the y-direction is used (Vikestad 2000). It is the author's opinion that this controversy is unnecessary as both are simple modeling methods that cannot account for the range of synchronization or the self-limiting nature of the amplitude of oscillation. Both just model the response of the cylinder in FIM for a specific flow velocity and a given experiment. On the other hand, both approaches are useful in post-processing data and understanding this complex

phenomenon better. For better understanding of the complexity of the added mass expression in forces and moments on a small body moving in a 3-D unsteady flow, the reader is referred to the detailed analysis by Foulhoux and Bernitsas (1993). The dependence of the inertia terms on absolute and/or relative velocity as well as the form of convective terms is explained thoroughly putting into perspective the simplicity of the inertia term used in Morison's equation.

3.2. Power Conversion Efficiency

The power in a fluid is calculated based on the area swept by the cylinders.

$$P_{Fluid}(W) = \frac{1}{2} \rho U^3 (2A_{max} + D)L \quad (12)$$

where A_{max} is the largest amplitude among all cylinders in a multiple-cylinder converter. Obviously, the denominator depends on the cylinder response, unlike propellers or wind turbines where the flow area is determined by the fixed geometry of the area swept by the blades. The Betz limit is the theoretical maximum power that can be extracted from an open flow and is equal to 59.26% (=16/27). Based on the Betz limit, the power conversion efficiency, $\eta_{convert}$, and power harness efficiency, $\eta_{harness}$, can be calculated as

$$\eta_{convert}(\%) = \frac{P_{convert}}{P_{Fluid} \times Betz\ Limit} \times 100 \quad (13)$$

$$\eta_{harness}(\%) = \frac{P_{harness}}{P_{Fluid} \times Betz\ Limit} \times 100 \quad (14)$$

Commercial scale wind turbines achieve peak efficiency of 75% to 80% of the Betz limit (Burton *et al.* 2011).

4. RESULTS AND DISCUSSION

4.1. Test Matrix

A series of experiments were conducted to measure the flow induced motions of two cylinders in tandem with the following parametric ranges: Spring-stiffness range of $400\text{N/m} \leq K \leq 1,200\text{N/m}$; damping ratio range of $0 \leq \zeta_{\text{Harness}} \leq 0.24$; longitudinal center- to-center spacing ratio (L/D) values of 1.57, 2.0 and 2.57; mass ratio m^* of 1.343. The experimental parameter values are as shown in Table 3. The values of stiffness K selected are accurate in form and value and exact as they are defined by the calibrated motor-torque and its relation to rotation. That is, the K value is not subjected to fabrication error or degradation in time due to fatigue. Similarly, the values of ζ_{Harness} and ζ_{Total} are exact and the corresponding linear viscous function is accurate as explained in the discussion on the Vck system (Section 2.3). The flow velocity range is $0.35\text{m/s} \leq U \leq 1.35\text{m/s}$ Reynolds number range is $30,000 \leq Re \leq 120,000$, and the corresponding reduced velocity range is $2.92 \leq U^* \leq 15.33$. The particulars for each case are shown in Table 3. The upstream and downstream cylinders are set to be identical adjusted by two independent V_{ck} systems. In this research, the ζ_{harness} and $\zeta_{\text{structure}}$ are adjusted precisely to each mass and stiffness.

Table 3. Particulars of two cylinders in tandem

	<i>Test sets</i>
L/D	1.57, 2.0, 2.57
$k[\text{N/m}]$	400, 600, 800, 1,000, 1,200
<i>Temperature</i> ($^{\circ}\text{C}$)	18.5~20.5
$\mu (\text{N s/m}^2)$	1.004E-03
$\nu (\text{m}^2/\text{s})$	9.940E-07
$\rho[\text{kg/m}^3]$	999.729
$D[\text{m}]$	0.0889
$L[\text{m}]$	0.895
$m_{\text{displacement}}(\text{kg})$	5.425
$m_{\text{oscillation}}(\text{kg})$	7.286
$m_{\text{added}}(\text{kg})$	5.425
$\zeta_{\text{structure}}$	0.020
$f_{n,\text{water}}$	0.97 ~ 1.67

The combination of five spring-stiffness K , seven damping values $\zeta_{harness}$, and three spacing values L/D , results in 105 experiments conducted. Each experiment lasted about 4-5 hours with measurements taken for 30 values of flow velocity up to 1.35m/s.

The harnessed and converted power results are presented in Section 4.2 in Figures 6-8 for each spacing value. The power generation and oscillatory response of the optimal configurations for power generation at each spacing value are presented in Section 4.3 in Figures 9-14. The power envelopes for a two-cylinder Converter and for a single cylinder are derived and discussed in Section 4.4 and Figure 15.

4.2. Harnessed and Converted Power for Two Cylinders in Tandem

Figures 6, 7, 8, show the test-results for harnessed power for the three tested spacing values of $L/D = 1.57, 2.01, 2.57$, respectively. The independent variable is Reynolds number. To facilitate understanding the results, the absolute velocity, which is proportional to Re , is also show parallel to the x-axis. Since K is not constant, the reduced velocity U^* changes and is not just proportional to the flow velocity as in Figs. 9-16. In Figures 9-14 in this paper, the optimal converted power is listed in Tables 7,8 and 9; K is constant in these figures making it possible to also plot U^* parallel to the x-axis. The following observations can be made:

- (a) In general, power in the flow increases with the cube of the flow velocity. In the three L/D values tested, however, the harnessed power exhibits drops and jumps. Specifically: 1) In the VIV region for all spacing values ($35,000 > Re > 60,000$ for $L/D=1.57$, and $35,000 > Re > 78,000$ for $L/D=2.01$ and 2.57), the harnessed power increases as the flow velocity increases. Higher spring stiffness results in higher power harnessed at the VIV upper branch. Lower stiffness results in higher harnessed power at the VIV lower branch. 2) The first drop in harnessed power

occurs in the transition region ($72,000 > Re > 80,000$ for $L/D=1.57$, and $78,000 > Re > 87,000$ for $L/D=2.01$ and 2.57) as shown in Fig. 6-8. The power drop is obvious at $L/D=2.57$ (Fig. 8), which is consistent with the single-cylinder case, because in our studies, as the spacing increases to a certain point, the oscillatory response approaches that of the isolated cylinder (Sun et al. 2016). Figures 6, 7, and 8 show that higher damping cases of two cylinders exhibit gaps in the transition region, while the lower damping cases do not have gaps. 3). In galloping, the changes in harnessed power occur at $Re \approx 105,000$ for $L/D=2.01$ and 2.57 ; the harness power drops and then increases steadily (Fig. 7,8). At closer spacing, the drops occur at lower stiffness ($K < 600 \text{ N/m}$) (Fig. 6).; at the higher stiffness ($K=1,200 \text{ N/m}$), though, the power increases regardless without a drop (Fig. 6).

(b) As damping increases, a point is reached where a gap appears between the end of VIV and the beginning of galloping (Fig. 6-8). This is due to two factors. First, higher damping results in delay of the onset of galloping; and second, increase in damping results in reduced amplitude and range of VIV. The flow velocity and corresponding Reynolds number of the gap between VIV and galloping in harnessed power increases as the spacing L/D decreases, being, $Re \sim 68,000$ at $L/D=1.57$, $Re \sim 78,000$ at $L/D=2.01$, and $Re \sim 82,000$ at $L/D=2.57$.

(c) In VIV the power graphs exhibit a local maximum at the end of the upper branch, being, $P_{harness} = 12.49 \text{ w}$, 14.88 w and 16.75 w at $L/D=1.57$, 2.01 and 2.57 , respectively (Figs. 6, 7, 8, and Tables 7, 8, 9); that is, the optimal harnessed power increases as the spacing increases. Especially at lower spacing, the disturbances in between the two cylinders have a negative effect on the harnessed power. The vortices from the upstream cylinder are disturbed by the downstream cylinder, affecting the oscillation patterns, (Sun et al. 2016).

- (d) In galloping, the local maximum is at the highest velocity since galloping is not self-limiting and it is an instability phenomenon. The optimal harnessed power is $P_{harness}=41.50\text{w}$, 31.30w , and 33.20w for $L/D=1.57$, 2.01 and 2.57 , respectively (Figs. 6, 7, 8, and Tables 7, 8, 9). As the spacing decreases, the optimal power harnessed by the two cylinders increases at each specific L/D increases linearly with the flow velocity. This is counter-intuitive since somebody would expect power to increase with the cube of the flow velocity. On the other hand, the change in stiffness K should be considered in explaining this linear relation.
- (e) The optimal harnessed and converted power values occur at the highest velocity but require a different configuration. For spacing of $L/D=1.57$, $K=1,200\text{N/m}$ is required; for spacing of $L/D=2.01$, $K=800\text{N/m}$ is required; and for spacing of $L/D=2.57$, $K=400\text{N/m}$ is required. That is, as spacing increases, the highest harnessed/converted power is achieved with low spring stiffness/natural frequency.
- (f) Jumps and drops occur in the harnessed power in galloping. Specifically, at $L/D=1.57$, in the fully developed galloping range ($90,000 < Re$), the harnessed power shows unstable trends at most of the $\zeta_{harness}$ expect at $\zeta_{harness}=0.04$ (lowest) and 0.24 (highest); both points show stable harnessed power, due to the close spacing disturbing the vortex in between the cylinders. At $L/D=2.01$ and 2.57 , the harnessed power in galloping, however, shows a stable trend in harnessed power, expect at higher stiffness values. The variations in the harnessed power occur at $105,000 < Re < 115,000$ at lower spacing (1.57) and $110,000 < Re < 115,000$ at higher spacing (2.01 and 2.57). In our experiments, at lower spacing, the cylinders move as one single bluff body for most cases; at higher spacing (2.57), the vortex and gap-flow in between the cylinders may have a positive effect ($K=400\text{N/m}$, $\zeta_{harness}=0.24$), or a negative effect ($K=1,200\text{N/m}$, $\zeta_{harness}=0.24$) on the harnessed power (See Fig.15).

466 The oscillatory response of the systems that generate optimal power for each spacing and its
 467 respective K and $\zeta_{harness}$ are studied in the next subsection.

Figure 6. Harnessed power at $L/D=1.57$

4.3. Configuration for Optimal Power Conversion

For a given converter, the spacing L/D between the two cylinders would be fixed. Thus, the power output curves are considered separately for each of the three values of spacing. The amplitude ratio A/D and the frequency ratio $f^*=f_{osc}/f_{n,water}$ along with the optimal power output are presented in Figures 9 and 10 for $L/D=1.57$, in Figures 11 and 12 for $L/D=2.01$, in Figures 13 and 14 for $L/D=2.57$. The amplitude A of the cylinder is calculated by averaging the absolute values of the 30 highest values of positive and 30 highest negative peaks in 60 seconds of running time past the transient. The mean absolute deviation is denoted by vertical bars. The frequency ratios $f^*=f_{osc}/f_{n,water}$ of the corresponding spacing are plotted versus reduced velocity U^* . The oscillation frequency f_{osc} is calculated by FFT (Fast Fourier transform) of the time history of the cylinder with the exclusion of the beginning and end transients.

Tables 4, 5, and 6 present the comparison of converted power for two tandem cylinders to a single cylinder at optimal power for the three spacing values in the galloping range. Those are $K=1,200\text{N/m}$, 800N/m and 400N/m at $L/D=1.57$, 2.01 and 2.57 , respectively. The harnessed power and the efficiency of the upstream (1st), the downstream (2nd), and the isolated cylinder. The ratio of power harnessed by synergy of two tandem cylinders to the power of the isolated cylinder is also presented, proving the benefit of synergy in a tandem configuration. Based on Tables 4-5 and Figs. 9~14, the corresponding oscillatory response, harnessed power and efficiency are discussed in detail.

Figure 9. Amplitude A/D and frequency $f^*=f_{osc}/f_{n,water}$ response for $K=1,200N/m$ at $m^*=1.343$ and $L/D=1.57$

Figure 10. Power and efficiency curves for $m^*=1.343$, $L/D=1.57$, $\zeta_{harness}=0.24$ at $K=1,200N/m$ compared to a single cylinder

498 **Table 4. Comparison of converted power for two tandem cylinders to a single cylinder at**
 499 **$L/D=1.57$ of $K=1,200\text{N/m}$**

U (m/s)	Converted Power(Watts)				Ratio	Efficiency (%)		Ratio
	1st Cyl.	2nd Cyl.	Synergy	Isolate		Synergy	Isolate	
0.95	17.43	5.94	23.37	5.36	4.36	34.25	12.35	2.77
1.03	20.73	9.41	30.14	8.01	3.76	33.49	12.13	2.76
1.15	24.7	14.86	39.56	9.29	4.26	30.84	9.83	3.14
1.19	25.85	14.78	40.63	10.59	3.84	28.37	9.79	2.90
1.27	26.92	16.6	43.52	15.02	2.90	25.11	10.31	2.44
1.31	26.56	18.41	44.97	17.71	2.54	24.05	10.54	2.28

500

501 **Figure 11. Amplitude A/D and frequency $f^*=f_{osc}/f_{n,water}$ response for $K=800\text{N/m}$ at $m^*=1.343$**
 502 **and $L/D=2.01$**

Figure 12. Power and efficiency curves for $m^*=1.343$, $L/D=2.01$, $\zeta_{harness}=0.24$ at $K=800N/m$ compared to single cylinder

Table 5. Comparison of converted power for two cylinders in tandem to a single cylinder at $L/D=2.01$ of $K=800N/m$

U (m/s)	Converted Power(Watts)				Ratio	Efficiency (%)		Ratio
	1st Cyl.	2nd Cyl.	Synergy	Isolate		Synergy	Isolate	
0.95	12.25	2.56	14.81	3.77	3.93	27.48	8.2	3.35
1.03	16.57	2.73	19.3	6.48	2.98	24.44	8.63	2.83
1.15	17.81	4.35	22.16	9.33	2.38	19.58	7.65	2.56
1.19	19.57	4.86	24.43	10.01	2.44	18.83	7.6	2.48
1.27	24.8	7.8	32.6	12.29	2.65	18.58	6.96	2.67
1.31	26.62	7.29	33.91	13.4	2.53	17.12	6.12	2.80

Figure 13. Amplitude A/D and frequency $f^* = f_{osc}/f_{n,water}$ response for $K=400N/m$ at $m^*=1.343$ and $L/D=2.57$

Figure 14. Power and efficiency curves for $m^*=1.343$, $L/D=2.57$, $\zeta_{harness}=0.24$ at $K=400N/m$ compared to single cylinder

Table 6. Comparison of converted powers for two tandem cylinders to a single cylinder at $L/D=2.57$ of $K=400N/m$

U (m/s)	Converted Power(Watts)				Ratio	Efficiency (%)		Ratio
	1st Cyl.	2nd Cyl.	Synergy	Isolate		Synergy	Isolate	
0.95	9.41	4.35	13.76	1.02	13.49	14.96	2.24	6.68
1.03	15.05	7.45	22.5	6.12	3.68	15.87	6.13	2.59
1.15	18.11	9.35	27.46	8.49	3.23	12.98	5.53	2.35
1.19	18.67	9.64	28.31	9.6	2.95	11.97	5.47	2.19
1.27	21.94	12.08	34.02	12.84	2.65	11.17	5.45	2.05
1.31	22.69	13.27	35.96	13.65	2.63	10.63	5.18	2.05

4.3.1. Oscillatory response A/D

For the three optimal configurations identified in this section, it is worth studying the response of the cylinders in FIM. Reference is made to each important branch in the FIM range over VIV, transition from VIV to galloping, and galloping.

Upstream cylinder:

(a) $30,000 \leq Re \leq 45,000 \sim 50,000$: this is the initial branch of VIV. The upper end of initial branches increases as stiffness increases, it varies from $0.78D$ at $K=400\text{N/m}$, $L/D=1.57$ to $1.05D$ at $K=800\text{N/m}$ $L/D=2.01$. A drop in A/D can be observed for all L/D from $\zeta_{\text{harness}}=0.00$ to $\zeta_{\text{harness}}=0.24$, Fig. 9, 11 and 13. This is due to the added damping for energy harnessing.

(b) $45,000 \leq Re \leq 70,000$ is the upper branch for $L/D=2.01$ and 2.57 (Figs. 11 and 13); $45,000 \leq Re \leq 65,000$ is the upper branch for $L/D=1.57$ (Fig. 9). The latter has narrower upper branch due to the close proximity of the two cylinders, which affects the wake formation of both cylinders. Also, compared to $L/D=1.57$, the shorter error bars in the upper branch indicate that the FIM of the cylinder at $L/D=2.01$ and 2.57 is more stable. The amplitude in the upper branch at $L/D=2.57$ is similar to the response of a single PTC-cylinder (Kim 2014), which indicates that at higher spacing, the vortex formation can be fully developed.

(c) $68,000 \sim 70,000 \leq Re \leq 80,000 \sim 90,000$: This range includes the lower branch of VIV, the desynchronization range, the gap between VIV and galloping, and the onset of galloping. Not all four sub-ranges are obvious in all amplitude response curves, because the VIV range moves with changes in K and shrinks with increasing ζ_{harness} . In addition, the velocity at which the onset of galloping occurs increases with increasing ζ_{harness} . As ζ_{harness} increases, the VIV desynchronization becomes more obvious for all K values. In this region, which includes all four sub-ranges, the

amplitude drops dramatically and random oscillatory patterns appear as expected. Consequently, the error bars become significantly larger in this region as shown in Figs. 12, 14 and 16.

(d) $80,000 \sim 90,000 \leq Re$: This is the galloping region. Galloping is known to be a high-amplitude and low-frequency FIM. The galloping generation mechanism is based on the lift instability due to asymmetry. Non-circular cross-section cylinders could be subjected to high amplitude galloping oscillation due to their geometric asymmetry with respect to the flow. The PTC on the cylinders causes the geometric asymmetry. There is an additional source of asymmetry for the downstream cylinder due to the wake of the upstream cylinder. In this study, the upstream cylinder is also affected by the downstream cylinder at closer spacing. For $K=400\text{N/m}$, the amplitude reaches $2.85D$ ($L/D=1.57$) at $Re=85,000$ compared to $2.38D$ ($L/D=2.01$) and $2.18D$ ($L/D=2.57$) at the $\zeta_{harness}=0.00$. In the galloping region, the amplitude response decreases as the $\zeta_{harness}$ increases, and decreases as K increases.

Downstream cylinder:

(a) $30,000 \leq Re \leq 45,000 \sim 50,000$ is the initial branch of VIV, in spite of the amplitude response not showing typical feature in Figures 9, 11, 13. Strong suppression can be observed at all the spring stiffness values, at $L/D=1.57$. For $K=1,200\text{N/m}$, the suppression region is up to $Re=55,000$, at $K=400\text{N/m}$, the range is up to $Re=43,000$. The suppression region increases as the spring stiffness and damping increase.

(b) $45,000 \sim 50,000 \leq Re \leq 80,000$ is the upper branch and transition region. The lower branch does not appear in the downstream cylinder response except at high stiffness and high damping. The amplitude response is higher than the upstream cylinder for $\zeta_{harness} < 0.12$, which shows that the downstream cylinder is effected by the upstream more in higher damping.

(c) $80,000 \leq Re$, is the galloping region. The maximum amplitude varies from $0.52D$ ($K=1,200\text{N/m}$, $\zeta_{\text{Harness}}=0.24$, $L/D=1.57$) to $2.85D$ ($K=400\text{N/m}$, $\zeta_{\text{Harness}}=0.00$, $L/D=1.57$) at $Re=118,000$ ($U=1.31\text{m/s}$). The short error bar at higher spacing indicates there are fewer beating phenomenon.

4.3.2. Efficiency of the two cylinders in tandem

The optimal harnessed power for each spacing L/D is plotted versus Reynolds number and flow velocity in three separate Figs. 13, 15 and 17. The right y -axis is the scale for the total converted power, which includes both the harnessed power and the dissipated power, where the latter is due to $\zeta_{\text{structure}}$. Those figures also show the single cylinder power curve with the same configuration for comparison. The following observations can be made:

(a) Considering the efficiency charts, all the optimal power curves show similar trend.

(b) All three figures show two local optima; one near the beginning of the VIV upper branch and one at the beginning of the fully developed galloping. The former is clearly the global optimum.

(c) In the VIV range, the efficiency reaches optimum at $Re=60,000$ for $L/D=1.57$; at $Re=46,000$ for $L/D=2.01$; and $Re=38,000$ for $L/D=2.57$.

(d) Tables 4, 5 and 6 show the ratios of power generated by two cylinders in tandem to the corresponding single cylinder. , In most cases, the two-cylinder efficiency is higher than twice that of the single cylinder. , The ratios range from 6.68 at $U=0.95\text{m/s}$, $L/D=2.57$, to 2.05 at $U=1.31\text{m/s}$ at the same spacing.

(e) The power optima do not coincide with the efficiency optima. The explanation lies in the denominator of the efficiency in equations (12) and (13) as defined by equation (12). The power in the denominator is $0.5\rho U^3(D+2A)L$, that is, the power through the area swept by the oscillating

cylinder rather than the projected area of the cylinder. VIV is self-limiting yielding an interior design-space point as the optimum. Galloping, on the other hand, is an instability based phenomenon and the cylinder can move in one direction as long as the Converter frame allows it. In a very large frame, high power would be produced but a higher denominator would make the efficiency nearly zero.

4.4. The Effects of Damping, Stiffness and Tandem Spacing on Harnessed and Converted Power of Upstream and Downstream Cylinders

The dependence of harness power and efficiency of the parameters (L/D , $\zeta_{harness}$, K) are studied and discussed separately based on the experimental result being cross-plotted in the next three chapters.

4.4.1. Effect of damping ratio L/D on power and efficiency

In Fig.15, the effect of harnessing damping ratio is cross-plotted from Figs. 6-8 for stiffness $K=600$. The following observations can be made:

Figure 15. Harnessed power and efficiency of upstream and downstream cylinders at various damping ratios $\zeta_{harness}$ for $K=600\text{N/m}$ and $L/D=2.57$ (cross-plotted from Figs. 6-8)

(a) For both cylinders, as the damping ratio increases, the harnessed power and the efficiency increase considerably for $\zeta_{harness} < 0.20$. For instance, the efficiency of the upstream cylinder changes from 14.3% ($K=600\text{N/m}$, $\zeta_{harness}=0.04$) to 23.4% ($K=600\text{N/m}$, $\zeta_{harness}=0.08$) at $Re=46,024$; the efficiency of the downstream cylinder changes from 4.1% to 7.4% at the same Re . At the higher end of $\zeta_{harness}$, however, increasing the harnessing damping results in little increase in efficiency, where, the efficiency of the upstream and downstream cylinders changes from 43.3% to 47.2% and from 13.7% to 14.9%, respectively. In the VIV lower branch, in transition, and in galloping, the efficiency at the higher damping ratio $\zeta_{harness}=0.24$, in both VIV and galloping of the upstream cylinder, is lower than the corresponding values for $\zeta_{harness}=0.20$, which indicates that the optimal efficiency is achieved.

(b) For the downstream cylinder in galloping, the efficiency increases as the harnessing damping increases up to a certain value and is stable after $\zeta_{harness} > 0.08$. This is so because at this spacing, the rear cylinder oscillates along with the upstream one expect for the lower end of damping ratios.

For instance, the efficiency is 4.6%, 5.18% and 5.4% at $\zeta_{harness}=0.16$, 0.20 and 0.24, respectively, at $Re=103,263$ (Fig. 5d).

4.4.2. Effect of spacing ratio on power and efficiency

The power and efficiency with same stiffness $K=400\text{N/m}$ and harness damping $\zeta_{harness}=0.16$ at three different spacing ratios are shown in Fig. 16, to study the impact of L/D . Some observations can be drawn:

Figure 16. Harness power and efficiency of upstream and downstream cylinders at various spacing ratios $L/D=1.57$, 2.01 and 2.57, $\zeta_{harness}=0.16$, of $K=400\text{N/m}$ (cross-plotted from Figs. 6-8)

(a) At lower spacing $L/D=1.57$, in VIV and transition, due to the disruption of the flow between cylinders, the harness power and efficiency of upstream cylinder do not perform at the same level of higher spacing ratios; on the contrary, the downstream cylinder performs better.

(b) At higher spacing $L/D=2.01$ and 2.57 , in VIV and transition, the upstream cylinder performs better compared to the isolated cylinder in terms of power. The downstream cylinder, on the other hand, shows strong suppression in initial and upper branches. From VIV lower branch to transition, the power of the upstream cylinder drops drastically. The power of the downstream cylinder, however, does not decrease in those regimes.

(c) In galloping at higher spacing $L/D=2.57$, the upstream cylinder shows an enhancement in power, and the rear one is lower than that of $L/D=2.01$. The overall performance is better at 2.57 . At lower spacing $L/D=1.57$, the power and efficiency show intensive disruptions due to cylinder proximity. Specifically, at $95,000 < Re < 107,000$, the power drop is around 50% for both cylinders; the efficiency remains relatively stable and decreases as the flow velocity increases due to the amplitude increase which affects the efficiency denominator as defined in equation (12).

4.4.3. Effect of spring stiffness on power and efficiency

The power and efficiency with spacing $L/D=2.57$ and harness damping $\zeta_{harness}=0.20$ at various stiffness values are shown in Fig.17. Some observations can be drawn:

Figure 17. Harnessed power and efficiency of upstream and downstream cylinders at various spring stiffness values $K=400, 600, 800, 1,000$ and $1,200\text{N/m}$ for $L/D=2.57$ and $\zeta_{\text{harness}}=0.20$ (cross-plotted from Figs. 6-8)

(a) For the upstream cylinder, the power efficiency curves are shown as a function of stiffness K in Figs. 17a, 17c. The flow velocity at peak efficiency at different stiffness in the VIV region varies from 0.55 m/s to 0.65 m/s and the corresponding harnessing efficiency varies from 16% ($K=400\text{N/m}$) to 53% ($K=800\text{N/m}$), respectively. The optimal power efficiency can be clearly seen to occur for $K=800\text{N/m}$. In the galloping region, the efficiency shows a small constant decrease with velocity. Stiffness $K=1,200\text{N/m}$ has the highest harnessing efficiency due to the decrease of

its amplitude response. A slowdown in efficiency decrease as the stiffness K increases can be seen in the galloping region. VIV starts around $U^*=5.0$ - higher at higher stiffness. As the flow velocity increases, the harnessed power at higher stiffness $K=1,200\text{N/m}$ reaches its local optimum around $Re=75,000$.

(b) For the downstream cylinder in the galloping range, the efficiency of $K=400\text{N/m}$ is higher than for the other stiffness values. This can be explained by equations (13) and (14), as the denominator is lower when the amplitude decreases. At higher stiffness $K=1,200\text{N/m}$, the power drops significantly after $Re>100,000$, which contributes to the drops in synergy between the two cylinders. The lower stiffness cases are stable in both power and efficiency.

4.5. Power Envelope for Two Cylinders in Tandem

Fig. 18 shows the harnessed power envelopes for each L/D combined with the single cylinder power envelope for the same mass ratio. The optimal parameters for each flow velocity are listed in Tables 7, 8, 9 and 10 in the Appendix.

Figure 18. Optimal power curves for spacing $L/D=1.57$, 2.01 , 2.57 , and for single cylinder at $m^*=1.343$

A few observations can be drawn:

(a) At the VIV initial branch up to the upper branch ($U < 0.47 \text{ m/s}$), a steady growth of power can be observed. Softer spring stiffness with lower harness damping ratio initiate power harnessing at flow velocity as low as 0.3946 m/s at all spacing values tested. The combination of higher spring stiffness with higher mass ratio starts power harnessing at higher flow velocity, but yields higher power in the VIV upper branch. As the L/D increases, the power in this region increases. The highest power is harnessed at $L/D=2.57$ is 17.1 W compared to 12.5 W at $L/D=1.57$, which indicates that closer spacing disturbs the vortex formation and induced lift. As spacing increases, the oscillation becomes more stable yielding more harnessed power. There is enhancement of the ratio of the harnessed power of a two-cylinder converter to a single cylinder. The lower stiffness results

in lower $f_{n, water}$ and yields optimal harnessed power at the initial branch. This is true for all the tests regardless of the spacing, single or tandem settings.

(b) In the upper branch ($U=0.47\text{m/s}\sim 0.75\text{m/s}$), the harnessed power drops sharply for $L/D=2.57$, but maintains a steady growth for $L/D=1.57$. The former is due to the drop in amplitude caused by the separation of VIV and galloping for both the upstream and downstream cylinders. At $L/D<2.0$, as discussed in the Introduction, the oscillation/VIV for two tandem cylinders shows similarities to the case of one cylinder in FIM, which are subjected to lower disturbance by the interaction between cylinders.

The optimal harnessed power moves towards the higher stiffness/ $f_{n, water}$. This is consistent for all spacing values. In addition, as the spacing increases, the optimal harnessed power in this region also increases. At the end of upper branch, the optimal power is: $P_{harness}=8.70\text{W}$ ($L/D=1.57$, $K=1,200\text{ N/m}$, $\zeta_{harness}=0.24$), $P_{harness}=13.21\text{W}$ ($L/D=2.01$, $K=1,200\text{ N/m}$, $\zeta_{harness}=0.24$), and $P_{harness}=15.06\text{W}$ ($L/D=2.57$, $K=1,200\text{ N/m}$, $\zeta_{harness}=0.24$). The optimal setting in the upper branch should be high stiffness/ $f_{n, water}$ with high harness damping at $L/D=2.57$, albeit the fact that the upward trend in power harnessed is not as the spacing increases.

Comparing the above values to $P_{harness}=8.38\text{W}$ ($K=1,200\text{N/m}$, $\zeta_{harness}=0.20$), we find that the ratios are 1.04 at $L/D=1.57$, 1.58 at $L/D=2.01$, and 1.80 at $L/D=2.57$, respectively. In the upper branch, the configuration of two cylinders in tandem enhances the harnessed/converted power by less than twice, which is due to the nature of VIV. The wake of the upstream cylinder has a negative impact on the downstream cylinder, which can be observed through Figure 16 at $Re=30,000$. The wake/gap-flow of the upstream cylinder at lower flow velocity goes against the FIM direction of the downstream cylinder in the fluid velocity range causing suppression and thus, low harnessed power by the downstream cylinder. Fig. 19 briefly shows the gap-flow at VIV and galloping

regime. At the VIV regime (Left), the gap flow (blue arrow line) does not reach the top of downstream cylinder, in other words, the gap-flow goes against the motion of downstream. At the Galloping regime (Right), the gap-flow reaches the bottom of downstream cylinder, in other words, the gap-flow push the downstream to its motion direction and enhances it. Furthermore, the Gap-flow enhancement of harness power is higher in the beginning of galloping, as which indicates at Table 4, 5 and 6, at the beginning is around 3 times of ratio, and slowly drops to 2.6 times of ratio, which indicates that partial of the gap-flow/interaction may start moving to the opposite motion direction as the flow progress.

Figure 19. Gap-flow behavior in VIV (Left, $Re=36,000$) and Galloping (Right, $Re=90,000$), the yellow arrow line indicate motion direction of cylinders (visualization of the flow past cylinders uses two 5W argon lasers and aluminum oxide particles of $100\mu m$)

(c) In the transition from VIV to galloping region ($U=0.75\text{m/s}\sim 0.91\text{m/s}$), the harnessed power of the tandem cylinders is distinctively different from the corresponding one isolated cylinder. Harnessed power increases monotonically in close spacing $L/D=1.57$. On the contrary, harnessed power by the isolated cylinder and two cylinders at $L/D=2.01$ and $L/D=2.57$ spacing exhibit a significant drop. The isolated cylinder exhibits the strongest drop in harnessed power due to the desynchronization from VIV. The optimal harnessed power is: $P_{\text{harness}}=20.93\text{W}$ ($L/D=1.57$, $K=800\text{N/m}$, $\zeta_{\text{harness}}=0.16$), $P_{\text{harness}}=15.72\text{W}$ ($L/D=2.01$, $K=1,200\text{N/m}$, $\zeta_{\text{harness}}=0.24$), $P_{\text{harness}}=15.08\text{W}$ ($L/D=2.57$, $K=1,200\text{N/m}$, $\zeta_{\text{harness}}=0.24$). The optimal power is achieved at the closest spacing in this region. Compared to $P_{\text{harness}}=5.63\text{W}$ ($K=1,200\text{N/m}$, $\zeta_{\text{harness}}=0.12$), the ratios are 3.72 at $L/D=1.57$; 2.79 at $L/D=2.01$; and 2.68 at $L/D=2.57$, respectively. The ratio increases drastically due to the desynchronization. It should be noted that the single cylinder can harness more power by properly adjusting the stiffness and harness damping in this region (Sun, et.al. 2016). At close spacing, there is no trend of lowering the harnessed power.

(d) In the galloping region ($U>0.91\text{m/s}$), as shown in Table 4, 5 and 6, the power harnessed by the two tandem cylinders increases as the flow velocity increase. However, there is a critical flow velocity ($U=0.91\text{m/s}$) that the power decreases for $L/D=2.01$ and $L/D=2.57$. The optimal harnessed power values are $P_{\text{harness}}=41.50\text{W}$ ($L/D=1.57$, $K=1,200\text{N/m}$, $\zeta_{\text{harness}}=0.24$), $P_{\text{harness}}=33.91\text{W}$ ($L/D=2.01$, $K=800\text{N/m}$, $\zeta_{\text{harness}}=0.24$), $P_{\text{harness}}=33.20\text{W}$ ($L/D=2.57$, $K=400\text{N/m}$, $\zeta_{\text{harness}}=0.24$) for the three spacing values. In the galloping region, the overall optimal power is achieved at the closest spacing with the highest stiffness. Comparing to $P_{\text{harness}}=16.80\text{W}$ ($K=1,200\text{N/m}$, $\zeta_{\text{harness}}=0.20$), the ratio values are 2.47 at $L/D=1.57$, 2.02 at $L/D=2.01$, and 2.00 at $L/D=2.57$, respectively. The ratio of optimal harnessed power shows a downward trend as the velocity increases. This is related to the spacing and the velocity as their combination dictates the

timing the upstream cylinder shear layers or rolled-up vortices impinge on the downstream cylinder. This is a separate and extensive study underway in the MRELab.

CONCLUSIONS

The synergistic flow induced motions and the hydrokinetic power conversion capacity of two rigid, circular tandem-cylinders, with passive turbulence control, placed on end-springs, were investigated experimentally, for various values of stiffness, spacing, and harnessing damping, for Reynolds numbers ($30,000 < Re < 120,000$), which fall in the TrSL3 high-lift flow regime. The corresponding velocity range is $0.395\text{m/s} < U < 1.315\text{m/s}$. The corresponding Reduced Velocity range is ($2.92 < U^* < 17.73$). The optimal power envelope was generated. All experiments were conducted in the LTFSW Channel of the MRELab of the University of Michigan using the second generation of the Vck virtual oscillator developed in the MRELab. The results, including oscillatory response, harnessed and converted power, and the efficiency of the two-cylinder VIVACE Converter were analyzed and presented as function of the three proportional velocity measures: Reynolds number, flow velocity, and reduced velocity. The effect of harnessing damping, tandem spacing, and spring stiffness were presented and discussed separately. The following conclusions can be drawn from the tests conducted.

- (1) In the galloping range, the harnessed power and the converted power by the two-cylinder Converter show a beneficial synergy between the two cylinders. The power harnessed or converted by the two tandem cylinders is more than twice the corresponding single cylinder power for the same parameters. For the tested values of cylinder spacing, two cylinders harness power between 2.56 and 7.5 times the power of a single cylinder.

- 761 (2) In the galloping range, the MHK power harnessed by the upstream cylinder – as affected by
762 the presence of the downstream cylinder - is increased by up to 100% compared to a single
763 isolated cylinder.
- 764 (3) In the galloping range, the MHK power harnessed by the downstream cylinder as well as its
765 FIM are affected to a lesser extent by the synergy of the two cylinders.
- 766 (4) In the VIV range, the improvement due to the presence of the downstream cylinder is minimal
767 and not worth the cost of designing a more complex converter. On the contrary, both in
768 transition from VIV to galloping and in galloping, the power increase due to the synergy
769 between two cylinders increases the power by a factor of up to 15 times the power generated
770 by one cylinder at the same conditions.
- 771 (5) The two-cylinder Converter can harness energy from flows as slow as 0.4m/s at low harness
772 damping and lower stiffness. There is no upper limit on the velocity for hydrokinetic energy
773 conversion based on the alternating lift concepts implemented in the Converter because they
774 are scalable.
- 775 (6) Close spacing and high spring stiffness yield the highest harnessed power and converted
776 power.
- 777 (7) The optimal harnessed power shifts from high stiffness at lower spacing to softer springs at
778 higher spacing.
- 779 (8) To harness/convert power, damping is added to the system. That results in reduction of the
780 oscillator amplitude in both the VIV and the galloping ranges. In addition, damping delays
781 the onset of galloping. The combination of those two effects develops a gap in power
782 harnessing between VIV and galloping. That gap is minimized with small spacing between

the two cylinders. At spacing of 1.57 diameters center-to-center, the gap between lower branch and galloping can be closed.

(9) There are two local power optima, one at the end of VIV and the other at the end of galloping. Actually, it is the end of the experimental capacity of the facility as galloping is an instability phenomenon without upper limit in velocity.

(10) There are two local efficiency optima, one at the beginning of VIV and the other at the beginning of galloping. That is the two local power optima and the two local efficiency optima do not coincide because the area in the denominator of the efficiency is a function of the amplitude of oscillation. The latter depends on the flow velocity as well as damping.

(11) The highest efficiency reached in this set of tests is 63% of the Betz limit for converted power. It occurs at center-to-center spacing between the two cylinders of 2.57 diameters and spring stiffness $K=1,200\text{N/m}$.

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APPENDIX

Table 7. Parameters for optimal harnessed and converted power for two cylinders in tandem; spacing $L/D=1.57$, $m^*=1.343$

Test Condition		Optimal harnessed power [W]	Optimal converted power [W]	Stiffness K [N/m]	$\zeta_{harness} = \frac{c_{harness}}{2\sqrt{m_{osc}K}}$
U (m/s)	Re	Two cylinders in tandem with spacing $L/D=1.57$			
0.35	31,714.23	0.09	0.12	400	0.08
0.39	35,291.69	1.04	1.13	400	0.24
0.43	38,869.15	2.02	2.36	800	0.12
0.47	42,446.62	2.62	3.05	800	0.12
0.51	46,024.08	3.42	3.71	600	0.24
0.55	49,601.55	4.89	5.29	600	0.24
0.59	53,179.01	6.10	6.60	600	0.24
0.63	56,756.48	6.97	7.55	600	0.24
0.67	60,333.94	8.37	9.06	1,000	0.24
0.71	63,911.41	8.83	9.56	1,200	0.24
0.75	67,488.87	8.70	9.43	1,200	0.24
0.79	71,066.34	12.49	14.05	800	0.16
0.83	74,643.80	13.90	15.06	600	0.24
0.87	78,221.27	18.50	20.81	800	0.16
0.91	81,798.73	20.93	23.54	800	0.16
0.95	85,376.20	23.11	25.03	1,000	0.24
0.99	88,953.66	25.62	27.76	1,000	0.24
1.03	92,531.13	27.84	30.16	1,200	0.24
1.07	96,108.59	31.47	34.09	1,200	0.24
1.11	99,686.06	34.54	37.42	1,200	0.24
1.15	103,263.52	36.52	39.57	1,200	0.24
1.19	106,840.99	37.50	40.63	1,200	0.24
1.23	110,418.45	38.36	41.55	1,200	0.24
1.27	113,995.92	40.18	43.53	1,200	0.24
1.31	117,573.38	41.50	44.96	1,200	0.24

Table 8. Parameters for optimal harnessed and converted power for two cylinders in tandem; spacing $L/D=2.01$, $m^*=1.343$

Test Condition		Optimal harnessed power [W]	Optimal converted power [W]	Stiffness K [N/m]	$\zeta_{harness} = \frac{c_{harness}}{2\sqrt{m_{osc}K}}$
U (m/s)	Re	Two cylinders in tandem with spacing $L/D=2.01$			
0.35	31,714.23	1.05	1.15	400	0.20
0.39	35,291.69	1.83	2.01	400	0.20
0.43	38,869.15	2.60	2.82	400	0.24
0.47	42,446.62	3.27	3.54	600	0.24
0.51	46,024.08	4.57	4.95	600	0.24
0.55	49,601.55	5.69	6.17	600	0.24
0.59	53,179.01	6.80	7.37	800	0.24
0.63	56,756.48	8.20	8.88	800	0.24
0.67	60,333.94	9.43	10.22	1,200	0.24
0.71	63,911.41	11.50	12.46	1,200	0.24
0.75	67,488.87	13.21	14.31	1,200	0.24
0.79	71,066.34	14.50	15.71	1,200	0.24
0.83	74,643.80	14.88	14.48	1,200	0.20
0.87	78,221.27	15.72	17.03	1,200	0.24
0.91	81,798.73	11.14	12.53	800	0.16
0.95	85,376.20	20.29	22.32	1,000	0.20
0.99	88,953.66	19.16	21.08	1,000	0.20
1.03	92,531.13	19.73	21.38	400	0.34
1.07	96,108.59	24.40	26.43	1,200	0.24
1.11	99,686.06	22.79	24.69	400	0.24
1.15	103,263.52	21.66	23.83	1,000	0.20
1.19	106,840.99	21.71	24.43	800	0.16
1.23	110,418.45	24.34	27.39	800	0.16
1.27	113,995.92	30.27	32.80	800	0.24
1.31	117,573.38	31.30	33.91	800	0.24

Table 9. Parameters for optimal harnessed and converted power for two cylinders in tandem; spacing $L/D=2.57$, $m^*=1.343$

Test Condition		Optimal harnessed power [W]	Optimal converted power [W]	Stiffness K [N/m]	$\zeta_{harness} = \frac{c_{harness}}{2\sqrt{m_{osc}K}}$
U (m/s)	Re	Two cylinders in tandem with spacing $L/D=2.57$			
0.35	31,714.23	1.25	1.41	600	0.16
0.39	35,291.69	1.50	1.65	400	0.20
0.43	38,869.15	2.41	2.61	400	0.24
0.47	42,446.62	3.23	3.50	400	0.24
0.51	46,024.08	4.73	5.20	800	0.20
0.55	49,601.55	7.63	8.27	800	0.24
0.59	53,179.01	8.82	9.56	800	0.24
0.63	56,756.48	9.85	9.26	1,000	0.24
0.67	60,333.94	10.83	11.74	1,200	0.24
0.71	63,911.41	13.31	14.42	1,200	0.24
0.75	67,488.87	15.06	16.32	1,200	0.24
0.79	71,066.34	15.89	17.22	1,200	0.24
0.83	74,643.80	16.75	18.15	1,200	0.24
0.87	78,221.27	15.08	16.33	1,200	0.24
0.91	81,798.73	11.83	13.31	600	0.16
0.95	85,376.20	18.38	20.68	1,000	0.16
0.99	88,953.66	20.19	22.72	1,000	0.16
1.03	92,531.13	21.08	23.18	1,200	0.20
1.07	96,108.59	25.62	28.19	1,200	0.20
1.11	99,686.06	28.67	31.06	1,200	0.24
1.15	103,263.52	25.34	27.45	400	0.24
1.19	106,840.99	26.13	28.31	400	0.24
1.23	110,418.45	28.36	30.72	400	0.24
1.27	113,995.92	31.40	34.02	400	0.24
1.31	117,573.38	33.20	35.96	400	0.24

947 **Table 10. Parameters for optimal harnessed and converted power a single cylinder;**
948 **$m^*=1.343$**

Test Condition		Optimal harnessed power [W]	Optimal converted power [W]	Stiffness K [N/m]	$\zeta_{harness} = \frac{c_{harness}}{2\sqrt{m_{osc}K}}$
U (m/s)	Re	Single-isolated cylinder			
0.35	31,714.23				
0.39	35,291.69	1.04	1.21	400	0.12
0.43	38,869.15	1.34	1.51	400	0.16
0.47	42,446.62	1.68	1.90	400	0.16
0.51	46,024.08	3.02	3.32	600	0.20
0.55	49,601.55	4.45	4.89	1,000	0.20
0.59	53,179.01	6.35	6.99	1,000	0.20
0.63	56,756.48	8.22	8.90	1,000	0.24
0.67	60,333.94	9.28	10.05	1,000	0.24
0.71	63,911.41	9.24	10.01	1,200	0.24
0.75	67,488.87	8.38	9.22	1,200	0.20
0.79	71,066.34	4.89	5.70	1,200	0.12
0.83	74,643.80	4.20	5.25	1,200	0.08
0.87	78,221.27	3.55	4.44	1,200	0.08
0.91	81,798.73	5.63	6.57	1,200	0.12
0.95	85,376.20	6.80	7.93	1,200	0.12
0.99	88,953.66	5.47	6.15	755	0.16
1.03	92,531.13	5.84	6.54	755	0.08
1.07	96,108.59	9.08	9.84	600	0.24
1.11	99,686.06	9.92	10.91	1,200	0.20
1.15	103,263.52	9.54	10.33	600	0.24
1.19	106,840.99	9.50	10.45	600	0.20
1.23	110,418.45	11.98	13.18	1,200	0.20
1.27	113,995.92	14.21	15.63	1,200	0.20
1.31	117,573.38	16.80	18.48	1,200	0.20

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