A Model-Based Framework for Fault Estimation and Accommodation Applied to Distributed Energy Resources

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Abstract

This paper presents the development and approach of a model-based fault identification and accommodation framework applied to sampled-data controlled distributed energy resources subject to control actuator faults. The main objective of the proposed approach is to handle faults that degrade stability as well as performance, while remaining robust to false alarms. The proposed method allows for dual fault detection and estimation, through the use of an embedded system model that minimizes the residual between the estimated and sampled states at each sampling period by adjusting a fault parameter in the embedded model over a past horizon. The resulting fault parameter estimate is then used by the control system to find an optimal fault accommodation strategy by minimizing a predefined performance metric whilst ensuring closed-loop stability. The developed fault accommodation framework is then applied to a simulated model of a solid oxide fuel cell subject to both stability and performance degrading faults in the control actuators. A discussion of some of the practical implementation issues associated with the developed framework is also included.

Keywords: Fault estimation; Fault accommodation; Model-based control, Sampled-data systems, Distributed energy resources

1. Introduction

Distributed energy resources (DERs) are composed of modular energy generation units including, for example, micro-turbines, fuel cells, renewable energy

28 systems, battery storage and other such technologies deployed close to the point
29 of consumption. The modular nature of DERs allows them to be integrated with
30 existing grid infrastructure or implemented in a stand-alone manner. DERs offer
31 advantages over conventional grid electricity by offering end users a diversified
32 fuel supply; higher power reliability, quality, and efficiency; lower emissions and
33 greater flexibility to respond to changing energy needs. While DERs have aided
34 in the integration of sustainable energy resources into the power grid, they also
35 pose fundamental challenges, including fluctuations in generation from inter-
36 mittent availability of renewable resources as well as the resulting increased
37 communication between generation and loads.

38 These and other challenges have been the subject of a significant and growing
39 body of research work on the control of DERs (e.g., [1, 2, 3, 4, 5, 6]). Important
40 contributions in this direction include the use of conventional and model-based
41 feedback control algorithms to regulate various types of grid-connected DERs
42 in order to enhance power system stability (e.g., [7, 8, 9, 10]), mitigate power
43 quality problems (e.g., [11]) and improve the continuity of electricity supply
44 (e.g., [12]), the development of various distributed control and coordination
45 architectures using multi-agent systems (e.g., [13, 14]) and predictive control
46 approaches (e.g., [15, 16]).

47 While control is necessary to ensure that the load demand is met and that
48 economical operation of each DER is maintained, the stability and performance
49 of smart grid DERs have not been rigorously assessed in the presence of faults or
50 failures at the local and network levels. This is an important problem given the
51 fact that the distributed power market is driven by the need for reliable high-
52 quality power, and the fact that local faults and disruptions in power flow can
53 have a substantial impact, especially in situations when DERs are integrated to
54 support grid operations. The timely identification and mitigation of faults at the
55 local level before they cascade through the network are important capabilities
56 that ensure autonomous control and protection, which when coupled with proper
57 supervisory oversight enable distributed energy generation to provide highly
58 reliable services under all disturbance and fault scenarios. In this context, fault-

59 tolerant control is an important tool for reducing performance deterioration in
60 the face of faults and uncertainties in the system components, such as actuators
61 and sensors, resulting in increased reliability of the network.

62 In prior work ([17]), fault-tolerant control has been studied in the context
63 of a small-scale network of solid oxide fuel cells (SOFCs), where the focus has
64 been on the detection and handling of destabilizing failure events at the local
65 level without supervisory oversight. Local monitoring of the health status of
66 each DER took place through use of a time-varying alarm threshold on a prop-
67 erly designed observer-based output residual. Exploiting the inherent actuator
68 redundancy in SOFCs, three stabilizing controller configurations were designed,
69 and a methodology for active switching between them in the event of a thresh-
70 old breach was developed. The main contribution was the characterization of a
71 stability region within which each controller configuration could operate. How-
72 ever, the faults considered in that work were limited to total failure events, thus
73 negating the need for fault isolation or estimation. The proposed stability-based
74 scheme also was not designed to detect faults that only degrade performance
75 but do not compromise stability, and this can lead to sub-optimal performance.

76 Motivated by these considerations, this paper presents a framework for local
77 actuator fault-tolerant control that accounts for both performance and stability
78 degrading faults, while optimizing suitable performance metrics and maintaining
79 closed-loop stability. The developed framework is realized through an integrated
80 approach that brings together model-based control, data-based fault estimation
81 and performance-based fault accommodation. The rest of the paper is organized
82 as follows. A SOFC system is initially introduced in Section 2 to motivate the
83 development and application of the proposed framework. This model system is
84 used to design a local model-based state feedback controller that operates on
85 sampled data and characterize its stability properties in the presence of faults.
86 A fault estimation scheme that provides an estimate of the fault magnitude
87 by solving a data-based moving horizon optimization problem is then intro-
88 duced in Section 3. A performance index suitable for sampled-data systems is
89 then introduced in Section 4 and used to develop a performance-based fault

90 accommodation strategy that alters model and control parameters to minimize
91 post-fault performance losses while maintaining closed-loop stability. Practical
92 implementation issues, such as the need to minimize false alarms, are discussed
93 and guidelines for dealing with these issues are presented. Finally, simulation
94 results are presented in Section 5.

95 **2. Preliminaries**

96 *2.1. Motivating example: A solid oxide fuel cell*

97 Due to their modular and stable nature SOFCs are of particular interest
98 when it comes to decentralized distributed energy generation. A SOFC consists
99 of two porous electrodes, an anode and a cathode, in contact with a solid metal
100 oxide electrolyte between them. Hydrogen is fed along the surface of the anode
101 where it releases electrons that migrate externally towards the cathode. The
102 electrons combine with oxygen fed along the surface of the cathode to form oxide
103 ions. These ions diffuse through the electrolyte towards the anode where they
104 combine with the hydrogen ions to produce water and power. Under standard
105 modeling assumptions (see [18]), the following dynamic model of the SOFC
106 stack can be obtained from first principles:

$$\begin{aligned}
\dot{p}_{H_2} &= \frac{T_s}{\tau_{H_2}^* T^* K_{H_2}} (q_{H_2}^{in} - K_{H_2} p_{H_2} - 2K_r I) \\
\dot{p}_{O_2} &= \frac{T_s}{\tau_{O_2}^* T^* K_{O_2}} (q_{O_2}^{in} - K_{O_2} p_{O_2} - K_r I) \\
\dot{p}_{H_2O} &= \frac{T_s}{\tau_{H_2O}^* T^* K_{H_2O}} (q_{O_2}^{in} - K_{H_2O} p_{H_2O} + 2K_r I) \\
\dot{T}_s &= \frac{1}{m_s C_{ps}} \left[\sum q_i^{in} \int_{T_{ref}}^{T_{in}} C_{p,i}(T) dT \right. \\
&\quad \left. - \sum q_i^{out} \int_{T_{ref}}^{T_s} C_{p,i}(T) dT - 2K_r I \Delta \hat{H}_r^o - V_s I \right] \\
V_s &= N_0 \Delta E - r_0 \exp \left[\alpha \left(\frac{1}{T_s} - \frac{1}{T_0} \right) \right] I \\
\Delta E &= \left[\Delta E_0 + \frac{RT}{2F} \ln \left(\frac{p_{H_2} p_{O_2}^{(0.5)}}{p_{H_2O}} \right) \right]
\end{aligned} \tag{1}$$

107 For the component mass balances, p_i is the partial pressure of component i ,
 108 T_s is the stack temperature, τ_i is the time constant for component i , described
 109 by $\frac{V}{K_i R T^*}$ with V being the volume component i is contained in, K_i is the
 110 valve molar constant for component i , R is the gas constant, $\tau_i^* = \tau_i \Big|_{T_s=T^*}$,
 111 and T^* is the operating temperature. q_i^{in}, p_i are the inlet flow rate and partial
 112 pressure of component i , respectively, $K_r = \frac{N_O}{4F}$, where N_O is the number of
 113 cells in the stack and F is Faraday's constant. Lastly I is the load current. As
 114 for the energy balance, m_s and C_{ps} are the mass and heat capacity of the stack,
 115 T_{ref} and T_0 are the reference and feed temperatures, $C_{p,i}$ is the heat capacity
 116 of component i , q_i^{out} is the outlet flow rate of gas i , $\Delta \hat{H}_r^o$ is the specific heat of
 117 reaction, V_s is the stack voltage, r_0 is the internal resistance, α is the resistance
 118 slope, and ΔE_0 is the standard cell potential.

119 2.2. Control problem formulation with fault modeling

120 The SOFC system modeled in Eq.1 has three potential manipulated vari-
 121 ables. Previous work has exploited this fact to implement multiple control
 122 configuration in the case of actuator failure (see [17]). However, here the focus

will be on the accommodation of faults through model manipulation, and the main control configuration will be the one utilizing the hydrogen gas flow rate as the manipulated variable (this should be assumed unless otherwise noted). A similar approach can be used if other manipulated variables are chosen instead (the oxygen flow rate or the feed temperature).

The problem formulation is the same as in [17], where the controller design is based on the linearized system model about the desired operating set-point, where the time evolution of the system is given by:

$$\dot{x} = Ax + B\theta u \quad (2)$$

$$\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}\hat{\theta}u \quad (3)$$

$$u = K\hat{x} \quad (4)$$

where $x = [p'_{H_2} \ p'_{O_2} \ p'_{H_2O} \ T'_s]^T$ is the state vector of deviation variables from the desired steady state operation point ($x_{ss} = [0.3973 \ 0.3142 \ 0.2951 \ 1162.8]^T$), $\hat{x}$ is the model state which provides an estimate of x , $\hat{A}, \hat{B}$ are constant model matrices that approximate the linearized system matrices, A , and B allowing for consideration of plant-model mismatch (see Eq.5 for the values of the nominal system state matrices). The effect of faults is captured through θ which, in general, is a diagonal matrix whose diagonal elements represent the fault or health status of each control actuator, where a value of 1 represents a healthy actuator and a value of zero reflects total failure. $\hat{\theta}$ is an estimated fault parameter used in the control model and will serve as a key fault accommodation parameter. Finally, u denotes the control action, with K being the feedback controller gain.

Remark 1. *It should be noted that the dimension of the $\theta, \hat{\theta}$ matrices are dependent on the number of manipulated inputs, and that in this study they are both taken as scalars.*

$$\begin{aligned}
A &= \begin{bmatrix} -0.034998 & 0 & 0 & 0 \\ 0 & -0.3139 & 0 & 0 \\ 0 & 0 & -0.011666 & 0 \\ -8.9278 & -28.673 & -3.2569 & -0.010988 \end{bmatrix}, \\
B &= \begin{bmatrix} 0.0034721 \\ 0 \\ 0 \\ 0.63486 \end{bmatrix}, \quad K = \begin{bmatrix} -12.4 & 0 & 0 & 0.047 \end{bmatrix} \quad (5)
\end{aligned}$$

145 **Remark 2.** Note that in general $\hat{A} \neq A, \hat{B} \neq B$, etc. In the case of this study
146 a perfect model of the plant was assumed such that $\hat{A} = A, \hat{B} = B$, etc.

147 To analyze the stability and performance properties of the model-based con-
148 troller of Eqs.2-4, we define an augmented state vector of the form $\xi = [x \ e]^T$,
149 where $e = x - \hat{x}$ is the model estimation error. The sampled-data closed-loop
150 system can then be cast in the following form:

$$\begin{aligned}
\dot{\xi} &= \Lambda \xi \\
e(t_j) &= 0, \quad j \in \{0, 1, 2, \dots\} \quad (6)
\end{aligned}$$

151 where

$$\Lambda = \begin{bmatrix} A + B\theta K & -B\theta K \\ (A - \hat{A}) + (B\theta - \hat{B}\hat{\theta})K & \hat{A} - (B\theta - \hat{B}\hat{\theta})K \end{bmatrix} \quad (7)$$

152 Note that the model estimation error is reset to zero at each sampling time
153 due to the update of the model state using the actual sampled state. It can
154 then be shown that the solution to the augmented system takes the following
155 form:

$$\xi(t) = e^{\Lambda(t-t_j)} (I_o e^{\Lambda \Delta} I_o)^j \xi_0, \quad t \in [t_j, t_{j+1}) \quad (8)$$

156 where $\Delta = t_{j+1} - t_j$ is the sampling period and $I_o = \begin{bmatrix} I & O \\ O & O \end{bmatrix}$, with I
157 being the identity matrix. Due to the discrete sampled nature of the system it
158 can be shown that closed-loop stability is assured if the maximum eigenvalue
159 magnitude of the matrix $M := I_o e^{\Lambda \Delta} I_o$ is less than 1, i.e., $\lambda_{max}(M) < 1$.

160 **Remark 3.** *Note that in this study the linear controller was applied to the lin-*
161 *earized model of the system. When implemented on the nonlinear system, the*
162 *stability results are expected to hold locally (i.e., over a sufficiently small operat-*
163 *ing region). To expand the operating region of the controller, one may consider*
164 *a linear time-varying model, where the state and input matrices $A, B, \hat{A}, \hat{B}$, are*
165 *time-varying to allow for operation in multiple modes across multiple lineariza-*
166 *tion regimes. Another approach is to use nonlinear control techniques to design*
167 *the controller directly on the basis of the nonlinear system. In these approaches,*
168 *however, the stability analysis of the sampled-data closed-loop system tends to*
169 *be conservative in general, which typically hinders the ability to obtain an exact*
170 *characterization of the closed-loop stability region.*

171 Owing to the parametrization of the stability test matrix M in terms of the
172 various model, fault and control system parameters, the stability condition can
173 be used to systematically explore the balance between these various parameters
174 in influencing closed-loop stability. For example, for a fixed model and controller
175 design parameters, the stability condition can be used to determine the range
176 of tolerable fault magnitudes that do not compromise closed-loop stability. Al-
177 ternatively, for a given fault size, the same condition can be used to identify
178 the possible ranges for model and/or controller parameters that would ensure
179 stability in the presence of the fault. This link is the basis for stability-based
180 fault accommodation strategies.

181 3. Fault detection and estimation via moving horizon optimization

182 The first step in fault handling is the detection and estimation of faults. This
183 is done through a moving-horizon optimization problem formulation (inspired

184 by the formulation presented in [19]) in which an estimated value of θ (this will
 185 be referred to as θ^*) is computed at each sampling time by minimizing the error
 186 between the sampled plant state and the estimated value provided by the fault
 187 diagnoser model. The cost function to be minimized is given by:

$$J(\zeta_j, \bar{\theta}) = \sum_{p=j-N_I}^{j-1} \left(\|x[p+1] - \bar{x}(\zeta_j, \bar{\theta})[p+1]\|^2 \right) \quad (9)$$

188 where

$$\begin{aligned} \zeta_j &= \left(x[j-q], u[j-q], \hat{\theta}[j-q], \Delta[j-q] \right), \\ q &\in \{0, 1, 2, \dots, N_I\} \end{aligned} \quad (10)$$

189 and N_I is the size of the horizon window, where each point in N_I is a time at
 190 which the plant state is sampled. $\bar{x}$ is obtained through initializing an embedded
 191 model of the system at $\bar{x}[i] = x[i]$ and then simulating the plant response over
 192 the sampling period using the ζ_j parameters fed to J at which point $\bar{x}[p+1]$ is
 193 compared with $x[p+1]$, and then $\bar{x}$ is initialized at time $[p+1]$ and the process
 194 is repeated over the horizon window. By minimizing the cost function in Eq.9
 195 the value for the estimated fault parameter can be obtained at each time step
 196 $t_j \forall j \geq N_I$ as:

$$\theta^*[j] = \underset{\bar{\theta}}{\operatorname{argmin}} \{J(\zeta_j, \bar{\theta})\} \quad (11)$$

Remark 4. *It should be noted that the results of this work can be extended and generalized to the case when only output measurements are available at the sampling times. Specifically, in this case, a suitable state observer would need to be included in the control system to provide estimates of the system states from the available output measurements. These estimates can then be used to update the states of the controller and diagnoser models at the sampling times (e.g., see [20] for a similar approach used in the context of networked output feedback control). Fault estimation can be done by minimizing the difference*

(over the chosen horizon) between the measured output of the system available at the sampling times and the output of the diagnoser model embedded in the sensor (which depends on the tunable fault parameter). This leads to a cost function of the following form:

$$J(\zeta_j, \bar{\theta}) = \sum_{p=j-N_i}^{j-1} \left(\|y[p+1] - \bar{y}(\zeta_j, \bar{\theta})[p+1]\|^2 \right)$$

where ζ_j would no longer contain the exact states at the sampling times but the estimates generated by the state observer.

Unlike residual-based approaches (e.g., [17]) which allow only for fault detection, the above optimization-based approach provides capabilities for simultaneous detection, isolation and estimation of multiple faults, which is important for the selection of the appropriate fault accommodation strategy. The methodology in this section is also applicable to faults in measurement sensors or the plant itself through placement of a fault parameter matrix similar to that of the actuator fault matrices in this study. These types of faults can be modeled using the following general forms: $y = \theta Cx$ for measurement sensor faults and $\dot{x} = A\theta x + BKx$ for plant faults. The fault estimator would then undergo the same logic of minimizing a suitable cost function. In the case of plant faults, however, θ would not necessarily be a diagonal matrix.

4. Fault accommodation strategies

4.1. Performance metric characterization

In order to accommodate a fault from a performance standpoint, a performance metric must be in place. The advantages of performance-based accommodation are that through the use of a performance metric, different stabilizing configurations can be compared objectively to decide which one is better suited to the needs of the process. Due to the sampled nature of the control system considered here, the extended H_2 -norm is chosen as a suitable performance metric (e.g., see [21, 15]). The extended H_2 -norm is used for measuring the 2-norm

219 of a periodically sampled-data systems where continuous measurements are not
 220 available. It is a measure of the settling time of the performance output of a
 221 system after an impulse disturbance is introduced in a prescribed input.

222 To characterize the extended H_2 -norm as a function of the plant, model, fault
 223 and control system parameters, we reconsider the state-space representation
 224 in Eq.2, but now with disturbances and a performance output introduced as
 225 follows:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + B\theta u(t) + Ed(t) \\ z(t) &= Mx(t)\end{aligned}\tag{12}$$

226 where d is the disturbance input and z is the performance output. In the
 227 case of the SOFC plant, the performance output chosen is the power output
 228 of the system. After some manipulations, the sampled-data closed-loop system
 229 can be formulated as:

$$\begin{aligned}\dot{\xi}(t) &= \Lambda\xi(t) + Hd(t), t \in [t_j, t_{j+1}) \\ \xi(t_j) &= [x^T(t_j) \ 0]^T, j \in \{0, 1, 2, 3, \dots\} \\ z(t) &= N\xi(t)\end{aligned}\tag{13}$$

230 with $H = [E^T \ E^T]^T$ and $N = [M \ O]$. By solving Eq.13, it can be ver-
 231 ified that the closed-loop response of the performance output to an impulse
 232 disturbance $d = \delta(t - t_0)$ can be expressed as:

$$z(t) = Ne^{\Lambda(t-t_j)} \left(I_o e^{\Lambda\Delta} \right)^j H, t \in [t_j, t_{j+1})\tag{14}$$

233 and the extended H_2 -norm can be computed as follows:

$$\|G\|_{H_2} = \text{trace}(H^T X H)^{1/2}\tag{15}$$

234 where X is the solution of the following discrete Lyapunov equation:

$$M(\Delta)^T X M(\Delta) - X + W_0(0, \Delta) = 0\tag{16}$$

235 with $M(\Delta) = I_o e^{\Lambda \Delta}$ and $W_o(0, \Delta) = \int_0^\Delta e^{\Lambda^T t} N^T N e^{\Lambda t} dt$.

236 The extended H_2 -norm is of particular interest here seeing that it can be
 237 expressed in closed form and is also a function of the same parameters used to
 238 characterize closed-loop stability, namely θ , $\hat{\theta}$, Δ , and K . This allows a quan-
 239 titative assessment of the potential balance that exists between these various
 240 parameters in influencing closed-loop performance. For example, this charac-
 241 terization of the performance metric can be used to assess the extent of per-
 242 formance deterioration resulting from different fault situations. Similarly, for
 243 a given fault scenario, the impact of different choices of the control and model
 244 parameters on performance can be analyzed. Standard optimization techniques
 245 can be used, for example, to find the minimum extended H_2 -norm as a function
 246 of these variables. This is useful in determining the best plan of action for the
 247 accommodation of faults.

248 4.2. Performance-based fault accommodation

249 Once a fault in need of accommodation has been deemed to occur, the con-
 250 troller must take a suitable corrective action to compensate for the effect of
 251 the fault. This is achieved by minimizing the performance objective (over one
 252 or more possible fault accommodation parameters), whilst ensuring closed-loop
 253 stability of the system. Note here that the performance objective is minimized
 254 because it denotes the settling time of the system from an impulse disturbance.
 255 For example, if a fault is established at time t_j and $\hat{\theta}$ is chosen as the fault
 256 accommodation parameter, the value that $\hat{\theta}(t_{j+1})$ should be assigned is deter-
 257 mined by solving the following constrained optimization problem:

$$\begin{aligned} \hat{\theta}(t_{j+1}) &= \min_{\hat{\theta}} \{ \|G\|_{H_2} \} \\ \text{s.t.} \quad &\lambda_{\max} \left(M(\bar{A}, \bar{B}, \hat{A}, \hat{B}, K, \theta^*, \hat{\theta}, \Delta) \right) < 1 \end{aligned} \quad (17)$$

258 where $\bar{A}$ and $\bar{B}$ are the modeled values of A and B in the embedded fault
 259 estimator, and the constraint on the maximum eigenvalue magnitude is made to
 260 ensure closed-loop stability (see Section 2.2). In some cases, it may be necessary

261 to perform the minimization of the extended H_2 -norm over a constrained range
 262 of possible $\hat{\theta}$ values to avoid unreasonably large values that may result from the
 263 solution of the unconstrained problem.

264 **Remark 5.** *An accommodation strategy similar to the one given in Eq.17 can*
 265 *be used if other possible fault accommodation parameters are chosen (e.g., the*
 266 *feedback gain K , or the sampling period, Δ). One can also optimize the per-*
 267 *formance index over multiple fault accommodation parameters simultaneously;*
 268 *this, however, comes at the expense of additional computational complexity, and*
 269 *the stability of the accommodated system needs to remain ensured.*

270 4.3. Implementation issues

271 An important consideration in the implementation of fault-tolerant control is
 272 the need to minimize false detection alarms which may trigger unnecessary fault
 273 accommodation events that, at best, could lead to increased computational cost,
 274 and, at worst, could compromise performance and lead to instability. This is
 275 important, especially in the presence of unmodeled uncertainties and unknown
 276 perturbations that could impact the optimization-based fault estimation results.
 277 Determining whether or not a fault is in need of accommodation, therefore,
 278 requires careful consideration. For example, once a change in the estimated
 279 fault parameter has been observed, the controller must determine if the fault
 280 is new or one that has had previous accommodation. This determination can
 281 be made through the use of a reference fault parameter, θ_{ref}^* , which is updated
 282 every time a fault is accommodated. An example heuristic that one can employ
 283 here is to check if the mean of the estimated fault parameter over the past N_{θ^*}
 284 sampled data points, μ , has deviated by some pre-defined threshold, θ_{thresh}^* ,
 285 i.e.,

$$|\theta_{ref}^* - \mu| > \theta_{thresh}^* \quad (18)$$

$$\mu = \text{mean} \{ \theta^*[p - N_{\theta^*}], \theta^*[p - N_{\theta^*} + 1], \dots, \theta^*[p] \} \quad (19)$$

286 If such a deviation is observed, then the controller suspects that a new fault
 287 has occurred.

288 Another important aspect is the need to provide some robustness margin
 289 against unmodeled disturbances. Given that the performance of the system
 290 under impulse-like disturbances is considered (which is captured by the extended
 291 H_2 -norm), it is important that such unmodeled disturbances not be mistaken
 292 for faults and thus not trigger unnecessary fault accommodation measures. One
 293 approach to deal with this is to exploit the fact that any impulse-like disturbance
 294 on the system will rapidly shift the states, and will be met by the embedded
 295 model attempting to find an estimated value of the fault size that would cause
 296 this stark shift, thus creating a rapid change in the estimated fault parameter.
 297 With this knowledge, a secondary condition for the determination of possible
 298 faults would be to require that the estimated fault parameter not exceed some
 299 pre-defined threshold, $\theta_{\max}^*$, that would not be expected to be reached under
 300 normal operating conditions. In other words, if the following condition holds:

$$\max \{|\theta^*[p - N_{\theta^*}]|, |\theta^*[p - N_{\theta^*} + 1]|, \dots, |\theta^*[p]|\} < \theta_{\max}^* \quad (20)$$

301 as well as that in Eqs.18-19, then the controller decides that a fault has
 302 occurred that must be accommodated.

303 Since the fault accommodation strategy is based on the estimated fault size,
 304 it is important once a fault has been suspected to wait for the estimated fault
 305 value to settle prior to taking corrective action. This would have the benefit of
 306 not taking unnecessary action during disturbances, as well as reducing the com-
 307 putational load of calculating the accommodation action every time θ^* changes
 308 (recall that fault accommodation is based on solving the constrained optimiza-
 309 tion problem in Eq.17). The settling time of the estimated fault parameter is
 310 dependent on the size of the horizon used in the fault estimation scheme. The
 311 estimated fault value gradually changes as additional faulty data points are in-
 312 corporated into the data set used for estimation. Eventually the estimated value
 313 settles at a constant once all data points in the horizon are ones containing the

314 fault. An example heuristic that can be used here is to trigger an optimization
 315 of the control action only when the deviations in successive estimates of θ^* fall
 316 below a certain threshold, ϵ_{θ^*} , i.e.,

$$|\mu' - \theta^*[p]| < \epsilon_{\theta^*} \quad (21)$$

$$\mu' = \text{mean} \{ \theta^*[p - N_{\theta^*}], \theta^*[p - N_{\theta^*} + 1], \dots, \theta^*[p - 1] \} \quad (22)$$

317 where ϵ_{θ^*} is a tunable parameter that θ^* must remain within for the optimization-
 318 based fault accommodation to take place. A summary of the fault detection,
 319 estimation and accommodation framework is given in Fig.1.

320 5. Simulation study

321 The objective of this section is to illustrate the implementation of the de-
 322 veloped fault estimation and accommodation framework to the SOFC example
 323 introduced earlier in Section 2.

324 5.1. Performance gains through use of fault accommodation

325 In prior work ([17]), stability regions were shown to aid in understanding
 326 the impact uncertainty had on closed-loop stability of the SOFC plant and
 327 used to develop a stability-based control system reconfiguration strategy that
 328 maintains stability in the presence of controller failures. In this study, a different
 329 approach is taken, and that is to show explicitly the dependence of stability and
 330 performance on the actual and modeled faults in the system. To this end, Fig.2
 331 is a contour plot of $\lambda_{\max}(M)$ as a function of θ and $\hat{\theta}$. The uncolored region is
 332 the region enclosed by the unit contour line and is therefore the stable region of
 333 operation. It can be seen from this plot that for any given fault, where $\theta \gtrsim 7.44$
 334 and $\hat{\theta} = 1$, the closed-loop system will be unstable. Furthermore, there is a wide
 335 range of possible accommodation scenarios that can be implemented through
 336 altering $\hat{\theta}$ that lead to stable post-accommodation operation.

Figure 1: A flowchart of the integrated fault estimation and accommodation methodology.

Fig.3 shows the advantage of fault accommodation when a destabilizing fault of $\theta = 7.9$ occurs at $t = 500$ s. The closed-loop system with the accommodated fault ($\hat{\theta} = 15$) maintains a steady power output trajectory (black profile) in the face of a fault in the actuator manipulating the hydrogen flow rate, whereas the power output of the closed-loop system with no fault accommodation ($\hat{\theta} = 1$) is shown to be unstable for the same fault (red profile).

Figure 2: Characterization of the sampled-data controlled SOFC stability region: A contour plot of $\lambda_{\max}(\theta, \hat{\theta})$.

Given the wide range of values of $\hat{\theta}$ that can be used for stability-based accommodation, an optimal post-fault choice of the value of $\hat{\theta}$ would naturally be desirable, and this is where the performance criteria is applied. It is sought to find the accommodation strategy for the model fault parameter that gives the best performance while maintaining closed-loop stability. A contour plot of the extended H_2 -norm as a function of θ and $\hat{\theta}$ is shown in Fig.4, where the extended H_2 -norm is the contour value on this plot and the smaller this norm

Figure 3: Evolution of the power output in the presence of a destabilizing fault.

the better the system performs (i.e., the faster the recovery from disturbances). It can be seen that, for a given value of $\hat{\theta}$, as θ increases beyond 1 (i.e., the fault severity increases), extended H_2 -norm increases and thus the performance deteriorates.

Note that the performance criterion is only meaningful in the region where the closed-loop system is stable (see Fig.2). Due to the shape of the stability contour plot, simply setting $\hat{\theta} = \theta^*$ can be seen to place the system in a stable region of operation. This is referred to as “minimal accommodation” in the sense that the controller needs no knowledge of the stability or performance of the system as a function of $\hat{\theta}$. This minimal accommodation strategy assumes that if the model matches the plant, then this will be a suitable accommodation strategy. It can be seen with the aid of the performance contour plot of the extended H_2 -norm (Fig.4), however, that there is in fact a better solution to the accommodation problem from a performance standpoint than simply setting $\hat{\theta} = \theta^*$. It should be noted that the shape of this particular contour plot is unique to the chosen control configuration, and that, in general, setting $\hat{\theta} = \theta^*$ cannot be guaranteed to result in a stable post-fault system even if $\theta^* = \theta$.

To illustrate the performance-based accommodation, we consider a non-

Figure 4: Characterization of the performance levels of the SOFC plant: A contour plot of the extended H_2 -norm as a function of θ and $\hat{\theta}$.

destabilizing fault in the control actuator at $t = 1000$ s, as shown in Fig.5 (red profile). This figure also shows the evolution of the estimated fault parameter (blue profile) which is computed from Eq.11. It can be seen that the value of θ^* steps its way closer to the actual value of the fault parameter every sampling period ($\Delta = 20$ s). Once θ^* has settled in accordance with Eqs.18-19, Eq.20, and Eqs.21-22, then fault accommodation is triggered and $\hat{\theta}$ is adjusted per Eq.17 (green profile) to minimize performance losses while maintaining closed-loop stability. The impact of this fault accommodation strategy on closed-loop performance is shown in Fig.6 and Fig.7.

Figure 5: Fault parameter estimation for a performance-degrading fault.

Fig.6 shows the accommodation result with $\hat{\theta}$ being optimized to minimize the extended H_2 -norm. This shows the advantage of accommodating a performance-degrading fault, $\theta = 6.9$, occurring at $t = 1000$ followed by a disturbance of 10% of the steady state value of the O_2 flow rate. This figure shows the accommodated power output response in black (extended H_2 -norm = 3.23) and the response with no accommodation in red (extended H_2 -norm = 7.42). The minimal accommodation strategy, where $\hat{\theta} = \theta^*$ is not shown in the plot, but yields an extended H_2 -norm of 3.78. It can be seen that the system with fault accommodation has a much smoother and faster response back to the steady state post the fault and disturbance events.

One can also see in Fig.5 that the disturbances occurring at $t = 1400$ s do not trigger any change in $\hat{\theta}$ even though θ^* changes drastically. This is due to the precautions put in place from Eqs.18-20. It should be noted again that these were obtained from the knowledge of the type of disturbances that would be affecting the system.

Remark 6. Note that in Fig.5 θ^* tracks exactly with the fault (θ) once the horizon contains all faulted data. This is due to the fact that no plant-model mismatch was considered, i.e., $\hat{A} = A, \hat{B} = B$, etc. When uncertainties are introduced in the context of plant-model mismatch, where $\hat{A} \neq A, \hat{B} \neq B$, then the resulting θ^* would be a range of values that the actual fault, θ , would lie in, $\theta \in \theta^*$; the end result being an upper and lower bounds on the fault, $\min(\theta^*) \leq \theta \leq \max(\theta^*)$, instead of the exact estimate, $\theta^* = \theta$, obtained in the case with no uncertainty. Note that the accommodation parameter $\hat{\theta}$ must be stabilizing for all potential values of the fault parameter, $\theta \in \theta^*$.

Figure 6: Evolution of power output subject to a performance-degrading fault followed by an impulse disturbance in the oxygen feed flow rate.

Fig.7 shows a similar accommodation result as in Fig.6, but with a disturbance of 10% of the steady state value of the stack temperature introduced into

the feed temperature following the fault event. This figure shows the accommodated power output response in black (extended H_2 -norm = 0.012) and the response with no fault accommodation in red (extended H_2 -norm = 0.028). The minimal accommodation strategy with $\hat{\theta} = \theta^*$ is not shown in the plot, but has an extended H_2 -norm of 0.014. It can be seen that the system with accommodation has a much smoother and faster response back to the steady state value post the fault and disturbance events.

Figure 7: Evolution of power output subject to a performance-degrading fault followed by an impulse disturbance in the feed temperature.

5.2. Choice of control configuration

By examining the values of the extended H_2 -norm obtained from the accommodation results reported in the previous subsection, it can be observed that the achievable values were far lower for the configuration where the manipulated variable was the hydrogen feed flow rate and the disturbance was introduced in the inlet feed temperature, as opposed to the configuration where the oxygen feed flow rate was used as the disturbance variable. It should also be noted that the ability of the control system to accommodate faults was far greater in the latter configuration. This can be seen in Fig.8 and Fig.9, which depict the dependence of the extended H_2 -norm on the fault parameter for

420 both configurations. It can be seen that faults in the control actuator have a
 421 far greater impact on closed-loop performance, when performance is measured
 422 with respect to recovery from disturbances in the oxygen feed flow rate, where
 423 as performance with respect to disturbances in the feed temperature is barely
 424 sensitive to changes in the fault parameter (over the stabilizing range). In Fig.8,
 425 for example, it can be seen that various values of θ can yield quite stark differ-
 426 ences in the extended H_2 -norm magnitude. This is likely due to the underlying
 427 physical principles governing the system, where adjusting the hydrogen flow has
 428 a far greater impact on the power output when the oxygen flow rate is fluctuat-
 429 ing than if the inlet feed temperature changes. However, this analysis does give
 430 insight into the importance of selecting proper control variables when knowl-
 431 edge of the disturbance variables is available. This analysis of the ability of the
 432 control system to accommodate faults could also be applied in more complex
 433 systems where the relationships between the available manipulated variables
 434 and expected disturbance variables are not as clear.

Figure 8: Magnitudes of the maximum eigenvalue of M and the extended H_2 -norm as functions of θ (with $\hat{\theta} = 1$) for the case when performance is measured with respect to disturbances in the oxygen feed flow rate.

435 Finally, Fig.10 and Fig.11 show the extent to which fault accommodation
 436 via adjusting $\hat{\theta}$ can improve performance for both configurations. It can be

Figure 9: Magnitudes of the maximum eigenvalue of M and the extended H_2 -norm as functions of θ (with $\hat{\theta} = 1$) for the case when performance is measured with respect to disturbances in the feed temperature.

seen that the choice of $\hat{\theta}$ can make a substantial impact on performance recovery when disturbances are considered in the oxygen feed flow rate, where as it has minimal impact on performance when the disturbance influences the feed temperature (note that the extended H_2 -norm is only plotted over the range where $\lambda_{\max}(M) < 1$). The latter configuration appears to be inherently robust from a performance standpoint to the faults tested here as can be seen from Fig.9; however, the selection of $\hat{\theta}$ for the fault shown in Fig.11 has little effect on performance, as can be seen in the extended H_2 -norm values for the accommodation and no accommodation states pertaining to Fig.7.

5.3. Benefits of control system reconfiguration

So far, we have shown that fault accommodation measures, such as updating the model parameters, can be used to minimize performance losses following the occurrence of faults. In this subsection, we show that in some cases, reconfiguring the control system (e.g., by switching to an alternative fault-free manipulated input) can further improve closed-loop performance beyond what can be achieved with fault accommodation measures alone. To this end, we can

Figure 10: Magnitudes of the maximum eigenvalue of M and the extended H_2 -norm as functions of $\hat{\theta}$ (with $\theta = 6.9$) for the case when performance is measured with respect to disturbances in the oxygen feed flow.

Figure 11: Magnitudes of the maximum eigenvalue of M and the extended H_2 -norm as functions of $\hat{\theta}$ (with $\theta = 6.9$) for the case when performance is measured with respect to disturbances in the feed temperature.

453 see from Fig.10 that if the nominal system were to experience a fault such that
 454 $\theta = 6.9$, then the best case scenario for recovery through fault accommodation
 455 (by adjusting $\hat{\theta}$) would yield an extended H_2 -norm value of ≈ 3.3 . However,
 456 if the control system were to switch to an alternative configuration in which
 457 the feed temperature, rather than the hydrogen flow rate, is used as the ma-
 458 nipulated input, then as can be seen in Fig.12, an even smaller performance
 459 index (i.e., an improved performance) can be achieved for all values of $\hat{\theta}$. The
 460 best performance is attained near $\hat{\theta} = 1$. This result suggests that in this case,
 461 and from a performance point of view, control system reconfiguration may be a
 462 better option than fault accommodation.

Figure 12: Magnitude of the maximum eigenvalue and the extended H_2 -norm as functions of $\hat{\theta}$, with $\theta = 1$, when the manipulated input is chosen as the feed temperature, and the disturbances occur in the oxygen feed flow.

463 In addition to performance gains through control system reconfiguration, it
 464 can be seen from Fig.13 that if the aforementioned reconfiguration strategy is
 465 implemented, then the newly reconfigured system would exhibit some degree of
 466 robustness to potential future faults in the new actuator configuration. Specif-
 467 ically, it can be seen that the extended H_2 -norm profile remains relatively flat
 468 (and well below 2) over a certain range of θ values around the healthy (fault-
 469 free) value of 1, thus indicating minimal performance degradation for a certain

Figure 13: Magnitude of the maximum eigenvalue and the extended H_2 -norm as functions of θ , with $\hat{\theta} = 1$, when manipulated input is chosen as the feed temperature, and the disturbances occur in the oxygen feed flow.

range of faults. This can be compared to the result in Fig.8 for the range of values that θ can take and still have a reasonable extended H_2 -norm value.

Finally, it should be noted that for Figs.8-13 the extended H_2 -norm is only plotted for values of θ or $\hat{\theta}$ (whichever is relevant) when $\lambda_{\max}(M) < 1$, that is to say for values at which the closed-loop system in question is stable. This is due to the fact that there is no meaning in the performance index for an unstable system. This restriction is the cause for any discontinuities/asymptotes that may appear in the various plots.

6. Concluding remarks

In this work, a methodology for the design of local model-based fault-tolerant control systems for DERs was developed and illustrated using a simulated model of a SOFC system. The framework integrates model-based feedback control, data-based fault estimation and performance-based fault accommodation. Fault estimation was realized via a moving-horizon optimization scheme that used past data to obtain an estimate of the fault magnitude. The fault estimation

485 scheme was complemented by a set of practical implementation guidelines aimed
486 at minimizing false detection alarms. Based on a suitably chosen performance
487 metric for the closed-loop system, a performance-based fault accommodation
488 strategy was then devised to minimize fault-induced performance losses, while
489 maintaining closed-loop stability.

490 When placed in the wider context of fault-tolerant control of DER networks,
491 the focus of the current study has been on the detection and handling of faults
492 solely at the local (i.e., the DER unit) level, without consideration of possible
493 supervisory oversight or intervention. This emphasis is motivated by the fact
494 that timely fault mitigation and protection measures at the local level are essential
495 (when feasible) for preventing the effects of local faults from cascading through
496 the system. However, in cases where local fault accommodation is no longer
497 feasible, an intervention by a higher-level supervisor that takes into account the
498 network-level dynamics becomes necessary to ensure fault-tolerance. The design
499 of a supervisory fault-tolerant control system for DERs is the subject of ongoing
500 research work.

501 **7. Acknowledgments**

502 Financial support by NSF, CBET-1438456, is gratefully acknowledged.

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