A semiparametric spatio-temporal model for solar irradiance data

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Abstract

We evaluate semiparametric spatio-temporal models for global horizontal irradiance at high spatial and temporal resolution. These models represent the spatial domain as a lattice and are capable of predicting irradiance at lattice points, given data measured at other lattice points. Using data from a 1.2 MW PV plant located in Lanai, Hawaii, we show that a semiparametric model can be more accurate than simple interpolation between sensor locations. We investigate spatio-temporal models with separable and non-separable covariance structures and find no evidence to support assuming a separable covariance structure. Our results indicate a promising approach for modeling irradiance at high spatial resolution consistent with available ground-based measurements. Such modeling may find application in design, valuation, and operation of fleets of utility-scale photovoltaic power systems.

Keywords: Irradiance, Spatio-temporal model, Nonseparability, Lattice

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1. Introduction

Accurate modeling of power output from photovoltaic (PV) power plants requires accurate estimates of the plane-of-array (POA) irradiance over each plant's footprint. For single, spatially contiguous plants, power correlates reasonably well with the aggregate irradiance over the plant [1]. Currently plant aggregate irradiance is estimated by either 1) averaging measurements from many irradiance sensors [2] or 2) transforming data from a single measurement point [3]. Various transformations of single measurements to plant-aggregate irradiance are proposed [4, 5, 6] all of which attempt to produce a time series of values representative of a spatial average over an area of interest by smoothing, in some fashion, a time series of measurements at a single point. However, these smoothing transformations are themselves calibrated by a set of spatially diverse measurements and are not inherently capable of estimating the spatial variability of irradiance within the area of interest at each point in time. Areal average irradiance can also be estimated from satellite data (e.g., [7]) albeit with somewhat coarse spatial and temporal resolution on the order of 1km^2 and 30 minutes.

Past research efforts [summarized in 2] have focused on accurate representation of the temporal variability in spatially-averaged irradiance. Increasing concerns about the effects of large numbers of PV systems connected to an electrical grid have led to significant interest in better understanding of the spatial variability of irradiance as well as the temporal variability of the spatial average. For example, spatial variability of irradiance, and hence PV power, over a utility's service area, or over the service area of a distribution feeder circuit, causes variable levels of power to be injected at different locations which may require mitigation efforts to maintain circuit voltages within acceptable limits. Contracts for purchase of power from large PV plants often require after-the-fact estimation of the potential energy production (in addition to measurement of actual production) over short time periods to accurately compensate for utility-directed limits on power and for equipment outages; these estimates would benefit from irradiance data with high spatial and temporal resolution. To better understand spatial variability and to more accurately determine potential production from large plants, utilities have deployed networks of irradiance sensors across service territories

[e.g. 8] or are requiring deployment of many sensors at large plants [e.g. 9]. Suppliers of satellite-based irradiance estimates are upscaling in time by simulating cloud movements [10] or running numerical weather simulations [11], although spatial coarseness remains. Statistical simulations have been employed to create representative irradiance data at high time resolution over large areas [12] but with coarse spatial resolution. Here, we explore statistical modeling of global horizontal irradiance (GHI) at high resolution spatial and temporal scales, i.e., on the order of 100 m^2 and one minute.

A challenge in modeling irradiance data is incorporating the interaction between time and space. Especially in the presence of advecting clouds, the irradiance observed at one location is likely to also be observed at other locations but with a time shift. Thus we anticipate that irradiance will exhibit a spatial autocorrelation that varies with time. Spatio-temporal models explicitly account for this autocorrelation and thus may predict aggregate irradiance more accurately than does a simple spatial average.

In spatio-temporal modeling, the time and spatial covariances structures are classified as either separable or nonseparable. Assuming separability allows for simpler methods for estimating the model but this assumption is not realistic for the complex interactions between the space and time components of irradiance data. We propose a spatio-temporal model that does not assume a separable covariance structure to the data. Our model incorporates a semiparametric method for modeling the time series component and a lattice structure for the spatial component.

The remainder of this paper is organized as follows. In Section 2, we present a survey of the literature on time series and spatio-temporal approaches for irradiance data. In Section 3, we discuss how the time series structure is modeled via a semiparametric model fitted with a method known as spline-backfitted kernel (SBK) estimation. In Section 3.3, we introduce the spatio-temporal model, and compare the model’s performance assuming either a separable or a nonseparable covariance structure to evaluate whether separability can be assumed. In Section 4, we apply the model to irradiance data from the La Ola photovoltaic plant in Lanai, HI. Finally, we provide discussion and conclusions in Section 5.

2. Literature survey

Literature reports several efforts at modeling time-series of irradiance at individual locations, and substantially fewer attempts to construct spatio-

temporal models of irradiance considering several proximal locations. The literature on individual time-series modeling includes approaches based on autoregressive integrated moving average (ARIMA) analysis [e.g., 13], non-linear autoregressive analysis [14], regression analysis [15], artificial neural networks analysis [16], k -Nearest Neighbors algorithm [16] and Bayesian inference [16]. These approaches focus on forecasting irradiance considering only the measurements at a selected location separately from measurements at other locations. Paoli et al. [16] considers a type of artificial neural network known as Multi-Layer Perceptron (MLP) network and finds their method performs as well or better than other methods such as ARIMA analysis, Bayesian inference, and k -Nearest Neighbors. Yang et al. [13] introduce an ARIMA model that incorporates low-resolution, ground-based cloud cover data to obtain next hour solar irradiance. The authors state that their ARIMA model outperforms all other time series forecasting methods in four of the six stations they tested. In both the MLP and ARIMA methods, the model does not incorporate a spatial component but only models irradiance in time.

The literature on spatio-temporal modeling of solar irradiance and/or power from PV systems comprise three general types: 1) lattice and geo-statistical models, which may predict irradiance at individual spatial locations; 2) variability models which estimate statistics for the time series of power from a large PV system or from a fleet of PV system; and 3) models to estimate the aggregate output of a collection of PV systems from measurements at a single location. We discuss each category in turn.

Among lattice models, [17] model irradiance data from ten sensors roughly at 5km spacing using a spatio-temporal autoregressive moving average (STARMA) model. The STARMA model incorporates the Euclidean distances between two points in order to model the spatial structure of the data. However, the STARMA model used in [17] assumes a separable covariance structure, an assumption which we find to be questionable at the scale of a single PV plant. Glasbey et al. [18] model irradiance with a nonseparable spatio-temporal covariance structure for data taken at 22 sites in the city of Edinburgh, Scotland, with distances between sites ranging from 0.1 to 20 Km. They found the nonseparable model performed marginally better with a mean square difference between estimated and fitted covariances of 0.0032 as compared to 0.0033 for the separable model. Spatio-temporal covariance structures were explored in [19] and [20]. In both papers, a geographical plane transformation by multidimensional scaling is used to make the process isotropic. After the transformation, a separable or nonseparable

109 covariance model is used for time-forward kriging in [19] and an empirical
 110 covariance structure is used to describe the correlation between sites based
 111 on distance in [20]. The methods used are then applied to data taken over
 112 Singapore (an area of about 700 km²).

113 Literature reports many studies intended to quantify and/or model the
 114 variability in the output of a large PV plant or the aggregate output of a fleet
 115 of PV plants. Typically, variability is quantified by computing statistics for
 116 the time series of changes in irradiance over a fixed time interval. Generally,
 117 variability models either estimate the variance of the time series of aggregate
 118 output from a fleet of PV systems, or the correlation between time series of
 119 changes in irradiance, or PV output power as a function of distance between
 120 the sites. Both types of results are of great importance for managing and
 121 balancing energy on an electrical grid with connected PV systems. However,
 122 most of these variability models do not allow for prediction of the time series
 123 of irradiance at a location of interest from measurements at nearby locations.

124 A model for maximum output variability was presented in [10] with
 125 changes assumed to be uncorrelated. Hoff and Perez [21] extended this model
 126 by incorporating correlation coefficients between the variability of individual
 127 plants of a PV fleet based on site distance and implied cloud speed. Perez
 128 et al. [22] used time-shifted data from a single location to examine the po-
 129 tential variability of clearness index by examining site pair correlation for
 130 distances ranging from 100 m to 100 km and for time intervals of 1 min, 5
 131 min, and 15 min. Smaller site pair distances are examined in [23] in which
 132 coherence spectra and wavelet analysis are used for quantifying geographic
 133 smoothing. Using the coherence spectra approach they found that strong
 134 correlations exist for station pairs within 3 km at time intervals as short as
 135 10 min. For longer time intervals, they found the correlation decreases with
 136 distance, although not uniformly in all directions. After applying the wavelet
 137 transform to one station and comparing to the average of six stations, the
 138 authors found that at time intervals shorter than about 5 minutes, the six
 139 stations are independent, resulting in reduced variability. For longer time in-
 140 tervals the reduction in variability lessens with almost no reduction at about
 141 68 min.

142 Moving beyond isotropic models for correlation, i.e., based only on time
 143 interval and distance between locations, [24] presented an anisotropic model
 144 which describes correlation for power production and for changes in power
 145 production as a function of spatial location in an X,Y plane and time in-
 146 terval. They consider forecasting in time for a fleet of PV systems spread

over a $50 \text{ km} \times 50 \text{ km}$ area. A mathematical model for the spatio-temporal correlations between changes in irradiance is presented in [25]. This model is based on a spatial Poisson process that is used to simulate the probability that advecting clouds cover two sites. This correlation model is inherently anisotropic since it takes into account the cross-wind distance, the along-wind distance, and the length of the motion path of the clouds.

Finally, aggregate power from a large system or fleet of systems over a defined area is estimated from measurements at a single point by transforming the time series of measurements: [6] describes a moving average technique; [5] proposes a low-pass filter; and [4] uses wavelet decomposition. The wavelet method uses a variability model for correlation among locations as described above to moderate each wavelet model’s amplitude to estimate the time series of aggregate power. The authors validate their simulation method at two plants, one of which is a $1 \text{ km} \times 1 \text{ km}$ distributed generation plant and the other is a $3 \text{ km} \times 2 \text{ km}$ utility scale PV plant. The wavelet method permits estimation of the aggregate output but not of the contribution to the aggregate from individual systems.

The spatio-temporal model that we propose improves upon the works of [13] and [14] because we do not assume a parametric form for the time component of the model, and improves on [17] through the nonseparable covariance structure. While the works of [23] and [4] examined correlation as a function of distance between sites, we incorporate a lattice structure and utilize the simultaneous autoregressive spatial (SAR) model in which the sites are connected through a “neighborhood.” Our model explains the correlation through this neighborhood structure and does not take as inputs the wind distance and cloud motion paths as in [25]. The use of the SAR model is based on viewing the sites within a utility scale PV plant as fixed locations instead of geostatistical points such as in [19] and [20].

3. Modeling Irradiance

Let $Q_{s,t}$ represent an observable process, e.g., measured GHI, at time t and location s for $t = 1, 2, \dots, T$ and $s = 1, 2, \dots, S$. If there is no interaction in time and space, the covariance function of $Q_{s,t}$ can be written as a product of two functions where one function is dependent on time only, and the other on location alone. Such a covariance function is called “separable.” However, when interactions in space and time are present, the covariance function is “nonseparable;” i.e., it cannot be factored into two separate func-

183 tions. Spatio-temporal models with separable covariance are much easier to
 184 implement. But in the presence of space-time interaction, separable models
 185 do not perform well, and can lead to misleading or incorrect conclusions.

186 For modeling $Q_{s,t}$, consider

$$Q_{s,t} = R_{s,t} + Z_{s,t}, \quad t = 1, \dots, T, \quad s = 1, \dots, S, \quad (1)$$

187 where, at time t and location s , $R_{s,t}$ represents the true irradiance signal and
 188 $Z_{s,t}$ is a noise process. Furthermore, decompose the noise process into a sum
 189 of three terms,

$$Z_{s,t} = X_{s,t} + Y_{s,t} + \varepsilon_{s,t}, \quad (2)$$

190 where $X_{s,t}$ is a time series process at location s , $Y_{s,t}$ is a spatial process at
 191 time t , and $\varepsilon_{s,t}$ is a multivariate error process with mean zero and $TS \times TS$
 192 covariance matrix $\Sigma(s, t)$. If the process is separable, then the covariance
 193 matrix can be written as $\Sigma(s, t) = \Lambda(t) \otimes \Gamma(s)$, where $\Lambda(t)$ is a $T \times T$
 194 temporal covariance matrix, $\Gamma(s)$ is an $S \times S$ spatial covariance matrix, and
 195 $\otimes$ is the Kronecker product [26].

196 There are a variety of methods for fitting separable spatio-temporal mod-
 197 els to space-time data. A review of space-time analysis methods and their
 198 computational counterparts can be found in [27] or [28]. We consider three
 199 approaches to fitting model (2). The first approach fits a spatial model at
 200 each time. Then spatial residuals are computed, and for each location a time
 201 series model is fitted to the spatial residuals at each location. The second
 202 approach models the time series at each location, computes time residuals,
 203 and then fits a spatial model at each time to the residuals. Both of these
 204 approaches carry the assumption that the covariance structure is separable
 205 in space and time. The third approach removes the separability assumption,
 206 jointly modeling time and space using the spatio-temporal model introduced
 207 in Section 3.4.

208 3.1. Modeling the Time Series Component

209 For modeling time series data arising from a dynamic process, such as
 210 solar irradiance, nonlinear models often out-perform linear models [29]. Al-
 211 though the class of nonlinear time series models is infinitely large, there are
 212 many popular parametric nonlinear models including the bilinear model [30],
 213 the exponential autoregressive model [31], and a variety of threshold autore-
 214 gressive models [32, 33]. When one of these parametric models is known to
 215 be appropriate for analyzing the time series, it should be used for analyzing

the series. However in the analysis of solar irradiance, no specific class of parametric nonlinear model has been shown to be generally applicable, and therefore we pursue a semiparametric approach. In this section, we examine only the time component of the model. So to ease notation, for the remainder of this section, we consider the location fixed, and suppress the s subscript; that is $X_{s,t} = X_t$ for a fixed value of s .

A highly versatile semiparametric model is the functional coefficient autoregressive model of order p (FCAR(p)), first introduced by [34]. The FCAR(p) model has an additive autoregressive structure, but with coefficients that vary as a function of some variable, u say, which can be exogenous to the series X_t . In the pure time series context, u is a lagged value of the series, and we write $u_t = X_{t-d}$. In this paper, we restrict the FCAR(p) models to those with $u_t = X_{t-d}$, and so define the FCAR(p) model as

$$X_t = m_0(u_t) + \sum_{j=1}^p m_j(u_t)X_{t-j} + \omega_t, \quad t = p+1, \dots, T \quad (3)$$

where $u_t = X_{t-d}$, $d \leq p$, $m_j(\cdot)$, $j = 0, 1, 2, \dots, p$ are measurable functions of u , and $\{\omega_t\}$ is a sequence of independent and identically distributed (IID) random variables with mean zero and constant variance.

Reasonable use of the FCAR(p) model requires only that the model is additive, and places few restrictions on the functional coefficients. To illustrate the versatility of the FCAR(p) model, note that if $m_0(u_t) = 0$, and $m_j(u_t) = \alpha_j$, $j = 1, 2, \dots, p$ are constants, then the FCAR(p) reduces to a linear autoregressive model of order p , $X_t = \alpha_1 X_{t-1} + \dots + \alpha_p X_{t-p} + \omega_t$. Another example is, for each $j = 1, \dots, p$, the coefficients are of the form $m_j(X_{t-d}) = \alpha_j + \beta_j \exp\{-\delta X_{t-d}^2\}$. Then the FCAR(p) model reduces to the exponential autoregressive model of [31]. Moreover, the FCAR(p) formulation allows for a mixture of models; for example, $m_1(X_{t-d}) = \alpha_1$ and $m_2(X_{t-d}) = \alpha_2 + \beta_2 \exp\{-\delta X_{t-d}^2\}$. For applications, the versatility of the FCAR(p) model is shown in [35], [34], and [36]. In [34], an FCAR(2) model is fit to the monthly records of cases of chickenpox in New York City and an FCAR(8) model is applied to Wolf's annual sunspot numbers data set. An ecological application can be found in [35] in which an FCAR(2) model is fit to the annual Canadian lynx fur auction sales. The estimated coefficient functions in this application have a nice interpretation of predator-prey interaction. An economic application can be found in [36] in which a bivariate FCAR(p) model is fit to U.S. GNP and unemployment rate.

250 Fan and Yao [37] contains a review of methods for fitting the FCAR(p)
 251 model, and related inferential procedures. In the following section, we pro-
 252 pose a more recent, improved method for fitting the FCAR(p) model.

253 3.2. Spline-Backfitting Kernel Estimation

254 With no presupposed form for the functional coefficients, we propose
 255 a semiparametric method for finding pointwise estimates of the functions
 256 $m_j(u)$, $j = 0, 1, 2, \dots, p$. A number of methods are proposed in the statistics
 257 literature. Chen and Liu [38] and [35] propose a kernel regression approach
 258 to fitting the model. Harvill and Ray [39] extend the procedure to the case
 259 when the series is a vector process. More recently, spline-backfitted kernel
 260 (SBK) estimation has been proposed as a means for fitting semiparametric
 261 models like the FCAR(p) model. SBK estimation is an adaptation of the
 262 backfitting algorithm of [40]. By using backfitting, we take advantage of
 263 the additive structure of the model to reduce the dimensionality. Instead
 264 of estimating the p coefficient functions simultaneously, we pre-estimate the
 265 functions, backfit, and then estimate the functions separately.

266 The SBK method uses an under-smoothed centered standard spline pro-
 267 cedure to pre-estimate the $m_j(u)$, $j = 0, 1, 2, \dots, p$. These pre-estimates,
 268 also called “oracle” estimates, are used to find pseudo-responses. Then the
 269 pseudo-responses are used to estimate the $m_j(u)$ through a kernel estimator;
 270 e.g., the Nadaraya-Watson estimator. The SBK method was first proposed
 271 by [41] for estimating nonlinear additive autoregressive models. Wang and
 272 Yang [42] adapt the SBK method for IID data, [43] adapt it to generalized
 273 additive models, and [44] to partially linear additive models. Liu and Yang
 274 [45] propose the SBK method for additive coefficient models.

275 The ability to estimate $m_j(u)$, $j = 0, 1, 2, \dots, p$ relies on the good approx-
 276 imation properties of spline estimators. For any $j = 0, 1, 2, \dots, p$, assume
 277 $m_j(\cdot)$ is sufficiently smooth. Without loss of generality, u can be defined
 278 on the compact interval $[0, 1]$. Define the integer $N \approx T^{2/5} \log T$, and let
 279 $H = (N + 1)^{-1}$ [see 41, Appendix]. Let $0 = \xi_0 < \xi_1 < \dots < \xi_N < \xi_{N+1} = 1$
 280 denote a sequence of equally spaced knots. There is a set of basis functions
 281 $b_0(u), b_1(u), \dots, b_{N+1}(u)$ and a set of constants $\tilde{\lambda}_{0,j}, \tilde{\lambda}_{1,j}, \dots, \tilde{\lambda}_{N+1,j}$ such that
 282 the spline estimator of the j -th coefficient is

$$m_j(u) \approx \tilde{m}_j(u) = \sum_{k=0}^{N+1} \tilde{\lambda}_{k,j} b_k(u). \quad (4)$$

For the basis functions, we choose the linear B -spline basis, defined by

$$b_k(u) = \left(1 - \frac{|u - \xi_k|}{H}\right)_+ = \begin{cases} (N+1)u - k + 1, & \xi_{k-1} \leq u < \xi_k, \\ k + 1 - (N+1)u, & \xi_k \leq u < \xi_{k+1}, \\ 0, & \text{otherwise.} \end{cases}$$

283 The coefficients $\tilde{\lambda}_{0,j}, \tilde{\lambda}_{1,j}, \dots, \tilde{\lambda}_{N+1,j}$ are estimated via least squares; that is,
 284 the $\tilde{\lambda}_{k,j}$, $k = 0, 1, \dots, N+1$, $j = 0, 1, 2, \dots, p$ are the values of $\lambda_{k,j}$ that
 285 minimize the sum of squares

$$\sum_{t=p+1}^T \left[X_t - \sum_{k=0}^{N+1} \lambda_{k,0} b_k(u) - \sum_{j=1}^p \left\{ \sum_{k=0}^{N+1} \lambda_{k,j} b_k(u) \right\} X_{t-j} \right]^2. \quad (5)$$

The spline-estimated functional coefficients are then used to compute “pseudo-responses.” Specifically, for each $j' = 0, 1, 2, \dots, p, j' \neq j$, the pseudo-responses are defined by

$$\tilde{W}_{t,j'} = X_t - \sum_{j=1, j \neq j'}^p \tilde{m}_j(u) X_{t-j}, \quad t = p+1, p+2, \dots, T.$$

For each $j' = 0, 1, 2, \dots, p$, let $\tilde{\mathbf{W}}_{j'} = (\tilde{W}_{p+1,j'}, \dots, \tilde{W}_{T,j'})'$ represent the vector of pseudo-responses, and define the matrix

$$\mathbf{M} = \text{diag} \{K_h(X_{p+1-d} - u), \dots, K_h(X_{T-d} - u)\},$$

286 where $K_h(\cdot) = h^{-1}K(\cdot/h)$, $K(\cdot)$ is a kernel function, and $h > 0$ is a band-
 287 width. Then the SBK estimator of $m_{j'}(u)$ is

$$\hat{m}_{j'}(u) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \left(\frac{1}{T} \mathbf{C}' \mathbf{M} \mathbf{C} \right)^{-1} \frac{1}{T} \mathbf{C}' \mathbf{M} \tilde{\mathbf{W}}_{j'}, \quad (6)$$

where

$$\mathbf{C}' = \begin{bmatrix} X_{p+1} & X_{p+2} & \cdots & X_T \\ X_{p+1}(X_{p+1-d} - u) & X_{p+2}(X_{p+2-d} - u) & \cdots & X_T(X_{T-d} - u) \end{bmatrix}$$

288 The idea behind SBK estimation is to under-smooth in the pre-estimates in
 289 order to reduce the bias. This under-smoothing leads to a larger variance
 290 which is reduced in the kernel estimation step. The use of splines for the

Figure 1: Mean-centered realization of length $T = 500$ from an EXPAR(2) model given in equation (7).

pre-estimates is computationally fast while using kernel smoothing provides convenient asymptotic results [45].

To illustrate, consider a series of $T = 500$ observations from the exponential autoregressive model of order $p = 2$ (EXPAR(2)) given by

$$X_t = \left\{ 0.5 - 1.1e^{-50X_{t-1}^2} \right\} X_{t-1} + \left\{ 0.3 - 0.5e^{-50X_{t-1}^2} \right\} X_{t-2} + 0.2\omega_t, \quad (7)$$

where the ω_t are standard normal errors. A time plot of a mean-centered realization of length 500 of such a series is given in Figure 1. Since X_{t-1} is the functional variable, and is one of the autoregressive lags, the model in (7) must be rewritten and treated as

$$X_t = m_1(X_{t-1}) + m_2(X_{t-1})X_{t-2} + 0.2\omega_t.$$

Consequently the functional coefficients of the autoregressive terms are

$$m_1(u_t) = 0.5u_t - 1.1u_te^{-50u_t^2} \quad \text{and} \quad m_2(u_t) = 0.3 - 0.5e^{-50u_t^2},$$

where $u_t = X_{t-1}$.

To estimate the functional coefficients, begin by accounting for the variability in the response due to the term $m_1(u_t)$. Remove that variability, and use the pseudo-responses to estimate $m_2(u_t)$. Noting that the maximum lag is 2, we have

- 300 1. For $j = 1, 2$ in equation (4), fit a spline to the mean-centered data.
 301 Note that there is no $m_0(u_t)$ in this example. The result is an estimate
 302 $\tilde{m}_1(u_t)$ of $m_1(u_t)$ and $\tilde{m}_2(u_t)$ of $m_2(u_t)$.
2. Compute pseudo-responses $\tilde{W}_{t,2}$, $t = 3, \dots, T$ using

$$\tilde{W}_{t,2} = X_t - \tilde{m}_1(u_t), \quad t = 3, 4, \dots, T.$$

303 These pseudo-responses are a proxy for the original realization, but
 304 with the effect of the $m_1(u_t)$ removed.

- 305 3. Fit a kernel regression to the pseudo-responses to get the SBK estimate
 306 $\hat{m}_2(u_t)$ of $m_2(u_t)$.

307 Repeat steps 2 and 3, reversing the roles of m_1 and m_2 .

308 Figure 2 shows the estimation results of a simulated series from the ex-
 309 ponential autoregressive model in equation (7) with IID standard normal ω_t
 310 and 500 samples. The dark curves of dots are the estimated functions, and
 311 the solid (thin) lines are the true functions. The dashed lines are the 95%
 312 pointwise confidence bands.

313 3.3. Spatial Modeling for Lattice Data

For a fixed time t , consider a lattice process Y_s , $s = 1, 2, \dots, S$. In this section, to ease notation, the time index t is suppressed. Let $\mathcal{N}_s$ represent a neighborhood around location s . The simultaneous autoregressive (SAR) model is defined as

$$Y_s = \sum_{j' \in \mathcal{N}_s} \beta_{s,j'} Y_{j'} + \delta_s,$$

314 where $\beta_{s,j'}$ is a set of coefficients that induces the spatial autocorrelation
 315 between locations j' and s in $\mathcal{N}_s$, and δ_s are independent, zero-mean, constant
 316 variance errors. The SAR model was first introduced by [46]. The adjective
 317 “simultaneous” describes the S autoregressions that occur simultaneously at
 318 each data location in the formulation. To fit this model in Section 4, we will
 319 employ a two nearest neighbor structure to define $\mathcal{N}_s$. The model is fitted
 320 using maximum likelihood estimators which are obtained using the R package
 321 **spdep** [47].

322 3.4. Spatio-temporal Modeling

323 We now introduce two spatio-temporal models both of the form in (1).
 324 The first model we consider uses the noise process defined in (2) and assumes

Figure 2: Spline-backfitted kernel estimates of the coefficients $m_0(u) = 0.5u - 1.1u \exp\{-50u^2\}$ (left panel) and $m_1(u) = 0.3 - 0.5 \exp\{-50u^2\}$ (right panel). Heavy curves are the estimates $\hat{m}_0(u)$ and $\hat{m}_1(u)$; thinner lines are the true functions $m_0(u)$ and $m_1(u)$; dashed lines are 95% confidence bands.

325 a separable covariance structure for this process. The time series structure,
 326 $X_{s,t}$, is modeled as an FCAR model using the SBK method. Values of p and
 327 d in (3) are allowed to vary between locations, and the spatial structure, $Y_{s,t}$,
 328 is modeled separately using a SAR model at each time. By modeling the time
 329 and space components separately, we are implicitly assuming separability. If
 330 this assumption is appropriate, then the order in which the two models are
 331 fit (time-then-space, or space-then-time) should not matter.

332 The second spatio-temporal model does not assume separability. Com-
 333 bining the FCAR(p) model with a generalized version of the SAR model,
 334 we define the space-time functional coefficient simultaneous autoregressive
 335 (FCSAR) model as

$$Z_{s,t} = \sum_{w=1}^b \sum_{\ell \in \mathcal{N}_s} \beta_{s,\ell,w} Z_{\ell,t-w} + \sum_{k=1}^{p_s} m_{k,s}(Z_{s,t-d_s}) Z_{s,t-k} + \varepsilon_{s,t}, \quad (8)$$

336 where b is the spatial time order for the spatial component in the model,
 337 $\beta_{s,\ell,w}$ is the spatial autocorrelation between locations s and ℓ at a time lag of
 338 w , $Z_{s,t-d_s}$ is a delay variable, and $\varepsilon_{s,t}$ are IID with mean zero and constant
 339 variance σ_ε^2 . We allow the values of p_s and d_s to vary among locations. The
 340 FCSAR model is based on the space-time simultaneously specified autore-
 341 gressive model of [26].

342 In [26], the model in (8) has a linear autoregressive structure to the time
 343 series term for which a Bayesian approach is used for estimation. For our
 344 model, we estimate the $\beta_{s,\ell,w}$'s using maximum likelihood. Define $I_S \in \mathbb{R}^{S \times S}$
 345 as an identity matrix, $\mathbf{D} \in \mathbb{R}^{S \times S}$ as a matrix of weights with elements $\delta_{ij} =$
 346 $1/(2S)$ if the j th sensor is one of the two nearest neighbors of sensor i
 347 and $\delta_{ij} = 0$ otherwise, and ρ as a spatial autocorrelation parameter. Let
 348 $\mathbf{Z}_t = (Z_{1,t}, Z_{2,t}, \dots, Z_{S,t})'$ and $\mathbf{Z} = \mathbf{D}(\mathbf{Z}_{t-1}, \dots, \mathbf{Z}_{t-b})$. The log-likelihood
 349 function is

$$\ln L = C + \sum_{t=1}^T \ln |I_S - \rho \mathbf{D}| - \sum_{t=1}^T \left(\frac{S}{2} \right) \ln (\mathbf{e}' \mathbf{e}) \quad (9)$$

350 where C represents a constant not involving the parameters,

$$\mathbf{e} = \mathbf{e}_\beta - \rho \mathbf{D} \mathbf{Z}_t + \mathbf{Z} (\mathbf{Z}' \mathbf{Z})^{-1} \mathbf{Z}' \mathbf{D} \mathbf{Z}_t,$$

351

$$\mathbf{e}_\beta = \mathbf{Z}_t - \mathbf{Z} \boldsymbol{\beta}_s,$$

352

$$\boldsymbol{\beta}_s = (\mathbf{Z}'\mathbf{Z})^{-1} \mathbf{Z}'\mathbf{Z}_t,$$

353 and $\boldsymbol{\beta}_s = (\beta_{s,\ell,1}, \dots, \beta_{s,\ell,b})'$. Note that the weight matrix we use forces each
 354 neighboring parameter to be the same, i.e., $\beta_{s,1,w} = \beta_{s,2,w}$ in the two nearest
 355 neighbors structure. We maximize (9) with respect to $\boldsymbol{\beta}_s$ for each location s .
 356 For more detail on estimating a SAR model using maximum likelihood see
 357 [48]. The functional coefficients $m_{k,s}(Z_{s,t-d_s})$ are estimated using the SBK
 358 method defined in Section 3.2. The values of p_s and d_s are chosen for each
 359 site s by minimizing the mean square error of the fit to the observed time
 360 series at s . The value of the bandwidth h for the kernel smoothing in the
 361 SBK method is chosen via a modified, multifold cross-validation procedure
 362 at each site s [see 39]

363 4. Application and discussion

364 To illustrate the utility and compare the performance of the proposed
 365 models, we model GHI data at the 1.2 MV La Ola PV plant on the island
 366 of Lanai, Hawaii. The La Ola PV plant contains a grid of 12 single-axis
 367 tracked arrays arranged in three columns and four rows covering a total area
 368 of approximately 250m by 250m. The La Ola plant is located at 20.7669N
 369 156.9229W, near the southern end of the island of Lanai, atop the plateau
 370 in a subtropical dry forest climate [49]. Lanai lies in the rainfall shadow of
 371 Maui and thus is relatively dry, yet measured irradiance shows that vari-
 372 able irradiance due to cloud movement are the dominant pattern [50]. We
 373 chose to model GHI rather than POA irradiance to illustrate a more general
 374 application of our method. However, the La Ola data are POA irradiance
 375 rather than GHI. Sandia National Laboratories and SunPower Corporation
 376 designed an irradiance measurement system in part to study the effects of
 377 the movement of cloud shadows across the PV arrays on the power output
 378 of the plant [1]. Plane-of-array (POA) irradiance (in W/m²) is measured at
 379 the midpoint of each tracking array using LiCor-200 pyranometers.

380 Before fitting the models it is necessary to remove the diurnal trend, a
 381 step which we found somewhat difficult. Clear sky models are available for
 382 removing trends from measured GHI data; a review of some of these models
 383 can be found in [51]. We set out to use clear sky models to remove the
 384 diurnal trend, which would present no great difficulty for measured GHI.
 385 We know of no equivalent “clear-sky” model for POA irradiance (although,
 386 if the tracking algorithm is known with sufficient precision, such a model

could be assembled by applying a GHI-to-POA translation model, e.g., the DISC model of [52] to the output of a clear-sky model). We translated POA irradiance to GHI by assuming the isotropic sky model for the sky diffuse irradiance and using concurrent measurements of diffuse horizontal irradiance (DHI) and direct normal irradiance (DNI) from a nearby rotating shadowband radiometer (RSR) operated by the National Renewable Energy Laboratory. Because tracker rotations are not measured we estimated the angle of incidence on the modules using a generic algorithm for single-axis tracking [53]. Even with the use of measured DHI and DNI, the estimated GHI profiles were not well-matched with the output of available clear-sky models, and the clear sky models performed poorly in removing the trend. Consequently, we removed the diurnal trend in the estimated GHI by using a local polynomial kernel regression implemented in the `KernSmooth` package [54] in the R programming software.

We selected one year (i.e., January 1, 2010 to December 31, 2010) of POA irradiance measurements, which are recorded every second. We observed little to no variability from one irradiance measurement to the next at one second intervals and consequently reduced the data by time averaging. We investigated time-averages of lengths of 30 seconds, 1 minute, 5 minutes, and 10 minutes. Longer time averages (e.g., 15 and 20 minutes) were also considered but did not appear to be significantly different from the 10 minute averages. We chose to use 1-minute average data for our exploratory work.

The top time plot in Figure 3 contains the 1-minute time averages of estimated GHI in solid black superimposed with the local polynomial kernel regression estimate in dashed red for March 10. The bottom time plot contains the residuals, hereafter referred to as “transformed irradiance,” obtained after removing the diurnal trend by subtracting the kernel fit.

We examined a large number of time plots of GHI to find days with different variability characteristics. For each day, the weather condition was classified visually as being in one of three categories: clear, partly cloudy, and mostly cloudy, by the variability and magnitude of GHI. Figure 4 shows an example of how the different categories looked visually. We show the measured GHI for March 8 - 11 for which March 8 is considered a clear day due to its smoothly changing irradiance similar in magnitude to that predicted a clear sky model. March 9 and 10 are considered as partly cloudy. These days show variation in GHI due to moving clouds with a general trend that is similar to a clear sky day (e.g., March 8). March 11 is considered a mostly cloudy day because the general trend is much lower than the magnitude of

Figure 3: Top graph is the time plot of 1-minute averages (solid black) of irradiance measurements for March 10 with the local polynomial kernel estimate (dashed red) superimposed. The bottom plot is transformed irradiance (residuals after using local polynomial kernel regression to remove the diurnal trend).

Figure 4: Time plot of 10-minute time averaged GHI for March 8 - 11, 2010. These days illustrate the three cloud cover categories we used for the Lanai data. March 8 is clear, March 10 is partly cloudy, and March 11 is mostly cloudy.

425 irradiance during a clear sky day. Days with irradiance that changed among
 426 these categories were not considered in this paper. In Section 5 we discuss
 427 future work in which a weather covariate can be used to allow the model to
 428 be fitted to days with varying weather conditions.

429 For 2010, in Lanai, HI, only six days could be classified as “clear” through-
 430 out the entire day. For partly cloudy and mostly cloudy conditions, we found
 431 many days. For both of these weather conditions, six days in 2010 were ran-
 432 domly selected.

433 For each selected day, we explored whether assuming space-time covari-
 434 ance separability in (1) would be justified. Using the separable models we
 435 fit the data in two ways: space-then-time and time-then-space. If the sepa-
 436 rability assumption is appropriate, then the two models are equivalent and
 437 should yield similar results. For the space-then-time approach, we first fit
 438 the SAR model to the 16 sensors for each time, t . We obtain the residuals
 439 from the fitted SAR model, and then apply the FCAR model to each sensor
 440 separately. For the time-then-space model, we first fit the FCAR model to
 441 the detrended irradiance for each of the 16 sensors, and then the residuals
 442 from the fitted FCAR model are fit with the SAR model at each time point.
 443 For each approach the root mean square errors (RMSEs) (over all sensor
 444 locations and times) for eighteen days with three different weather condi-

tions are found in the first two columns of Table 1. For all days considered in this study, the RMSE for the model that fit space first is considerably smaller than when time was fit first. This is a strong indication that the assumption of separable covariance structures is not supported and that nonseparable models should be employed.

For a fixed time t , because the sensors are at fixed locations, the spatial structure can properly be considered a lattice. Consequently, for the nonseparable model we fit the FCSAR model in (8) for spatial time orders $b = 1, 2$. The last two columns in Table 1 contain values of RMSE for these two fits. The FCSAR model with $b = 2$ has the smallest RMSE for all 18 days, indicating the best fit among the models considered. For cloudy and partly cloudy conditions RMSE decreases substantially from $b = 1$ to $b = 2$ indicating that a lagged model is needed for greater prediction accuracy. For comparing accuracy between days with different irradiance levels, we provide the percentage of GHI that the RMSE represents. These are shown in the parentheses in Table 1.

Figures 5 through 7 contain six plots, grouped in three pairs. Each figure displays one sensor location for one day: a clear day (October 21, Figure 5); a cloudy day (April 1, Figure 6); and a partly cloudy day (August 3, Figure 7). For any one pair of plots, the top graph contains the GHI data represented by a solid black line, and the modeled GHI represented by a red dashed line. The bottom graph contains the detrended GHI data (solid black line) and the detrended modeled GHI (red dashed line). For all three days, the set of two plots labeled (a) were fit using a separable time-then-space approach; the two sets of plots labeled (b) were fit using a separable space-then-time approach; and the plots labeled (c) were fit using the nonseparable FCSAR model with $b = 2$. The collection of figures illustrates the nonseparable approach yields the best fit, regardless of the weather conditions, which is in agreement with minimum RMSE in Table 1. However, where RMSE is an aggregate measure of goodness-of-fit, the plots illustrate that at individual time points, the goodness-of-fit is uniformly better for the nonseparable model.

Forecasting the FCSAR model in time is largely dependent on using the SBK method for forecasting the FCAR term in (8). In [55], methodology is presented for forecasting a FCAR model using the SBK method. In this paper, we examine the performance of forecasting (8) in space for unobserved locations by using cross-validation. We simulated unobserved locations by omitting one or several sensors from our data set, and compared FSCAR model performance with a commonly used interpolation technique to judge

Table 1: Root mean squared error (RMSE, in W/m^2) for the four spatio-temporal models of the days with clear, partly cloudy, and mostly cloudy conditions. Columns S-T and T-S contain the RMSE for the separable spatio-temporal models. Column S-T contains RMSE for data with the spatial component fit first, then time; Column T-S contains RMSE with the time component fit first, then space. The last two columns contain RMSE for the nonseparable FCSAR model with spatial time orders $b = 1$ and $b = 2$, respectively. The values in parentheses are the percentages of the mean GHI for that day that the RMSE represents.

	Date	Separable		FCSAR	
		S-T	T-S	$b = 1$	$b = 2$
Clear	Feb. 3	0.64 (0.1)	2.32 (0.5)	0.38 (0.1)	0.16 (0.1)
	Feb. 16	3.58 (0.8)	15.03 (3.2)	2.28 (0.5)	1.36 (0.3)
	Mar. 8	0.62 (0.1)	1.99 (0.4)	0.37 (0.1)	0.22 (0.1)
	Mar. 19	0.60 (0.1)	6.33 (1.1)	0.36 (0.1)	0.22 (0.1)
	Oct. 21	0.94 (0.2)	4.01 (0.9)	0.65 (0.1)	0.42 (0.1)
	Dec. 16	1.13 (0.2)	6.17 (1.2)	0.87 (0.2)	0.44 (0.1)
Partly Cloudy	Mar. 7	8.26 (2.6)	63.68 (19.8)	6.79 (2.1)	3.35 (1.0)
	Apr. 1	7.88 (2.2)	99.53 (27.3)	6.96 (1.9)	4.51 (1.2)
	May 10	6.18 (2.1)	56.02 (19.1)	5.84 (2.0)	3.72 (1.3)
	Jun. 4	9.69 (2.8)	50.91 (14.7)	6.34 (1.8)	3.36 (1.0)
	Jun. 28	4.07 (1.4)	55.00 (18.8)	2.78 (1.0)	1.76 (0.6)
	Nov. 15	10.70 (3.0)	58.14 (16.3)	9.07 (2.5)	6.00 (1.7)
Mostly Cloudy	Feb. 1	8.61 (2.4)	62.60 (17.7)	7.80 (2.2)	4.88 (1.4)
	Mar. 15	13.26 (4.0)	116.71 (35.0)	10.65 (3.2)	5.50 (1.6)
	Apr. 6	6.00 (1.9)	48.16 (15.0)	3.93 (1.2)	2.43 (0.8)
	May 31	11.23 (2.2)	91.11 (17.8)	8.53 (1.7)	5.08 (1.0)
	Aug. 3	6.05 (2.0)	64.36 (21.0)	5.10 (1.7)	2.82 (1.0)
	Oct. 27	4.54 (1.2)	93.69 (25.4)	3.23 (0.9)	1.89 (0.5)

Figure 5: Time plots of 1-minute time averaged irradiance and of transformed irradiance with predicted values superimposed in red for a clear day, October 21. Forecasting was conducted using the (a) time-then-space fitting (b) space-then-time fitting (c) nonseparable model.

(a) April 1, Time-Space.

(b) April 1, Space-Time.

(c) April 1, Nonseparable.

Figure 6: Time plots of 1-minute time averaged irradiance and of transformed irradiance with predicted values superimposed in red for a partly cloudy day, April 1. Forecasting was conducted using the (a) time-then-space fitting (b) space-then-time fitting (c) nonseparable model.

(a) August 3, Time-Space.

(b) August 3, Space-Time.

(c) August 3, Nonseparable.

Figure 7: Time plots of 1-minute time averaged irradiance and of transformed irradiance with predicted values superimposed in red for a mostly cloudy day, August 3. Forecasting was conducted using the (a) time-then-space fitting (b) space-then-time fitting (c) nonseparable model.

483 the potential improvement offered by the FCSAR model.

484 Unobserved data are often estimated by interpolating between nearby
 485 sensors; one such technique is natural neighbor interpolation which comprises
 486 a weighted average with weights determined by a Voronoi partition [56]. A
 487 Voronoi partition divides the space that contains the sensors into regions.
 488 Each sensor will have a corresponding region consisting of all points closer
 489 to that sensor than to any other. We constructed a Voronoi partition on
 490 the set of training sensors along with the location of the missing sensor. For
 491 cross-validation, we took the weighted average of the training sensors where
 492 the weights are determined by the size of the regions. This weighted average
 493 is used for the prediction for the missing sensors.

494 We fit the FCSAR model to the training set of sensors with $b = 2$ and
 495 using a two nearest neighbor structure for $\mathcal{N}_s$. For each missing sensor, we
 496 determined the two nearest neighbors and predicted the irradiance by using
 497 the estimated β 's for those neighbors.

For our set of 16 sensors, we calculated the predictions with $k = 1, 2, 3, 4$ missing sensor locations. For $k > 1$, we predicted for each missing location one at a time. We calculated the root mean prediction error (RMPE) as

$$RMPE_{\Omega_i} = \frac{1}{Tk} \sum_{s \in \Omega_i} \left(\sum_{t=1}^T (\tilde{Z}_{s,t} - Z_{s,t})^2 \right),$$

where Ω_i is the i th set of k missing sensors, $\tilde{Z}_{s,t}$ is the predicted irradiance for the s th sensor at time t , and $Z_{s,t}$ is the observed irradiance. The $RMPE_{\Omega_i}$ is calculated for all $K = \binom{16}{k}$ possible combinations of k missing sensors. The mean RMPE is calculated as

$$\overline{RMPE}_k = \frac{1}{K} \sum_{i=1}^K RMPE_{\Omega_i}.$$

To compare the FCSAR model to the interpolation method, we take the ratio

$$\frac{\overline{RMPE}_k \text{ for FCSAR}}{\overline{RMPE}_k \text{ for interpolation}}.$$

498 The ratios for all 18 days are plotted in Figure 8. All ratios are less than one
 499 indicating that the FCSAR model performs better at predicting the missing
 500 sensors.

(a) Clear days.

(b) Partly cloudy days.

(c) Mostly cloudy days.

Figure 8: Plots of the ratios of the RMPE for (a) the clear days, (b) the partly cloudy days, and (c) the mostly cloudy days. The ratios are calculated as the RMPE of the FCSAR model divided by the RMPE of linear interpolation.

501 To examine the effect of different time averaging windows on the FSCAR
 502 model's performance we fit the model for a range of time averaging windows,
 503 from 10 minutes down to 30 seconds. For each day, we calculated the model's
 504 RMSE as well as the adjusted coefficient of determination R_a^2 . The adjusted
 505 coefficient of determination R_a^2 quantifies the level of agreement between the
 506 data and a fitted model taking into account the number of variables used in
 507 the model:

$$R_a^2 = 1 - \frac{SS_{\text{Fit}} / (TS - \nu_{\text{Fit}})}{SS_{\text{Total}} / (TS)},$$

508 where

$$SS_{\text{Fit}} = \sum_{s=1}^S \sum_{t=1}^T \left(\tilde{Z}_{s,t} - Z_{s,t} \right)^2,$$

509

$$SS_{\text{Total}} = \sum_{s=1}^S \sum_{t=1}^T \left(Z_{s,t} - \bar{Z} \right)^2,$$

510

$$\bar{Z} = \frac{1}{TS} \sum_{s=1}^S \sum_{t=1}^T Z_{s,t},$$

511 $\tilde{Z}_{s,t}$ is the predicted irradiance for the s th sensor at time t , $Z_{s,t}$ is the observed
 512 irradiance, and ν_{Fit} is the number of parameters used in the fit. Since we are
 513 using kernel regression to fit the time series, we must estimate the number of
 514 parameters associated with that regression. For the SBK estimate of the k th
 515 coefficient function in (8), the effective number of parameters is the trace of
 516 the smoother matrix

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \left(\frac{1}{T} \mathbf{C}' \mathbf{M} \mathbf{C} \right)^{-1} \frac{1}{T} \mathbf{C}' \mathbf{M}$$

517 in (6) [see 40, 57]. We calculate the total number of parameters as the sum
 518 of the parameters in the first double sum in (8) plus the sum of the effective
 519 number of parameters for the FCAR term. The values of R_a^2 and RMSE
 520 are shown in Table 2. We show the fits of three days for the different time
 521 averages in Figures 9-11.

Table 2: Root mean squared error (RMSE, W/m^2) for the nonseparable spatio-temporal models with $b = 2$ of the days with clear, partly cloudy, and mostly cloudy conditions. The columns are the RMSE for data at 30-second, 1-minute, 5-minute, and 10-minute averages. The values in parenthesis are the R_a^2 .

Condition	Date	30-sec	1-min	5-min	10-min
Clear	Feb. 3	0.38 (0.990)	0.30 (0.993)	0.19 (0.999)	0.16 (0.999)
	Feb. 16	4.98 (0.952)	3.68 (0.970)	1.82 (0.988)	1.36 (0.963)
	Mar. 18	1.79 (0.970)	0.99 (0.989)	0.42 (0.999)	0.22 (0.992)
	Mar. 19	0.81 (0.999)	0.63 (0.998)	0.32 (0.952)	0.22 (0.993)
	Oct. 21	3.94 (0.846)	2.21 (0.926)	0.67 (0.991)	0.42 (0.983)
	Dec. 16	0.67 (0.998)	0.61 (0.998)	0.49 (0.952)	0.44 (0.993)
Partly Cloudy	Mar. 7	25.66 (0.920)	17.68 (0.960)	5.42 (0.991)	3.35 (0.997)
	Apr. 1	31.75 (0.932)	24.65 (0.960)	6.99 (0.996)	4.51 (0.998)
	May 10	25.91 (0.927)	19.68 (0.954)	7.70 (0.987)	3.72 (0.999)
	June 4	22.05 (0.907)	20.60 (0.920)	7.21 (0.986)	3.36 (0.995)
	June 28	13.85 (0.935)	10.12 (0.966)	3.88 (0.994)	1.76 (0.999)
	Nov. 15	27.69 (0.908)	22.56 (0.935)	9.24 (0.979)	6.00 (0.987)
Mostly Cloudy	Feb. 1	26.46 (0.899)	24.12 (0.917)	9.27 (0.982)	4.88 (0.993)
	Mar. 15	27.85 (0.938)	20.29 (0.968)	6.05 (0.996)	5.50 (0.997)
	Apr. 6	24.22 (0.883)	19.44 (0.930)	4.01 (0.997)	2.43 (0.997)
	May 31	42.26 (0.899)	30.26 (0.943)	9.60 (0.988)	5.08 (0.997)
	Aug. 3	25.92 (0.948)	20.59 (0.966)	6.15 (0.995)	2.82 (0.998)
	Oct. 27	7.29 (0.980)	5.89 (0.985)	2.77 (0.996)	1.89 (0.999)

Table 2 shows that as the time averaging window decreases, RMSE increases and R_a^2 decreases, both indicating increasing disagreement between data and model. However, as the time averaging window decreases, variance in time averaged data at any individual location increases substantially (Figures 9-11). As ramps in the data increase in both magnitude and frequency the largest residuals of the fitted model also increase. Similar patterns are evident in the spatially-averaged data. Figure 12 compares distributions for the spatially averaged detrended irradiance data and corresponding distributions residuals for the fitted FSCAR model for a partly cloudy day. As the averaging window decreases, outliers increase in both the data and the model residuals also increase, leading to the increasing RMSE and decreasing R_a^2 evident in Table 2. However, the FSCAR model continues to fit the bulk of the data equally well across all time averaging windows, as is demonstrated by the relatively constant boxes and whiskers across the different time averages. Thus, the FSCAR model follows time averaged data equally well for various averaging windows.

5. Conclusion

We have presented a novel nonseparable spatio-temporal model for GHI data. This approach, termed the FSCAR model, outperforms a natural neighbor interpolation when predicting GHI at unobserved locations over the footprint of a PV system. We compared the nonseparable FSCAR model with simpler, separable models, and find little support for models that assume a separable covariance structure. The FSCAR model integrates an FCAR form for the time series component of the model and a SAR form for the spatial component. The FCAR(p) form of the time series component of our nonseparable model makes the FSCAR model flexible and reliable, and may be suitable for fitting irradiance data in general. Currently, the model is fit separately on each day. Further research will consider validating the fitted models by comparing predicted aggregate irradiance with generated power for a much larger solar power plant than La Ola. Future work may also explore adding a weather condition covariate that will allow the model to be fit over days with different weather conditions, by permitting the coefficient functions in the time series structure to vary based on weather condition. The use of the SBK method for estimating the time series structure has an assumption that the second derivative of the coefficient function is continuous [e.g. see 41, 55]. The use of a weather condition covariate may result in

(a)

(b)

(c)

(d)

Figure 9: Plots of the detrended observed irradiance and the fit of the FCSAR model for sensor 1 from 11:00 to 13:00 on October 21 (a clear day). The different plots are for different averages: (a) 10 minutes, (b) 5 minute, (c) 1 minute, and (d) 30 second.

Figure 10: Plots of the detrended observed irradiance and the fit of the FCSAR model for sensor 1 from 11:00 to 13:00 on April 1 (a partly cloudy day). The different plots are for different averages: (a) 10 minutes, (b) 5 minute, (c) 1 minute, and (d) 30 second.

Figure 11: Plots of the detrended observed irradiance and the fit of the FCSAR model for sensor 1 from 11:00 to 13:00 on August 3 (a mostly cloudy day). The different plots are for different averages: (a) 10 minutes, (b) 5 minute, (c) 1 minute, and (d) 30 second.

(a)

(b)

Figure 12: Boxplots for (a) the observed detrended data and (b) the residuals of the fit of the FCSAR model with $b = 2$ for April 1 (a partly cloudy day).

558 coefficient functions that do not satisfy this assumption. Patrick et al. [55]
559 examine an example for which the assumption is relaxed and shows the SBK
560 method still performs well in estimation and forecasting. Thus, the use of an
561 weather condition covariate will be worth exploring.

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