Energy harvesting from a piezoelectric biomimetic fish tail

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Abstract

Understanding fish migratory patterns and movements often relies on tags that are externally or internally implanted. Energy harvesting from fish swimming may benefit the state of the art of fish-tags, by increasing their battery lifetime and expanding their sensory capabilities. Here, we investigate the feasibility of underwater energy harvesting from the vibrations of a biomimetic fish tail through piezoelectric materials. We propose and experimentally validate a modeling framework to predict the underwater vibration of the tail and the associated piezoelectric response. The tail is modeled as a geometrically tapered beam with heterogeneous physical properties, undergoing large amplitude vibration in a viscous fluid. Fluid-structure interactions are described through a hydrodynamic function, which accounts for added mass and nonlinear hydrodynamic damping. To demonstrate the practical benefit of energy harvesting, we assess the possibility of powering a wireless communication module using the underwater vibration of the tail hosting the piezoelectrics. The electrical energy generated by the piezoelectrics during the undulations of the tail is stored and used to power the wireless communication device. This preliminary test offers compelling evidence for future technological developments toward self-powered fish-tags.

Keywords: Energy harvesting, Fish swimming, Piezoelectric, Self-powered device, Underwater vibration

1. Introduction

The design of miniature self-sustained devices powered by renewable energy sources have received considerable attention in the last few years [1, 2, 3]. Energy harvesting from the environment can especially benefit field applications, where the deployment of sensors is often hampered by limited lifetime, high maintenance, and large battery weight [4, 5, 6]. For example, the study of wildlife through externally or internally implanted sensing systems can greatly benefit from energy harvesting technologies, which could afford long observations and expand the sensory output [7, 8, 9].

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Smart materials have been recently proposed to be a promising technology toward self-powered animal monitoring [7, 9]. Energy harvesting of mechanical energy through piezoelectrics [10, 11, 12, 13], ionic polymer metal composites (IPMCs) [14, 15, 16, 17, 18], magnetostrictive materials [19, 20], and shape memory alloys [21] has been extensively studied [4, 5, 6]. Based on the current state of technology, piezoelectric materials are generally preferred for their efficiency and reliability [22, 23, 24, 25]. For example, energy harvesting from the human body through the use of piezoelectric materials has been demonstrated in the context of walking [26], limb motions [27], jaw movements [28], and heartbeats [29]. Beyond the human body, energy harvesting from avian flight using piezoelectrics has been considered in [7].

While most of these studies have focused on in-air applications, a few recent efforts have considered the possibility of underwater energy harvesting through piezoelectric materials [30, 31, 32, 33, 34, 35]. For example, the so-called energy harvesting eel converting energy from a fluid flow has been demonstrated in [30, 31]. Energy harvesting from underwater base excitation of piezoelectric beams has been investigated in [32, 35]. Finally, hydraulic pressure fluctuations as a source of usable energy for piezoelectric harvesting have been studied in [33, 34]. Here, we seek to investigate the possibility of harvesting mechanical energy from the undulations of a fish tail toward the design of self-powered fish-tags.

Specifically, we consider the biomimetic fish tail developed by our group in [9] to simulate the swimming of a thresher shark. The tail is fabricated through injection molding of Silicone, and a slender stainless steel beam is inserted in the mold to increase its stiffness. The active tail hosts two piezoelectric composites that are assembled following [32, 35] and are attached to both sides of the tail in a bimorph configuration. As the tail vibrates under base excitation, the piezoelectric composites generates a relatively large voltage output, which can be rectified to power external batteries or other devices [22, 23]. Such voltage output depends on the amplitude and frequency of base excitation, which ultimately control the underwater vibration of the biomimetic tail. We propose a tractable modeling framework to describe the underwater vibration of the tail and, ultimately, predict the voltage output for energy harvesting. The tail is modeled as a geometrically tapered beam with heterogeneous physical properties due to both the presence of the piezoelectric composites and the stainless steel beam. The effect of the surrounding water is modeled through a complex hydrodynamic function [36], which accounts for added mass and nonlinear hydrodynamic damping associated with vortex shedding and advection under large amplitude vibration. From the model of underwater vibrations, we compute the average curvature in the region covered by the piezoelectric composites, which is, in turn, used to predict the output voltage [32, 35].

The model is validated through a series of experiments, in which the amplitude and frequency of base excitation are systematically varied to assess the robustness of our predictions with respect to the open-circuit voltage and the vibration at the tip. To demonstrate the possibility of converting some of the vibration energy into usable electrical energy, we design a laboratory setup to simulate a plausible instance of a self-powered fish-tag. In such a setup, we connect the piezoelectric composites to an energy storage

module, which powers a wireless unit with a temperature sensor. As the biomimetic tail vibrates underwater, charge builds up in the storage module until temperature can be measured and transmitted wirelessly to an external receiver.

From practical and methodological points of view, the main contributions of this effort are: i) addressing the feasibility of energy harvesting devices for fish tags using a piezoelectric biomimetic fish tail; ii) demonstrating the entire energy harvesting and usage process, from capturing mechanical energy to sensing and powering a wireless communication module; and iii) developing a mathematical framework to model the entire process, including the underwater large amplitude vibrations of the biomimetic fish tail, the energy conversion through piezoelectric composites, and the storage of the harvested electrical energy.

This paper is organized as follows. In Section 2, we introduce the modeling framework to predict the undulations of the tail and the voltage output of the piezoelectrics. In Section 3, we demonstrate the accuracy of the model through a series of experiments in which the base excitation is parametrically varied. In Section 4, we discuss the practical implementation of the approach through the ad-hoc experimental setup. Conclusions are summarized in Section 5.

Figure 1: Biomimetic fish tail hosting two piezoelectric composites. White markers on the black electrical tape aid in tracking the tail motion through video processing.

2. Modeling

2.1. Underwater base excitation

The passive biomimetic fish tail used in this study is equivalent to that designed in [9], see Figure 1. To simplify the analysis and compensate for imperfections in the realization of the clamp, we model the thick portion of the tail that is not covered by the piezoelectrics and is adjacent to the base as a rigid pendulum connected to the rest of the structure through a torsional spring. As displayed in Figure 2, we place the origin of the material abscissa x in correspondence of the torsional spring of stiffness κ and focus on the base excitation of a beam with varying geometrical and physical properties. The base excitation is controlled through the rotation $\theta_B(t)$, so that the x -axis rotates with respect to the otherwise quiescent fluid and the elastic deflection is measured along the rotating z -axis.

Next, by assuming that the portion of the tail covered by the piezoelectric composites is considerably stiffer than the rest of the tail, we neglect its dynamics. Specifically, we hypothesize that for $0 < x < L_p$, the tail elastic deflection $w_p(x, t)$ can be approximated by:

$$w_p(x, t) = A_1(t)x^3 + A_2(t)x^2 + A_3(t)x + A_4(t) \quad (1)$$

where $A_1(t)$, $A_2(t)$, $A_3(t)$, and $A_4(t)$ are unknown functions of time. Note that Eq. (1) is the solution of the elastica with a null distributed load and constant stiffness [37]. These constants are determined from the following boundary conditions:

$$w_p(0, t) = 0 \quad (2a)$$

$$\kappa \frac{\partial w_p(x, t)}{\partial x} \Big|_{x=0} = k_p \frac{\partial^2 w_p(x, t)}{\partial x^2} \Big|_{x=0} \quad (2b)$$

$$w_p(L_p, t) = w(L_p, t) \quad (2c)$$

$$k_p \frac{\partial w_p(x, t)}{\partial x} \Big|_{x=L_p} = k(L_p) \frac{\partial w(x, t)}{\partial x} \Big|_{x=L_p} \quad (2d)$$

$$k_p \frac{\partial^2 w_p(x, t)}{\partial x^2} \Big|_{x=L_p} = k(L_p) \frac{\partial^2 w(x, t)}{\partial x^2} \Big|_{x=L_p} \quad (2e)$$

$$k_p \frac{\partial}{\partial x} \left[\frac{\partial^2 w_p(x, t)}{\partial x^2} \right] \Big|_{x=L_p} = \frac{\partial}{\partial x} \left[k(x) \frac{\partial^2 w(x, t)}{\partial x^2} \right] \Big|_{x=L_p} \quad (2f)$$

where k_p is an equivalent bending stiffness for the portion of the tail that is covered by the piezoelectrics, $k(x)$ is the bending stiffness for $L_p < x < L$, and $w(x, t)$ is the elastic deflection for $L_p < x < L$. Equations (2a) and (2b) impose that the elastic deflection is zero at the spring and the bending moment equal to the spring torque. Similarly, Eqs. (2c)–(2f) imposes the continuity of the displacement, rotation, bending moment, and shear at $x = L_p$. Note that both Eqs. (2b) and (2e) discards the direct piezoelectric effect, whereby they assume that the bending moment in the portion of the tail covered by the piezoelectric composites is independent of the voltage across their electrodes due to the large elastic stiffness [5, 38]. For convenience, we term the elastic deflection and the rotation at $x = L_p$ as $\delta(t)$ and $\theta(t)$, respectively, and we combine Eqs. (1) and (2a)–(2f) to find

$$A_4(t) = 0 \quad (3a)$$

$$A_3(t)\kappa = 2A_2(t)k_p \quad (3b)$$

$$A_1(t)L_p^3 + A_2(t)L_p^2 + A_3(t)L_p = \delta(t) \quad (3c)$$

$$3A_1(t)L_p^2 + 2A_2(t)L_p + A_3(t) = \theta(t) \quad (3d)$$

$$6A_1(t)L_p + 2A_2(t) = \frac{k(L_p)}{k_p} \frac{\partial^2 w(x, t)}{\partial x^2} \Big|_{x=L_p} \quad (3e)$$

$$6A_1(t) = \frac{1}{k_p} \frac{\partial}{\partial x} \left[k(x) \frac{\partial^2 w(x, t)}{\partial x^2} \right] \Big|_{x=L_p} \quad (3f)$$

Figure 2: Schematic of the undulation of the biomimetic fish tail.

The elastic deflection for $L_p < x < L$ is given by the following partial differential equation [9]:

$$\frac{\partial^2}{\partial x^2} \left[k(x) \frac{\partial^2 w(x, t)}{\partial x^2} \right] + \rho(x) \frac{\partial^2}{\partial t^2} [w(x, t) + \theta_B(t)(x + L_0)] = H(x, t) + s(x, t) \quad (4)$$

where $\rho(x)$ is the mass density per unit length, $H(x, t)$ is the hydrodynamic loading exerted on the tail by the encompassing fluid, and $s(x, t)$ is the structural damping. Both $H(x, t)$ and $s(x, t)$ are related to the tail deflection field. The remaining boundary conditions for the tail at $x = L_p + L$ are

$$\frac{\partial^2 w(x, t)}{\partial x^2} \Big|_{x=L_p+L} = 0 \quad (5a)$$

$$\frac{\partial}{\partial x} \left[k(x) \frac{\partial^2 w(x, t)}{\partial x^2} \right] \Big|_{x=L_p+L} = 0 \quad (5b)$$

Following [9], the bending stiffness is

$$k(x) = \begin{cases} E_M \frac{b_M h_M^3}{12} + E_S \frac{b(x) h^3(x)}{12} & L_p \leq x \leq L_M \\ E_S \frac{b(x) h^3(x)}{12} & L_M < x \leq L_p + L \end{cases} \quad (6)$$

109 and the mass density per unit length is

$$\rho(x) = \begin{cases} \rho_M b_M h_M + \rho_S b(x) h(x) & L_P \leq x \leq L_M \\ \rho_S b(x) h(x) & L_M < x \leq L_P + L \end{cases} \quad (7)$$

110 where E_M , ρ_M , b_M , h_M , and L_M are the Young's modulus, mass density per unit volume, width, height,
 111 and length of the metal reinforcement, respectively, and E_S , ρ_S , $b(x)$, and $h(x)$ are the Young's modulus,
 112 mass density per unit volume, width, and height of the silicone, respectively. Equations (6) and (7) assume
 113 that the tail cross-section is rectangular. While this assumption is likely inadequate to accurately resolve
 114 the flow physics generated by the vibrating tail, we have shown in [9] that it is appropriate to closely
 115 predict low-frequency structural vibration of the tail both in air and in water. For a slender body, structural
 116 vibrations are expected to be controlled by averaged physical properties and fluid-structure interactions at
 117 each cross-section.

118 By defining the relative abscissa X and elastic deflection $W(X, t)$ such that $x = X + L_P$ and $w(x, t) =$
 119 $W(X, t) + \delta(t) + \theta(t)X$ into Eq. (4), we obtain

$$\frac{\partial^2}{\partial X^2} \left[k(X) \frac{\partial^2 W(X, t)}{\partial X^2} \right] + \rho(X) \frac{\partial^2}{\partial t^2} [W(X, t) + \delta(t) + \theta(t)X + \theta_B(t)(X + L_0 + L_P)] = H(X, t) + s(X, t) \quad (8)$$

120 From Eqs. (2c), (2d), (5a), and (5b), we derive the following homogeneous boundary conditions for $W(X, t)$:

$$W(0, t) = 0 \quad (9a)$$

$$\left. \frac{\partial W(X, t)}{\partial X} \right|_{X=0} = 0 \quad (9b)$$

$$\left. \frac{\partial^2 W(X, t)}{\partial X^2} \right|_{X=L} = 0 \quad (9c)$$

$$\left. \frac{\partial}{\partial X} \left[k(X) \frac{\partial^2 W(X, t)}{\partial X^2} \right] \right|_{X=L} = 0 \quad (9d)$$

121 Eqs. (8) and (9a)–(9d) cannot be solved independently since $\delta(t)$ and $\theta(t)$ are yet unknown; Eqs. (3a)–(3f)
 122 are required to close the system.

123 We assume that the base excitation $\theta_B(t)$ is sinusoidal with radian frequency ω and amplitude $\hat{\theta}_B(\omega)$,
 124 that is,

$$\theta_B(t) = \hat{\theta}_B(\omega) \sin(\omega t) \quad (10)$$

125 Here and in what follows, we use a superimposed hat to indicate phasors. In addition, we hypothesize
 126 that the relative displacements are harmonic and share the same frequency of the base excitation under
 127 steady-state conditions [9], that is,

$$w(x, t) = \text{Im}[\hat{w}(x, \omega) e^{i\omega t}] \quad (11)$$

128 where i is the imaginary unit and $\text{Im}[\cdot]$ is the imaginary part. Specifically, we use $\hat{A}_1(\omega)$, $\hat{A}_2(\omega)$, $\hat{A}_3(\omega)$, and
 129 $\hat{A}_4(\omega)$ for $0 < x < L_P$ and $\hat{W}(X, \omega)$ for $0 < X < L$.

130 The hydrodynamic force per unit length is [36]

$$\hat{H}(X, \omega) = \frac{\pi}{4} \rho_f \omega^2 b^2(X) \Theta(\beta(X, \omega), \epsilon(X, \omega), \alpha(X)) [\hat{W}(X, \omega) + \hat{\delta}(\omega) + \hat{\theta}(\omega)X + \hat{\theta}_B(\omega)(X + L_0 + L_p)] \quad (12)$$

131 where ρ_f is the fluid mass density per unit volume, $\beta(X, \omega) = \rho_f \omega b^2(X)/(4\mu)$ is the oscillatory Reynolds
 132 number with μ being the dynamic viscosity of the fluid, $\epsilon(X, \omega) = |\hat{W}(X, \omega) + \hat{\delta}(\omega) + \hat{\theta}(\omega)X + \hat{\theta}_B(\omega)(X + L_0 +$
 133 $L_p)|/b(X)$ is the local absolute displacement of the beam cross-section, and $\alpha(X) = h(X)/b(X)$ is the cross-
 134 section aspect ratio. The complex hydrodynamic function $\Theta(\beta, \epsilon, \alpha) = \Gamma(\beta, \alpha) + \Delta(\beta, \epsilon, \alpha)$ is composed of the
 135 added mass effect $\Gamma(\beta, \alpha)$ and the hydrodynamic damping $\Delta(\beta, \epsilon, \alpha)$. $\Gamma(\beta, \alpha)$ is only applicable for infinitely
 136 small vibrations, while $\Delta(\beta, \epsilon, \alpha)$ is a nonlinear correction to account for the effect of finite amplitude
 137 vibration. Following [36], we use $\Gamma(\beta, \alpha) = (1.02 + 0.995\alpha - 0.496\alpha^2) + (3.07 + 1.83\alpha - 1.245\alpha^{0.5})\beta^{-0.5} -$
 138 $i(3.12 + 1.495\alpha - 0.52\alpha^{0.5})\beta^{-0.5}$ and $\Delta(\beta, \epsilon, \alpha) = -i0.626e^{-1.143\alpha}\beta^{0.75/(1+5.942\alpha)}\epsilon^{2/(1+1.192\alpha)}$.

139 The structural damping $\hat{s}(X, \omega)$ in the frequency domain is approximated with a simple hysteretic model,
 140 where the real bending stiffness $k(X)$ is replaced with the complex bending stiffness $(1 + i\eta)k(X)$ with loss
 141 factor η [39].

142 We use the Galerkin method to compute an approximate solution for $\hat{W}(X, \omega)$, see for example [9, 36, 40].
 143 Specifically, we express $\hat{W}(X, \omega)$ as a linear combination of the in-vacuum mode shapes of a cantilever of
 144 length L with constant mass per unit length and bending stiffness $\{\psi_m(X)\}_{m=1}^\infty$, that is,

$$\psi_m(X) = \zeta_m \left[\sin \frac{\lambda_m X}{L} - \sinh \frac{\lambda_m X}{L} - \frac{\sin \lambda_m + \sinh \lambda_m}{\cos \lambda_m + \cosh \lambda_m} \left(\cos \frac{\lambda_m X}{L} - \cosh \frac{\lambda_m X}{L} \right) \right] \quad (13)$$

145 for $m = 1, 2, \dots$. Here, λ_m is the m -th solution of the characteristic equation $\cos \lambda \cosh \lambda = -1$ and ζ_m is
 146 selected to normalize the mode shapes $\frac{1}{L} \int_0^L \psi_m^2(X) dX = 1$. Therefore, the relative elastic deflection can be
 147 written as

$$\hat{W}(X, \omega) = \sum_{m=1}^{\infty} q_m(\omega) \psi_m(X) \quad (14)$$

148 where $q_m(\omega)$ is the m -th complex modal amplitude coefficient for $m = 1, 2, \dots$

149 By substituting Eq. (14) into Eqs. (3b), (3e) and (3f), we solve for unknown functions $\hat{A}_1(\omega)$, $\hat{A}_2(\omega)$,
 150 and $\hat{A}_3(\omega)$ in terms of $q_m(\omega)$

$$\hat{A}_1(\omega) = \frac{k(0)}{6k_p} \sum_{m=1}^{\infty} q_m(\omega) \left. \frac{d^2 \psi_m(X)}{dX^2} \right|_{X=0} \quad (15a)$$

$$\hat{A}_2(\omega) = \frac{k(0)}{2k_p} \sum_{m=1}^{\infty} q_m(\omega) \left(\left. \frac{d^2 \psi_m(X)}{dX^2} \right|_{X=0} - L_p \left. \frac{d^3 \psi_m(X)}{dX^3} \right|_{X=0} \right) \quad (15b)$$

$$\hat{A}_3(\omega) = \frac{k(0)}{\kappa} \sum_{m=1}^{\infty} q_m(\omega) \left(\left. \frac{d^2 \psi_m(X)}{dX^2} \right|_{X=0} - L_p \left. \frac{d^3 \psi_m(X)}{dX^3} \right|_{X=0} \right) \quad (15c)$$

151 Therefore, $\hat{\delta}(\omega)$ and $\hat{\theta}(\omega)$ can be expressed as functions of $q_m(\omega)$ by using Eqs. (3c) and (3d).

Finally, by substituting Eq. (14) into Eq. (8), multiplying by $\psi_n(x)$ with $n = 1, 2, \dots$, and integrating from $X = 0$ to L , we find

$$\begin{aligned}
& \sum_{m=1}^{\infty} q_m(\omega) \int_0^L \left\{ (1 + i\eta) \frac{d^2}{dX^2} \left[k(X) \frac{d^2 \psi_m(X)}{dX^2} \right] \right. \\
& \quad \left. - \omega^2 \psi_m(X) \left[\rho(X) + \frac{\pi}{4} \rho_f b^2(X) \Gamma(\beta(X, \omega), \alpha(X)) \right] \right\} \psi_n(X) dX = \\
& \omega^2 \int_0^L \left\{ \left[\hat{\delta}(\omega) + \hat{\theta}(\omega)X + \hat{\theta}_B(\omega)(X + L_0 + L_p) \right] \left[\rho_2(X) + \frac{\pi}{4} \rho_f b^2(X) \Gamma(\beta(X, \omega), \alpha(X)) \right] \right. \\
& \quad \left. + \frac{\pi}{4} \rho_f b^2(X) \Delta(\beta(X, \omega), \epsilon(X, \omega), \alpha(X)) \right. \\
& \quad \left. \times \left[\sum_{m=1}^{\infty} q_m(\omega) \psi_m(X) + \hat{\delta}(\omega) + \hat{\theta}(\omega)X + \hat{\theta}_B(\omega)(X + L_0 + L_p) \right] \right\} \psi_n(X) dX \quad (16)
\end{aligned}$$

Due to the fact that the mode shapes in Eq. (13) form a basis for the solution space, solving the infinite set of algebraic equations with $n = 1, 2, \dots$ in Eq. (16) would yield the exact solution of the problem. An approximate solution of Eq. (16) with $\hat{\delta}(\omega)$ and $\hat{\theta}(\omega)$ given by Eqs. (3c), (3d), and (15a)–(15c), at the select radian frequency ω , is obtained by truncating the summation to the first five mode shapes, approximating the integrals by using 51 Gaussian quadrature points, and solving the resulting nonlinear complex algebraic system using the MATHEMATICA built-in function “FindRoot” (www.wolfram.com). We comment that a similar modeling framework is presented in [9], where we have studied IPMC-based energy harvesting from the undulation of a biomimetic tail. However, different from piezoelectrics, the IPMCs are highly compliant materials that have a negligible effect on the vibrations of the host structure. Thus, the analysis in [9] did not account for their presence, and, as a result did not require the detailed treatment proposed here to effectively handle the stepped nature of the biomimetic tail hosting the piezoelectrics.

With respect to Figure 2, the approach in [9] did not distinguish between the tail portions that were or were not covered by smart materials and hypothesized that the rigid pendulum was perfectly clamped to the rest of the structure. The present approach extends the model in [9] through the inclusion of the functions of time $A_1(t)$, $A_2(t)$, $A_3(t)$, and $A_4(t)$ to describe the tail elastic deflection in the region covered by piezoelectric composites, see Eq. (1), and considers the possibility of a relative rotation between the pendulum and the flexible structure through a torsional spring, see Eq. (2b). These extensions, in turn, modify the governing equation for underwater large amplitude vibration, see Eq. (8), where the elastic deflection and rotation at $x = L_p$, namely, $\delta(t)$ and $\theta(t)$, appear as additional unknowns.

2.2. Electrical output

The open-circuit voltage from the piezoelectrics is related to the mean curvature of the region covered by piezoelectrics [41, 42], that is,

$$V(t) = \gamma k_{\text{mean}}(t) \quad (17)$$

176 where γ is the electromechanical gain and $k_{\text{mean}}(t)$ is the tail mean curvature. Specifically,

$$k_{\text{mean}}(t) = \frac{1}{x_2 - x_1} \left. \frac{\partial w_p(x, t)}{\partial x} \right|_{x=x_1}^{x=x_2} \quad (18)$$

177 where x_1 and x_2 are the beginning and ending abscissas of the active part of the piezoelectric composite,
 178 which is a portion of L_p due to the additional coating provided by the manufacturer. By substituting Eq. (1)
 179 into Eq. (18), we obtain the value of the mean curvature in the frequency domain as

$$\hat{k}_{\text{mean}}(\omega) = 3\hat{A}_1(\omega)(x_1 + x_2) + 2\hat{A}_2(\omega) \quad (19)$$

180 Here, $\hat{k}_{\text{mean}}(\omega)$ is calculated by using the solution from Eq. (16) and replacing $\hat{A}_1(\omega)$ and $\hat{A}_2(\omega)$ with
 181 Eqs. (15a) and (15b), respectively.

182 The short-circuit current through the piezoelectrics can be computed from the open-circuit voltage in
 183 Eq. (17)

$$\hat{I}(\omega) = i\omega C\hat{V}(\omega) \quad (20)$$

184 where C is the internal capacitance. Notably, both the electromechanical gain and the capacitance depends
 185 on how the piezoelectrics are interconnected, for example, in series or in parallel.

186 3. Results

187 3.1. Experimental setup

188 We use the same biomimetic fish tail in [9] for our experiments. Briefly, the tail is designed from
 189 the shark morphometric data in [43] for the fork, caudal fin, and dorsal and ventral lobes lengths. The
 190 heterocercal fish tail consists of a conical frustum with elliptical cross-section and the caudal fin. The tail is
 191 realized through injection molding of Silicone II produced by General Electric, and a slender stainless steel
 192 beam is inserted into the mold to increase its overall stiffness, see detailed descriptions in [9]. The tail has
 193 maximum dimensions of 250, 100, and 15 mm in length, width, and thickness, respectively, see Figure 1.
 194 The stainless steel beam has $L_M = 170$ mm, $b_M = 12.7$ mm, and $h_M = 0.4$ mm. The width and height of
 195 a simplified beam model of the tail are $b(x) = -0.163x + 54$ ($0 \leq x < 150$), $1.24x - 156$ ($150 \leq x < 170$),
 196 $0.73x - 69$ ($170 \leq x < 190$), and $0.666x + 169$ ($190 \leq x \leq 250$) and $h(x) = -0.025x + 11$ ($0 \leq x < 150$),
 197 $0.382x - 50$ ($150 \leq x < 170$), $-0.011x + 17$ ($170 \leq x < 190$), and $-0.158x + 45$ ($190 \leq x \leq 250$) in mm,
 198 respectively.

199 The piezoelectric composites used in this study are assembled following [32, 35]. We select waterproof
 200 macro-fiber composites (MFCs) M4007P1-PES (www.smart-material.com) with an active surface area of
 201 40×7 mm². Each MFC has a length of 52 mm and a width of 16 mm larger than its active part due to the
 202 waterproof coating. For each piezoelectric composite, two MFCs are glued on a thin aluminum lamina of

203 dimensions $40 \times 7 \times 0.127 \text{ mm}^3$, using epoxy 3M DP460. The total mass of the two piezoelectric composites
 204 is 6 g, which is approximately 5.7% of the mass of the biomimetic tail.

205 Two piezoelectric composites are attached to the sides of the fish tail by positioning their active regions
 206 from $x_1 = 30 \text{ mm}$ to $x_2 = 70 \text{ mm}$. Specifically, the piezoelectric composites are interposed between two
 207 mylar layers and fastened by 3M electrical tape to the tail to avoid detachment during vibration. The tail
 208 is connected to a rigid steel bar of 13 mm diameter connected to a fixed base through a frictionless bearing.
 209 We measure the lengths of the three regions in the tail as $L_0 = 50 \text{ mm}$, $L_p = 70 \text{ mm}$, and $L = 180 \text{ mm}$.
 210 The motion of the tail is controlled via a servo motor Futaba S3306MG programmed using an Arduino
 211 microcontroller. This is used in conjunction with an external gear with 1/3 gear ratio to allow for precise
 212 adjustments of the base excitation amplitude. The mount for the apparatus is fabricated via ABS plastic
 213 using a Stratasys Dimension Elite 3D Printer. The setup is installed in a glass test-pool of dimensions
 214 $1200 \times 1200 \times 150 \text{ mm}^3$ at room temperature, for which the nominal values of the mass density per unit
 215 volume of the water is $\rho_f = 1000 \text{ kg m}^{-3}$, and its dynamic viscosity equals $\mu = 10^{-3} \text{ Pa s}$. Figure 3 displays
 216 a computer-aided design of the experimental setup.

Figure 3: Computer-aided design of the experimental setup used for investigating energy harvesting from a biomimetic tail hosting piezoelectric composites.

217 To investigate the role of the base excitation on underwater vibration, we explore three different nominal
 218 base excitation amplitudes of 0.005, 0.022, and 0.043 rad and the excitation frequency is varied from 0.5
 219 to 2.5 Hz with increments of 0.1 Hz. The base excitation and the tip motion of the tail are estimated by
 220 tracking a few markers painted on as selected fiducial points. Specifically, the trajectories of the mark-
 221 ers are recorded through a IDT/Y3C high speed camera, positioned above the tail, with Motion studio

222 (www.idtvision.com). The camera sampling rate is set to 100 frames per second. We use Xcitex Proana-
 223 lyst software (www.xcitex.com) to extract the trajectory and a MATLAB (www.mathworks.com) script to
 224 obtain the magnitude and phase of the phasors of the tail tip motion with respect to the base excitation.

225 During these experiments, the piezoelectrics are connected in series, see Figure 4, to produce a large
 226 voltage in the whole frequency range for ease of measurement. Such a voltage is referred to as $V_{\text{ser}}(t)$ in
 227 Eq. (17) and is acquired using National Instrument data acquisition board USB-6221 and a custom-made
 228 code in Labview 8.6 (www.ni.com). The input impedance of the data acquisition board is $10\text{ G}\Omega$ in parallel
 229 with 100 pF , so that the piezoelectrics are practically open-circuited. The measured voltages are processed
 230 using another MATLAB script that removes the DC offset and extracts the fundamental harmonics. The
 231 associated internal capacitance C_{ser} is measured using a Protek B940 True RMS Multimeter.

Figure 4: Series connection of the piezoelectric composites.

232 3.2. Mechanical and electrical properties

233 Following [9], we use $E_M = 212\text{ GPa}$, $\rho_M = 8030\text{ kg m}^{-3}$, $E_S = 0.23\text{ MPa}$, $\rho_S = 1024\text{ kg m}^{-3}$, and
 234 $\eta = 0.13$ for the tail. We estimate κ , k_p , and the electromechanical gain for the MFC is in series connection
 235 γ_{ser} through a series of in-air experiments with a base excitation of 0.005 rad in the frequency range from 1.5
 236 to 4.0 Hz with a step of 0.1 Hz . Figure 5 displays the tip deflection with respect to the base excitation for
 237 short-circuit electrodes, with reference to Figure 4. The spring torsional stiffness κ and the bending stiffness
 238 k_p are identified from the log-scaled transfer function at the tip through least squares error minimization [40],
 239 considering both the real and imaginary parts of the transfer function. The coefficient of determination of
 240 the fitting, R -squared, is 0.8398 for $\kappa = 4\text{ N m}$ and $k_p = 1.8\text{ N m}^2$. Notably, the maximum value of k along
 241 the tail is 0.018 N m^2 , which is only a small fraction of k_p . The in-air resonance frequency is 3.6 Hz and
 242 increases by as much as 1.7 Hz due to the variation of the stiffness by the attachment of the piezoelectric
 243 composites [9]. For open-circuit conditions, we perform the same experiment, see Figure 6. Finally, through
 244 least square error minimization for the log-scaled magnitude of the voltage, we find $\gamma_{\text{ser}} = 162\text{ V m}$ with
 245 R -squared of 0.8391 , and from the multimeter reading we have $C_{\text{ser}} = 0.4\text{ nF}$.

Figure 5: (Top) magnitude and (bottom) phase of the frequency response defined as the ratio of the phasor of the tail deflection at tip to the amplitude of base rotation in air for short-circuited electrodes. Dots are experimental data and solid lines are predictions from the theoretical model in Section 2. The nominal base excitation amplitude is 0.005 rad.

Figure 6: Output voltage per unit base rotation for in-air vibrations and open-circuited electrodes. Dots are experimental data and solid lines are predictions from the theoretical model in Section 2. The nominal base excitation amplitude is 0.005 rad.

3.3. Underwater vibrations

Figure 7 displays the transfer functions for the tip deflection for underwater vibrations at three different base excitation amplitudes of 0.005, 0.022, and 0.043 rad. Experimental data are compared with predictions from Eq. (16), which accounts for nonlinear fluid-structure interactions. The model closely predicts the trends of both the magnitude and phase. The resonance frequency is largely independent of the level of the base excitation and is approximately 1.6 Hz as seen from both the amplitude and phase diagrams. This value is compatible with the tail beat frequency of live fish swimming at approximately one body length per second [44]. Specifically, for steady swimming the cruise speed of several fish is well approximated by the product between the body length and the tail beat frequency, multiplied by a constant factor 0.63 [45]. This suggests that a tail beat frequency between 0.5 and 2.5 Hz, see Figure 7, would result in a swimming speed between 0.3 and 1.6 body lengths per second, which is in line with observations on sharks acquired using tags [46]. The model is successful in anticipating the large added mass effect, which causes the resonance frequency to reduce by approximately 55% from in-air to underwater vibrations. Notably, when compared with the IPMC-based biomimetic tail considered in [9], we observe a resonance frequency that is 0.7 Hz larger, due to the remarkable stiffening effect from the piezoelectric composites.

Figure 7: (Top) magnitude and (bottom) phase of the frequency response defined as the ratio of the phasor of the tail deflection at the tip to the amplitude of base excitation in water for open-circuited electrodes. Dots are experimental data and solid lines are predictions from the theoretical model in Section 2. Red, green, and blue are the nominal base excitation amplitudes 0.005, 0.022, and 0.043 rad, respectively.

In the vicinity of such a resonance, the magnitude of the transfer function for the large base excitation (0.043 rad) is considerably lower than the magnitude for small base excitation (0.005 rad), due to the non-

Figure 8: Output voltage per unit base excitation for underwater vibrations and open-circuited electrodes. Dots are experimental data and solid lines are predictions from the theoretical model in Section 2. Red, green, and blue are the nominal base excitation amplitudes 0.005, 0.022, and 0.043 rad, respectively.

linear hydrodynamic damping [40]. Specifically, nonlinear hydrodynamic damping associated with vorticity shedding from the tail cross-section [36] is responsible for a two-fold reduction in the amplitude of the frequency response as the base excitation is varied from 0.005 rad to 0.043 rad. Some discrepancy between experiments and theoretical predictions for the small base excitation may be ascribed to the difficulty of accurately detecting the minute deflection of the tail.

3.4. Electrical response

In Figure 8, we report the voltage output from the piezoelectric composites with respect to the three base excitations. The presence of the strong nonlinear hydrodynamic damping is also evident. Although the voltage output for the large base excitation is higher than the output for the small base excitation, the voltage per unit base excitation is considerably smaller. To demonstrate the predictive capabilities of the model, we compare experimental data with model predictions computed from Eqs. (16) and (17). Comparison of theoretical and experimental results indicate that the model is accurate in predicting the piezoelectric voltage output across a broad range of base excitation amplitudes and frequencies.

Notably, the voltage output from the piezoelectric composites is as large as 9 V, which is six orders of magnitude larger than the output of IPMCs [9], thus allowing for rectification with off-the-shelf electronics [22, 23]. The difference in the voltage outputs between the piezoelectric composites and the IPMCs is in line with experimental values from [24]. We comment that even if the voltage outputs from piezoelectrics and IPMCs are largely different, the short-circuit currents can be highly comparable due to the large capacitance of IPMCs, which is associated with their high surface electrodes [47, 48].

4. Energy harvesting

To demonstrate the possibility of self-powering a fish-tag from the tail undulation, we consider the setup depicted in Figure 9. We utilize an Advanced Linear Devices EH300A module for energy storage from the piezoelectrics. This module includes a rectifying circuit and two capacitors of 3.3 mF so that

the total capacitance is $C_{\text{stor}} = 6.6 \text{ mF}$. The energy storage module powers a Texas Instruments eZ430-RF2500T communication unit that includes a temperature sensor. The communication unit transmits to another eZ430-RF2500T, which is connected to a computer through Texas Instruments RF USB debugging interface. The SimpliciTI network protocol (www.ti.com) is utilized for the communication, and the Texas Instruments eZ430-RF2500 Sensor Monitor software is used to monitor the communication status and the measured temperature. The voltage V_{stor} across the capacitor in the storage module is simultaneously measured using the data acquisition board USB 6221, described in Section 3.1. As illustrated in Figure 9, during the experiment, the four MFCs are connected in parallel to increase the internal capacitance and produce a larger current, see Eq. (20). The capacitance of the parallel connection of the four MFCs is $C_{\text{par}} = 3.8 \text{ nF}$, which is ten times larger than C_{ser} . The associated electromechanical gain also varies with respect to the series connection presented in Figure 4, whereby, following an identical procedure, we find $\gamma_{\text{par}} = 55 \text{ V m}$ with R -squared of 0.8106. Thus, the electromechanical gain is reduced by 60% of its value for the series connection.

Figure 9: Experimental setup for testing the possibility of powering a communication device from the undulations of the biomimetic tail.

The experiment is as follows. The capacitance of the storage is pre-charged to $V_i = 1.8 \text{ V}$, which is the minimum voltage value at which the communication module can operate. Next, the base of the tail is excited at the maximum amplitude that the motor can sustain at the underwater resonance frequency. Specifically, the base excitation is 0.085 rad and the frequency is 1.6 Hz . When the output voltage of the storage module reaches 2.5 V , the communication module is activated and remain active until the voltage output is above V_i .

The experiment is run for approximately 14.7 hours to charge into the module to 2.5 V . Figure 10 displays the voltage across the capacitance in the storage module along with the communication log and the temperature at the transmitter. Thus, the persistent tail beating enables the wireless communication of the temperature sensor reading. Specifically, the energy harvested from the tail over the 14.7 hours is 9.9 mJ , estimated from the variation of the energy in the capacitor in the storage module $\frac{1}{2} C_{\text{stor}} V_{\text{stor}}^2|_{V_{\text{stor}}=2.5} -$

Figure 12: Circuit diagram for charging the storage module.

By considering the pre-charged voltage V_i as an initial condition, the solution of Eq. (21) is

$$V_{\text{stor}}(t) = |\hat{V}_{\text{par}}(\omega)| - \left(|\hat{V}_{\text{par}}(\omega)| - V_i \right) e^{-\frac{2\omega C_{\text{par}}}{\pi C_{\text{stor}}} t} \quad (22)$$

Further, we use $|\hat{V}_{\text{par}}(\omega)| = 5.7 \text{ V}$ from Eqs. (16) and (17). Figure 11 displays model predictions against experimental findings, demonstrating the accuracy of the proposed approach and its potential to aid in the design of feasible fish-tags. Specifically, the model yields a time to reach the set voltage of 2.5 V, which is only 15 minutes more than the experimental value.

5. Conclusions

In this paper, we studied underwater energy harvesting from the undulations of a biomimetic fish tail designed after a thresher shark and hosting two piezoelectric composites in a bimorph configuration. We developed a tractable modeling framework to predict the underwater vibration of the tail and the associated voltage output as a function of the amplitude and frequency of base excitation. The model accounted for important multiphysics phenomena, such as nonlinear fluid-structure interactions, highly heterogeneous physical and geometric properties of the tail, and piezoelectric transduction. Model predictions were found to be in good agreement with experimental findings across a broad range of base excitations, thus offering a solid base for the design of self-sustained fish-tags. Toward this latter aim, we designed a laboratory setup to simulate the battery-less operation of a fish-tag powered by the undulation of a fish tail. Using this setup, we demonstrated the possibility of measuring temperature in-situ and transmitting the reading wirelessly using only energy converted by the piezoelectrics. Importantly, the modeling framework closely predicted the operation of the device.

When comparing the performance of the piezoelectric composites with the IPMCs in [9], we observe two main differences. While the IPMCs did not cause any measurable change in the vibrations of the biomimetic tail, the piezoelectric composites produced a significant stiffening effect, which caused an increase in the fundamental underwater vibration frequency of almost 80%. Future work should investigate the implications of such an increase in the swimming efficiency of the potential live animal. With respect to the output voltage, our data suggest that the piezoelectric composites are able to produce voltage levels on the order of few volt, which is six orders of magnitudes larger than the typical output of IPMCs. As

a result, the piezoelectric voltage can be rectified using off-the-shelf electronics, as we have demonstrated in this work. On the other hand, considerable technological advancements are required to store energy converted by IPMCs. To transition this work to live animals, practical research is underway to: i) identify feasible mechanisms for attaching the energy harvesting module on the animal, minimizing discomfort and ii) optimize the power consumption.

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