Chaotic hash function based on circular shifts with variable parameters

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Abstract

We propose a chaotic hash algorithm based on circular shifts with variable parameters in this paper. We exploit piecewise linear chaotic map and one-way coupled map lattice to produce initial values and variable parameters. Circular shifts are introduced to improve the randomness of hash values. We evaluate the proposed hash algorithm in terms of distribution of the hash value, sensitivity of the hash value to slight modifications of the original message and secret keys, confusion and diffusion properties, robustness against birthday and meet-in-the-middle attacks, collision tests, analysis of speed, randomness tests, flexibility, computational complexity, and the results demonstrate that the proposed algorithm has strong security strength. Compared with the existing chaotic hash algorithms, our algorithm shows moderate statistical performance, better speed, randomness tests, and flexibility.

Keywords:

Chaos, Hash function, Variable parameters, Piecewise linear chaotic map, One-way coupled map lattice

1. Introduction

A hash function is a one-way function that can be used to map arbitrary length of digital data to digital data with fixed length. The digital data returned by a hash function are referred to as hash value, or message digest. Hash functions have been used in a wide range of applications, such as integrity protection [1], message authentication [2] and digital signature [3], which have the properties of sensitivity to

initial conditions, diffusion and confusion, collision resistance. Traditional hash functions such as MD5 and SHA-1 are mainly based on logical operations, modular arithmetic operations or digital algebraic operations, which greatly impact the security, since attacks on these algorithms have been discovered [4-9]. For instance, X.Y. Wang found an effective method to reduce the complexity of collisions of SHA-1, issued as a Federal Information Processing Standard by NIST [5]. Chaos has some inherent merits of one way, sensitivity to tiny modifications in initial conditions and parameters, mixing property and ergodicity, which can be used for designing chaotic hash functions. K.W. Wong is the first to propose the chaotic hash function, which is built on the number of iterations of one-dimensional logistic map needed to reach the region corresponding to the character, along with a lookup table updated dynamically [10]. After then, chaotic hash functions are gradually attracting more and more researchers to study ranging from the use of simple maps, such as tent map [11-13] and logistic map [14,15], to the use of more complicated maps of the sine map [16], standard map [17], piecewise linear or nonlinear chaotic maps [18-21], and high-dimensional chaotic maps [22-24].

Since these chaotic maps in cryptanalytic studies reveal security weakness [25-32], we propose a circular shift based chaotic hash algorithm with variable parameters in this paper. We exploit piecewise linear chaotic map and one-way coupled map lattice to produce initial values and variable parameters. Circular shifts are introduced to improve the randomness of hash values. We evaluate the proposed hash algorithm in terms of distribution of the hash value, sensitivity of the hash value

to slight modifications of the original message and secret keys, confusion and diffusion properties, robustness against birthday and meet-in-the-middle attacks, collision tests, analysis of speed, randomness tests, flexibility, computational complexity, and the results demonstrate that the proposed algorithm has strong security strength. Compared with the existing chaotic hash algorithms, our algorithm shows better statistical performance and strong collision resistance.

The remainder of this paper is organized as follows: Section 2 briefly introduces the preliminaries of piecewise linear chaotic map and one-way coupled map lattice used in our algorithm. In Section 3, we design the circular shift based chaotic hash function with variable parameters in detail, which is composed of parameter initialization, message processing and hash value generation. We extensively evaluate the performance of the proposed hash algorithm in Section 4 and present conclusions in Section 5.

2. Preliminaries

In this section, we briefly depict the one-dimensional piecewise linear chaotic map and the four-dimensional one-way coupled map lattice used in our hash algorithm, respectively.

2.2 Piecewise linear chaotic map (PWLCM)

We select the one-dimensional piecewise linear chaotic map in the proposed hash algorithm, which is expressed as:

$$x_{i+1} = \text{PWLCM}(u, x_i) = \begin{cases} x_i / u & 0 \leq x_i < u \\ (x_i - u) / (0.5 - u) & u \leq x_i < 0.5 \\ (1 - u - x_i) / (0.5 - u) & 0.5 \leq x_i < 1 - u \\ (1 - x_i) / u & 1 - u \leq x_i \leq 1 \end{cases} \quad (1)$$

where x_i represents the iteration trajectory value, and u denotes the control parameter.

When u is assigned values in $(0, 0.5)$, x_i evolves into a chaotic state in range of $(0, 1)$.

The PWLCM has properties of uniform distribution, good ergodicity, confusion and diffusion, therefore, it can provide chaotic random sequences. To show the quality of PWLCM, we plot the Lyapunov exponents and the bifurcation diagram for different values of control parameter u in Figs. 1 and 2. As illustrated in Figs. 1 and 2, for $0 < u < 0.5$, PWLCM can exhibit chaotic behavior.

Fig.1. Lyapunov exponents of PWLCM for different control parameter u ($u \in (0, 0.5)$)

Fig.2. Bifurcation diagram of PWLCM for different control parameter u ($u \in (0, 0.5)$)

2.3 One-way coupled map lattice (OCML)

We select the four-dimensional one-way coupled map lattice in the proposed hash algorithm, which is expressed as:

$$\begin{bmatrix} x_1(i+1) \\ x_2(i+1) \\ x_3(i+1) \\ x_4(i+1) \end{bmatrix} = OCML(e, x_1(i), x_2(i), x_3(i), x_4(i)) = \begin{cases} x_1(i+1) = (1-e)g(x_1(i)) + eg(x_4(i)) \\ x_2(i+1) = (1-e)g(x_2(i)) + eg(x_1(i)) \\ x_3(i+1) = (1-e)g(x_3(i)) + eg(x_2(i)) \\ x_4(i+1) = (1-e)g(x_4(i)) + eg(x_3(i)) \end{cases}$$

where $g(x) = 4x(1-x)$, and e denotes a coupling constant in the range of $(0, 1)$. The four inputs $x_1(i), x_2(i), x_3(i)$, and $x_4(i)$ are in the range of $[0, 1]$, and the corresponding four outputs $x_1(i+1), x_2(i+1), x_3(i+1)$, and $x_4(i+1)$ belong to the range of $[0, 1]$ as well [34].

3. Description of hash algorithm

In this section, we design the circular shift based chaotic hash function with variable parameters, which is composed of parameter initialization, message processing and hash value generation. In addition, we describe the proposed algorithm in Algorithm 1 and illustrate the structure of the proposed hash function in Figs. 3 and 4. As designed, the input is an arbitrary length of message M' and the output is $h = 128$ bits of hash value.

3.1 Parameter Initialization

We first convert the original message M' into ASCII code values based on the ASCII code chart and then save them in an array M , where the length of M is denoted as l . Then, with initial value $x_0 = 0.8$ and parameter $u_0 = 0.232323$, we iterate the PWLCM $(128+l)$ times to obtain an array X . Then we round the first 128 elements of X to the nearest integers, and assign them to the initial hash value H_0 ($H_0 = (X[1], X[2], \dots, X[128])$), where $H(i)$ ($i=1, 2, \dots, 128$) are binary values. Finally, we assign the rest elements of the array X to a new array U ($U = (X[129], X[130], \dots, X[128+l])$), where $U(i)$ ($i=1, 2, \dots, l$) are pure decimals.

3.2 Message processing

Since each element (character) of array M is processed in the same mode, we choose $M(n)$ ($n=1, 2, \dots, l$) as a representative to illustrate the generation process of the n th middle hash value H_n . For element $M(n)$, we iterate PWLCM $i = 132$ times to generate an array uX with the initial value $ux_0 = \frac{M(n)}{256}$, and variable

parameters
$$uu_n = \frac{uu_0 + ux_{n-1}}{3} (i = 1, 2, \dots, 132) ,$$

where $uu_0 = \left| w - \lfloor w \rfloor - 0.5 \right|$ and $w = \frac{(n+1) \times (M(n) + 2)}{(n+2) \times (M(n) + 1)} + U(n)$. Then, we round the

first 128 elements of uX into the nearest integers, and assign them to the four initial buffers A, B, C, D as: $A = (uX(1), uX(2), \dots, uX(32))$,

$$B = (uX(33), uX(34), \dots, uX(64)),$$

$$C = (uX(65), uX(66), \dots, uX(96)),$$

$$D = (uX(97), uX(98), \dots, uX(128)).$$

We assign the rest four elements of uX as: $x_1 = uX(129)$, $x_2 = uX(130)$, $x_3 = uX(131)$, and $x_4 = uX(132)$, which are the input parameters of OCML. With above parameters,

we iterate $OCML(uX(i), x_1, x_2, x_3, x_4)$ $i = 128$ times and we obtain four sequence values of x_1, x_2, x_3, x_4 , each with length of 128. Then, we assign values

$$\begin{aligned} \text{as: } ch_1(i) &= \text{mod}(\lfloor x_1(i) \rfloor \times 1000, 32) + 1 ; \quad cl_1(i) = \text{mod}(\lfloor x_1(i) \rfloor \times 10000, 32) ; \\ ch_2(i) &= \text{mod}(\lfloor x_2(32+i) \rfloor \times 1000, 32) + 1 ; \quad cl_2(i) = \text{mod}(\lfloor x_2(32+i) \rfloor \times 10000, 32) ; \\ ch_3(i) &= \text{mod}(\lfloor x_3(64+i) \rfloor \times 1000, 32) + 1 ; \quad cl_3(i) = \text{mod}(\lfloor x_3(64+i) \rfloor \times 10000, 32) ; \\ ch_4(i) &= \text{mod}(\lfloor x_4(96+i) \rfloor \times 1000, 32) + 1 ; cl_4(i) = \text{mod}(\lfloor x_4(96+i) \rfloor \times 10000, 32) . \end{aligned}$$

For buffers A, B, C , and D , we conduct i -time ($i=1, 2, \dots, 32$) value switching and left circular shifts as: swap $A(i)$ with $A(ch_1(i))$, and then apply $cl_1(i)$ -bit left circular shifts; swap $B(i)$ with $B(ch_2(i))$, and then apply $cl_2(i)$ -bit left circular shifts; swap $C(i)$ with $C(ch_3(i))$, and then apply $cl_3(i)$ -bit left circular shifts; swap $D(i)$ with $D(ch_4(i))$, and then apply $cl_4(i)$ -bit left circular shifts. Then, we obtain updated buffer values A, B, C , and D , which are then cascaded to generate the middle hash value H_n . Meanwhile, we

store values as: $chf_1(n) = ch_1(1) + ch_2(1) + ch_3(1) + ch_4(1)$,

$chf_2(n) = ch_1(32) + ch_2(32) + ch_3(32) + ch_4(32)$, and $clf(n) = cl_1(32) + cl_2(32) + cl_3(32) + cl_4(32)$.

3.3 Have value generation

For current array $M(n)$ ($n=1,2,\dots,l$), $H_n = H_n \oplus H_{n-1}$, we swap $H_n(chf_1(n))$ with $H_n(chf_2(n))$, and then apply $clf(i)$ -bit left circular shifts. After all the messages are processed, we obtain the final hash value H_l .

Fig.3. Structure of the proposed chaotic hash function

Fig.4. Processing details of message $M(n)$

Algorithm 1 The circular shift based hash function construction

Input: Secret keys (x_0, u_0)

Output: 128-bit hash value H_l

1: Initialize H_0 ($H_0 = (X[1], X[2], \dots, X[128])$) and U ($U = (X[129], X[130], \dots, X[128+l])$) by iterating

PWLCM $(128+l)$ times with secret keys (x_0, u_0) *// Parameter initialization*

2: **for** $n = 1$ to l **do** *// Message processing*

3: Generate array uX by iterating PWLCM i ($i = 132$) times with initial value $ux_0 = \frac{M(n)}{256}$ and variable parameters $uu_n = \frac{uu_0 + ux_{n-1}}{3}$, where $uu_0 = |w - \lfloor w \rfloor - 0.5|$ and $w = \frac{(n+1) \times (M(n) + 2)}{(n+2) \times (M(n) + 1)} + U(n)$;

4: $A = (uX(1), uX(2), \dots, uX(32))$;

5: $B = (uX(33), uX(34), \dots, uX(64))$;

6: $C = (uX(65), uX(66), \dots, uX(96))$;

7: $D = (uX(97), uX(98), \dots, uX(128))$;

8: Generate new x_1, x_2, x_3, x_4 by iterating $OCML(uX(i), x_1, x_2, x_3, x_4)$ i ($i = 128$) times with inputs of

$x_1 = uX(129)$, $x_2 = uX(130)$, $x_3 = uX(131)$, $x_4 = uX(132)$;

9: $ch_1(i) = \text{mod}(\lfloor x_1(i) \rfloor \times 1000, 32) + 1$; $cl_1(i) = \text{mod}(\lfloor x_1(i) \rfloor \times 10000, 32)$;

10: $ch_2(i) = \text{mod}(\lfloor x_2(32+i) \rfloor \times 1000, 32) + 1$; $cl_2(i) = \text{mod}(\lfloor x_2(32+i) \rfloor \times 10000, 32)$;

11: $ch_3(i) = \text{mod}(\lfloor x_3(64+i) \rfloor \times 1000, 32) + 1$; $cl_3(i) = \text{mod}(\lfloor x_3(64+i) \rfloor \times 10000, 32)$;

12: $ch_4(i) = \text{mod}(\lfloor x_4(96+i) \rfloor \times 1000, 32) + 1$; $cl_4(i) = \text{mod}(\lfloor x_4(96+i) \rfloor \times 10000, 32)$;

11: **for** $i = 1$ to 32 **do**

12: swap $A(i)$ with $A(ch_1(i))$, and apply $cl_1(i)$ -bit left circular shifts;

13: swap $B(i)$ with $B(ch_2(i))$, and apply $cl_2(i)$ -bit left circular shifts;

14: swap $C(i)$ with $C(ch_3(i))$, and apply $cl_3(i)$ -bit left circular shifts;

15: swap $D(i)$ with $D(ch_4(i))$, and apply $cl_4(i)$ -bit left circular shifts;

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16:    end for

17:     $H_n = \text{cascaded } A, B, C, D; \quad H_n = H_n \oplus H_{n-1};$            //Middle hash value generation

18:     $chf_1(n) = ch_1(1) + ch_2(1) + ch_3(1) + ch_4(1);$ 

19:     $chf_2(n) = ch_1(32) + ch_2(32) + ch_3(32) + ch_4(32);$ 

20:     $clf(n) = cl_1(32) + cl_2(32) + cl_3(32) + cl_4(32);$ 

21:    swap  $H_n(chf_1(n))$  with  $H_n(chf_2(n))$ , and apply  $clf(i)$ -bit left circular shifts;

22: end for

23: Return  $H_l$                                            //Hash value generation

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4. Performance analysis

We evaluate the performance of the circular shift based chaotic hash function in terms of distribution of the hash value, sensitivity of the hash value to slight modifications of the original message and secret keys, confusion and diffusion properties, robustness against birthday and meet-in-the-middle attacks, collision tests, analysis of speed, randomness tests, flexibility, computational complexity, and comparison with other algorithms. To evaluate the performance of the proposed hash algorithm, we randomly choose an arbitrary of length of message as:

“Southwest University (SWU) is a key comprehensive university, under the direct administration of the Ministry of Education. It was newly established in July 2005 through the incorporation of former Southwest China Normal University and Southwest Agricultural University upon the approval of the Ministry of Education. SWU is situated nearby the beautiful Jialing River, and

is located at the foot of Jinyun Mountain, a state level scenic spot, in Beibei District, Chongqing Municipality.”

4.1 Distribution of the hash value

The characteristic of uniform distribution of hash value directly relates to the security of a hash function. With the randomly chosen message, we simulate the proposed hash algorithm to generate the hash value, and the distribution of both the message and the corresponding hash value are plotted. As depicted in Fig. 5(a), the random chosen message spreads in a small area. This is because the ASCII code values of printable characters (message) in ASCII code chart locate in the range of [32, 126]. Nevertheless, the hexadecimal hash value spreads around randomly, as shown in Fig. 5(b). In contrast, we perform the same experiment on a particular “blank space” message with same length, and the distribution of the particular message and the hash value are plotted as well. As illustrated in Fig. 6, even under this very extreme condition, the distribution of hash value is uniform as well. Therefore, the hash value of the proposed hash algorithm has a good characteristic of uniform distribution.

Fig.5. Distribution of randomly chosen message and hash value: (a) distribution of the message in ASCII code; (b) distribution of the hexadecimal hash value (FE5A293B5CDABC7E7EC9B89538D8A0A1)

Fig.6. Distribution of the particular “blank space” message and hash value: (a) distribution of the “blank space” message; (b) distribution of the hexadecimal hash value (4E980477437174B15C93833BD443BAAA)

4.2 Sensitivity to the original message and secret key

Hash function is a many-to-one map since its domain is much larger than its range. According to the characteristic of a hash function, it is hard to find the original message if the hash value is known, which indicates that a hash function must be sensitive to tiny modifications in original message or secret key. In particular, any slight modifications on message or secret key will lead to a 50% difference in hash value, according to a Hamming distance of approximately $n/2$ (n denotes the length of hash value) between the two hash values. We simulate the hash algorithm under eight different conditions to evaluate the sensitivity of tiny modifications to the original message and secret key.

C1: The original message is the randomly chosen message;

C2: Modify the first character “S” in original message to “s”;

C3: Add a number “1” in front of the original message;

C4: Modify “July” in the middle of the message to “June”;

C5: Swap the “China Normal University” and “Southwest Agricultural University” in the original message;

C6: Modify the full stop “.” at the end of original message to a comma “,”;

C7: Add a “blank space” at the end of the original message;

C8: Modify the secret key $x_0 = 0.232323$ to $x_0 = 0.2323230000000001$.

We list the corresponding hash values in hexadecimal formats associated with the eight conditions, followed by the bit differences of hash values comparing with Condition 1 in the parenthesis:

C1: FE5A293B5CDABC7E7EC9B89538D8A0A1

C2: DBC3A7DCBEDE048F4F35AFDD0B7EFEF8 (63)

C3: B1C87925F2448D08B7A8270C661285DB (66)

C4: 9BDA624D5463F2F4CB6BAC13C05A9E10 (56)

C5: 7B3366ADDE7E280EDCB92C7171655BB7 (59)

C6: F8055075C0437A6E7BDC4ED72DD3620E (58)

C7: 7C8CD1DA337133AB6D17814EB03C8D6D (70)

C8: 7B23D05E1C84CDC48FDF800852FBBFA7 (63)

Fig.7. Hash values under eight different conditions

We illustrate the corresponding binary sequences of hash values in Fig.7. As depicted in Fig.7, the sensitivity property of the hash algorithm is so good that any tiny modification of the message (C1 to C7) causes huge difference in the hash value. In addition, a tiny change on the key (C8) leads to 63-bit difference, which is close to

the ideal value of 64. Based on the experiment, the sensitive precision of secret key x_0 to hash value is 10^{-16} , and the sensitive precision of the parameter u is 10^{-16} as well. Considering both $x_0 \in (0,1)$ and $u \in (0,0.5)$, we can derive that the secret key space is approximately larger than 2^{106} .

4.3 Statistical analysis of confusion and diffusion

Shannon introduces confusion and diffusion as two of the key features of a secure cipher, which are regarded as basic design criteria for any cryptographic algorithm [35]. In cryptography, confusion refers to as the relationship between a message and its corresponding hash value must be complex and unpredictable, while diffusion refers to as the hash value is highly dependent on the message. In particular, with good diffusion, a 1-bit modification in a message leads to a significant change in the hash value.

In order to capture qualitative characteristics of the diffusion and confusion capabilities of the proposed hash algorithm, we conduct the following experiment: first, we obtain the hash value of a randomly chosen message. Then, we randomly modify a bit in the original message, and generate the corresponding hash value. Finally, we compare the two hash values and count the number of different bits at the same position in the hash values denoted as B_i . Besides, in order to capture the statistic characteristics of diffusion and confusion, we introduce six statistical measures:

Minimum changed bit number: $B_{\min} = \min\{B_1, B_2, \dots, B_N\}$,

Maximum changed bit number: $B_{\max} = \max\{B_1, B_2, \dots, B_N\}$,

Mean changed bit number: $\bar{B} = \frac{1}{N} \sum_{i=1}^N B_i$,

Mean changed probability: $P = (\bar{B} / h) \times 100\%$,

Standard variance of the changed bit number: $\Delta B = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (B_i - \bar{B})^2}$,

Standard variance: $\Delta P = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (B_i / h - P)^2} \times 100\%$, where N indicates the test

time of experiments, and h denotes the length of hash value.

The experiment is performed N times on our hash algorithm, where $N = 256, 512, 1024, 2048, \text{ and } 10000$. The corresponding results of $B_{\min}$, $B_{\max}$, $\bar{B}$, P , ΔB , and ΔP are tabulated in Table 1. The corresponding distribution of changed bit number $\bar{B}$, is illustrated in Fig. 8.

Table 1 Statistics of number of changed bits

N	$N = 256$	$N = 512$	$N = 1024$	$N = 2048$	$N = 10000$	Mean
$B_{\min}$	48	47	47	47	47	47.2
$B_{\max}$	77	81	79	81	81	79.8
$\bar{B}$	63.8627	64.0783	64.0479	64.1700	64.0415	64.0401
P (%)	49.89	50.06	50.04	50.13	50.03	50.03
ΔB	5.4974	5.7628	5.8880	5.7488	5.7929	5.7380
ΔP (%)	4.29	4.50	4.60	4.49	4.53	4.48

Fig.8. Spread of changed bit number: (a) Plot of B_i , (b) Histogram of B_i .

Based on the statistical experiment results, the proposed hash function has a mean changed bit number $\bar{B} = 64.0401$ and mean changed probability $P = 50.03$ that are very close to the ideal values of 64 bits and 50%, respectively. The values of the

standard variances are also very small, which indicate a strong capability for confusion and diffusion. Besides that, as illustrated in Fig. 8(a), the plot of B_i shows that its value is evenly distributed, and as shown in Fig. 8(b), the histogram of B_i has a normal distribution centering on the ideal value of 64. These results indicate that the proposed hash function has near-ideal statistical qualities in terms of confusion and diffusion strength, whereby any change of even 1 bit of the plaintext will lead to a completely different message digest.

4.4 Collision analysis

A collision refers to as two distinct messages produce the same hash value, while collision attack indicates that it tries to find two arbitrary messages that collide. Collision resistance is a property of a hash function: a hash function h is collision resistant if it is hard to find any two distinct messages M and M' that hash to the same output, such that $h(M)=h(M')$. In this section, we first perform qualitative analysis on birthday and meet-in-the-middle attacks and then conduct quantitative analysis on collision resistance.

4.4.1 Birthday and meet-in-the-middle attacks

Birthday attack is an algorithm-independent attack, where a set of random messages are inspected for a possible pair of messages whose hash values collide [49,50]. For an h -bit hash function, the complexity of this attack is about $2^{\frac{h}{2}}$. Since the length of hash value of the proposed algorithm $h \geq 128$, birthday attacks are rendered impractical.

Meet-in-the-middle attack is another generic attack applied to cryptographic systems composed of different layers. The attack is to find collisions on intermediate hash results rather than the final hash value. For example, assuming that the original message is described as $M = (M_1, M_2, \dots, M_{l-1}, M_l)$, then the expected contradicted message can be represented as $M' = (M_1, M_2, \dots, M_{l-1}, M'_l)$. As we tested in Section 4.2, any character modification will cause huge difference in the final hash value. Therefore, the proposed hash algorithm is robust against meet-in-the-middle attacks.

4.4.2 Collision tests

In order to show quantitative analysis on collision resistance, we conduct the experiment: first, we generate the hash value of a randomly chosen message and then store it in ASCII code format. Then, we randomly modify a bit in the message, obtain a new hash value, and store it in the same format as well. Finally, we compare the two produced hash values by counting the number of the same ASCII character at the same location, that is, the number of hits. We conducted $N=10000$ times of the experiment, and plot the distribution of the number of hits in Fig.9. As depicted in Fig.9 and Table 2, 505 tests hit once, while in 9495 tests, no hit occurs. The maximum number of equal characters is only 1, therefore, the collision on our hash algorithm is very low.

Fig.9. Distribution of the number of the same ASCII characters at the same location in hash value

Table 2 Number of hits

Number of equal characters	0	1	2	3	4	5	6	7	8
The proposed algorithm	9495	505	0	0	0	0	0	0	0

In addition, we compare the hash values by sequences of bytes, which can be represented by an ASCII character. We denote the decimal value of an ASCII character e by $t(e)$. For two messages M and M' , we calculate the hash values h and h' . Let e_i denote the i th ASCII character of hash value h while e'_i represents the i th ASCII character of hash value h' . Then, we compute the absolute difference between the two hash values h and h' by the equation: $d = \sum_{i=1}^N |t(e_i) - t(e'_i)|$. We perform the collision test $N=10000$ times, and the corresponding results of the maximum, minimum, mean and

mean/character values of d are listed in Table 3. As described in Table 3, the mean/character of absolute difference d of two hash values is 78.31. For the absolute difference d , the theoretical mean/character value computed in [36,48,50,58] is 83.3333, therefore, our mean/character value is close to the theoretical value and the analysis on collision shows that our hash algorithm has strong collision resistance. Moreover, as described in [18,58,59], the ideal value of sum of absolute difference for two uniform distributions is $2/3$ of the mean value for the distribution. The ideal mean value of the absolute difference for 128-bit hash value is equal to 1360, while our mean value is 1253.

Table 3 Absolute differences d of two hash values

Maximum	Minimum	Mean	Mean/Character
1972	685	1253	78.31

4.5 Analysis of speed

The time required to generate the hash value is directly related to the message processing procedure, according to the structure of the proposed hash algorithm. To analyze the compression speed, we implement the proposed hash algorithm in C99 on a PC with 2.50GHz Intel Pentium IV Dual-core, 2G Memory and Ubuntu 10.10 operation system and the test message is composed of 10000 ASCII codes. Moreover, we implement the well-known and widely used MD5 [36] and SHA-1 [51] algorithms in C99 with optimized codes on the same settings. Finally, we present the average compression speed comparison based on specified settings with some representative

hash algorithms [18,34,38,43,49,52,53] in Table 4. As shown in Table 4, the average compression speed of the proposed algorithm (132.07 Mbps) is higher than SHA-1 (114.45 Mbps) and is very close to MD5 (132.10 Mbps). In addition, the algorithm of [18] shows higher compression speed, and Yi's algorithm [52] illustrates interesting compression speed, since the settings of the implementation are far behind ours.

Table 4 Average compression speed comparison

Algorithms	Platform (on PC)	Average compression speed
Akhavan's[18]	Visual C++.Net, Pentium IV 2.0GHz Dual-Core CPU, 140Mbps 512MB RAM	
Wang'[34]	Visual C++, Pentium IV 2.4GHz CPU, 256MB RAM	156.86Kbps
MD5[36]	C99, Pentium IV 2.50GHz Dual-core CPU, 2G RAM under Ubuntu 10.10	132.10Mbps
Li's [38]	C99, Pentium IV 2.50GHz Dual-core CPU, 2G RAM under Ubuntu 10.10	131.07Mbps
Zhang's[43]	Visual C++, Pentium IV 2.4GHz CPU, 256MB RAM	1.54Kbps
Li's[49]	C99, Pentium IV 2.50GHz Dual-core CPU, 2G RAM under Ubuntu 10.10	132.05Mbps
SHA-1[51]	C99, Pentium IV 2.50GHz Dual-core CPU, 2G RAM under Ubuntu 10.10	114.45Mbps

Yi's[52]	A software on Pentium III 667MHz CPU under Win98	8.42Mbps
Guo's[53]	C99, Pentium IV 2.50GHz Dual-core CPU, 2G RAM under Ubuntu 10.10	131.25Mbps
This algorithm	C99, Pentium IV 2.50GHz Dual-core CPU, 2G RAM under Ubuntu 10.10	132.07Mbps

4.6 Randomness tests

In order to show the randomness of the proposed hash algorithm, we perform a preliminary security analysis, consisting in examining the statistical properties of its output over a low-entropy input. By randomly modifying one bit of the original message each time, we generate a sequence of hash values constructed a file of 250 MB to be analyzed with four batteries of statistical tests: NIST [54], Diehard [55], and ENT [56], and the most stringent testing TestU01 [57,58]. The results obtained with NIST, Diehard, ENT, and TestU01 are presented in Tables 5, 6, 7, and 8, respectively. In Table 5, we calculate 100 p -values for every test in the statistical suite and present the proportion of successful ones. If this proportion is lower than 0.96, it is considered that the whole test failed. In Table 6, we first calculate several p -values in the same test, and then summarize them by a Kolmogorov-Smirnov p -value (marked with *), where if it is greater than 0.05, it is considered successful. In Table 8, we evaluate three different crush type batteries: Small Crust (which consists of 10 tests), Crust (96 tests), and Big Crust (106 tests).

From the results in Tables 5, 6, 7, and 8, we can conclude that the hash values of the proposed hash algorithm successfully pass these randomness tests, even with the

hash values being obtained from an extremely low-entropy input.

Table 5 Proportion results obtained with the NIST suite

Test	Proportion	Test	Proportion
Frequency	0.97	Random-excursions	0.98, 0.97
Block-frequency	0.97		0.99, 0.98
Cumulative-sums	0.97, 0.97		1.00, 0.99
Runs	1.00		0.98, 1.00
Longest-run	1.00	Random-excursions-variant	1.00, 0.99, 1.00
Rank	0.98		0.99, 1.00, 1.00
FFT	0.99		1.00, 1.00, 1.00
Overlapping-templates	0.97		1.00, 1.00, 1.00
Universal	0.97		0.99, 0.99, 0.98
Apen	0.98		0.97, 1.00, 0.98
Linear-complexity	0.99	Serial	0.98, 0.99

Table 6 Test results obtained with the Diehard suite

Test	p -value	Test	P -value
Birthday Spacings	0.224	Count the 1s in a Stream of Bytes	0.665
GCD	0.776*	Count the 1s in Specific Bytes	0.587*

Overlapping Permutations	0.425	Parking Lot Test	0.446
Ranks of 31x31 and 32x32 Matrices	0.699	Minimum Distance Test	0.433
Ranks of 6x8 Matrices	0.532	Random Spheres Test	0.799
Monkey Tests on 20-bit Words	0.321*	The Squeeze Test	0.902
Monkey Test OPSO	0.453*	Overlapping Sums Test	0.201
Monkey Test OQSO	0.738*	Runs up and down Test	0.192
Monkey Test DNA	0.688*	The Craps Test	0.443*
Overall p-value		0.532	

Table 7 Test results obtained with ENT

Test	Results
Entropy	7.999996 bit/byte
Compression Rate	0%
χ^2 Statistic	256.10 (50%)
Arithmetic Mean	127.7705
Monte Carlo π Estimation	3.140523711 (0.00%)
Serial Correlation Coefficient	-0.000053

Table 8 Test results obtained with TestU01 test suite

Battery	Parameters	Number of statistics	Results
Small Crush	Standard	10	Pass

Crush	Standard	96	Pass
Big Crush	Standard	106	Pass

4.7 Flexibility

To show the flexibility of the proposed hash algorithm, if we modify the viable parameter generation method of PWLCM for each message block to be a constant value, then we will obtain a parallel message processing algorithm. Since the focus of the proposed algorithm is a variable parameter for each message block which is close related to previous message block to keep message information, then we will leave the parallel solution to future work.

4.8 Computational complexity

The computational complexity of the proposed hash algorithm depends on the iteration operations of PWLCM and OCML and message length to be processed. First, there are about $128+l$ times of iteration of PWLCM for parameter initialization, and the time complexity is $O(n)$. Then, for each message block, the following steps will be executed $O(l)$ times: 132 times of iteration of PWLCM, 128 times of iteration of OCML, 13 times of value assignments, $O(32)$ times of value swapping and circular shifts, and then the corresponding time complexity is $O(n^2)$. Therefore, the total computational complexity of the proposed algorithm is $O(n^2)$.

4.9 Comparison with other hash algorithms

We perform a comparison between the proposed hash function and some significant chaos-based hash functions as well as MD5, which is based on statistical performance and collision resistance.

Tables 9 and 10 describe the comparison of statistical performance between our algorithm and selected existing excellent algorithms. Note that the results reported in Table 9 are based on $N = 2048$ random tests and 128-bit hash value, while the results of Table 10 focus on $N = 10000$ random tests and 128-bit hash value. Based on the results, our algorithm shows better statistical performance.

Table 9 Comparison on statistical performance with $N=2048$ random tests

Schemes	Maximum hit numbers	Statistical performance of the schemes			
		$\bar{B}$	$P(\%)$	ΔB	$\Delta P(\%)$
MD5[36]	N/A	64.03	50.02	5.66	4.42
Li's[19]	2	63.5684	49.66	7.4260	5.80
Xiao's[20]	2	63.9185	49.94	5.6172	4.39
Xiao's[21]	2	64.0918	50.07	5.4832	4.28
Kanso's[22]	N/A	63.94	49.95	5.69	4.44
Deng's[25]	2	63.84	49.88	5.88	4.59
Guo's[29]	N/A	63.399	49.53	7.127	6.3492
Wang's[34]	N/A	63.98	49.98	5.53	4.33
Li's[37]	2	63.8076	49.8497	5.7563	4.4971
Li's[38]	2	63.89	49.91	5.64	4.41
Ren's[39]	2	63.92	49.94	5.78	4.52
Xiao's[40]	2	64.0098	50.01	5.7236	4.47
Xiao's[41]	2	64.1777	50.15	5.6700	4.405

Zhang's[42]	N/A	63.91	49.92	5.58	4.36
Zhang's[43]	4	64.43	49.96	5.568	4.506
Kanso's[44]	2	64.01	50.01	5.61	4.38
Teh's[45]	N/A	64.0078	50.0061	5.6631	4.2558
Wang's[46]	N/A	64.153	50.119	5.767	4.506
This paper	1	64.1700	50.13	5.7488	4.49

Table 10 Comparison on statistical performance with $N=10000$ random tests

Schemes	Maximum hit numbers	Statistical performance of the schemes			
		$\bar{B}$	$P(\%)$	ΔB	$\Delta P(\%)$
Kanso's[22]	1	63.94	49.95	5.64	4.41
Wang's[34]	2	63.90	49.91	5.58	4.36
Ren's[39]	2	64.004	50.003	5.620	4.391
Zhang's[42]	2	63.96	49.97	5.52	4.32
Teh's[45]	1	63.8518	49.8842	5.6701	4.4298
Wang's[46]	2	63.986	49.988	5.616	4.388
This paper	1	64.0415	50.03	5.7929	4.53

In addition, Tables 11 and 12 present the comparison of the number of the same ASCII characters at the same location and absolute difference d in 128-bit hash values between our algorithm and selected algorithms. Note that the reported results of Table 11 are based on $N = 2048$ random tests, while the results of Table 12 focus on $N =$

10000 random tests. Based on the results, the proposed algorithm shows better collision resistance. However, the ideal mean value of the absolute difference for 128-bit hash value is equal to 1360 [18, 58, 59], while our mean value is 1255 when $N = 2048$ and 1253 when $N = 10000$, which shows a moderate collision resistance of the proposed algorithm comparing to some algorithms.

Table 11 Comparison on collision resistance with $N=2048$ random tests

Schemes	Number of hits				Absolute difference d			
	0	1	2	3	Min.	Max.	Mean	Mean/Char.
Xiao's[20]	1924	120	4	0	658	2156	1431.1	89.46
Xiao's[21]	1915	132	1	0	812	2034	1349.1	84.32
Deng's[25]	1940	104	4	0	583	2206	1399.8	87.49
Xiao's[40]	1926	120	2	0	605	1952	1227.8	76.74
Xiao's[41]	1932	114	2	0	573	2224	1401.1	87.56
Li's[37]	1928	118	2	0	687	2220	1432.1	89.51
Kanso's[44]	1954	92	2	0	731	2230	1368	85.5
Luo's[47]	1928	117	3	0	796	2418	1598.6	99.91
This paper	1945	103	0	0	685	1972	1255	78.44

Table 12 Comparison on collision resistance with $N=10000$ random tests

Schemes	Number of hits				Absolute difference d			
	0	1	2	3	Min.	Max.	Mean	Mean/Char.

Akhavan's[18]	9434	553	13	0	744	2431	1371	85.69
Wang's[34]	9377	607	16	0	689	2295	1526	95.38
Ren's[39]	9373	614	13	0	599	2455	1439	89.94
Zhang's[42]	9416	572	12	0	565	2022	1257	78.56
Kanso's[44]	9431	557	12	0	656	2391	1364	85.25
Teh's[45]	9969	31	0	0	575	2390	1379	86.81
Wang's[46]	9359	626	15	0	655	2046	1367	85.44
This paper	9495	505	0	0	685	1972	1253	78.31

5. Conclusion

In this paper, we propose a circular shift based chaotic hash algorithm with variable parameters. We exploit piecewise linear chaotic map and one-way coupled map lattice to produce initial values and variable parameters. Circular shifts are introduced to improve the randomness of hash values. We evaluate the proposed hash algorithm in terms of distribution of the hash value, sensitivity of the hash value to slight modifications of the original message and secret keys, confusion and diffusion properties, robustness against birthday and meet-in-the-middle attacks, collision tests, analysis of speed, randomness tests, flexibility, computational complexity, and the results demonstrate that the proposed algorithm has strong security strength. Compared with the existing chaotic hash algorithms, our algorithm shows moderate statistical performance and better speed, randomness tests, and flexibility.

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