Closed-loop supply chains under reward-penalty mechanism: retailer collection and asymmetric information^{*1}

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Abstract: Being motivated by the issue of waste electrical and electronic equipment (WEEE) collection, we consider a contract design problem for a manufacturer with entrusting the collection of WEEE to a retailer. However, the manufacturer has asymmetric information on the collection effort level of the retailer. This paper designs an information screening contract for the manufacturer to obtain the information of collection effort level, and the optimal decision-making with several properties of contract parameters are derived. The results indicate that the manufacturer would offer lower wholesale price and higher buy-back price for the H-type retailer while charge more franchise fee to the H-type retailer. Considering the government intervention, reward-penalty mechanism (RPM) is developed to stimulate the asymmetric information closed-loop supply chain (CLSC). We also analyze the impacts of RPM by comparing the cases whether or not RPM is implemented. The comparison results show that the RPM can lower the wholesale price and retail price meanwhile raise buy-back price and collection quantity. Finally, several numerical studies are conducted for more managerial insights.

Keywords: Information screening contract; Closed-Loop Supply Chain (CLSC); Reward-Penalty Mechanism (RPM); Waste Electrical and Electronic Equipment (WEEE)

1. Introduction

The rapidly updating and shortening life cycles of products have led to industries seemingly producing new electric and electronic products almost daily, which can account for the increase in growth of waste electrical and electronic equipment(WEEE). It is estimated that, by 2017, the WEEE of the whole world will reach about 65 million tons which will increase by 33 percent compared with 49 million tons in 2012. There are more than 700 chemical materials in WEEE,

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and half of them are hazardous to human health, while WEEE may derive 80–1500 g of gold and 350–1850 g of silver per ton, valued at several thousand US dollars. (<http://www.tijinw.com/>)

Instead of incineration and landfill, the used products should be taken into collecting, refurbishing and remanufacturing (Thierry et al., 1995). Remanufacturing, as a significant part of closed-loop supply chain (CLSC), is the process of taking the used product to an as-good-as-new condition by repairing, replacing and restoring its components as necessary (Aras et al., 2011). Remanufacturing has both economic and environmental benefits (Huysman et al, 2015. Wang et al, 2015). From the economic point of view, remanufacturing is lucrative since the cost of remanufacturing is typically 40–60% of the cost of manufacturing a new product with only 20% of the effort, and the remanufactured product is of the same quality as a new product sold with the same warranty (Mitra, 2007). Remanufacturing is not only an economic viable solution, but also positively contributing to the environment by conserving resources (Sharma et al, 2016). Meanwhile, the manufacturer in such a CLSC usually faces the risk of asymmetric information, for example, the collection cost parameter of the collector (Li et al, 2014) and the collection effort cost of the retailer (Zhang et al, 2014). From another aspect to achieve sustainable development, many governments have taken steps to stimulate the collection of the used products, especially for WEEE. For instance, a number of legislations with respect to product recovery, such as the Paper Recycling Directive, the End-of-Life Vehicle Directive and the WEEE Directive etc, have been implemented in the European Union (Giri and Sharma, 2015). China also has started to carry out the WEEE Recycling Management Regulation and adopt subsidization to the manufacturer to stimulate collection. The governmental financial intervention on CLSC can improve the social welfare and chain-based profits (Sheu and Chen, 2012). Combined with the government intervention, it's a challenging problem for the manufacturer to attain the complete information in the asymmetric information CLSC.

Research on CLSC has gained considerable attention since it arose, and various strategic and operational aspects of CLSC were studied, such as CLSC network designing and planning, production planning and inventory management, etc. Interested reader can refer to Ataus et al (2008), Ilgin and Gupta (2010), Souza (2013) and Govindan et al. (2015) for a comprehensive review. In our paper, literature mainly related to three research streams: contract design problem in CLSC, government intervention and its impact on the CLSC and the collection channel of CLSC.

Contract design is a crucial factor to coordinate and manage the CLSC members. Savaskan et al. (2004) proposed three decentralized models and find retailer-based collection much more effective, then proposed a coordination mechanism if the retailer is incentivized to induce collection. Savaskan and Wassenhove (2006) further extended the above model to multiple setting that the retailing is competitive and showed that the manufacturer should entrust the collection of used products to the retailers under certain conditions. Hammond and Beullens (2007) formulated an equilibrium model for WEEE with variable inequality in a Cournot pricing game. Kaya (2010)

studied an incentive production model for remanufacturing decisions in a stochastic demand setting. Kenné et al. (2012) addressed a production planning model to minimize the sum of the holding and backlog costs for manufacturing and remanufacturing products in a continuous time stochastic context. Özkır and Başlıgil (2013) proposed a multiple objective optimization model in uncertain environment regarding three recovery options.

Some studies have focused on the collection channel of the CLSC. Thierry et al. (1995) presented products recovery management that the recovery options were classified into repairing, refurbishing, remanufacturing and collecting. Atasu and Wassenhove (2010) extended it to product take-back (collection) issues. Huang et al. (2013) examined optimal strategies of a CLSC with dual collecting channel, in which the retailer and the third party collect WEEEs competitively. Giovanni and Zaccour (2014) studied whether a remanufacturer should manage his own waste collection or outsource it to a collector. He (2015) modeled a closed-loop supply chain (CLSC) with a manufacturer and its supply channels – recycle channel and reliable supply channel.

Since the government act as a significant role in the CLSC, government intervention has also received much more attention from researchers. Atasu and Subramanian (2012) investigated the implications of collective and individual producer responsibility models of product take-back laws for WEEE. Vlachos et al. (2007) tackled the development of efficient capacity planning policies for remanufacturing facilities in reverse supply chains, taking into account the take-back obligation imposed by legislation. Webster and Mitra (2007) proposed a two-period manufacturer-remanufacturer competitive model to study the collective versus individual product collection under the take-back laws. Mitra and Webster (2008) examined the influence of government's remanufacturing subsidy on the manufacturer and the remanufacturer based on a two-period manufacturer-remanufacturer competitive model. Georgiadis and Besiou (2008) examined the impact of ecological motivation manifesting through legislation and green image factor. Plambeck and Wang (2009) found that adopting the "fee-upon-disposal" policy will motivate manufacturers to design for recyclability. Aksen et al. (2009) developed models to guide the government to subsidize manufacturers, while Atasu et al. (2009) discussed effective conditions for government to legislate WEEE recycling and remanufacturing policy. Atasu and Wassenhove (2012) proved that it is necessary to enforce collection and recycling of WEEE from the operations perspective. Rahman and Subramanian (2012) found that the government legislation can motivate the computer recycling operations. Govindan et al. (2013) found the CLSC members are not only directed by the value of the WEEEs but also by regulations. Atasu et al. (2013) compared two forms of product take-back legislation with manufacturer-operated and state-operated systems, and Ma et al. (2013) investigated how consumption-subsidy influences dual-channel CLSC.

The aforementioned literature makes an important contribution to the theory development of our research. But, most literatures adopt the government intervention policy and study CLSC under symmetric information environment. In contrast to the above references, this paper investigates the asymmetric information CLSC. In this paper, we consider that in the CLSC the

manufacturer deputs the collection of WEEE to the retailer. The collection quantity of WEEE relies strictly on the retailer's collection effort level which is the retailer's private information unavailable to the manufacturer. The retailer can choose to be high collection effort level or low collection effort level while the manufacturer only knows the probability of the retailer being H collection effort level. In the asymmetric information CLSC, we design an information screening contract to screen the retailer's collection effort level and the optimal decision-making and properties of contract parameters are derived. In addition, a reward-penalty mechanism (RPM) for the government to intervene in the asymmetric information CLSC is developed. The RPM prescribes a targeted collection quantity and a reward-penalty intensity. If the collection quantity is more or less than the targeted collection quantity, the manufacturer will receive a reward or a penalty. The impacts of the RPM are studied by comparing the cases with or without the implementation of the RPM.

The remainder of this paper is organized as follows. Section 2 formalizes the problem. Section 3 investigates the decision-making and contract parameters of the asymmetric information CLSC in which the manufacturer designs an information screening contract. Decision-making of the asymmetric information CLSC under RPM is addressed in Section 4, while Section 5 compares the two models and addresses the key findings. A numerical study is given to validate the models and develop managerial insights in Section 6. Section 7 remarks the conclusions and future research.

2. Problem formulation

2.1. Model description

Figure 1. The closed-loop supply chain structure with retailer collection under government's RPM

The closed-loop supply chain structure considered in this paper is shown as Figure 1 which gives the general structure of the CLSC under RPM, and the solid line represents the direction of forward logistics while the dotted line indicates the direction of reverse logistics. The CLSC has one manufacturer, one retailer and the consumer. As the Stackelberg leader, the manufacturer entrusts the collection of WEEE to the retailer. According to assumptions, the retailer chooses high collection effort level or low collection effort level, denoted as H-type retailer or L-type retailer respectively. The retailer obtains WEEE from consumers at the collection price r_{ij} and

sells them back to the manufacturer at buy-back price b_j . The manufacturer gives priority to WEEE-recycled materials and components in production due to the lower cost. The manufacturer sells the production at wholesale price w_j to the retailer, who then sells the products to final consumers at the retail price p_{ij} . For the manufacturer, how to seek the retailer's private information with the information screening contract is as follows: (1) the manufacturer designs a set of contracts according to the probability of being H-type retailer; (2) the retailer decides to accept which contract; (3) the manufacturer identifies the collection effort level of the retailer through the contract choice of the retailer.

2.2. Notations

In this paper, the collection effort level of the retailer is divided into high and low level. The parameters with superscript H are related to high collection effort level while parameters with superscript L are referred to low collection effort level. Superscript $*$ indicates the optimal decision results in the scenario. The specific parameters are expressed as follows:

c_n : Manufacturer's production cost of using new component ;

c_r : Manufacturer's production cost of using recycled WEEE's component ;

Δ : Unit saving cost is expressed as $\Delta = c_n - c_r$, differences of production cost of new materials or recycled components ;

e_i : Collection effort level of the retailer, $i \in \{H, L\}$, e_H denotes high collection effort level, e_L represents low collection effort level, and $e_H > e_L$;

r_{ij} : Collection price when the retailer chooses j contract to collect WEEE while her real collection effort level is i , where $i, j \in \{H, L\}$;

b_j : Buy-back price when the retailer chooses contract j , $j \in \{H, L\}$;

w_j : Wholesale price of new product when the retailer chooses contract j , $j \in \{H, L\}$;

p_{ij} : Retail price when the retailer chooses contract j to sell new products while her real collection effort level is i , $i \in \{H, L\}$, $j \in \{H, L\}$;

T_{rj} : Franchise fee paid by the retailer who chooses contract j to the manufacturer under her real collection effort level i , $j \in \{H, L\}$. Franchise fee is a common phenomenon in practice, such as franchised store ;

v : The probability of the retailer being H collection effort level, which is symmetric information.

Q_{ij} : Collection quantity when the retailer choose contract j to collect WEEE under her real collection effort level $i, i \in \{H, L\}, j \in \{H, L\}$;

$E(\pi_{mi})$: The expected profit of the manufacturer when the retailer chooses contract $i, i \in \{H, L\}$;

$E(\pi_{rij})$: The expected profit when the retailer chooses contract j under her collection effort level $i, i \in \{H, L\}, j \in \{H, L\}$;

π_r^0 : The reserved profit which is available to the retailer without contract ;

k : The reward-penalty intensity established by government;

Q_0 : The target collection quantity set by government.

2.3. Basic assumptions

- (i) There are no differences between remanufactured products and new products.
- (ii) We assume $\Delta = c_n - c_r > 0$, where unit production cost c_n is higher than c_r . We also assume that the unit cost of remanufactured product is consolidated, i.e. c_r , which can avoid tedious calculation but won't influence the management insights in the model.
- (iii) We assume that the new components take precedence over collected WEEE materials and components in production.
- (iv) In this model, the manufacturer and the retailer are risk neutral, without considering risk aversion or risk preference.
- (v) The market demand is $D = \phi - p_{ij} + \theta, i, j \in \{H, L\}$, where ϕ is the base market size, p_{ij} is the retail price, and θ is a random variable and satisfies the uniform distribution $U(0, a), (a > 0)$, whose probability density function is $f(\cdot)$ and probability distribution function is $F(\cdot)$.
- (vi) We assume the retailer collection quantity Q_{ij} is a linear function of the collection effort level and the collection price, $Q_{ij} = e_i + r_{ij}$. Due to the dependence of collection quantity on the retailer's collection effort level and the collection price, the expression of the collection quantity can be given by the formula above.

3. Decision-making of CLSC without the RPM (Case 1)

In this section, we investigate the information screening contract designed for the manufacturer to attain the private information of the retailer in the CLSC. Figure 1 shows that the reverse buy-back and forward wholesale connect the manufacturer with retailer. So the information screening contract can be expressed as $\{G_H(b_H, w_H, T_{rH}), G_L(b_L, w_L, T_{rL})\}$ composed of the wholesale price w , the buy-back price b and franchise fee T .

In accordance with the assumptions (v) and (vi), the expected profit of the manufacturer when the retailer chooses contract G_H can be expressed as:

$$E(\pi_{mH}) = \int_0^{z_H} (\phi - p_{HH} + x)(w_H - c_r)f(x)dx + \int_{z_H}^a (\phi - p_{HH} + x)(w_H - c_n)f(x)dx \\ + \int_{z_H}^a (e_H + r_{HH})(c_n - c_r)f(x)dx - b_H(e_H + r_{HH}) + T_{rH} \quad (1)$$

In Eq.(1), $z_H = e_H + r_{HH} - \phi + p_{HH}$, the new remanufactured products can satisfy the market demand if $0 < x \leq z_H$, if $z_H < x < a$, market demand is more than the collection quantity of WEEEs, new remanufactured products and new-manufactured products meet the market demand.

Similarly, when the retailer chooses contract G_L , the expected profit of manufacturer can be formulated as:

$$E(\pi_{mL}) = \int_0^{z_L} (\phi - p_{LL} + x)(w_L - c_r)f(x)dx + \int_{z_L}^a (\phi - p_{LL} + x - e_L - r_{LL})(w_L - c_n)f(x)dx \\ + \int_{z_L}^a (e_L + r_{LL})(w_L - c_r)f(x)dx - b_L(e_L + r_{LL}) + T_{rL} \quad (2)$$

In Eq. (2), $z_L = e_L + r_{LL} - \phi + p_{LL}$, if $0 < x \leq z_L$, the collection quantity of WEEEs is more than the demand of new products, so the new remanufactured products can satisfy market demand. However, if $z_L < x < a$, market demand is more than the collection quantity of WEEEs, new remanufactured products and new-manufactured products meet market demand together.

If the retailer exerts H level of collection effort and chooses contract G_H , her expected profit is:

$$E(\pi_{rHH}) = (b_H - r_{HH})(e_H + r_{HH}) - e_H^2 / 4 \\ + (p_{HH} - w_H) \int_0^a (\phi - p_{HH} + x)f(x)dx - T_{rH} \quad (3)$$

where $e_H^2 / 4$ is the cost of collection effort.

When the retailer adopts H level of collection effort but chooses contract G_L , in this case her expected profit is:

$$E(\pi_{rHL}) = (b_L - r_{HL})(e_H + r_{HL}) - e_H^2 / 4 \\ + (p_{HL} - w_L) \int_0^a (\phi - p_{HL} + x)f(x)dx - T_{rL} \quad (4)$$

If the retailer makes L level of collection effort adopting contract G_L , the profit of the retailer is given as follows:

$$E(\pi_{rLL}) = (b_L - r_{LL})(e_L + r_{LL}) - e_L^2 / 4 + (p_{LL} - w_L) \int_0^a (\phi - p_{LL} + x) f(x) dx - T_{rL} \quad (5)$$

Where $e_L^2 / 4$ denotes the cost of collection effort.

As the retailer exerts L level of collection effort but chooses the contract G_H , the expected profit of the retailer is denoted as:

$$E(\pi_{rLH}) = (b_H - r_{LH})(e_L + r_{LH}) - e_L^2 / 4 + (p_{LH} - w_H) \int_0^a (\phi - p_{LH} + x) f(x) dx - T_{rH} \quad (6)$$

In the CLSC under random demand environment without the government intervention, the retailer's collection effort level information is asymmetric. The manufacturer seeks to maximize the expected profit which can be expressed as:

$$\max E(\pi_m) = vE(\pi_{mH}) + (1 - v)E(\pi_{mL}) \quad (7)$$

$$s.t. \quad (p_{HH}^*, r_{HH}^*) = \arg \max E(\pi_{rHH}) \quad (8)$$

$$(p_{LL}^*, r_{LL}^*) = \arg \max E(\pi_{rLL}) \quad (9)$$

$$E(\pi_{rHH}) \geq E(\pi_{rHL}) \quad (10)$$

$$E(\pi_{rLL}) \geq E(\pi_{rLH}) \quad (11)$$

$$E(\pi_{rHH}) \geq \pi_r^0 \quad (12)$$

$$E(\pi_{rLL}) \geq \pi_r^0 \quad (13)$$

The optimal profit of the retailer without the contract is π_r^0 called conserved profit. Eqs. (12) and (13) are participation constraints that can guarantee the retailer's profit no lesser than her conserved profit. Eq. (10) indicates the profit of retailer exerting H level and G_H contract is higher than the profit of choosing H level and G_L contract. Meanwhile, Eq. (11) shows the profit of the retailer with L level and G_L contract is much more than the profit under L level and G_H contract. Eqs. (10) and (11) are both incentive compatibility constraints to avoid the false information from the retailer, by which the retailer can maximize her profit. In Eq. (7), $E(\pi_{mH})$ represents the expected profit of the manufacturer when the retailer with H level accepts G_H contract, while $E(\pi_{mL})$ denotes the expected profit of the manufacturer if the retailer adopted L level accepts contract.

Solving the above programming model with Lagrange multiplier method, we find that:

$$w_H^* = \frac{4\Delta(\phi - e_H) - 3\Delta^2 + (\Delta + 2a)(c_r + 3c_n)}{8(\Delta + a)} \quad (14)$$

$$w_L^* = \frac{4\Delta\phi + (\Delta + 2a)(c_r + 3c_n) - 3\Delta^2}{8(\Delta + a)} - \frac{\Delta(e_L - ve_H)}{2(1-v)(\Delta + a)} \quad (15)$$

$$b_H^* = \frac{\Delta(\phi - e_H - w_H^* + 3a/2)}{\Delta + 2a} \quad (16)$$

$$b_L^* = \frac{\Delta(\phi - w_L^* + 3a/2)}{\Delta + 2a} - \frac{[\Delta(1-v) - 2av]e_L + 2ave_H}{(1-v)(\Delta + 2a)} \quad (17)$$

Substituting w_H^*, w_L^* into Eqs. (16) and (17), we result in the optimal buy-back prices of the manufacturer without the government intervention:

$$T_{rH}^* = \frac{(b_L^* + e_L)^2 - e_L^2 + (\phi - w_L^* + a/2)^2}{4} - \pi_r^0 \quad (18)$$

$$T_{rH}^* = \frac{(b_H^* + e_H)^2 + (\phi - w_H^* + a/2)^2 - (b_L^* + e_H)^2 - (\phi - w_L^* + a/2)^2}{4} + T_{rL}^* \quad (19)$$

Substituting w_H^*, w_L^*, b_H^* and b_L^* into Eqs. (18) and (19), we obtain the optimal franchise fees

$$p_{HH}^* = \frac{\phi + w_H^* + a/2}{2} \quad (20)$$

$$p_{LL}^* = \frac{\phi + w_L^* + a/2}{2} \quad (21)$$

The optimal retail price of the CLSC without the RPM can be attained by substituting w_H^*, w_L^* into Eqs. (20) and (21):

$$r_{HH}^* = \frac{b_H^* - e_H}{2} \quad (22)$$

$$r_{LL}^* = \frac{b_L^* - e_L}{2} \quad (23)$$

Similarly, after the b_H^*, b_L^* have been substituted into Eqs. (22) and (23), we obtain the optimal collection prices based on the retailer's collection effort level.

By solving Case 1, we can derive Proposition 1.

Proposition 1. (a) The optimal information screening contract G_L are given by

$$b_H^* = \frac{\Delta(\phi - e_H - w_H^* + 3a/2)}{\Delta + 2a}, w_H^* = \frac{4\Delta(\phi - e_H) - 3\Delta^2 + (\Delta + 2a)(c_r + 3c_n)}{8(\Delta + a)}$$

$$T_{rH}^* = \frac{(b_H^* + e_H)^2 + (\phi - w_H^* + a/2)^2 - (b_L^* + e_H)^2 - (\phi - w_L^* + a/2)^2}{4} + T_{rL}^*$$

(b) The optimal information screening contract G_L are given by

$$b_L^* = \frac{\Delta(\phi - w_L^* + 3a/2)}{\Delta + 2a} - \frac{[\Delta(1-v) - 2av]e_L + 2ave_H}{(1-v)(\Delta + 2a)}$$

$$w_L^* = \frac{4\Delta\phi + (\Delta + 2a)(c_r + 3c_n) - 3\Delta^2}{8(\Delta + a)} - \frac{\Delta(e_L - ve_H)}{2(1-v)(\Delta + a)},$$

$$T_{rL}^* = \frac{(b_L^* + e_L)^2 - e_L^2 + (\phi - w_L^* + a/2)^2}{4} - \pi_r^0$$

(c) The optimal retail price and collection price are given by

$$p_{HH}^* = \frac{\phi + w_H^* + a/2}{2}, r_{HH}^* = \frac{b_H^* - e_H}{2}; p_{LL}^* = \frac{\phi + w_L^* + a/2}{2}, r_{LL}^* = \frac{b_L^* - e_L}{2}.$$

Proposition 1 implies that the information screening contract can ensure the manufacturer obtain the retailer's collection effort level.

Proposition 2. We have the following observations about information screening contracts.

$$w_H^* < w_L^*, b_H^* > b_L^*, T_{rH}^* > T_{rL}^*, p_{HH}^* < p_{LL}^*, r_{HH}^* < r_{LL}^*$$

Proof. $w_L^* - w_H^* = \frac{\Delta(e_H - e_L)}{2(\Delta + a)(1-v)}$, in terms of $e_H > e_L$, we obtain $w_L^* - w_H^* > 0$, thus

$$w_H^* < w_L^* \quad ; \quad b_H^* - b_L^* = \frac{\Delta(1-v)(w_L^* - w_H^* - e_H)}{\Delta + 2a} + \frac{\Delta(1-v) + 2av(e_H - e_L)}{(1-v)(\Delta + 2a)}, \text{ we have known}$$

$$w_L^* - w_H^* > 0 \text{ and } e_H > e_L, \text{ so } b_H^* > b_L^*; T_{rH}^* - T_{rL}^* = \frac{(b_H^{*2} - b_L^{*2}) + (w_L^* - w_H^*)[2\phi + a - (w_H^* + w_L^*)]}{4}, \text{ fr}$$

om assumption (v) we know $\phi > p > w$, so $2\phi + a - (w_H^* + w_L^*)$ and we get

$$T_{rH}^* - T_{rL}^* > 0; p_{LL}^* - p_{HH}^* = \frac{w_L^* - w_H^*}{2} > 0, \text{ thus } p_{LL}^* > p_{HH}^*, \text{ Similarly, we can get } r_{HH}^* < r_{LL}^*.$$

Proposition 2 indicates that the manufacturer would offer lower wholesale price and higher buy-back price for the H-type retailer to reward and stimulate the H-type retailer. Meanwhile, to ensure its own profits, the manufacturer charge more franchise fee to H-type retailer than L-type retailer. Because of the lower wholesale price, the H-type retailer sales products at a lower retailer price. In addition, the H-type retailer can collect the waste products from consumer at a lower price than the L-type retailer.

4. Decision-making of the CLSC with RPM(Case 2)

In this section, we investigate the impacts of RPM established by the government on the CLSC. Same as Case 1, the retailer's collection effort level is privately asymmetric information and the information screening contract is denoted as $\{G_H(b_H, w_H, T_{rH}), G_L(b_L, w_L, T_{rL})\}$ in case 2. The RPM established by government is expressed as (k, Q_0) , where Q_0 is the target collection quantity and k is the reward-penalty intensity to enhance the WEEE collection. Thus, the manufacturer be rewarded if the collection target is fulfilled; otherwise he will be penalized. In

this case, the reward intensity is assumed the same as the penalty intensity for avoiding complicated calculation.

When the retailer with H level chooses contract G_H , the expected profit of manufacturer can be expressed as:

$$\begin{aligned} E(\pi_{mH}) = & \int_0^{z_H} (\phi - p_{HH} + x)(w_H - c_r)f(x)dx + T_{rH} + k(Q_{HH} - Q_0) \\ & + \int_{z_H}^a (\phi - p_{HH} + x - e_H - r_{HH})(w_H - c_n)f(x)dx \\ & + \int_{z_H}^a (e_H + r_{HH})(w_H - c_r)f(x)dx - b_H(e_H + r_{HH}) \end{aligned} \quad (24)$$

In Eq. (24), $z_H = e_H + r_{HH} - \phi + p_{HH}$, if $0 < x \leq z_H$, the new products can satisfy the market demand. Otherwise, if $z_H < x < a$, the market demand exceeds the collection quantity of WEEE, the remanufactured and new-manufactured products satisfy the market demand jointly.

Similarly, if the retailer with L level chooses contract G_L , the manufacturer's expected profit can be formulated as:

$$\begin{aligned} E(\pi_{mL}) = & \int_0^{z_L} (\phi - p_{LL} + x)(w_L - c_r)f(x)dx + T_{rL} + k(Q_{LL} - Q_0) \\ & + \int_{z_L}^a (\phi - p_{LL} + x - e_L - r_{LL})(w_L - c_n)f(x)dx \\ & + \int_{z_L}^a (e_L + r_{LL})(w_L - c_r)f(x)dx - b_L(e_L + r_{LL}) \end{aligned} \quad (25)$$

In Eq. (25), $z_L = e_L + r_{LL} - \phi + p_{LL}$, if $0 < x \leq z_L$, the remanufactured products are in sufficient quantities to satisfy the market demand. However, if $z_L < x < a$, remanufactured and new-manufactured products meet the market demand conjunctly.

If the retailer opts to contract G_H under H level, the expected profit of the retailer is

$$\begin{aligned} E(\pi_{rHH}) = & (b_H - r_{HH})(e_H + r_{HH}) - e_H^2 / 4 \\ & + (p_{HH} - w_H) \int_0^a (\phi - p_{HH} + x)dx - T_{rH} \end{aligned} \quad (26)$$

When the retailer is H collection effort level but choosing contract G_L , the retailer's expected profit is denoted as:

$$\begin{aligned} E(\pi_{rHL}) = & (b_L - r_{HL})(e_H + r_{HH}) - e_H^2 / 4 \\ & + (p_{HL} - w_L) \int_0^a (\phi - p_{HL} + x)dx - T_{rL} \end{aligned} \quad (27)$$

If the retailer chooses contract G_L under L collection effort level, we find her expected profit can be expressed as:

$$\begin{aligned} E(\pi_{rLL}) = & (b_L - r_{LL})(e_L + r_{LL}) - e_L^2 / 4 \\ & + (p_{LL} - w_L) \int_0^a (\phi - p_{LL} + x)dx - T_{rL} \end{aligned} \quad (28)$$

If the retailer's collection effort is L level, and chooses contract G_H , we find her expected profit is

$$E(\pi_{rLH}) = (b_H - r_{LH})(e_L + r_{LH}) - e_L^2 / 4 + (p_{LH} - w_H) \int_0^a (\phi - p_{LH} + x) dx - T_{rH} \quad (29)$$

In the CLSC under random demand environment with RPM, when the retailer's collection effort level information is asymmetric, the manufacturer seeks to maximize the expected profit which can be expressed as:

$$\max E(\pi_m) = vE(\pi_{mH}) + (1-v)E(\pi_{mL}) \quad (30)$$

$$s.t. \quad (p_{HH}^{**}, r_{HH}^{**}) = \arg \max E(\pi_{rHH}) \quad (31)$$

$$(p_{LL}^{**}, r_{LL}^{**}) = \arg \max E(\pi_{rLL}) \quad (32)$$

$$E(\pi_{rHH}) \geq E(\pi_{rHL}) \quad (33)$$

$$E(\pi_{rLL}) \geq E(\pi_{rLH}) \quad (34)$$

$$E(\pi_{rHH}) \geq \pi_r^0 \quad (35)$$

$$E(\pi_{rLL}) \geq \pi_r^0 \quad (36)$$

π_r^0 is the optimal profit of the retailer without the contract, which is called conserved profit. Eqs. (35) and (36) are participation constraints ensuring the retailer accept the contract. Eqs. (33) and (34) are incentive compatibility constraints restricting the retailer only when she chooses the same type contract as the collective effort level can she gains higher profit than exerting other contracts. The incentive compatibility constraints enable the manufacturer obtain the collection effort level information from the retailer. Eq. (33) indicates the profit of the retailer with H collection effort level and G_H contract is much more than the profit from choosing H collection effort level but G_L contract. Meanwhile, Eq. (34) shows the profit of the retailer exerting L collection effort level and G_L contract is higher than the case under L collection effort level but G_H contract.

From the solution of the above programming model with Lagrange multiplier method, we have

$$w_H^{**} = \frac{4\Delta(\phi - e_H - k) - 3\Delta^2 + (\Delta + 2a)(c_r + 3c_n)}{8(\Delta + a)} \quad (37)$$

$$w_L^{**} = \frac{4\Delta(\phi - k) - 3\Delta^2 + (\Delta + 2a)(c_r + 3c_n)}{8(\Delta + a)} - \frac{\Delta(e_L - ve_H)}{2(1-v)(\Delta + a)} \quad (38)$$

$$b_H^{**} = \frac{\Delta(\phi - e_H - w_H^{**} + 3a/2) + 2ak}{\Delta + 2a} \quad (39)$$

$$b_L^{**} = \frac{\Delta(\phi - w_L^{**} + 3a/2) + 2ak}{\Delta + 2a} - \frac{[\Delta(1-v) - 2av]e_L + 2ave_L}{(1-v)(\Delta + 2a)} \quad (40)$$

The substitution of w_H^{**} , w_L^{**} into Eqs. (39) and (40) gets the optimal buy-back price of the manufacturer with RPM.

$$T_{rL}^{**} = \frac{(b_{LL}^{**} + e_L)^2 - e_L^2 + (\phi - w_L^{**} + a/2)^2}{4} - \pi_r^0 \quad (41)$$

$$T_{rH}^{**} = \frac{(b_H^{**} + e_H)^2 + (\phi - w_H^{**} + a/2)^2 - (b_L^{**} + e_H)^2 - (\phi - w_L^{**} + a/2)^2}{4} + T_{rL}^{**} \quad (42)$$

Substituting w_H^{**} , w_L^{**} , b_H^{**} and b_L^{**} into Eqs.(41) and(42) results in the optimal franchise fee with RPM :

$$p_{HH}^{**} = \frac{\phi + w_H^{**} + a/2}{2} \quad (43)$$

$$p_{LL}^{**} = \frac{\phi + w_L^{**} + a/2}{2} \quad (44)$$

Substitution of w_H^{**} , w_L^{**} into Eqs.(43)and (44) gets the optimal retail price of the retailer according to the real collection effort level with the RPM:

$$r_{HH}^{**} = \frac{b_H^{**} - e_H}{2} \quad (45)$$

$$r_{LL}^{**} = \frac{b_L^{**} - e_L}{2} \quad (46)$$

Lastly, substituting b_H^{**} , b_L^{**} , into Eqs. (45) and (46) gets the optimal collection price based on the retailer's collection effort level when the government establishes the RPM for the manufacturer.

By solving Case 2, we can derive Lemma 2.

Proposition 3. (a) Under PRM, the optimal information screening contract G_H are given by

$$w_H^{**} = \frac{4\Delta(\phi - e_H - k) - 3\Delta^2 + (\Delta + 2a)(c_r + 3c_n)}{8(\Delta + a)}, b_H^{**} = \frac{\Delta(\phi - e_H - w_H^{**} + 3a/2) + 2ak}{\Delta + 2a}$$

$$T_{rH}^{**} = \frac{(b_H^{**} + e_H)^2 + (\phi - w_H^{**} + a/2)^2 - (b_L^{**} + e_H)^2 - (\phi - w_L^{**} + a/2)^2}{4} + T_{rL}^{**}$$

(b) Under PRM, the optimal information screening contract G_L are given by

$$w_L^{**} = \frac{4\Delta(\phi - k) - 3\Delta^2 + (\Delta + 2a)(c_r + 3c_n)}{8(\Delta + a)} - \frac{\Delta(e_L - ve_H)}{2(1-v)(\Delta + a)}$$

$$b_L^{**} = \frac{\Delta(\phi - w_L^{**} + 3a/2) + 2ak}{\Delta + 2a} - \frac{[\Delta(1-v) - 2av]e_L + 2ave_L}{(1-v)(\Delta + 2a)}$$

$$T_{rL}^{**} = \frac{(b_{LL}^{**} + e_L)^2 - e_L^2 + (\phi - w_L^{**} + a/2)^2}{4} - \pi_r^0$$

5. Analysis of the two cases

Proposition 4. If the government establishes RPM for the manufacturer, the wholesale price and retail price are lower than those without RPM.

Proof. Comparing Eq. (37) with Eq.(14), we have $w_H^{**} - w_H^* = -\Delta k / 2(\Delta + a) < 0$. The $w_L^{**} - w_L^* = -\Delta k / 2(\Delta + a) < 0$ can be got from the comparison of Eq.(38) with Eq.(15). We can find from $w_H^{**} - w_H^* < 0$ and $w_L^{**} - w_L^* < 0$ that the wholesale price is lower than that in the case which the government establishes RPM for the manufacturer. Then again, we have $p_{HH}^{**} - p_{HH}^* = (w_H^{**} - w_H^*) / 2 < 0$ from the comparison of Eq. (43) with Eq. (20) and $p_{LL}^{**} - p_{LL}^* = (w_L^{**} - w_L^*) / 2 < 0$ from the comparison of Eq. (44) with Eq. (21). So the price of the retail lower than that in the condition which the government establishes RPM for the manufacturer can be obtained.

Proposition 4 shows that under RPM the manufacturer offers lower wholesale price to the retailer and the retailer to sell more products will provides a lower retailer to consumers. The RPM can reduce the wholesale price and retail price on the market.

Proposition 5. The buy-back price, collection price and collection quantity under RPM are higher than those without RPM.

Proof. $b_H^{**} - b_H^* = k(\Delta + 2a) / 2(\Delta + a) > 0$ and $b_L^{**} - b_L^* = k(\Delta + 2a) / 2(\Delta + a) > 0$ can be got by the comparison of Eq. (39) with Eq. (16) and Eq. (40) with Eq. (17) respectively. Then the buy-back price is higher than the case without RPM. Comparing Eq. (45) with Eq. (22) and Eq. (46) with Eq. (23) respectively, the $r_{HH}^{**} - r_{HH}^* = (b_H^{**} - b_H^*) / 2 > 0$ and $r_{LL}^{**} - r_{LL}^* = (b_L^{**} - b_L^*) / 2 > 0$ can be attained, hence the collection price is higher than that in the case under RPM. Based on Assumption (vi), we know $Q_{HH}^{**} - Q_{HH}^* = r_{HH}^{**} - r_{HH}^*$, so $Q_{HH}^{**} - Q_{HH}^* > 0$. Similarly, $Q_{LL}^{**} - Q_{LL}^* = r_{LL}^{**} - r_{LL}^*$, thus $Q_{LL}^{**} - Q_{LL}^* > 0$, so the collection quantity is higher than that under the case without RPM.

Proposition 5 indicates that under the PRM, the manufacturer would raise the buy-back price to stimulate the retailer's collection effort and the retailer would raise the collection price to collect more waste products corresponding. Thus, the implementation of RPM can increase the collection quantity.

Proposition 6. The franchise fee in the case with RPM is higher than that without RPM.

Proof. From. (41) and (18), we have

$$T_{rL}^{**} - T_{rH}^* = [(b_L^{**} + e_L)^2 - (b_L^* + e_L)^2 + (\phi - w_L^{**} + a/2)^2 - (\phi - w_L^* + a/2)^2] / 4,$$

since $b_L^{**} > b_L^*$ and $w_L^{**} < w_L^*$, $T_{rL}^{**} - T_{rL}^* > 0$. Similarly, subtract Eq.(19) from Eq.(42) and we get

$$T_{rH}^{**} - T_{rH}^* = \frac{K(\Delta+2a)(b_H^{**}+b_H^*+2e_L)/2(\Delta+a)+(\phi-w_H^{**}+a/2)^2-(\phi-w_H^*+a/2)^2}{4}$$

$$T_{rH}^{**} - T_{rH}^* = \frac{K(\Delta+2a)(b_H^{**}+b_H^*+2e_L)/2(\Delta+a)+(\phi-w_H^{**}+a/2)^2-(\phi-w_H^*+a/2)^2}{4},$$

due to $w_H^{**} < w_H^*$ and $T_{rH}^{**} - T_{rH}^* > 0$. The franchise fee with RPM for the manufacturer is higher than that without RPM can be obtained in accordance with the above results.

From Proposition 4 and 5, we know that the manufacturer reduce the wholesale price and increase the buy-back price under the RPM. So Proposition 6 indicates the manufacturer would ensure its own profits by charging more franchise fee to the retailer under the RPM.

6. Numerical examples

6.1. Analysis of Case 1

We now study the asymmetric information CLSC further. Let

$$c_n = 60, c_r = 45, a = 4, \phi = 120, e_H = 10, e_L = 7, \pi_r^0 = 400, \Delta = c_n - c_r = 15;$$

and $v \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$. Table 1 shows the optimal decision-making of CLSC with different v and we can find:

(1) The manufacturer offers a fixed wholesale price to w_H^* the H retailer no matter how the v changes. But, the manufacturer will provide the L retailer w_L^* considering the change of v . In Eq. (14), taking the first derivative of w_H^* respect to v and get $\frac{dw_H^*}{dv} = 0$, which shows that w_H^* won't change with v . In Eq. (15), taking the first derivative of w_L^* with respect to v and we have $\frac{dw_L^*}{dv} = \frac{\Delta(e_H - e_L)}{2(\Delta + a)(1 - v)^2} > 0$, $\frac{dw_L^*}{dv} > 0$ due to $e_H > e_L$, which implies the wholesale price w_L^* increases with v . So when the manufacturer offers wholesale price w_L^* for the L retailer, the manufacturer will take the probability of being H retailer. Under the same v , $w_H^* < w_L^*$ means the retailer chooses H collection effort level can enjoy lower wholesale price. From Eqs. (20) and (21), we know the retail price is the positive function of the wholesale price which implies the changing trend of the retail price is the same as that of the wholesale price.

Table 1. The optimal decision results changing with the parameter v

	v								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
w_H^*	73.026	73.026	73.026	73.026	73.026	73.026	73.026	73.026	73.026

w_L^*	74.342	74.420	74.507	74.605	74.718	74.848	75.000	75.179	75.395
b_H^*	28.026	28.026	28.026	28.026	28.026	28.026	28.026	28.026	28.026
b_L^*	29.009	28.890	28.757	28.605	28.432	28.233	28.000	27.725	27.395
T_{rH}^*	479.727	475.751	471.290	466.249	460.508	453.911	446.250	437.246	426.512
T_{rL}^*	492.593	492.771	492.971	493.198	493.458	493.757	494.106	494.519	495.014
r_{HH}^*	97.513	97.513	97.513	97.513	97.513	97.513	97.513	97.513	97.513
r_{LL}^*	98.171	98.210	98.253	98.303	98.359	98.424	98.500	98.590	98.697
p_{HH}^*	9.013	9.013	9.013	9.013	9.013	9.013	9.013	9.013	9.013
p_{LL}^*	11.004	10.945	10.878	10.803	10.716	10.616	10.500	10.362	10.197
Q_{HH}^*	19.013	19.013	19.013	19.013	19.013	19.013	19.013	19.013	19.013
Q_{LL}^*	18.004	17.945	17.878	17.803	17.716	17.616	17.500	17.362	17.197
$E(\pi_{ml}^*)$	541.270	541.448	541.648	541.875	542.134	542.434	542.783	543.196	543.691
$E(\pi_{ml}^*)$	541.838	541.813	541.770	541.704	541.605	541.460	541.250	540.944	540.493
$E(\pi_{rHH}^*)$	443.513	443.335	443.135	442.908	442.649	442.349	442.000	441.587	441.092
$E(\pi_{rLL}^*)$	400.000	400.000	400.000	400.000	400.000	400.000	400.000	400.000	400.000

(2) The manufacturer provides a fixed buy-back price b_H^* to the H retailer no matter how v changes. When the retailer exerts L collection effort level, w_L^* will change with v . In Eq. (16), taking the first derivative of b_H^* respect to v , we have $\frac{db_H^*}{dv} = 0$ that implies v does not impact b_H^* .

From Eq. (17), taking the first derivative of b_L^* respect to v obtains $\frac{db_L^*}{dv} = -\frac{2a(e_H - e_L)}{(\Delta + 2a)(1 - v)^2} < 0$, so

the buy-back price b_L^* decreases with v when the retailer chooses L collection effort level. For the L retailer, the manufacturer offers lower buy-back price as the v increase. Based on Eqs. (22), (23) and Assumption (vi), we know the collection price and collection quantity are the positive correlation of the buy-back price which means the collection price and collection quantity has the same changing trend with the buy-back price.

(3) The franchise fee T_{rH}^* increases with v , while T_{rL}^* decreases with v .

(4) When the retailer chooses G_H , the expected profit of the manufacturer increases with v , however, the expected profit of the retailer decreases with v . When the retailer accepts G_L , the

expected profit of the manufacturer decreases with v , and the expected profit of the retailer is the conserved profit.

6.2. Analysis of Case 2

To indicate the influence of RPM on the WEEE collection, we employ the same parameters as that in Section 6.1:

$c_n = 60, c_r = 45, a = 4, \phi = 120, e_H = 10, e_L = 7, \pi_r^0 = 400, \Delta = c_n - c_v$, let $k \in \{10, 20, 30, 40\}$, $Q_0 = 25$. The numerical results of the CLSC with RPM are shown in Table 2 and Figure 3-10.

(1) Table 2 shows the optimal decision-making of the CLSC with RPM. Compared with Table 1, we can find that: $w_H^{**} < w_H^*, w_L^{**} < w_L^*, p_{HH}^{**} < p_{HH}^*, p_{LL}^{**} < p_{LL}^*$. Figure 3 and Figure 4 show that the wholesale price w_H^{**} and w_L^{**} , retail price p_{HH}^{**} and p_{LL}^{**} decrease with reward-penalty intensity k . It can be proofed as follows. Taking the first derivative of Eq. (37) with respect to k results in $\frac{dw_H^{**}}{dk} = -\frac{\Delta}{2(\Delta + a)} < 0$; Similarly, $\frac{dw_L^{**}}{dk} = -\frac{\Delta}{2(\Delta + a)} < 0$, $\frac{dp_{HH}^{**}}{dk} = -\frac{\Delta}{2(\Delta + a)} < 0$, $\frac{dp_{LL}^{**}}{dk} = -\frac{\Delta}{2(\Delta + a)} < 0$ can be got by taking the first derivative of Eqs. (38), (43) and (44) respectively.

Combining the results above, we have that the wholesale price and the retail price decrease with reward-penalty intensity and they are lower than that without RPM. The decrease in retail price benefits the consumer and enlarges the demand of new products. Therefore, the RPM can decrease retail price and benefit the consumers.

Table 2. The optimal decision results vs. k and v

$k \backslash v$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
w_H^{**}									
10	69.079	69.079	69.079	69.079	69.079	69.079	69.079	69.079	69.079
20	65.132	65.132	65.132	65.132	65.132	65.132	65.132	65.132	65.132
30	61.184	61.184	61.184	61.184	61.184	61.184	61.184	61.184	61.184
40	57.237	57.237	57.237	57.237	57.237	57.237	57.237	57.237	57.237
w_L^{**}									
10	70.395	70.559	70.771	71.053	71.447	72.039	73.026	75.000	80.921
20	66.447	66.612	66.823	67.105	67.500	68.092	69.079	71.053	76.974
30	62.500	62.664	62.876	63.158	63.553	64.145	65.132	67.105	73.026
40	58.553	58.717	58.929	59.211	59.605	60.197	61.184	63.158	69.079
b_H^{**}									
10	34.079	34.079	34.079	34.079	34.079	34.079	34.079	34.079	34.079
20	40.132	40.132	40.132	40.132	40.132	40.132	40.132	40.132	40.132
30	46.184	46.184	46.184	46.184	46.184	46.184	46.184	46.184	46.184
40	52.237	52.237	52.237	52.237	52.237	52.237	52.237	52.237	52.237
b_L^{**}									
10	35.061	34.809	34.485	34.053	33.447	32.539	31.026	28.000	18.921
20	41.114	40.862	40.538	40.105	39.500	38.592	37.079	34.053	24.974

30	47.167	46.914	46.590	46.158	45.553	44.645	43.132	40.105	31.026
40	53.219	52.967	52.643	52.211	51.605	50.697	49.184	46.158	37.079
T_{rH}^{**}									
10	708.306	708.684	709.17	709.819	710.727	712.089	714.358	718.898	732.516
20	950.127	950.505	950.992	951.64	952.548	953.91	956.18	960.719	974.338
30	1218.056	1218.435	1218.921	1219.57	1220.477	1221.839	1224.109	1228.649	1242.267
40	1512.094	1512.472	1512.958	1513.607	1514.515	1515.877	1518.146	1522.686	1536.304
T_{rL}^{**}									
10	695.816	686.291	674.111	657.988	635.64	602.606	548.856	446.25	177.595
20	938.014	927.401	913.823	895.834	870.875	833.925	773.648	657.988	350.171
30	1206.319	1194.619	1179.642	1159.789	1132.218	1091.353	1024.548	895.834	548.856
40	1500.733	1487.945	1471.569	1449.851	1419.67	1374.888	1301.556	1159.789	773.648
r_{HH}^{**}									
10	12.039	12.039	12.039	12.039	12.039	12.039	12.039	12.039	12.039
20	15.066	15.066	15.066	15.066	15.066	15.066	15.066	15.066	15.066
30	18.092	18.092	18.092	18.092	18.092	18.092	18.092	18.092	18.092
40	21.118	21.118	21.118	21.118	21.118	21.118	21.118	21.118	21.118
r_{LL}^{**}									
10	14.031	13.905	13.742	13.526	13.224	12.77	12.013	10.5	5.961
20	17.057	16.931	16.769	16.553	16.25	15.796	15.039	13.526	8.987
30	20.083	19.957	19.795	19.579	19.276	18.822	18.066	16.553	12.013
40	23.11	22.984	22.821	22.605	22.303	21.849	21.092	19.579	15.039
p_{HH}^{**}									
10	95.539	95.539	95.539	95.539	95.539	95.539	95.539	95.539	95.539
20	93.566	93.566	93.566	93.566	93.566	93.566	93.566	93.566	93.566
30	91.592	91.592	91.592	91.592	91.592	91.592	91.592	91.592	91.592
40	89.618	89.618	89.618	89.618	89.618	89.618	89.618	89.618	89.618
p_{LL}^{**}									
10	96.197	96.28	96.385	96.526	96.724	97.02	97.513	98.5	101.461
20	94.224	94.306	94.412	94.553	94.75	95.046	95.539	96.526	99.487
30	92.25	92.332	92.438	92.579	92.776	93.072	93.566	94.553	97.513
40	90.276	90.359	90.464	90.605	90.803	91.099	91.592	92.579	95.539
Q_{HH}^{**}									
10	22.039	22.039	22.039	22.039	22.039	22.039	22.039	22.039	22.039
20	25.066	25.066	25.066	25.066	25.066	25.066	25.066	25.066	25.066
30	28.092	28.092	28.092	28.092	28.092	28.092	28.092	28.092	28.092
40	31.118	31.118	31.118	31.118	31.118	31.118	31.118	31.118	31.118
Q_{LL}^{**}									
10	21.031	20.905	20.742	20.526	20.224	19.77	19.013	17.5	12.961
20	24.057	23.931	23.769	23.553	23.25	22.796	22.039	20.526	15.987
30	27.083	26.957	26.795	26.579	26.276	25.822	25.066	23.553	19.013
40	30.11	29.984	29.821	29.605	29.303	28.849	28.092	26.579	22.039
$E(\pi_{ml}^{**})$									
10	487.454	487.832	488.319	488.967	489.875	491.237	493.507	498.046	511.664
20	463.901	464.28	464.766	465.414	466.322	467.684	469.954	474.493	488.112
30	470.612	470.99	471.477	472.125	473.033	474.395	476.664	481.204	494.822
40	507.586	507.964	508.45	509.099	510.007	511.368	513.638	518.178	531.796
$E(\pi_{ml}^{**})$									
10	488.023	487.954	487.789	487.434	486.678	484.975	480.625	466.25	377.73
20	464.47	464.402	464.237	463.882	463.125	461.423	457.072	442.697	354.178
30	471.181	471.112	470.947	470.592	469.836	468.133	463.783	449.408	360.888
40	508.154	508.086	507.921	507.566	506.809	505.107	500.757	486.382	397.862

$E(\pi_{HH}^{**})$									
10	452.592	452.214	451.727	451.079	450.171	448.809	446.539	442	428.382
20	461.671	461.293	460.806	460.158	459.25	457.888	455.618	451.079	437.461
30	470.75	470.372	469.885	469.237	468.329	466.967	464.697	460.158	446.539
40	479.829	479.451	478.964	478.316	477.408	476.046	473.776	469.237	455.618
$E(\pi_{LL}^{**})$									
10	400	400	400	400	400	400	400	400	400
20	400	400	400	400	400	400	400	400	400
30	400	400	400	400	400	400	400	400	400
40	400	400	400	400	400	400	400	400	400

(2) Figures 5, 6 and 7 show that the buy-back price b_H^{**} and b_L^{**} , the collection price r_{HH}^{**} and r_{LL}^{**} , and the collection quantity Q_{HH}^{**} and Q_{LL}^{**} increase with reward-penalty intensity k . The proof is as follows: Taking the first derivative of Eq. (39) obtains $\frac{db_H^{**}}{dk} = \frac{\Delta + 2a}{2\Delta + 2a} > 0$. Similarly, taking the first derivative of Eq. (40) obtains $\frac{db_L^{**}}{dk} = \frac{\Delta + 2a}{2\Delta + 2a} > 0$. From Eqs. (45) and (46), we know the collection price is the positive correlation of the buy-back price and the collection quantity is the positive function of the collection price according to Assumption (vi). Obviously, the collection price, the collection quantity and buy-back price have the same trends as reward-penalty intensity changes. Comparing Table 2 with Table 1, we find $b_H^{**} > b_H^*$, $b_L^{**} > b_L^*$, $r_{HH}^{**} > r_{HH}^*$, $r_{LL}^{**} > r_{LL}^*$, $Q_{HH}^{**} > Q_{HH}^*$ and $Q_{LL}^{**} > Q_{LL}^*$ which indicate that the buy-back price, the collection price and the collection quantity with RPM are higher than that without RPM. The phenomenon shows the manufacturer enhances the buy-back price while the retailer increases the collection price, which improves the collection quantity. Thus, the RPM can lead the collection of WEEE effectively.

Figure 3. The wholesale price w_H^{**} and w_L^{**} vs. k and v

Figure 4. The retail price p_{HH}^{**} and p_{LL}^{**} vs. k and v

(3) Figure 8 shows the franchise fee T_{rH}^{**} and T_{rL}^{**} increase with reward-penalty intensity.

Since $\partial T_{rL}^{**} / \partial k = [(b_L^{**} + e_L)(\Delta + 2a) + \Delta(\phi - w_L^{**} + a/2)] / (4\Delta + 4a) > 0$ and $\partial T_{rH}^{**} / \partial k = [(b_H^{**} + e_L)(\Delta + 2a) + \Delta(\phi - w_H^{**} + a/2)] / (4\Delta + 4a) > 0$, the franchise fee T_{rH}^{**} and T_{rL}^{**} increase with k . Comparing Table 2 with Table 1, we find $T_{rL}^{**} > T_{rL}^*$, $T_{rH}^{**} > T_{rH}^*$ which indicate the manufacturer charges more franchise fee under the RPM than that without RPM. In the context of RPM, the manufacturer not only provides more buy-back price but also charges more franchise fee to maximize his profit.

(4) Figure 9 demonstrates the manufacturer's expected profit $E(\pi_{mH}^{**})$ and $E(\pi_{mL}^{**})$ decrease firstly and then increase with reward-penalty intensity. Figure10 indicates the expected profit of the retailer $E(\pi_{rHH}^{**})$ increase with reward-penalty intensity while $E(\pi_{rLL}^{**})$ does not change with reward-penalty intensity.

In the context of RPM, the manufacturer's expected profit decreases because the manufacturer faces a penalty when the collection quantity is below the collection quantity target. However, the manufacturer's expected profit increases as the manufacturer gains rewards since the collection quantity surpasses the collection quantity target. Furthermore, the expected profit $E(\pi_{rHH}^{**})$ of the retailer exerting H level of collection effort increases with reward-penalty intensity for the higher buy-back price, however, the expected profit of the retailer exerting L level of collection effort only acquires the conserved profit with the screening contract. Comparing Table 2 with Table 1, we find $E(\pi_{mH}^{**}) < E(\pi_{mH}^*)$, $E(\pi_{mL}^{**}) < E(\pi_{mL}^*)$, $E(\pi_{rHH}^{**}) > E(\pi_{rHH}^*)$, and $E(\pi_{rLL}^{**}) = E(\pi_{rLL}^*)$. We can conclude with the comparison that the manufacturer is sensitive to the RPM, whose profit is influenced heavily. If the reward-penalty intensity is high enough, his profit is more than that without RPM. The expected profit of H level retailer with RPM increases, whose profit is more than that without RPM; the expected profit of L level retailer remains acquiring the conserved profit since her collection effort level is lower.

Figure 5. The buyback price b_H^{**} and b_L^{**} vs. k and v

Figure 6. The recovery price r_{HH}^{**} and r_{LL}^{**} vs. k and v

Figure 7. The recovery volume Q_{HH}^{**} and Q_{LL}^{**} vs. k and v

Figure 8. The franchise fee T_{rH}^{**} and T_{rL}^{**} vs. k and v

Figure 9. The expected profit of manufacturer $E(\pi_{mH}^{**})$ and $E(\pi_{mL}^{**})$ vs. k and v

Figure 10. The expected profit of retailer $E(\pi_{rHH}^{**})$ and $E(\pi_{rLL}^{**})$ vs. k and v

7. Conclusions

This paper explores the information screening contract in the asymmetric information CLSC with or without RPM, where a manufacturer delegates the WEEE collection to a retailer. The retailer can choose to be H-type retailer or L-type retailer while the collection effort level as its own private information unobservable to the manufacturer. We develop the information screening contract in the asymmetric information CLSC with or without RPM. The contract parameters and optimal decision-making results are obtained by solving the models and the main finding are as

follows:

(1) The information screening contract can enable the manufacturer to obtain the retailer's collection effort level. With the information screening contract, the H-type retailer can only choose the contract G_H and the L-type retailer can only choose the contract G_L . With the information screening contract, the manufacturer would offer lower wholesale price and higher buy-back price for the H-type retailer while charge higher franchise fee to the H-type retailer. Under the contract G_H , the H-type retailer can collect the waste products and sell products at a lower price.

(2) The RPM can reduce the wholesale price and retail price. Under the RPM, the wholesale price and retail price are lower than those without RPM. The RPM can increase the collection quantity effectively. Under the RPM, the manufacturer would offer higher buy-back price and the retailer would also higher collection price to increase the collection quantity.

(3) The franchise fee in the case with RPM is higher than that without RPM. The buy-back price, collection price, franchise fee and collection quantity increase with reward-penalty intensity while the wholesale price and the retail price decrease with reward-penalty intensity. The expected profit of the H-type retailer increases with reward-penalty intensity, while the L-type retailer's profit remains constant with reward-penalty intensity. The expected profit of the manufacturer decreases before increasing with reward-penalty intensity and with higher reward-penalty intensity, it still will surpass the profit without RPM.

Our paper is among the first efforts to investigate the asymmetric information closed-loop supply chain under the government intervention by developing RPM. There are several possible extensions for the future research. First, in this paper, we assume that the retailer has private information. An important aspect is to look at other CLSC members having private information as was the case with our main focus (retailer). Secondly, we can further perform an analysis by investigating retailer-led CLSC model. Finally, we can explore other objective functions and incorporating decision variables for further research directions.

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Appendix

A1. Proof of Case 1

Proof.

Assume $z_H = e_H + r_{HH} - \phi + p_{HH}$, $z_L = e_L + r_{LL} - \phi + p_{LL}$, then we have

$$\begin{aligned} E(\pi_{mH}) &= \int_0^{z_H} (\phi - P_{HH} + x)(\omega_H - c_r) f(x) dx \\ &\quad + \int_{z_H}^a (\phi - P_{HH} + x - e_H - r_{HH})(\omega_H - c_n) f(x) dx \\ &\quad + \int_0^a (e_H + r_{HH})(\omega_H - c_r) f(x) dx - b_H(e_H + r_{HH}) + T_{rH} \end{aligned} \quad (1)$$

$$\begin{aligned} E(\pi_{mL}) &= \int_0^{z_L} (\phi - P_{LL} + x)(\omega_L - c_r) f(x) dx \\ &\quad + \int_{z_L}^a (\phi - P_{LL} + x - e_L - r_{LL})(\omega_L - c_n) f(x) dx \\ &\quad + \int_0^a (e_L + r_{LL})(\omega_L - c_r) f(x) dx - b_L(e_L + r_{LL}) + T_{rL} \end{aligned} \quad (2)$$

$$\begin{aligned} E(\pi_{rHH}) &= (b_H - r_{HH})(e_H + r_{HH}) - e_H^2 / 4 \\ &\quad + (P_{HH} - \omega_H) \int_0^a (\phi - P_{HH} + x) f(x) dx - T_{rH} \end{aligned} \quad (3)$$

$$\begin{aligned} E(\pi_{rHL}) &= (b_L - r_{HL})(e_H + r_{HL}) - e_H^2 / 4 \\ &\quad + (P_{HL} - \omega_L) \int_0^a (\phi - P_{HL} + x) f(x) dx - T_{rL} \end{aligned} \quad (4)$$

$$\begin{aligned} E(\pi_{rLL}) &= (b_L - r_{LL})(e_L + r_L) - e_L^2 / 4 \\ &\quad + (P_{LL} - \omega_L) \int_0^a (\phi - P_{LL} + x) f(x) dx - T_{rL} \end{aligned} \quad (5)$$

$$\begin{aligned} E(\pi_{rLH}) &= (b_H - r_{LH})(e_L + r_{LH}) - e_L^2 / 4 \\ &\quad + (P_{LH} - \omega_H) \int_0^a (\phi - P_{LH} + x) f(x) dx - T_{rH} \end{aligned} \quad (6)$$

$$\max E(\pi_m) = \nu E(\pi_{mH}) + (1 - \nu) E(\pi_{mL}) \quad (7)$$

$$(P_{HH}^*, r_{HH}^*) = \arg \max E(\pi_{rHH}) \quad (8)$$

$$(P_{LL}^*, r_{LL}^*) = \arg \max E(\pi_{rLL}) \quad (9)$$

$$E(\pi_{rHH}) \geq E(\pi_{rHL}) \quad (10)$$

$$E(\pi_{rLL}) \geq E(\pi_{rLH}) \quad (11)$$

$$E(\pi_{rHH}) \geq \pi_r^0 \quad (12)$$

$$E(\pi_{rLL}) \geq \pi_r^0 \quad (13)$$

In accordance with Eqs (8) and (9), we define

$$\begin{cases} \partial E(\pi_{rHH}) / \partial P_{HH} = 0 \\ \partial E(\pi_{rHH}) / \partial r_{HH} = 0 \\ \partial E(\pi_{rLL}) / \partial P_{LL} = 0 \\ \partial E(\pi_{rLL}) / \partial r_{LL} = 0 \end{cases},$$

thus, we have $P_{HH} = \frac{\phi + \omega_H + a/2}{2}$, $r_{HH} = \frac{b_H - e_H}{2}$, $P_{LL} = \frac{\phi + \omega_L + a/2}{2}$ and $r_{LL} = \frac{b_L - e_L}{2}$

Similarly, based on Eqs (4) and (6), we define

$$\begin{cases} \partial E(\pi_{rHL}) / \partial P_{HL} = 0 \\ \partial E(\pi_{rHL}) / \partial r_{HL} = 0 \\ \partial E(\pi_{rLH}) / \partial P_{LH} = 0 \\ \partial E(\pi_{rLH}) / \partial r_{LH} = 0 \end{cases},$$

so we get $P_{HL} = \frac{\phi + \omega_H + a/2}{2}$, $r_{HL} = \frac{b_L - e_H}{2}$, $P_{LH} = \frac{\phi + \omega_L + a/2}{2}$ and $r_{LH} = \frac{b_H - e_L}{2}$.

Taking P_{HH} , P_{LL} , r_{HH} , r_{LL} , P_{HL} , P_{LH} , r_{HL} , r_{LH} , into $E(\pi_{mH})$, $E(\pi_{mL})$, $E(\pi_{rHH})$, $E(\pi_{rLL})$, $E(\pi_{rHL})$ and $E(\pi_{rLH})$, we obtain

$$\begin{aligned} E(\pi_{mH}) &= \int_0^{\bar{z}_H} \left(\frac{\phi}{2} - \frac{1}{4}a - \frac{\omega_H}{2} + x \right) (\omega_H - c_r) f(x) dx \\ &\quad + \int_{\bar{z}_H}^a \left(\frac{\phi}{2} - \frac{1}{4}a - \frac{\omega_H}{2} + x - \frac{e_H + b_H}{2} \right) (\omega_H - c_n) f(x) dx \\ &\quad + \int_{\bar{z}_H}^a \left(\frac{e_H + b_H}{2} \right) (\omega_H - c_r) f(x) dx - b_H \left(\frac{e_H + b_H}{2} \right) + T_{rH} \end{aligned}$$

$$\begin{aligned} E(\pi_{mL}) &= \int_0^{\bar{z}_L} \left(\frac{\phi}{2} - \frac{1}{4}a - \frac{\omega_L}{2} + x \right) (\omega_L - c_r) f(x) dx \\ &\quad + \int_{\bar{z}_L}^a \left(\frac{\phi}{2} - \frac{1}{4}a - \frac{\omega_L}{2} + x - \frac{e_L + b_L}{2} \right) (\omega_L - c_n) f(x) dx \\ &\quad + \int_{\bar{z}_L}^a \left(\frac{e_L + b_L}{2} \right) (\omega_L - c_r) f(x) dx - b_L \left(\frac{e_L + b_L}{2} \right) + T_{rL} \end{aligned}$$

$$E(\pi_{rHH}) = \left(\frac{e_H + b_H}{2} \right) \left(\frac{e_H + b_H}{2} \right) - \frac{e_H^2}{4} + \left(\frac{\phi}{2} + \frac{1}{4}a - \frac{\omega_H}{2} \right) \int_0^a \left(\frac{\phi}{2} - \frac{1}{4}a - \frac{\omega_H}{2} + x \right) f(x) dx - T_{rH}$$

$$E(\pi_{rLL}) = \left(\frac{e_L + b_L}{2} \right) \left(\frac{e_L + b_L}{2} \right) - \frac{e_L^2}{4} + \left(\frac{\phi}{2} + \frac{1}{4}a - \frac{\omega_L}{2} \right) \int_0^a \left(\frac{\phi}{2} - \frac{1}{4}a - \frac{\omega_L}{2} + x \right) f(x) dx - T_{rL}$$

$$\begin{aligned} E(\pi_{rHL}) &= \left(\frac{e_H + b_L}{2} \right) \left(\frac{e_H + b_L}{2} \right) - \frac{e_H^2}{4} \\ &\quad + \left(\frac{\phi}{2} + \frac{1}{4}a - \frac{\omega_L}{2} \right) \int_0^a \left(\frac{\phi}{2} - \frac{1}{4}a - \frac{\omega_L}{2} + x \right) f(x) dx - T_{rL} \end{aligned}$$

$$\begin{aligned} E(\pi_{rLH}) &= \left(\frac{e_L + b_H}{2} \right) \left(\frac{e_L + b_H}{2} \right) - \frac{e_L^2}{4} \\ &\quad + \left(\frac{\phi}{2} + \frac{1}{4}a - \frac{\omega_H}{2} \right) \int_0^a \left(\frac{\phi}{2} - \frac{1}{4}a - \frac{\omega_H}{2} + x \right) f(x) dx - T_{rH} \end{aligned}$$

The original objective function and constraints can be converted as follows:

$$\max E(\pi_m) = \nu E(\pi_{mH}) + (1 - \nu) E(\pi_{mL}) \quad (14)$$

$$E(\pi_{rHH}) \geq E(\pi_{rHL}) \quad (15)$$

$$E(\pi_{rLL}) \geq E(\pi_{rLH}) \quad (16)$$

$$E(\pi_{rHH}) \geq \pi_r^0 \quad (17)$$

$$E(\pi_{rLL}) \geq \pi_r^0 \quad (18)$$

From Eq (14), we find $E(\pi_{rHH}) \geq E(\pi_{rLL}) + \frac{b_L}{2}(e_H - e_L) > E(\pi_{rLL}) \geq \pi_r^0$, thus Eq (17)

can be omitted.

Defining λ, μ, γ are correspond to the Lagrange multiplier of the constraints, we have

$$\frac{\partial L}{\partial b_H} = \frac{1}{2}v \left[\int_{z_H}^a (c_n - c_r) f(x) dx - e_H - 2b_H \right] + \lambda \left(\frac{e_H + b_H}{2} \right) - \mu \left(\frac{e_H + b_L}{2} \right) = 0$$

$$\frac{\partial L}{\partial b_L} = \frac{1}{2}(1-v) \left[\int_{z_L}^a (c_n - c_r) f(x) dx - e_L - 2b_L \right] + (\gamma + \mu - \lambda) \left(\frac{e_L + b_L}{2} \right) = 0$$

$$\begin{aligned} \frac{\partial L}{\partial \omega_H} = & -\frac{1}{2}v \left[\int_0^{z_H} \left(\varphi - c_r - \frac{1}{2}a + 2x \right) f(x) dx - \int_{z_H}^a \left(\varphi - c_n - \frac{1}{2}a + 2x \right) f(x) dx \right] \\ & - \frac{1}{2}(\lambda + \mu) \left[(\varphi - \omega_H) \int_0^a f(x) dx + \int_0^a x f(x) dx \right] = 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial \omega_L} = & -\frac{1}{2}(1-v) \left[\int_0^{z_L} \left(2\omega_L - c_r - \varphi + \frac{1}{2}a \right) f(x) dx + \int_{z_L}^a \left(2\omega_L - c_n - \varphi + \frac{1}{2}a \right) f(x) dx + 2 \int_{z_L}^a x f(x) dx \right] \\ & + \frac{1}{2}(\lambda - \mu - \gamma) \int_0^a (\varphi - \omega_L + x) f(x) dx = 0 \end{aligned}$$

$$\frac{\partial L}{\partial T_{rH}} = v - \lambda + \mu = 0$$

$$\frac{\partial L}{\partial T_{rL}} = 1 - v + \lambda - \mu - \gamma = 0$$

From $\frac{\partial L}{\partial T_{rH}} = 0$ and $\frac{\partial L}{\partial T_{rL}} = 0$, we get $\lambda = v + \mu \geq 0$, $\gamma = 1 - v \geq 0$ and have $\mu = 0$ further.

(Disproof: we assume $\mu > 0$, the corresponding constraint (16) must get equal interms of K-T. Similarly, $\lambda = v + \mu \geq 0$ indicates the constraint (15) is proper. Combining constraints (15) and (16), we have $E(\pi_{rHH}) = E(\pi_{rHL})$ and $E(\pi_{rLL}) = E(\pi_{rLH})$ and know that assumption $\mu > 0$ is not reasonable, thus, we get $\mu = 0$.)

When $\gamma > 0$, $E(\pi_{rLL}) = \pi_r^0$. When $\lambda > 0$, $E(\pi_{rHH}) = E(\pi_{rHL})$, so

$$E(\pi_{rHH}) = \pi_r^0 + \frac{b_L}{2}(e_H - e_L);$$

From $\frac{\partial L}{\partial \omega_H} = 0$, we have $\omega_H^* = \frac{4\Delta(\phi - e_H) - 3\Delta^2 + (\Delta + 2a)(c_r + 3c_n)}{8(\Delta + a)}$,

From $\frac{\partial L}{\partial \omega_L} = 0$, we get

$$\omega_L^* = \frac{4\Delta(\phi - e_H) - 3\Delta^2 + (\Delta + 2a)(c_r + 3c_n)}{8(\Delta + a)} - \frac{\Delta(e_L - v e_H)}{2(1-v)(\Delta + a)};$$

From $\frac{\partial L}{\partial b_H} = 0$, we have $b_H^* = \frac{\Delta(\phi - e_H - \omega_H^* + 3a/2)}{\Delta + 2a}$;

$$\text{From } \frac{\partial L}{\partial b_L} = 0, \text{ we have } b_L^* = \frac{\Delta(\phi - \omega_L^* + 3a/2)}{\Delta + 2a} - \frac{[\Delta(1-\nu) - 2a\nu]e_L + 2ave_H}{(1-\nu)(\Delta + 2a)}$$

We already know $\omega_H^*, b_H^*, E(\pi_{rHH}) = \pi_r^0 + \frac{b_L}{2}(e_H - e_L), \omega_L^*, b_L^*$, thus, we get

$$T_{rH}^* = \frac{(b_H^* + e_H)^2 + (\phi - \omega_H^* + a/2)^2 - (b_L^* + e_H)^2 - (\phi - \omega_L^* + a/2)^2}{4} + T_{rL}^*$$

$$\text{Similarly, we have } \omega_L^*, b_L^* \notin E(\pi_{rLL}) = \pi_r^0, \text{ so } T_{rL}^* = \frac{(b_L^* + e_L)^2 - e_L^2 + (\phi - \omega_L^* + a/2)^2}{4} - \pi_r^0$$

For further, we obtain

$$P_{HH}^* = \frac{\phi + \omega_H^* + a/2}{2}, P_{LL}^* = \frac{\phi + \omega_L^* + a/2}{2}, r_{HH}^* = \frac{b_H^* - e_H}{2} \text{ and } r_{LL}^* = \frac{b_L^* - e_L}{2}.$$

A2. Proof of Case 2

Proof.

We assume $z_H = e_H + r_{HH} - \phi + P_{HH}, z_L = e_L + r_{LL} - \phi + P_{LL}$, then

$$\begin{aligned} E(\pi_{mH}) &= \int_0^{z_H} (\phi - P_{HH} + x)(\omega_H - c_r)f(x)dx + \int_{z_H}^a (\phi - P_{HH} + x - e_H - r_{HH})(\omega_H - c_n)f(x)dx \\ &\quad + \int_0^a (e_H + r_{HH})(\omega_H - c_r)f(x)dx - b_H(e_H + r_{HH}) + T_{rH} + k(Q_{HH} - Q_0) \end{aligned} \quad (19)$$

$$\begin{aligned} E(\pi_{mL}) &= \int_0^{z_L} (\phi - P_{LL} + x)(\omega_L - c_r)f(x)dx + \int_{z_L}^a (\phi - P_{LL} + x - e_L - r_{LL})(\omega_L - c_n)f(x)dx \\ &\quad + \int_0^a (e_L + r_{LL})(\omega_L - c_r)f(x)dx - b_L(e_L + r_{LL}) + T_{rL} + k(Q_{LL} - Q_0) \end{aligned} \quad (20)$$

$$E(\pi_{rHH}) = (b_H - r_{HH})(e_H + r_{HH}) - e_H^2/4 + (P_{HH} - \omega_H) \int_0^a (\phi - P_{HH} + x)f(x)dx - T_{rH} \quad (21)$$

$$E(\pi_{rHL}) = (b_L - r_{HL})(e_H + r_{HL}) - e_H^2/4 + (P_{HL} - \omega_L) \int_0^a (\phi - P_{HL} + x)f(x)dx - T_{rL} \quad (22)$$

$$E(\pi_{rLL}) = (b_L - r_{LL})(e_L + r_{LL}) - e_L^2/4 + (P_{LL} - \omega_L) \int_0^a (\phi - P_{LL} + x)f(x)dx - T_{rL} \quad (23)$$

$$E(\pi_{rLH}) = (b_H - r_{LH})(e_L + r_{LH}) - e_L^2/4 + (P_{LH} - \omega_H) \int_0^a (\phi - P_{LH} + x)f(x)dx - T_{rH} \quad (24)$$

$$\max E(\pi_m) = \nu E(\pi_{mH}) + (1-\nu)E(\pi_{mL}) \quad (25)$$

$$(P_{HH}^{**}, r_{HH}^{**}) = \arg \max E(\pi_{rHH}) \quad (26)$$

$$(P_{LL}^{**}, r_{LL}^{**}) = \arg \max E(\pi_{rLL}) \quad (27)$$

$$E(\pi_{rHH}) \geq E(\pi_{rHL}) \quad (28)$$

$$E(\pi_{rLL}) \geq E(\pi_{rLH}) \quad (29)$$

$$E(\pi_{rHH}) \geq \pi_r^0 \quad (30)$$

$$E(\pi_{rLL}) \geq \pi_r^0 \quad (31)$$

For Eqs (26) and (27), let $\partial E(\pi_{rHH})/\partial P_{HH} = 0$, $\partial E(\pi_{rHH})/\partial r_{HH} = 0$,

$\partial E(\pi_{rLL})/\partial P_{LL} = 0$ and $\partial E(\pi_{rLL})/\partial r_{LL} = 0$, so we have

$$P_{HH} = \frac{\phi + \omega_H + a/2}{2}, r_{HH} = \frac{b_H - e_H}{2}, P_{LL} = \frac{\phi + \omega_L + a/2}{2}, r_{LL} = \frac{b_L - e_L}{2}.$$

For Eqs 22) and (24), let $\partial E(\pi_{rHL})/\partial P_{HL} = 0$, $\partial E(\pi_{rHL})/\partial r_{HL} = 0$,

$\partial E(\pi_{rLH})/\partial P_{LH} = 0$ and $\partial E(\pi_{rLH})/\partial r_{LH} = 0$, we get

$$P_{HL} = \frac{\phi + \omega_H + a/2}{2}, r_{HL} = \frac{b_L - e_H}{2}, P_{LH} = \frac{\phi + \omega_L + a/2}{2}, r_{LH} = \frac{b_H - e_L}{2}.$$

Substituting $P_{HH}, P_{LL}, r_{HH}, r_{LL}, P_{HL}, P_{LH}, r_{HL}$ and r_{LH} into $E(\pi_{mH}), E(\pi_{mL}), E(\pi_{rHH}),$

$E(\pi_{rLL}), E(\pi_{rHL})$ and $E(\pi_{rLH})$, we have

$$\begin{aligned} E(\pi_{mH}) &= \int_0^{z_H} \left(\frac{\phi}{2} - \frac{1}{4}a - \frac{\omega_H}{2} + x \right) (\omega_H - c_r) f(x) dx + \int_{z_H}^a \left(\frac{\phi}{2} - \frac{1}{4}a - \frac{\omega_H}{2} + x \right) (\omega_H - c_n) f(x) dx \\ &\quad - \int_{z_H}^a \left(\frac{e_H + b_H}{2} \right) (c_n - c_r) f(x) dx - b_H \left(\frac{e_H + b_H}{2} \right) + T_{rH} + k \left(\frac{e_H + b_H}{2} - Q_0 \right) \\ E(\pi_{mL}) &= \int_0^{z_L} \left(\frac{\phi}{2} - \frac{1}{4}a - \frac{\omega_L}{2} + x \right) (\omega_L - c_r) f(x) dx + \int_{z_L}^a \left(\frac{\phi}{2} - \frac{1}{4}a - \frac{\omega_L}{2} + x \right) (\omega_L - c_n) f(x) dx \\ &\quad - \int_{z_L}^a \left(\frac{e_L + b_L}{2} \right) (c_n - c_r) f(x) dx - b_L \left(\frac{e_L + b_L}{2} \right) + T_{rL} + k \left(\frac{e_L + b_L}{2} - Q_0 \right) \\ E(\pi_{rHH}) &= \left(\frac{e_H + b_H}{2} \right) \left(\frac{e_H + b_H}{2} \right) - \frac{e_H^2}{4} + \left(\frac{\phi}{2} + \frac{1}{4}a - \frac{\omega_H}{2} \right) \int_0^a \left(\frac{\phi}{2} - \frac{1}{4}a - \frac{\omega_H}{2} + x \right) f(x) dx - T_{rH} \\ E(\pi_{rLL}) &= \left(\frac{e_L + b_L}{2} \right) \left(\frac{e_L + b_L}{2} \right) - \frac{e_L^2}{4} + \left(\frac{\phi}{2} + \frac{1}{4}a - \frac{\omega_L}{2} \right) \int_0^a \left(\frac{\phi}{2} - \frac{1}{4}a - \frac{\omega_L}{2} + x \right) f(x) dx - T_{rL} \\ E(\pi_{rHL}) &= \left(\frac{e_H + b_L}{2} \right) \left(\frac{e_H + b_L}{2} \right) - \frac{e_H^2}{4} + \left(\frac{\phi}{2} + \frac{1}{4}a - \frac{\omega_L}{2} \right) \int_0^a \left(\frac{\phi}{2} - \frac{1}{4}a - \frac{\omega_L}{2} + x \right) f(x) dx - T_{rL} \\ E(\pi_{rLH}) &= \left(\frac{e_L + b_H}{2} \right) \left(\frac{e_L + b_H}{2} \right) - \frac{e_L^2}{4} + \left(\frac{\phi}{2} + \frac{1}{4}a - \frac{\omega_H}{2} \right) \int_0^a \left(\frac{\phi}{2} - \frac{1}{4}a - \frac{\omega_H}{2} + x \right) f(x) dx - T_{rH} \end{aligned}$$

The original objective function and constraints can be converted as follows:

$$\max E(\pi_m) = \nu E(\pi_{mH}) + (1 - \nu) E(\pi_{mL}) \quad (32)$$

$$E(\pi_{rHH}) \geq E(\pi_{rHL}) \quad (33)$$

$$E(\pi_{rLL}) \geq E(\pi_{rLH}) \quad (34)$$

$$E(\pi_{rHH}) \geq \pi_r^0 \quad (35)$$

$$E(\pi_{rLL}) \geq \pi_r^0 \quad (36)$$

From Eqs (33) and (36), we have

$$\mathbb{E}(\pi_{rHH}) \geq \mathbb{E}(\pi_{rLL}) + \frac{b_L}{2}(e_H - e_L) > \mathbb{E}(\pi_{rLL}) \geq \pi_r^0,$$

which can lead to ignoring Eq (35).

Defining λ, μ, γ are correspond to the Lagrange multiplier of the constraints, we have

$$\begin{aligned} \frac{\partial L}{\partial b_H} &= \frac{1}{2}v \left[\int_{z_H}^a (c_n - c_r)f(x)dx - e_H - 2b_H \right] + \lambda \left(\frac{e_H + b_H}{2} \right) - \mu \left(\frac{e_H + b_L}{2} \right) + \frac{k}{2} = 0 \\ \frac{\partial L}{\partial b_L} &= \frac{1}{2}(1-v) \left[\int_{z_L}^a (c_n - c_r)f(x)dx - e_L - 2b_L \right] + (\mu + \gamma - \lambda) \left(\frac{e_L + b_L}{2} \right) + \frac{k}{2} = 0 \\ \frac{\partial L}{\partial \omega_H} &= \frac{1}{2}v \left[\int_0^{z_H} \left(\varphi - \frac{1}{2}a - 2\omega + 2x + c_r \right) f(x)dx + \int_{z_H}^a \left(\varphi - \frac{1}{2}a - 2\omega + 2x + c_n \right) f(x)dx \right] \\ &\quad + \frac{1}{2}(\mu - \lambda) \int_0^a (\varphi - \omega_H + x)f(x)dx = 0 \\ \frac{\partial L}{\partial \omega_L} &= \frac{1}{2}(1-v) \left[\int_0^{z_L} \left(\varphi - \frac{1}{2}a - 2\omega_H + 2x + c_r \right) f(x)dx + \int_{z_L}^a \left(\varphi - \frac{1}{2}a - 2\omega_H + 2x + c_n \right) f(x)dx \right] \\ &\quad + \frac{1}{2}(\lambda - \mu - \gamma) \int_0^a (\varphi - \omega_L + x)f(x)dx = 0 \\ \frac{\partial L}{\partial T_{rH}} &= v - \lambda + \mu = 0 \\ \frac{\partial L}{\partial T_{rL}} &= 1 - v + \lambda - \mu - \gamma = 0 \\ \text{From } \frac{\partial L}{\partial T_{rH}} = 0 \text{ and } \frac{\partial L}{\partial T_{rL}} = 0, \text{ we have } \lambda = v + \mu \geq 0 \text{ \& } \gamma = 1 - v \geq 0 \end{aligned}$$

For further, we have $\mu=0$ (Disproof: assume $\mu > 0$, the corresponding constraint (34) must get equal in terms of K-T. Similarly, from $\lambda = v + \mu \geq 0$, the constraint (2) gets equal. Combining constraints (33) and (34), we have $\mathbb{E}(\pi_{rHH}) = \mathbb{E}(\pi_{rHL}) \neq \mathbb{E}(\pi_{rLL}) = \mathbb{E}(\pi_{rLH})$, the assumption that $\mu > 0$ is not proper, thus, we have $\mu=0$)

$$\text{When } \gamma > 0, \mathbb{E}(\pi_{rLL}) = \pi_r^0; \text{ When } \lambda > 0, \mathbb{E}(\pi_{rHH}) = \pi_r^0 + \frac{b_L}{2}(e_H - e_L).$$

$$\text{From } \frac{\partial L}{\partial b_H} = 0, \text{ we have } b_H = \frac{\Delta(\phi - e_H - \omega_H + 3a/2) + 2ak}{\Delta + 2a};$$

$$\text{From } \frac{\partial L}{\partial b_L} = 0, \text{ we get } b_L = \frac{\Delta(\phi - \omega_L + 3a/2) + 2ak}{\Delta + 2a} - \frac{[\Delta(1-v) - 2av]e_L + 2av e_H}{(1-v)(\Delta + 2a)}$$

$$\text{From } b_H, \frac{\partial L}{\partial \omega_H} = 0, \text{ we obtain}$$

$$w_H^{**} = \frac{4\Delta(\phi - e_H - k) - 3\Delta^2 + (\Delta + 2a)(c_r + 3c_n)}{8(\Delta + a)};$$

From $b_L, \frac{\partial L}{\partial \omega_L}=0$, we have

$$w_L^{**} = \frac{4\Delta(\phi - k) - 3\Delta^2 + (\Delta + 2a)(c_r + 3c_n)}{8(\Delta + a)} - \frac{\Delta(e_L - \nu e_H)}{2(1 - \nu)(\Delta + a)};$$

Combining above results, we get

$$b_L^{**} = \frac{\Delta(\phi - \omega_L^{**} + 3a/2) + 2ak}{\Delta + 2a} - \frac{[\Delta(1 - \nu) - 2a\nu]e_L + 2a\nu e_H}{(1 - \nu)(\Delta + 2a)}$$

From $\omega_L^{**}, b_L^{**}, E(\pi_{rLL}) = \pi_r^0$, we have $T_{rL}^{**} = \frac{(b_L^{**} + e_L)^2 - e_L^2 + (\phi - \omega_L^{**} + a/2)^2}{4} - \pi_r^0$.

From $\omega_H^{**}, b_H^{**}, E(\pi_{rHH}) = \pi_r^0 + \frac{b_L}{2}(e_H - e_L), \omega_L^{**}, b_L^{**}, T_{rL}^{**}$, we obtain

$$T_{rH}^{**} = \frac{(b_H^{**} + e_H)^2 + (\phi - \omega_H^{**} + a/2)^2 - (b_L^{**} + e_H)^2 - (\phi - \omega_L^{**} + a/2)^2}{4} + T_{rL}^{**}$$

Furthermore, we get $p_{HH}^{**} = \frac{\phi + \omega_H^{**} + a/2}{2}, p_{LL}^{**} = \frac{\phi + \omega_L^{**} + a/2}{2}, r_{HH}^{**} = \frac{b_H^{**} - e_H}{2}$ and $r_{LL}^{**} = \frac{b_L^{**} - e_L}{2}$.