MV benchmark estimation based on high-frequency test signal

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Abstract

This work is concerned with the minimum variance (MV) benchmark estimation without a prior knowledge of time-delay. The idea is to use a small high-frequency test signal in order to obtain the accurate time-delay estimation (TDE) result, then the MV benchmark can be calculated from the closed-loop data. Based on the analysis, a well designed test signal for control performance monitoring (CPM) is not only possible but also necessary, as it increases the identifiability of the closed-loop data without having a large effect on routine operation. A new TDE algorithm by using high-order auto regressive exogenous (ARX) model is proposed as well as the multivariable issue is investigated. Simulation examples of performance monitoring with low signal-to-noise ratio (SNR) are employed to demonstrate the effectiveness of the proposed method.

Keywords: minimum variance benchmark, time-delay estimation, control performance monitoring, optimal test signal, high-order ARX

1. Introduction

Control performance monitoring (CPM) aims at assessing the performance of control loops, which has become one of the key components in process control. A successful CPM would help directly or indirectly in product quality

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maintenance, operation safety, material and energy consumption, and thus the economic performance. This area has got considerable attention from both the academic and industrial communities in the past decades. Some relevant references can be found in, for example, [1, 2, 3, 4].

The fundamental research of CPM started by Harris in 1989; see [5]. This research developed a performance assessment technique using routine operating data for univariate control loops. Assuming that reducing the output variance is the control object, the minimum variance control is naturally used as a standard to compare with the current output variance. This index is called the minimum variance (MV) benchmark or Harris index since then. Afterwards, a number of researches have been developed based on this idea. For example, [6] used the squared correlation coefficient to estimate the Harris index, along with some of the statistical properties of the estimator; [7] developed an efficient, stable filtering and correlation (FCOR) method to estimate the MV benchmark; a statespace framework for MV control and assessment was proposed by [8, 9]; the assessment of internal model control (IMC) structure was developed by [1]; [10, 7] proposed algorithms for the multivariable control performance monitoring. The comprehensive analysis of the MV benchmark could be seen in [2]. Some other extensions for different control situations were also considered in many literatures such as feedforward and feedback (FF/FB) controller performance assessment [11, 12], non-minimum-phase systems [13], cascade controls [14], and time-variant processes [15].

For applying the MV benchmark in univariate control loops, one essential information is the time-delay. Most of the MV based monitoring methods assume that the time-delay is known a priori. However, in many industrial cases, time-delay is unknown or varying after a period of operation. Therefore, time-delay usually needs to be estimated from the closed-loop data at first.

It is a well known fact that the time-delay estimation (TDE) is a part of the system identification problem. A general method for determining the time-delay is typically a trial-and-error procedure for the prediction-error methods. One basic idea is to try correlation analysis and/or test reasonable delay values

in a medium size auto regressive exogenous (ARX) model. Choose delay that provides the best model fit based on prediction errors or other fit criterion. Comprehensive surveys and comparative (simulation) studies of TDE problem can be seen in [16, 17]. It is important to realize that time-delay cannot be estimated from routine operating data without external excitations or abrupt changes in the control signals [4]. This fact has been well proven in the theory of system identification literature; see [18].

For the control performance monitoring, there are generally two steps for applying MV benchmark: 1, estimate the time-delay; 2, calculate the MV benchmark based on the time-delay. Within these two steps, it is not fair to say it can be done based on routine operating data without excitation. Actually, it has been shown in [2] that high-frequency random test signal or white noise may have advantages on estimating time-delay and thus can be used in MV benchmark calculation.

The test signal in CPM increases the identifiability of the closed-loop data, which considers not only possible in real industrial application but also necessary. The magnitude of the test signal for CPM purpose should be small such that it has a weak effect on the process output relative to the existing process disturbances. This property is quite different from the test signal which is used in system identification perspective. For identification purpose, test signal tends to fully excite the process in the entire frequency range in order to get a reliable result in both dynamic and static. On the contrary, CPM usually requires a part of process model such as time-delay in MV benchmark. In this case, the test signal for CPM usually concentrates on a narrower frequency range and therefore reduces the effect of extra disturbance. This idea was applied in the model error detection problem for model prediction control (MPC) [19], where three frequencies sinusoid signal was used to estimate the true frequency response and compare the estimations with the MPC model. Examples in [19] also showed that the diagnosis test can be weak such that the test signal almost hides in the unmeasured disturbance.

In this paper, a test-signal-based method for estimating the MV bench-

mark in multivariable control loops is proposed. The optimal test signal for the TDE problem is given. Based on the non-disturbing high-frequency experiment, properties of high-order ARX model are used to determine the time-delay and followed by the MV benchmark estimation and its multivariable solution.

The remainder of this paper is organized as follows. The problem formulation and motivation are in Section 2. Optimal test signal for TDE problem is presented in Section 3. Section 4 discusses the time-delay determination based on the high-order ARX model and a group of hypothesis tests. Then the MV benchmark calculation is given in Section 5 and 6 with multivariable solutions. Examples and comparative studies are given in Section 7. The paper ends with conclusions in Section 8.

2. Problem Formulation and Motivation

Assuming a SISO general linear system with time-delay T_d is denoted as

$$y(t) = \tilde{G}(q)u(t - T_d) + v(t) \quad (1)$$

$$v(t) = H(q)e(t) \quad (2)$$

where $y(t)$, $u(t)$, $v(t)$ and $e(t)$ are output, input, unmeasured disturbance and white noise sequence at sampling time $t = 1, \dots, N$, respectively. The discrete transfer functions with the backshift operator q^{-1} are denoted as

$$\tilde{G}(q) = \frac{B(q)}{A(q)} = \frac{b_1 + b_2q^{-1} + \dots + b_nq^{-n+1}}{1 + a_1q^{-1} + \dots + a_nq^{-n}}, \quad (3)$$

$$H(q) = \frac{C(q)}{D(q)} = \frac{1 + c_1q^{-1} + \dots + c_nq^{-n}}{1 + d_1q^{-1} + \dots + d_nq^{-n}}. \quad (4)$$

where $\tilde{G}(q)$ is known as the delay-free model. The process model is

$$G(q, T_d) = \tilde{G}(q) \cdot q^{-T_d}. \quad (5)$$

Considering the process $G(q, T_d)$ is in feedback control shown in Fig. 1, where $C(q)$ is the controller transfer function. The test signal for estimating time-delay and MV benchmark is denoted as $u_{opt}(t)$, which will introduce specifically in

the following sections. For the routine feedback control ($u_{opt}(t) = 0$), Fig. 1 follows

$$y(t) = \frac{H(q)}{1 + G(q)C(q)} e(t). \quad (6)$$

Figure 1: Closed-loop system structure

If T_d is known, one is able to model the closed-loop output data $y(t)$ by a moving average (MA) model such as

$$y(t) = \underbrace{f_1 e(t) + f_2 e(t-1) + \cdots + f_{T_d} e(t-T_d+1)}_{y_{mv}} + f_{T_d+1} e(-T_d) + f_{T_d+2} e(-T_d-1) + \cdots \quad (7)$$

where y_{mv} is the portion representing the MV control output, independent of feedback control [2]. Although the MV cannot be achieved in most circumstances because of the physical constraints, it still offers a reference that tells us the distance between the current control performance and the best.

The MV benchmark can be calculated based on the minimum variance σ_{mv}^2 and the current output variance σ_y^2

$$\eta = \frac{\sigma_{mv}^2}{\sigma_y^2}. \quad (8)$$

The MV benchmark requires the prior knowledge of time-delay; however, as discussed in the introduction, the time-delay may not be available for most cases and the TDE algorithm usually needs to be applied first.

One way to look at the TDE problem is from the frequency domain perspective, the frequency response of $\tilde{G}(q)$ is

$$\begin{aligned}\tilde{G}(e^{i\omega}) &= \frac{b_1 + b_2 e^{-i\omega} + \dots + b_n e^{-(n-1) \cdot i\omega}}{1 + a_1 e^{-i\omega} + \dots + a_n e^{-n \cdot i\omega}} \\ &= |\tilde{G}(e^{i\omega})| e^{i\angle \tilde{G}(e^{i\omega})}.\end{aligned}$$

The frequency response of the process model is

$$G(e^{i\omega}, T_d) = |\tilde{G}(e^{i\omega})| e^{i(\angle \tilde{G}(e^{i\omega}) - T_d \cdot \omega)}. \quad (9)$$

We can see that the changing of time-delay has nothing to do with the amplitude in frequency domain. It only affects the phase of $G(e^{i\omega}, T_d)$. More specifically, if we investigate the changing in time-delay ΔT_d , the difference in phase can be presented by

$$\angle G(e^{i\omega}, T_d + \Delta T_d) - \angle G(e^{i\omega}, T_d) = -\omega \cdot \Delta T_d. \quad (10)$$

We can see from Eq. (10) that the frequency ω has a large effect on the difference of phase. If $\omega = 0$, the difference is zero. However, with the increase of ω , the difference becomes larger with the rate $-\omega$. When $\omega = \pi$, the difference of phase reaches the largest value $-\pi \cdot \Delta T_d$. In this point of view, it may have some advantages for us to determine the time-delay with high-frequency test signal.

3. The Optimal Test Signal for TDE Problem

For the sake of estimating time-delay from input-output data efficiently, the optimal test signal needs to be investigated. Considering the frequency response of the process model shown in Eq. (9), one takes the first order derivative with respect to T_d and yields the filter $F(q)$ in frequency domain as

$$F(e^{i\omega}) = \left[\frac{\partial G(e^{i\omega}, T_d)}{\partial T_d} \right]^{\frac{1}{2}} \quad (11)$$

$$= [-i\omega \cdot G(e^{i\omega}, T_d)]^{\frac{1}{2}} \quad (12)$$

where T_d is an integer while applying the derivative in Eq. (11). The amplitude values are

$$|F(e^{i\omega})| = \sqrt{\omega} \cdot |\tilde{G}(e^{i\omega})|^{\frac{1}{2}}. \quad (13)$$

We can see that the filter $F(q)$ is generally a high-pass filter. In low-frequency zone, we have $\omega \rightarrow 0$, $|F(e^{i\omega})| \rightarrow 0$; but in high-frequency area when $\omega \rightarrow \pi$, $|F(e^{i\omega})| \rightarrow \sqrt{\pi \cdot \tilde{G}(-1)}$. An example of $|F(e^{i\omega})|$ can be seen in Fig. 2.

Figure 2: The amplitudes of the optimal filter for delay-free model $\tilde{G}(q) = \frac{1+0.5q^{-1}}{1-1.5q^{-1}+0.7q^{-2}}$

According to [20, 18], the optimal test signal $u_{opt}(t)$ for enhancing the TDE result could be generated by filtering a white noise sequence $\epsilon(t)$

$$u_{opt}(t) = \mu \cdot F(q^{-1})\epsilon(t) \quad (14)$$

where μ is a constant which is used to adjust the excitation such that the amplitude or the energy is limited.

By using an arbitrary input signal $u(t)$, the estimation of time-delay $\hat{T}_d$ has the approximated variance [18]:

$$var\hat{T}_d \approx C_1 \left[\int_{-\pi}^{\pi} \frac{\partial G(e^{i\omega}, T_d)}{\partial T_d} \cdot \frac{\partial G(e^{-i\omega}, T_d)}{\partial T_d} \Phi_u(\omega) d\omega + C_2 \right]^{-1} \quad (15)$$

where C_1 and C_2 are constants and $\Phi_u(\omega)$ is the spectrum of the input. This expression shows the mechanism of how the input spectrum influences the TDE; see [20] for detail discussions. The optimal test signal $u_{opt}(t)$ generated from Eq. (14) can obviously reduce the variance of the estimation. Several remarks for the input signal design are given next.

Remark 1 In order to obtain a satisfaction TDE result, the test signal should excite the process in high frequency area. Based on the analysis in Eqs. (10), (15) and Fig. 2, it is not necessary to focus on low frequency. This fact is quite different from the regular system identification in which more efforts are put in for estimating the process gain. In practice, one may not be able to obtain the optimal input signal shown in Eq. (14) as the true delay-free system should be known for doing that. In this case, a general high-pass filter can be used for the replacement of $F(q)$. Alternatively, white noise is another choice, although it may waste some energy in low frequency. In Section 7.1, the simulation example shows the accuracy of estimations based on different types of test signal.

Remark 2 The TDE experiment is for the purpose of control performance monitoring, which indicates that the process is operating in closed-loop condition. The high-frequency test signal could be added into the system through the manipulated variable as the location of $u_{opt}(t)$ shown in Fig. 1. One should keep in mind that the added test signal should not disturb the normal operating significantly. In other words, the signal-noise-ratio (SNR) is limited. As previous research [19] shows that we recommend to apply a non-disturb test as the influence of test signal can be covered by noise. Based on the experience and simulations in [19], $\text{SNR} < 1$ seems a good choice. Notice that in the general identification test, the range of SNR is usually between 3 to 5, which is considerably larger than the test for CPM purpose.

Remark 3 In closed-loop MV benchmark estimation, the time-delay may be estimated based on the closed-loop system $G_{cl}(q) = \frac{G(q)}{1+G(q)C(q)}$; see Sec-

tions 5 and 6 for detail discussion. The corresponding optimal test signal need to be changed as well. Similar as the open-loop situation, one can take the first order derivative for $G_{cl}(q)$ with respected to T_d which results in the frequency response of the filter:

$$F_{cl}(e^{i\omega}) = \frac{[-i\omega \cdot G(e^{i\omega}, T_d)]^{\frac{1}{2}}}{1 + G(e^{i\omega}, T_d)C(e^{i\omega})} \quad (16)$$

From Eq. (16) we can see that the filter also shows the high-pass property. However, in order to achieve the optimal, one may need to know both process model and sensitivity function, which is a demanding condition in real industrial application. In this case, a general high-pass filter is recommended as an approximation.

4. The Time-delay Estimation Algorithm

In this section, we introduce the TDE method by using a high-order ARX model. The high-order ARX model was first introduced in [21] for multivariable system identification, which involves solving the experiment design, model structure selection, parameter estimation, and model validation.

For the TDE problem, a high-order ARX model could be used to detect the length of zeros for the first set of estimated parameters. In detail, the model can be presented as

$$A_N^h(q)y(t) = B_N^h(q)u(t) + e(t) \quad (17)$$

where the model order h and the total sampling points N satisfy

$$h \rightarrow \infty, \text{ and } h^2/N \rightarrow 0 \text{ as } N \rightarrow \infty. \quad (18)$$

$B_N^h(q)$ and $A_N^h(q)$ are polynomials, which are denoted as

$$B_N^h(q) = \underbrace{b_1 + b_2q^{-1} + \dots + b_{T_d}q^{-T_d+1}}_{T_d \text{ terms}} + \underbrace{b_{T_d+1}q^{-T_d} + \dots + b_hq^{-h+1}}_{h-T_d \text{ terms}} \quad (19)$$

$$A_N^h(q) = 1 + \underbrace{a_1q^{-1} + \dots + a_hq^{-h}}_{h \text{ terms}} \quad (20)$$

where the first T_d terms in $\hat{B}_N^h(q)$ are considered as delay coefficients and the rests are non-delay coefficients. Based on the asymptotic theory [18], the high-order ARX model is consistent in frequency perspective:

$$\hat{G}_N^h(e^{i\omega}) = \frac{\hat{B}_N^h(e^{i\omega})}{\hat{A}_N^h(e^{i\omega})} \rightarrow G(e^{i\omega}). \quad (21)$$

One can reform the ARX structure by

$$y(t) = \varphi(t)\theta + e(t) \quad (22)$$

where the parameters are denoted by

$$\theta = \begin{bmatrix} b_1 & \cdots & b_h & a_1 & \cdots & a_h \end{bmatrix}^T \quad (23)$$

and the corresponding data matrix is

$$\varphi(t) = \begin{bmatrix} u(t) & \cdots & u(t-h+1) & -y(t-1) & \cdots & -y(t-h) \end{bmatrix}. \quad (24)$$

The delay and non-delay parameters can also be separated, i.e.

$$\theta_D = \begin{bmatrix} b_1 & \cdots & b_{T_d} \end{bmatrix}^T \quad (25)$$

$$\tilde{\theta} = \begin{bmatrix} b_{T_d+1} & \cdots & b_h & a_1 & \cdots & a_h \end{bmatrix}^T. \quad (26)$$

It is obvious that the true time-delay parameters are zeros:

$$b_1 = \cdots = b_{T_d} = 0 \quad (27)$$

and we have the vector form for true parameters

$$\theta = \begin{bmatrix} 0^T & \tilde{\theta}^T \end{bmatrix}^T. \quad (28)$$

Eq. (22) can be solved based on minimizing the lost function

$$V(\theta) = \frac{1}{2} \sum_{t=1}^N [y(t) - \varphi(t)\theta]^2. \quad (29)$$

Then the parameter estimation result can be obtained by using the least square method

$$\hat{\theta} = \left[\sum_{t=1}^N \varphi^T(t)\varphi(t) \right]^{-1} \left[\sum_{t=1}^N \varphi^T(t)y(t) \right]. \quad (30)$$

Furthermore, to analyse this result, we bring in Eq. (22) and the true parameters shown in Eq. (28), which results in

$$\begin{bmatrix} \hat{\theta}_D \\ \hat{\tilde{\theta}} \end{bmatrix} = \left[\sum_{t=1}^N \varphi^T(t) \varphi(t) \right]^{-1} \left[\sum_{t=1}^N \varphi^T(t) \begin{bmatrix} 0 \\ \tilde{\theta} \end{bmatrix} + e(t) \right] \quad (31)$$

$$= \begin{bmatrix} 0 \\ \tilde{\theta} \end{bmatrix} + \left[\sum_{t=1}^N \varphi^T(t) \varphi(t) \right]^{-1} \left[\sum_{t=1}^N \varphi^T(t) e(t) \right]. \quad (32)$$

Denote that

$$Z = \left[\sum_{t=1}^N \varphi^T(t) \varphi(t) \right]^{-1} \left[\sum_{t=1}^N \varphi^T(t) e(t) \right]. \quad (33)$$

Once the persistently excitation has been applied as shown in Eq. (14), we obtain that the expectation of Z is zero, i.e.

$$E(Z) = 0 \quad (34)$$

and the covariance matrix is

$$\Sigma = \text{cov}(Z) = \lambda^2 \left[\sum_{t=1}^N \varphi^T(t) \varphi(t) \right]^{-1} \quad (35)$$

where the unbiased estimation of λ^2 can be calculated by

$$s^2 = 2V(\hat{\theta})/(N - 2h). \quad (36)$$

In Eq. (32), the time-delay parameters are the first T_d entries of $\hat{\theta}$, which can be presented by

$$\hat{\theta}_D = \hat{\theta}_{1:T_d} = Z_{1:T_d}. \quad (37)$$

where the subscript $1 : T_d$ indicates the entries of vector. The corresponding covariance matrix for $\hat{\theta}_D$:

$$\Sigma_D = \text{cov}(\hat{\theta}_D) = \Sigma_{1:T_d, 1:T_d}. \quad (38)$$

If we further assume that the white noise sequence $e(t)$ is Gaussian distributed with zero mean and variance λ^2 , then the estimated delay parameters are Gaussian distributed as well [22]:

$$\hat{\theta}_D \sim \mathcal{N}(\mathbf{0}, \Sigma_D). \quad (39)$$

Eq. (39) tells us the distribution of $\hat{\theta}_D$. Since Σ_D is the covariance matrix of the joint distribution of the vector $\hat{\theta}_D$, we have useful information about the covariance and correlation between the different components of $\hat{\theta}_D$. Based on the definition of χ^2 distribution, considering the true value that $\theta_D = \mathbf{0}$, we are able to find a statistic:

$$z_d = \hat{\theta}_D^T \Sigma_D^{-1} \hat{\theta}_D \quad (40)$$

which has a χ^2 distribution with $\dim \hat{\theta}_D = d$ degrees of freedom,

$$z_d \sim \chi^2(d). \quad (41)$$

The probability that

$$z_d = \hat{\theta}_D^T \Sigma_D^{-1} \hat{\theta}_D \geq \alpha_d \quad (42)$$

is thus $\chi_{\alpha_d}^2(d)$, the α_d significance level of the $\chi^2(d)$ distribution. The detail analysis can also be seen in [18].

The time-delay can be determined by using the parameter confidence level. We can form $\hat{\theta}_D$ in different dimension and then go through a group of hypothesis tests as following

$$H_0 : \hat{\theta}_D = \mathbf{0}, \quad H_1 : \hat{\theta}_D \neq \mathbf{0} \quad (43)$$

The dimension d can go through 2 (we assume the process should have at least one time-delay) to h . When $d = h$, hypothesis H_1 must be held. In this case, the TDE problem has been addressed to find the first d such that z_d exceeds its confidence level α_d . Based on the simulation, 99% confidence level appears to be a good choose.

The hypothesis tests from $d = 2$ to h are to determine the first estimated parameter of $\hat{B}_N^h(q)$ that changes from zero to nonzero. For example, if the process has 10 samples of time-delay, then the parameters $\hat{\theta}_D$ with dimension $d = 2$ to 10 should hold true for the hypothesis H_0 , i.e. $z_d < \alpha_d$. When $d = 11$ to h , the parameters $\hat{\theta}_D$ contains nonzero values, and the hypothesis H_1 must hold true, i.e. $z_d > \alpha_d$.

The threshold at the 99% confidence level is determined by χ^2 distribution, which has fixed values corresponding to its freedom d . The statistic z_d is calculated based on the TDE test. We find that if the excitation test runs in an appropriate way, the point that z_d passes through α_d should be clear. And from that breaking point on, the z_d increases fast and goes far beyond α_d .

The hypothesis on the ending point $z_{d=h}$ should have H_1 to hold true. $z_{d=h}$ is generated by assuming all the parameters in $\hat{B}_N^h(q)$ are zeros. The excitation level significantly influences the value of $z_{d=h}$, because a better excitation usually leads to the larger difference between $\hat{B}_N^h(q)$ and $\mathbf{0}$. Thus, the ending point $z_{d=h}$ directly indicates the excitation level for TDE. With this property, one is able to judge whether the excitation is sufficient and how reliable the estimation can be. One can refer to Section 7.1 which discusses this property in detail with a simulation.

In this way, there are several advantages of the new approach of TDE method. First, as the asymptotic theory has been proved in [18], the TDE method works in closed-loop tests with open-loop tests as a special case. Second, this approach requires neither specified parameters nor initial values, which makes it easier to apply in industry. Third, since the analytical solution of ARX model can always be obtained as shown in Eq. (30), the computational burden can be significantly reduced comparing to other methods such as numerical searching based, also there is no risk of local minimum problem. Finally, one is able to determine whether the TDE result is reliable through the curve of hypothesis tests.

5. MV Benchmark Estimation

With the TDE result, one can easily apply the MV benchmark by using the test-free data. This requires some extra routine data collection before or after the experiment. An alternative approach is provided in this section in order to estimate time-delay as well as the MV benchmark by using the closed-loop experiment data. The closed-loop test can be operated as shown in Fig. 1. Take

the SISO system as an example, with the experiment data we have

$$y(t) = \frac{G(q)}{1 + G(q)C(q)} u_{opt}(t) + \frac{H(q)}{1 + G(q)C(q)} e(t). \quad (44)$$

As it has been proved in [2] that the delay structure in closed-loop system is the same as the open-loop one, the time-delay can be estimated based on the closed-loop high-order ARX model from the data set

$$\varphi_{cl}(t) = \begin{bmatrix} u_{opt}(t) & \cdots & u_{opt}(t-h+1) & -y(t-1) & \cdots & -y(t-h) \end{bmatrix} \quad (45)$$

which yields

$$y(t) = \frac{\hat{B}_{cl}^h(q)}{\hat{A}_{cl}^h(q)} u_{opt}(t) + \frac{1}{\hat{A}_{cl}^h(q)} e(t) \quad (46)$$

where $\hat{B}_{cl}^h(q)$ and $\hat{A}_{cl}^h(q)$ are high-order polynomials for closed-loop system. Based on the asymptotic theory we have

$$\hat{G}_{cl}^h(q) = \frac{\hat{B}_{cl}^h(q)}{\hat{A}_{cl}^h(q)} \rightarrow \frac{G(q)}{1 + G(q)C(q)}. \quad (47)$$

The estimated parameter vector for closed-loop ARX model is denoted as

$$\hat{\theta}_{cl} = \begin{bmatrix} \hat{b}_1^{cl} & \cdots & \hat{b}_h^{cl} & \hat{a}_1^{cl} & \cdots & \hat{a}_h^{cl} \end{bmatrix}^T \quad (48)$$

and the covariance matrix of parameters is

$$\Sigma_{cl} = \lambda^2 \left[\sum_{t=1}^N \varphi_{cl}^T(t) \varphi_{cl}(t) \right]^{-1}. \quad (49)$$

The same methodology proposed in Section 4 can be used again. The first set elements in $\hat{\theta}_{cl}$ that close to zeros are determined as time-delay through the hypothesis tests.

When $\hat{T}_d$ has been determined, one is able to calculate the test-free data by using

$$v(t) = y(t) - \frac{\hat{B}_{cl}^h(q)}{\hat{A}_{cl}^h(q)} u_{opt}(t). \quad (50)$$

Based on the procedures of MV benchmark estimation introduced in [23], the test-free data need to fit into a time series model (ARMA). The estimated

ARMA model can be presented by the following transfer function

$$\frac{v(t)}{e(t)} = \frac{1 + \hat{c}_1 q^{-1} + \dots + \hat{c}_\gamma q^{-\gamma}}{1 + \hat{a}_1 q^{-1} + \dots + \hat{a}_p q^{-p}}. \quad (51)$$

Use the long division of Eq. (51) and then separate the delay terms from delay-free terms

$$\frac{v(t)}{e(t)} = \underbrace{\hat{f}_1 + \hat{f}_2 q^{-1} + \dots + \hat{f}_{\hat{T}_d} q^{-\hat{T}_d+1}}_{\hat{T}_d \text{ terms}} + \hat{f}_{\hat{T}_d+1} q^{-\hat{T}_d} + \dots. \quad (52)$$

Based on the MV benchmark definition, we have

$$\hat{\sigma}_{mv}^2 = [\hat{f}_1^2 + \hat{f}_2^2 + \dots + \hat{f}_{\hat{T}_d}^2] \hat{\sigma}_e^2 \quad (53)$$

where $\hat{\sigma}_e^2$ can be estimated as the variance of residuals after fitting Eq. (51).

Then the MV benchmark can be calculated by

$$\hat{\eta} = \frac{\hat{\sigma}_{mv}^2}{\hat{\sigma}_v^2}. \quad (54)$$

Note that the true time-delay and the theoretical MV are irrelevant to controller tunings. But the estimation results may be sensitive to different controller parameters. This fact usually comes from the excitation that is generated from open-loop is not optimal for its closed-loop system. When we collected $u_{opt}(t)$, $u(t)$ and $y(t)$ from the closed-loop system, the TDE algorithm can be applied for data pair $\varphi_{cl}(t) = \{y(t), u_{opt}(t)\}$ or $\varphi(t) = \{y(t), u(t)\}$. If we choose to use $\varphi_{cl}(t)$, then we are dealing the entire closed-loop system like an open-loop one, which may benefit to the test signal design. The proposed TDE algorithm is also available for using data $\varphi(t)$, where similar results can be achieved. For both estimations, one can always check the statistic curve and the value of $z_{d=h}$ to know whether the excitation is sufficient. If the estimation result is not reliable, one can improve the excitation by increasing the amplitude, waiting for a longer time, or changing the $u_{opt}(t)$ character.

In summary, the MV benchmark estimation for univariate process is as follows:

- a) Generate a high-frequency test signal $u_{opt}(t)$ for TDE problem based on Eq. (14) or (16). If the delay-free model is not available, replace $F(q)$ by a general high-pass filter.
- b) Apply the experiment as shown in Fig. 1 and collect the process data.
- c) Estimate a high-order ARX model with zero delay based on data set $y(t)$ and $u_{opt}(t)$ as shown in Eq. (46).
- d) Extract the first d dimension subset of $\hat{\theta}$. Calculate the statistic value $z_d = \hat{\theta}_D^T \Sigma_D^{-1} \hat{\theta}_D$ and its 99% confidence level α_d . Go through the dimension from $d = 2$ to h . The first d that makes $z_d > \alpha_d$ afterwards should be determined as the first non-delay value.
- e) Obtain the test-free output data by using Eq. (50) and fit them into an ARMA model.
- f) Apply the long division and calculate the MV benchmark via Eqs. (53) and (54).

6. MV Benchmark for Multivariable Process

In the multivariable MV benchmark estimation, the interactor matrix is usually required [2]. The interactor matrix is denoted as $\mathbf{D}(q)$, which is the generalization of the SISO time-delay for the MIMO case. The interactor matrix has the properties

$$|\mathbf{D}(q)| = q^r \quad (55)$$

$$\lim_{q^{-1} \rightarrow 0} \mathbf{D}(q)\mathbf{G}(q) = \mathbf{K} \quad (56)$$

where $\mathbf{K}$ is a full rank constant matrix, and the integer r is defined as the number of infinite zeros of $\mathbf{G}(q)$.

Two special forms of the interactor matrix are *diagonal interactor matrix* i.e. $\mathbf{D}(q) = \text{diag}(q^{d_1}, q^{d_2}, \dots)$ and *general interactor matrix*. It has been shown that the diagonal interactor matrix is relatively easy to obtain based on the TDE method. In many cases, multiloop control design is based on decoupling and pairing, where a complex control problem is divided into simpler ones based

on some principle, for instance, the input-output pairing could be such that the diagonal elements have smaller delays. This leads to the observation that the occurrence of a diagonal interactor matrix is not rare. For the general interactor matrix, first few impulse response coefficient matrices are required.

The same methodology can be extended to multivariable MV benchmark estimation straightforwardly. The high-order multivariable ARX model can be estimated from closed-loop data in which the independent high frequency test signal is added to excite each input of process. The high-order multivariable ARX model for closed-loop system is denoted as

$$\hat{\mathbf{A}}_{cl}(q)\mathbf{y}(t) = \hat{\mathbf{B}}_{cl}(q)\mathbf{u}_{opt}(t) + \mathbf{e}(t) \quad (57)$$

where the number of outputs and inputs are p and m respectively; $\hat{\mathbf{A}}_{cl}(q)$ is a diagonal $p \times p$ polynomial matrix and $\hat{\mathbf{B}}_{cl}(q)$ is the matrix with a dimension $p \times m$. In this form, the MIMO closed-loop system is decoupled into p MISO models for simplicity. After the estimation, we have the similar results

$$\hat{\mathbf{G}}_{cl}(q) = \hat{\mathbf{A}}_{cl}^{-1}(q)\hat{\mathbf{B}}_{cl}(q) \rightarrow [\mathbf{I} + \mathbf{G}(q)\mathbf{C}(q)]^{-1}\mathbf{G}(q) \quad (58)$$

$$\hat{\mathbf{A}}_{cl}^{-1}(q) \rightarrow [\mathbf{I} + \mathbf{G}(q)\mathbf{C}(q)]^{-1}\mathbf{H}(q). \quad (59)$$

and the test-free output data can be estimated by

$$\mathbf{v}(t) = \mathbf{y}(t) - \hat{\mathbf{A}}_{cl}^{-1}(q)\hat{\mathbf{B}}_{cl}(q)\mathbf{u}_{opt}(t). \quad (60)$$

It is a well known fact that the delay structure or the interaction matrix is feedback-control invariant [24]. This implies that although the Markov parameters of the open-loop and closed-loop transfer function matrix are different, their linear combination yields the same interactor matrix. Exploiting this result, the interactor matrix of an open-loop transfer function can be estimated from the closed-loop system.

It is also recommended to use the data set $\boldsymbol{\varphi}_{cl}(t) = \{\mathbf{y}(t), \mathbf{u}_{opt}(t)\}$ for the multivariable closed-loop model estimation as some other identification methods usually use the data set $\boldsymbol{\varphi}(t) = \{\mathbf{y}(t), \mathbf{u}(t)\}$ to get an open-loop model. Regarding the interactor matrix estimation, the open and closed-loop system

yield the same results as discussed before. It is easier to obtain the closed-loop model from $\varphi_{cl}(t)$ with the diagonal ARX structure when correlations between different outputs exist.

The TDE method for MIMO system can be likewise achieved by detecting the first zero of each entry of $\hat{\mathbf{B}}_{cl}(q)$ through hypothesis tests based on χ^2 distribution. The estimated time-delay matrix is denoted as $\hat{\mathbf{T}}_d$, in which each entry represents the direct input-to-output time-delay. If the interactor matrix cannot be presented as a diagonal form, the long division can be performed for the ARX model excluding the time-delay in order to obtain first few impulse response coefficient matrices:

$$\hat{\mathbf{A}}_{cl}^{-1}(q)\hat{\mathbf{B}}_{cl}(q) = \hat{\mathbf{F}}_{\hat{\mathbf{T}}_d+1}^{cl}q^{-\hat{\mathbf{T}}_d-1} + \hat{\mathbf{F}}_{\hat{\mathbf{T}}_d+2}^{cl}q^{-\hat{\mathbf{T}}_d-2} + \dots \quad (61)$$

The interactor matrix $\mathbf{D}(q)$ and MV benchmark can then be calculated by following the procedure given in [2]. For simplicity, one can apply a practical solution proposed in [25] to calculate bounds on the MV with the order of interactor matrix.

7. Simulation Examples

In this section, four simulation examples are employed to verify the proposed approach. The first simulation focuses on the different excitations for TDE problem. The second simulation compares the new algorithm with the traditional one. Then the MV benchmark estimation in closed-loop systems is presented in the third and fourth examples.

7.1. Time-delay Estimation based on different test signals

Consider a simulated open-loop test with the following process

$$G(q) = \frac{3 - 2.05q^{-1} - 2.45q^{-2} + 1.37q^{-3} + 0.28q^{-4}}{1 - 3.36q^{-1} + 4.64q^{-2} - 3.26q^{-3} + 1.16q^{-4} - 0.17q^{-5}}q^{-12} \quad (62)$$

where the time-delay is 12 samples.

Three different test signals for estimating time-delay are compared in this example. They are high-frequency signal (Fig. 3.a), white noise (Fig. 4.a), and

low-frequency signal (Fig. 5.a). The unmeasured disturbance $v(t)$ is generated by filtering a white noise using the following low-pass filter

$$H(q) = \frac{1}{1 - 0.9q^{-1}}. \quad (63)$$

The variance of noise is adjusted to 9 times larger than the noise-free output, i.e. the SNR is $1/9 = 0.11$, which means the unmeasured disturbance is roughly 3 times larger than the noise-free output on average. As in a regular system identification test, the SNR value is usually greater than 3, which indicates that the CPM experiment is more than 27 times weaker than the identification test in this example.

Figure 3: High-frequency test signal and its TDE result

- (a) High-frequency test signal is generated by filtering a white noise through filter $1 - 0.9q^{-1}$
- (b) The TDE result is 12

Three simulations are run in the same condition and 1,000 sampling points for each simulation are collected. Three high order ARX models are estimated, followed by applying the procedure d) in Section 5. Figs. 3.b, 4.b, and 5.b show the TDE results, where the x-axis is the different dimension of $\hat{\theta}_D$ which extracts from the polynomial $\hat{B}_N^{30}(q)$ ($\hat{b}_1, \hat{b}_2, \dots, \hat{b}_{30}$). In these figures, the blue triangle curve is the statistic z_d and the red circle curve is the 99%-level confidence intervals. For the first several points, z_d are beneath the α_d , and we confirm that $\hat{\theta}_D = \mathbf{0}$. Once z_d goes above α_d from a specific point, we can confirm the

Figure 4: White noise test signal and its TDE result

(a) White noise test signal

(b) The TDE result is 13, as the true time-delay is 12

Figure 5: Low-frequency test signal and its TDE result

(a) Low-frequency test signal is generated by filtering a white noise through filter $\frac{1}{1-0.9q^{-1}}$

(b) The TDE result is 15, as the true time-delay is 12

time-delay is found.

With three test signals, we obtain different results. By using the high frequency test signal, the proposed TDE algorithm has the correct result of 12 samples as the time-delay. However, by using the white noise and the low frequency test signal, the estimation results are 13 and 15, respectively.

Actually, the statistic results shown in Figs. 3.b, 4.b, and 5.b tell us more than the delay estimation values. In the low-frequency signal test (in Fig. 5.b), we observe that z_d and α_d get closer to each other when comparing to Fig. 3.b and Fig. 4.b. The z_d barely cross α_d at 16 and it does not rise up significantly afterwards. This phenomenon indicates the excitation in Fig. 5.a for TDE is relatively weak and therefore it gives us confusing result. Take the ending point $d = 30$ for example, the z_d values are 192, 122 and 91, corresponding to high-frequency signal test, white noise test and low-frequency signal, respectively. As we have discussed before, the higher value at the ending point indicates the better excitation and more reliable result. From this point of view, it is clear that the high-frequency test signal results in the best TDE performance, which is consistent with our discussion in Section 2 and 3.

7.2. Comparison of time-delay Estimation algorithm

In this section, we compare the proposed TDE algorithm with the MATLAB command `delayest`. The simulation is in open-loop and the system model is the same as the previous one, where the process model and disturbance model are shown in Eqs. (62) and (63). The test signal is white noise. The SNR is adjusted to 1. Since the command `delayest` requires to assign the order of the delay-free model, we use different orders from 1 to 15 for estimation. In each assigned order, we run `delayest` for 1000 times and count the number that achieves the correct time-delay and then compare them with the proposed TDE algorithm.

The result can be seen in Fig. 6. We can see from the figure that `delayest` is very sensitive to the assignment of model order. For example, at the orders 3, 7, and 8, they achieve good performance as the rate of the correct estimations is around 998 times out of 1000. However, if we choose order 1, 4, and 10, the correct number reduces to 0, 749 and 494 out of 1000. In practice, it is difficult to determine the true order of the delay-free system, thus the `delayest` may give us confusing results. Whereas the proposed TDE algorithm does not need to assign the order of delay-free model, and the correct estimation number is

consistently at 998 times out of 1000.

Figure 6: 1000 times simulation result

7.3. SISO MV benchmark estimation in closed-loop system

The same process $G(q)$ in Eq. (62) is used in a feedback control loop. A simple PI controller is employed as

$$\Delta u(t) = K_p \Delta \varepsilon(t) + K_i \varepsilon(t) \quad (64)$$

where $K_p = 0.0017$, $K_i = 0.0034$, and $\varepsilon(t)$ is the control error between the setpoint and process variable. The closed-loop step response can be seen in Fig. 7.

The same high-frequency test signal as shown in Fig. 3.a is used to excite the process in closed-loop condition. One can adjust the magnitude of test signal such that its effect on experiment is minor. In this simulation, we have the following two conditions: 1), the experiment-free condition, which means the system operates in closed-loop with no test signal added i.e. $u_{opt}(t) = 0$ in Fig. 1; 2), the experiment condition, which means that the nonzero test signal u_{opt} is added into the closed-loop system through the process input. Fig. 8 shows the experiment output against the experiment-free one in the first 500 samples.

Figure 7: SISO system closed-loop step response

In the figure, these two curves are very close to each other as it is hardly to see the difference. The standard deviation of the test output only increases 7.92%, i.e.

$$\frac{\text{std}(y_{\text{test}}(t) - y_{\text{test-free}}(t))}{\text{std}(y_{\text{test-free}}(t))} \times 100\% = 7.92\%.$$

10,000 samples of closed-loop data are collected for estimating an ARX model with order $h = 30$. With the closed-loop test data, procedures c) and d) can then be applied. The TDE result can be seen in Fig. 9. It is clear to see that the first 12 coefficients in the polynomial $\hat{B}_{cl}(q)$ are within the 99% confidence threshold and thus $\hat{T}_d = 12$ can be determined. Also, we can see from the figure that the curve z_d rises up directly after $d = 12$ and then increases fast and appears to have a great difference from the threshold. The ending point is $z_{30} = 192$, which is a relatively large number when comparing to the previous open-loop examples. These facts show that the excitation for TDE is well performed and the result is reliable. Following the procedure f), the estimated MV benchmark is 5.01 as the theoretical MV is 4.92 in this example. Finally, the estimated MV

Figure 8: System output in closed-loop control. The excitation results in increasing the output *std* by 7.92%.

benchmark can be calculated as

$$\hat{\eta} = \frac{\hat{\sigma}_{mv}^2}{\hat{\sigma}_v^2} = \frac{5.01}{7.26} = 0.69. \quad (65)$$

7.4. MIMO MV benchmark estimation in closed-loop system

Consider a 2×2 minimum-phase multivariable process with the open-loop transfer function matrix $\mathbf{G}(q)$ and disturbance transfer function matrix $\mathbf{H}(q)$ [23]

$$\mathbf{G}(q) = \begin{bmatrix} \frac{1.2q^{-8}}{1-0.9q^{-1}} & \frac{0.5q^{-12}}{1-0.7q^{-1}} \\ \frac{2.3q^{-10}}{1-0.6q^{-1}} & \frac{0.5q^{-9}}{1-0.95q^{-1}} \end{bmatrix}$$

$$\mathbf{H}(q) = \begin{bmatrix} \frac{1}{1-0.95q^{-1}} & \frac{-q^{-1}}{1-0.71q^{-1}} \\ \frac{q^{-1}}{1-0.82q^{-1}} & \frac{1}{1-0.94q^{-1}} \end{bmatrix}$$

Figure 9: TDE result based on closed-loop data in SISO system

In this MIMO example, the interactor matrix is in the diagonal form and can be calculated as

$$\mathbf{D}(q) = \begin{bmatrix} q^8 & 0 \\ 0 & q^9 \end{bmatrix}$$

The white noise $\mathbf{e}(t)$ for generating the unmeasured disturbance is a two-dimensional normal-distributed white noise sequence with $\Sigma_e = \mathbf{I}$.

Consider that the following multiloop feedback controller is implemented on the process

$$\mathbf{C}(q) = \begin{bmatrix} \frac{0.0132-0.0093q^{-1}}{1-q^{-1}} & 0 \\ 0 & \frac{0.0983-0.092q^{-1}}{1-q^{-1}} \end{bmatrix}$$

and the closed-loop step response can be seen in Fig. 10.

Then the closed-loop transfer function from the white noise $\mathbf{e}(t)$ to the output $\mathbf{y}(t)$ can also be calculated as

$$\mathbf{y}(t) = [\mathbf{I} + \mathbf{G}(q)\mathbf{C}(q)]^{-1}\mathbf{H}(q)\mathbf{e}(t) \quad (66)$$

By following the procedures of the multivariable MV benchmark calculation,

Figure 10: MIMO system closed-loop step response

one can obtain the theoretical MV benchmark as

$$\boldsymbol{\sigma}_{mv} = \begin{bmatrix} 7.7421 & 1.4113 \\ 1.4113 & 8.6954 \end{bmatrix}$$

The high-frequency test signal is generated based on independent white noise sequences passing a linear filter:

$$\mathbf{u}_t(t) = \begin{bmatrix} 1 - 0.95q^{-1} & 0 \\ 0 & 1 - 0.95q^{-1} \end{bmatrix} \begin{bmatrix} \epsilon_1(t) \\ \epsilon_2(t) \end{bmatrix}$$

where the variance of $\epsilon_1(t)$ and $\epsilon_2(t)$ are 0.004 and 0.04. 10,000 samples of data are collected. The control outputs are shown in Fig. 11. The blue solid curve represents the experiment-free output and the red dashdotted one indicates the control output with excitations. The standard deviation increases 3.95% (output 1) and 4.80% (output 2) respectively. This is a small increase of output fluctuation. In practice, this level of excitation has no noticeable effect on the normal operation.

Fig. 12 shows the TDE results. Each subfigure corresponds to the hypothesis test result of $\hat{\mathbf{B}}_{ij}(q)$ polynomial. We can see that since a specific dimension of parameters, the statistic z_d sharply increases and passes through the 99% con-

Figure 11: System outputs in MIMO closed-loop control. The excitations result in increasing the *std* by 3.95% and 4.80%.

fidence threshold clearly. The ending points are relatively large which indicates a well performed excitation. The TDE result is

$$\hat{\mathbf{T}}_d = \begin{bmatrix} 8 & 12 \\ 10 & 9 \end{bmatrix}$$

Considering that the input-output pairing is based on the diagonal elements that have smallest delay, a diagonal interactor matrix can be obtained which is the same as the theoretical one. With the intermediate results, MV benchmark can be calculated by following procedures in [23] and results in

$$\hat{\boldsymbol{\sigma}}_{mv} = \begin{bmatrix} 7.4385 & 1.4282 \\ 1.4282 & 9.0130 \end{bmatrix}.$$

8. Conclusion

In this paper, a method for MV benchmark estimation without a prior knowledge of time-delay has been presented. Optimal test signal is introduced for

Figure 12: TDE results based on closed-loop data in MIMO system

CPM purpose, which on one hand minimizes the effect of disturbing the routine process operations and on the other hand increases the identifiability of closed-loop system. In our case, time-delay or interactor matrix are estimated based on high frequency test signal. The high-order ARX model estimation and parameter statistic test are employed. With these intermediate results, MV benchmark can be calculated based on standard procedure.

The proposed method is verified by simulations. The first example shows the advantage of using high-frequency test signal in TDE. In the second example, the proposed algorithm shows its consistency when comparing with the traditional TDE algorithm. In the closed-loop examples, the estimations of time-delays and MV benchmarks are obtained accurately with weak high-frequency test signals while the fluctuation of outputs increases approximately less than 10%, whereas it usually increases to 300% when applying a regular system identification test. The results demonstrate that the CPM can be significantly improved by introducing a well designed test signal.

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