Theoretical and experimental investigations of vibration waveforms excited by an electro-hydraulic type exciter for fatigue with a two-dimensional rotary valve

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Abstract

Conventional electro-hydraulic excitation is usually controlled by a servo valve employing a sliding spool type construction. However, the comparatively slow response of the servo valve will greatly limit the system's high-frequency performance. Therefore, a rotary valve with the rotary motion of the spool as a new excitation mechanism is proposed to obtain the desired excitation, especially a high-frequency excitation wave for fatigue. An electro-hydraulic exciter using a combination of a three-way two-dimensional rotary valve (2D rotary valve) and an unequal area piston is taken as an example. Analysis of the vibration output to a typical wave input yields an analytic solution of the vibration waveform excited by this electro-hydraulic system. The mathematical formulation of the harmonics is also derived. Additionally, an electro-hydraulic excitation test-bed is built to acquire an experimental excitation wave. Consequently, the analysis of the excitation waveform in an approximate analytical and experimental method is used to verify access to high-frequency excitation, even resonance excitation.

Key words

rotary valve; electro-hydraulic excitation waveform; high frequency; harmonics; analytical solution; hydraulic resonance

1. Introduction

Electro-hydraulic excitation created by hydraulic power focuses attention on stability, bandwidth and wave distortion [1-5]. Therefore, it is always necessary to analyse and improve the high-frequency properties of an electro-hydraulic excitation system. Because the electro-hydraulic servo valve as an important element has a direct influence on the performance and characteristics of the whole system, to obtain

satisfactory performance, a basic method is to develop a fast-response servo valve. For this reason, many corporations collaborate with universities or research institutes to design the structure of the control valve and its key components [6-8]. A hydraulically driven shaking table for shock absorbers has been developed by the Beijing Institute of Technology [9]. In this system, a type of D761 series servo valve of America Moog Co. is adopted to control the cylinder. This servo valve frequency at a -90° phase shift is larger than twice the natural frequency of the system, so it is also found in some vibrating test beds for waveform replication [10]. A special three-stage servo valve with

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all stages being of the spool type is produced by the Nanjing Engineering Research Center [11], and has a better dynamic response than a flapper valve used as the first stage in a two-stage electro-hydraulic servo valve. However, the bandwidth is just 200 Hz, resulting from the flow characteristics of the servo valve. It has been used by the Harbin Institute of Technology in a servo control system of a 6-DOF electro-hydraulic vibration table to simulate the vibration environment of flight space. MTS Co. has designed a two-stage servo valve with a voice coil pilot stage for use as a first-stage valve instead of a flapper-nozzle, while the second-stage valve is universally of the spool type [12]. This servo valve has the characteristics of a pressure feedback servo valve at higher frequencies. Therefore, it is used to control a high-frequency electro-hydraulic excitation system specifically for an elastomeric material test or crack growth test at 1000 Hz or above, which has been integrated at Michigan Technological University.

Electro-hydraulic excitation controlled by the linear motion of a spool is most common. This linear displacement of a spool is created by supplying a cyclically varied current to the electro-mechanical converter of the servo valve, which in turn varies the alternative flow rate into or out of the cylinder or motor, and subsequently, the piston or rotor is excited with respect to the desired vibrating wave form [13]. This results in the system performance largely depending on the comparatively slow response of the servo valve, so it is not possible in practice to obtain a high-frequency excitation wave or complicated excitation wave shapes. This factor limits the high performance improvement of the electro-hydraulic excitation type.

Compared to a servo valve with a multi-stage structure or an improved electrical-to-mechanical

transformer, the new structure of the other control valve is always developed instead of the servo valve in an electro-hydraulic excitation system. There have been some studies on the design feature and work performance of the rotary valve with the rotary motion of the spool [14-16]. A novel fluid-driven PWM on/off valve design is presented, which is based on a unidirectional rotary spool [17, 18]. Such a design reduces the valve actuation power from a cubic dependence on the PWM frequency to a square dependence by eliminating motion reversals during transition. In addition, a rotary valve has replaced the servo valve to be used in a special excitation mechanism, in which the hydraulic excitation source is caused by water hammer fluctuation. The vibration frequency of this system can be adjusted to 90 Hz [19], which is usually found in vibrating separation or compaction engineering applications characterized by large output forces. A similar scheme is also adopted by Southeast University and applied to marble block vibratory compaction [20]. However, this rotary valve is not only for improving the working frequency of the system, but also for supplying a sinusoidal pressure drop across the load by designing the valve orifice geometry.

Although the rotary valve also connects the electronic and hydraulic mechanical portions of a system that is similar to a servo valve, the rotary valve as a new excitation mechanism is more attractive in a high-frequency electro-hydraulic excitation system [21]. Therefore, a rotary valve-controlled electro-hydraulic exciter is designed and developed in this paper. A new type of two-dimensional rotary valve (called 2D valve [22, 23]) is redesigned as a power element of a three-way valve-piston combination in an electro-hydraulic exciter. The single spool of this 2D valve has both rotary and linear

motions. The two-dimensional motions are controlled independently which in turn produce cyclic flow or pressure to a hydraulic actuator to output a vibration. Because the rotary spool is lubricated with oil, it is very easy to achieve high vibrational frequencies by increasing the rotary speed of the spool.

In this paper, a 2D rotary valve-controlled high-frequency electro-hydraulic exciter is developed for the fatigue test of materials, and approximate analytical and experimental methods of the excitation waveform are proposed. It remains particularly significant for the theory, design, and application of a medium- or high-frequency electro-hydraulic excitation system, especially in the area of excitation type controlled by the rotary motion of the spool. The rest of the paper is organized as follows. Section 2 describes the system principle, including a load analysis, the basic equations and a math model. In section 3, the excited waveform to typical wave input is derived using an analytical method, and its characteristics are analysed. In section 4, a set of relational models for the orifice area, the geometry of the 2D rotary and the vibration wave form are established with mathematical formulations. The expression of the harmonics is also derived. Section 5 presents the experimental results of the 2D rotary valve controlled electro-hydraulic exciter. Lastly, conclusions are made to summarize the main work of this paper.

2. Hydraulic Power Element of the Electro-hydraulic Exciter

2.1 Valve Controlled Piston

The vibration waveform of the electro-hydraulic exciter displays more or less distortion because of hydraulic resonance, and the distortion becomes more

significant at some special frequencies [24, 25]. As the input frequency approaches the hydraulic resonant frequency, the output essentially becomes the hydraulic resonance. Therefore, the combination of the three-way valve and unequal piston areas applies to the electro-hydraulic exciter as a power element. Such a combination contributes an adjustable hydraulic natural frequency so that the working frequency of the exciter or its harmonics could avoid the resonant frequency. Consequently the influence of the hydraulic resonance for output is to be attenuated.

Spring loads are much more common with piston type actuators because of their limited travel [26]. This case is applicable to the electro-hydraulic exciter system in the fatigue test of materials or a structure strength test. At frequencies far below the natural frequency, the mass and viscous forces are small and may be neglected entirely. The load of electro-hydraulic exciter is determined just by the spring constant. A mode of this case is illustrated in Fig. 1.

Fig. 1 Hydraulic power elements of the electro-hydraulic exciter

However, actual loads including mass or viscous force are so complex that mathematical expressions that describe this system are quite involved. Although the development of digital computer programs could solve the governing equations [27], they only apply to

a particular system because of the given parameters. Hence, a describing function based on the dominance of the spring load force would be in order.

With no spring load on the piston, a steady-state control pressure acting on the head area is

$$p_{c0} = \frac{p_s}{2} \quad (1)$$

where p_{c0} is the steady-state control pressure, and p_s is system pressure.

This design relation allows the control pressure to rise or fall and to provide equal acceleration and deceleration capability. Equation (1) is satisfied by making the head area twice the rod area, that is,

$$A_h = 2A_r \quad (2)$$

where A_h is the head side area of piston, and A_r is rod side area of piston.

Assuming a constant supply pressure, the supply and return ports connect to the head chamber. When the supply port (valve orifice 1) is open or active, pressurized oil produces a hydrostatic force to drive the piston moving in the right-hand direction. Assuming that the load acts as a tensile spring at the initial position, the restoring force is in the same direction as the hydrostatic force. Thus, in this process, the spring load is essentially an overrunning load. Under the action of this applied load, the piston will be accelerated, as illustrated in fig. 2(a). As the piston is continuously moving, the direction of the hydrostatic remains the same as the proceeding process, but the elastic load turns into a compressed spring, which consumes part of the hydrostatic force. Therefore, the spring load becomes a normal load, as shown in fig. 2(b). At the point the valve opens and the flow rate drops to zero, the displacement of the piston reaches its peak value. When the return port (valve orifice 2) is open or active, a similar reasoning can also be applied to the following process from fig. 2(c) to (d). Consequently, the reciprocating vibration

of the piston is output as either valve orifice 1 or 2 is active.

Fig. 2 Load on the double-acting piston actuator

2.2 Basic Equations of the Exciter

For the critical centre of the three-way valve, the pressure-flow equation can be written directly as

$$Q_L = \begin{cases} C_d A_1 \sqrt{\frac{2(p_s - p_c)}{\rho}} & x \geq 0 \\ -C_d A_2 \sqrt{\frac{2p_c}{\rho}} & x < 0 \end{cases} \quad (3)$$

where Q_L is the load flow, C_d is called the discharge coefficient, A_1 is the area of valve orifice 1, A_2 is the area of valve orifice 2, p_c is the control pressure, ρ is the density of the oil, and x is the valve displacement from neutral.

Application of the continuity equation to the control volume in Fig. 1 yields

$$Q_L + C_f(p_s - p_c) = \frac{dV_h}{dt} + \frac{V_h}{E_h} \frac{dp_c}{dt} \quad (4)$$

where C_f is the cross-port leakage coefficient; V_h is the head chamber volume, ($V_h = V_0 + A_h y$, where V_0 is

the initial head chamber volume, and y is the displacement of the piston); and E_h is the effective bulk modulus of system.

Experience has shown that the displacement of the piston during excitation can be omitted because it is sufficiently small compared to the initial chamber volume. Usually, the amount of leakage accounting for the excitation system's pressure is much less than the system pressure, so it is also neglected. Therefore, Eq. (4) is simplified as

$$Q_L = \frac{A_h dy}{dt} + \frac{V_0}{E_h} \frac{dp_c}{dt} \quad (5)$$

The force equation resulting from analysing the principles of the electro-hydraulic exciter is

$$p_c A_h - p_s A_r = K_L y \quad (6)$$

where K_L is the load spring gradient.

Assuming that the control pressure could reach the system pressure, in Eq. (6) an extreme value may be given

$$y_{\max} = \frac{p_s A_r}{K_L} \quad (7)$$

where $y_{\max}$ is the vibration extremum.

Equations (3), (5) and (6) are three basic equations that may be solved simultaneously to obtain

$$C_0 A_1 dt = \frac{1}{\sqrt{p_s - K_L y / A_r}} dy \quad (8)$$

$$-C_0 A_2 dt = \frac{1}{\sqrt{p_s + K_L y / A_r}} dy \quad (9)$$

where $C_0 = C_d / \left[\sqrt{\rho} \left(A_h + \frac{V_0 K_L}{E_h A_h} \right) \right]$

Equations (8) and (9) give the piston response to the valve orifice area input, which are two different cases depending on whether the piston is moving to the right or the left. The equations are quite general and are applicable to the system where the spring force is dominant. A detailed derivation of the basic equations has been given in Appendix A.

3. Mathematical Solution of the Vibration to the Typical Wave Input

The orifice areas vary with the spool stroke (linear or rotary motion), as is usually the case, and flows through the orifices also vary because the flows are described by the orifice equation. This decides the output vibration of the piston. If the orifice areas are input as periodic waves, the mathematical formulations of the output vibration can be given for such an electro-hydraulic exciter system.

3.1 Sinusoidal Wave Input

Assuming that the orifice area varies with the sinusoidal wave, the expression is

$$A_v = A_1 - A_2 = W \sin\left(\frac{2\pi}{T} t\right) \quad (10)$$

where A_v is the orifice area, and W is the area gradient of the valve.

Substituting Eq. (10) into Eqs. (8) and (9) yields

$$\bar{Y} = \frac{y}{y_{\max}} = \frac{1}{p_s} \left\{ p_s - \left[\sqrt{p_s + \frac{K_L y_u}{A_r}} - \frac{K_L T C_0 W}{4\pi A_r} \left(1 - \cos\left(\frac{2\pi}{T} t\right) \right) \right]^2 \right\} \quad 0 \leq t < \frac{T}{2} \quad (11)$$

$$\bar{Y} = \frac{y}{y_{\max}} = \frac{1}{p_s} \left\{ -p_s + \left[\sqrt{p_s + \frac{K_L y_u}{A_r}} - \frac{K_L T C_0 W}{4\pi A_r} \left(1 + \cos\left(\frac{2\pi}{T} t\right) \right) \right]^2 \right\} \quad \frac{T}{2} \leq t < T \quad (12)$$

where $\bar{Y}$ is the non-dimensional displacement of the vibration, and y_u is amplitude of the vibration.

Assuming constant-amplitude vibration, the neutral position of the vibration is centred in the hydraulic cylinder, and at this point, the deformation (elastic or spring) force of the tested sample is just zero. The positive displacement is given at the head side, whereas the negative one is at the rod side.

Although this assumption is not true for real systems because of hydraulic nonlinearities, the stability problem of the vibration system, especially in the neutral position, can be solved by compensation for bias [23].

Using Eq. (11), the expression for the amplitude of the vibration can be given as

$$y_u = \frac{A_r}{K_L} \sqrt{p_s^2 - \left[p_s - \frac{1}{2} \left(\frac{K_L TC_0 W}{2\pi A_r} \right)^2 \right]^2} \quad (13)$$

For the relation of Eq. (7) to Eq. (13), $y_u \leq y_{\max}$, the area gradient of the valve becomes

$$W \leq \frac{2\pi A_r \sqrt{2p_s}}{K_L TC_0} \quad (14)$$

The valve area gradient is the principal parameter and, consequently, has a direct influence on the system output. If the valve area gradient is increased further, it causes a vibration wave that is saturated. Hence, the maximum area gradient in Eq. (14) is actually a critical value of the valid working space.

Fig. 3 Output waveform for a sinusoidal input

The vibration waveform for a sinusoidal input is shown in Fig. 3. Comparing the output waveform and the sinusoidal wave with a $T/4$ phase difference (dashed curve in Fig. 3), the output waveform displays a certain deflection because the piston is driven in the reciprocating motion so that the direction of elastic deformation changes. When the elastic deformation

acts on an overrunning load, the slope of the curve is somewhat steep and gentle, as it is a resistive load.

3.2 Triangular Wave Input

Assuming that the orifice area varies with a triangular wave, the expression is

$$A_r = A_1 - A_2 = \begin{cases} \frac{4W}{T}t & 0 \leq t < \frac{T}{4} \\ 2W - \frac{4W}{T}t & \frac{T}{4} \leq t < \frac{T}{2} \\ 2W - \frac{4W}{T}t & \frac{T}{2} \leq t < \frac{3T}{4} \\ -4W + \frac{4W}{T}t & \frac{3T}{4} \leq t < T \end{cases} \quad (15)$$

Substituting Eqs. (15) into Eqs. (8) and (9) and then integrating them, the displacement of vibration can be derived as (the analytical solution is given in Appendix B)

$$\bar{Y} = \frac{y}{y_{\max}} = \frac{1}{p_s} \left[p_s - \left(\sqrt{p_s + \frac{K_L y_u}{A_r}} - \frac{K_L C_0 W t^2}{A_r T} \right)^2 \right] \quad 0 \leq t < \frac{T}{4} \quad (16)$$

$$\bar{Y} = \frac{y}{y_{\max}} = \frac{1}{p_s} \left\{ p_s - \left[\sqrt{p_s + \frac{K_L y_u}{A_r}} - \frac{K_L C_0 W}{A_r T} \left(Tt - t^2 - \frac{T^2}{8} \right) \right]^2 \right\} \quad \frac{T}{4} \leq t < \frac{T}{2} \quad (17)$$

$$\bar{Y} = \frac{y}{y_{\max}} = -\frac{1}{p_s} \left\{ p_s - \left[\sqrt{p_s + \frac{K_L y_u}{A_r}} + \frac{K_L C_0 W}{A_r T} \left(Tt - t^2 - \frac{T^2}{4} \right) \right]^2 \right\} \quad \frac{T}{2} \leq t < \frac{3T}{4} \quad (18)$$

$$\bar{Y} = \frac{y}{y_{\max}} = -\frac{1}{p_s} \left\{ p_s - \left[\sqrt{p_s + \frac{K_L y_u}{A_r}} + \frac{K_L C_0 W}{A_r T} \left(t^2 - 2Tt + \frac{7T^2}{8} \right) \right]^2 \right\} \quad \frac{3T}{4} \leq t < T \quad (19)$$

Using Eq. (16), the expression for the amplitude of the vibration is

$$y_u = \frac{A_r}{K_L} \sqrt{p_s^2 - \left[p_s - \frac{1}{2} \left(\frac{K_L C_0 W T}{8A_r} \right)^2 \right]^2} \quad (20)$$

For the relation of Eq. (7) to Eq. (20), $y_u \leq y_{\max}$, the area gradient of the valve becomes

$$W \leq \frac{8A_r \sqrt{2p_s}}{K_L TC_0} \quad (21)$$

This valve area gradient is limited to a value, the right side of Eq. (21), and when it is equal to this value, the output excited vibration is in the critical working space, as shown in Fig. 4.

Fig. 4 Output waveform for a triangular input

Referring to Fig. 4, the vibration waveform is a sinusoidal wave with a certain deflection for a triangular input, which is analogous to a sinusoidal input. However, the orifice area varies with a triangular wave and is easier to control or machine.

3.3 Square Wave Input

Assuming that the orifice area varies with a square wave, the expression is

$$A_v = A_1 - A_2 = \begin{cases} W & 0 \leq t < \frac{T}{2} \\ -W & \frac{T}{2} \leq t < T \end{cases} \quad (22)$$

Substituting Eq. (22) into Eqs. (8) and (9) yields

$$\bar{Y} = \frac{y}{y_{\max}} = \frac{1}{p_s} \left[p_s - \left(\sqrt{p_s + \frac{K_L y_u}{A_r}} - \frac{K_L C_0 W t}{2A_r} \right)^2 \right] \quad 0 \leq t < \frac{T}{2} \quad (23)$$

$$\bar{Y} = \frac{y}{y_{\max}} = \frac{1}{p_s} \left\{ -p_s + \left[\sqrt{p_s + \frac{K_L y_u}{A_r}} + \frac{K_L C_0 W}{2A_r} \left(\frac{T}{2} - t \right) \right]^2 \right\} \quad \frac{T}{2} \leq t < T \quad (24)$$

Using Eq. (23), the expression for the amplitude of the vibration is

$$y_u = \frac{A_r}{K_L} \sqrt{p_s^2 - \left[p_s - \frac{1}{2} \left(\frac{K_L C_0 W T}{4A_r} \right)^2 \right]^2} \quad (25)$$

For the relation of Eq. (7) to Eq. (25), $y_u \leq y_{\max}$, the area gradient of the valve is limited to the value

$$W \leq \frac{4A_r \sqrt{2p_s}}{K_L TC_0} \quad (26)$$

At the maximum value, the output vibration is a critical working state, which is plotted in Fig. 5. Referring to this figure, the trace of critical output is very different from the trace of the standard function (dashed curve).

Fig. 5 Output waveform for a square input

3.4 Input Wave -Sinusoidal Output

Assuming that vibration output is sinusoidal wave,

$$y = y_u \sin\left(\frac{2\pi}{T}t - \frac{\pi}{2}\right) \quad (27)$$

Substituting Eq. (27) into Eqs. (8) and (9), the change of the valve orifice area can be obtain as

$$A_v = \frac{\frac{2\pi}{T} y_u \sqrt{A_r} \sin \frac{2\pi}{T} t}{C_0 \sqrt{p_s A_r + K_L y_u \cos \frac{2\pi}{T} t}} \quad 0 \leq t < \frac{T}{2} \quad (28)$$

$$A_v = \frac{\frac{2\pi}{T} y_u \sqrt{A_r} \sin \frac{2\pi}{T} t}{C_0 \sqrt{p_s A_r - K_L y_u \cos \frac{2\pi}{T} t}} \quad \frac{T}{2} \leq t < T \quad (29)$$

These two equations show that the wave of change of the valve orifice area contains other

frequency components, not just a single one. This requirement is not easily satisfied because of the limited response capability of the servo valve itself. Therefore, it is too difficult to obtain a high-frequency vibration or complex desired waveform, even for just a sinusoidal vibration output.

4. Electro-hydraulic Excitation Controlled by the Rotary Motion of the Spool

A conventional electro-hydraulic vibration exciter usually consists of a servo valve and a cylinder or motor. This is a threat to high-frequency vibration applications because of the limited response capability of the servo valve itself. Therefore, designing a new structural valve called a 2D valve instead of a servo valve is useful for applications in an electro-hydraulic excitation system. The desired electro-hydraulic excitation, especially high-frequency excitation, can be easily obtained by the controlling rotary motion of this rotary valve spool.

4.1 Three-way 2D Valve

Inspired by the axial flow distribution structure of the rotary valve [23], the three-way 2D valve is designed for realizing high-frequency excitation. The rotary motion of the spool is driven by a gear and hydraulic motor combination in a continuous rotary mode. The sliding motion of the spool is controlled by an eccentric mechanism driven by a servomotor, as shown in Fig. 6.

Fig. 6 Three-way 2D valve with rotary spool

1—gear; 2—valve body; 3—sleeve; 4—spool; 5—eccentric mechanism; 6—servomotor; 7—hydraulic motor

The working frequency in this electro-hydraulic excitation type depends on the rotary speed of the spool. However, if the speed exceeds a limited value, the rotary motion of the spool will be unstable because of spool whirl. Therefore, it is necessary to design hydrodynamic pressure lubrication to increase the critical speed of the spool whirl. Special grooves are created on the sleeve, as illustrated in Fig. 7, the construction of which is analogous to a bearing shell and uses a Cu-based alloy material. Such a lubricating structure causes the rotary motion of the spool to be controlled precisely within a great range of rotary speed. In addition, the case of instability is also due to flow force acting on the face of the 2D valve spool land. Therefore, a lateral angle between the groove on the spool and the window on the sleeve is designed to compensate lateral component of this flow force, which is shown in Fig. 8(a). The result is that the rotary precision of the spool is improved greatly. However, as the spool spins rapidly, a situation of cavitation may arise when the pressure surges in a large transient due to a valve closure. Thereby, a triangular groove is open on every window of the sleeve as shown in Fig. 8(b), which dissipates the kinetic energy and prevents cavitation by dumping fluid into the forward chamber. Consequently, the stability of the performance characteristic could be satisfied and not require some sacrifice in the rotary speed of the spool.

Fig. 7 Lubricating structure

Fig. 8 Matching between the groove and the window

The 2D valve-controlled excitation system consists of a three-way 2D rotary valve controlling the flow from a hydraulic power supply to the cylinder, as illustrated in Fig. 9. When the valve ports formed on shoulder I are open, A and P are interlinked, so that the hydraulic fluid flows into the head chamber. Thereby, the control pressure on the head area of the piston drives the single rod in the left-hand direction. As the spool is rotated at a certain angle, the ports formed on shoulder II are open, A and T are interlinked, and the supply pressure in the rod chamber drive the rod in the opposite direction. While the spool of the 2D valve continuously rotates, the pressure inside the control chamber of the cylinder changes alternately, and the piston is subsequently driven in a reciprocating motion.

Fig. 9 Three-way 2D valve-piston combination

1—2D valve; 2—single rod actuator; 3—load

4.2 Relevant Model between the Orifice Shape and the Output Wave

For the full port pattern of the three-way 2D valve, a series of grooves are designed to be distributed

symmetrically on the spool (central angle of every groove is α) and coordinated with the windows distributed uniformly on the sleeve. If the central angle between two adjoining grooves is 4α , then, geometrically, the groove has a 2α angle difference with respect to one on any two adjacent shoulders. Such a special arrangement will create an ‘alternately varied’ valve port area between grooves on the spool and windows on the sleeve. This area has a rectangular shape. The length of the lateral side x_v is the axial displacement of the spool, and the other side, y_v , is the peripheral width of the window overlapped by the groove on the spool, as shown in Fig. 10

Fig. 10 Rectangular valve port window

As the spool spins, the area of the rectangular valve orifice is

$$A_v = z x_v y_v = A_{v1} - A_{v2} = \begin{cases} z x_v r \theta & 0 \leq \theta < \alpha \\ 2 z x_v r \alpha - z x_v r \theta & \alpha \leq \theta < 2\alpha \\ 2 z x_v r \alpha - z x_v r \theta & 2\alpha \leq \theta < 3\alpha \\ -4 z x_v r \alpha + z x_v r \theta & 3\alpha \leq \theta < 4\alpha \end{cases} \quad (30)$$

where $A_{vi} (i=1,2)$ is, respectively, the orifice area on shoulder I or II, z is number of grooves on a single shoulder, x_v is the axial displacement of spool, y_v is the peripheral width of the window overlapped by the groove on the spool, r is the radius of the spool, and θ is the angular displacement of the spool, with the values ranging from 0 to 4α , θ is defined as $\omega_v t - \left\lfloor \frac{\omega_v t}{4\alpha} \right\rfloor 4\alpha$, where ω_v is the rotational speed of the spool, t is the working time, and α is the central angle of the groove, which follows from z by the

relationship $\alpha = \pi/(2z)$.

By contrasting Eq. (30) with Eq. (15), it is clear that the orifice area of the three-way 2D valve varies with a triangular wave (Fig. 11). As the three-way valve is a critical centre type, a positive or negative orifice area not only refers to the case in which either orifice 1 or 2 is active at one time, but also represents the direction of flow.

An electro-hydraulic excitation system controlled by a rotary valve is essentially a basic system of a three-way valve-controlled piston, so Eqs. (8) and (9) may be written as

$$\frac{C_0 A_v}{\omega_v} d\theta = \frac{1}{\sqrt{p_s - K_L y/A_r}} dy \quad 0 \leq \theta < 2\alpha \quad (31)$$

$$\frac{C_0 A_v}{\omega_v} d\theta = \frac{1}{\sqrt{p_s + K_L y/A_r}} dy \quad 2\alpha \leq \theta < 4\alpha \quad (32)$$

where $C_0 = C_d / \left[\sqrt{\rho} \left(A_h + \frac{V_0 K_L}{E_h A_h} \right) \right]$

Substituting Eq. (30) into Eq. (31) and (32), a precise mathematical solution of a given set of equations can be obtained as

$$\bar{Y} = \frac{y}{y_{\max}} = \frac{1}{p_s} \left[p_s - \left(\sqrt{p_s + \frac{K_L y_u}{A_r}} - \frac{K_L C_0 z x_v r \theta^2}{4 \omega_v A_r} \right)^2 \right] \quad 0 \leq \theta < \alpha \quad (33)$$

$$\bar{Y} = \frac{y}{y_{\max}} = \frac{1}{p_s} \left\{ p_s - \left[\sqrt{p_s + \frac{K_L y_u}{A_r}} - \frac{K_L C_0 z x_v r}{\omega_v A_r} \left(\alpha \theta - \frac{\theta^2}{4} - \frac{\alpha^2}{2} \right) \right]^2 \right\} \quad \alpha \leq \theta < 2\alpha \quad (34)$$

$$\bar{Y} = \frac{y}{y_{\max}} = -\frac{1}{p_s} \left\{ p_s - \left[\sqrt{p_s + \frac{K_L y_u}{A_r}} + \frac{K_L C_0 z x_v r}{\omega_v A_r} \left(\alpha \theta - \frac{\theta^2}{4} - \alpha^2 \right) \right]^2 \right\} \quad 2\alpha \leq \theta < 3\alpha \quad (35)$$

$$\bar{Y} = \frac{y}{y_{\max}} = -\frac{1}{p_s} \left\{ p_s - \left[\sqrt{p_s + \frac{K_L y_u}{A_r}} + \frac{K_L C_0 z x_v r}{\omega_v A_r} \left(\frac{\theta^2}{4} - 2\alpha\theta + \frac{7\alpha^2}{2} \right) \right]^2 \right\} \quad 3\alpha \leq \theta < 4\alpha \quad (36)$$

If the excitation output is a constant amplitude wave with no bias, the expression for the amplitude can be derived as

$$y_u = \frac{A_r}{K_L} \sqrt{p_s^2 - \left[p_s - \frac{1}{2} \left(\frac{C_0 K_L z x_v r \alpha^2}{2 \omega_v A_r} \right)^2 \right]^2} \quad (37)$$

As illustrated in Fig. 11, the ascent and descent processes of the output wave display some inconsistency. This is because changing the elastic load force causes the speed of the piston to increase and decrease at different rates. However, it should be noted that the slope value of the positive curve equals that of the negative curve at the same valve orifice opening.

Fig. 11 Input and output of the excitation system controlled by the 2D valve

4.3 Harmonic Analysis of the Vibration Wave

The output waveform excited by the 2D rotary valve represents a periodic vibration, the characteristics of which are relatively complicated. Hence, it is necessary to analyse the harmonic components of this excitation waveform.

Assuming that the spool of the 2D valve could rotate reversely, the workspace of $(-4\alpha, 0)$ is added to $[0, 4\alpha)$, that is, the analytical function of the vibration waveform could be determined in the interval of $(-4\alpha, 4\alpha)$ so that it has a series expansion. Consequently, the sine series are expanded by an odd continuation for the vibration waveform because sine is an odd function. The coefficient of the series can be given by

$$b_n = \frac{1}{2\alpha} \int_0^{4\alpha} \bar{Y} \sin \frac{n\pi\theta}{4\alpha} d\theta \quad (n=1, 2, 3, \dots) \quad (38)$$

where b_n is the Fourier coefficient, that is, the amplitude of the harmonic component.

Substituting Eq. (33), Eq. (34), Eq. (35) and Eq. (36) into Eq. (38), a precise mathematical solution of the Fourier coefficient is obtained as (analytical solution is given in Appendix C)

$$b_n = \frac{2K_L y_u}{p_s n \pi A_r} \left[-1 + 2 \cos\left(\frac{n\pi}{2}\right) - \cos(n\pi) \right] + \frac{B^2 \alpha^4}{2p_s} \left[\frac{-1}{n\pi} \cos(n\pi) - \frac{32}{(n\pi)^3} \cos(n\pi) - \frac{1536}{(n\pi)^5} \cos(n\pi) + \frac{1}{n\pi} \cos\left(\frac{n\pi}{2}\right) + \frac{32}{(n\pi)^3} \cos\left(\frac{n\pi}{2}\right) + \frac{3072}{(n\pi)^5} \cos\left(\frac{n\pi}{2}\right) - \frac{1536}{(n\pi)^5} - \frac{32}{(n\pi)^3} \cos\left(\frac{n\pi}{4}\right) + \frac{768}{(n\pi)^4} \sin\left(\frac{n\pi}{4}\right) + \frac{32}{(n\pi)^3} \cos\left(\frac{3n\pi}{4}\right) - \frac{768}{(n\pi)^4} \sin\left(\frac{3n\pi}{4}\right) \right] + \frac{AB\alpha^2}{2p_s} \left[\frac{128}{(n\pi)^3} \cos\left(\frac{n\pi}{4}\right) - \frac{64}{(n\pi)^3} + \frac{4}{n\pi} \cos(n\pi) - \frac{128}{(n\pi)^3} \cos\left(\frac{3n\pi}{4}\right) + \frac{64}{(n\pi)^3} \cos(n\pi) - \frac{4}{n\pi} \cos\left(\frac{n\pi}{2}\right) \right] \quad (39)$$

where $A = \sqrt{p_s + \frac{K_L y_u}{A_r}}$, $B = \frac{K_L C_0 z x_v r}{\omega_v A_r}$

Applying the Fourier series expansion to the vibration waveform yields

$$\bar{Y} = b_n \sin \frac{n\pi\theta}{4\alpha} \quad (n=1, 2, 3, \dots) \quad (40)$$

Although the expression of the coefficient for the harmonic components of the vibration waveform is very complicated, it can be compared with a standard function using this same analytical procedure.

The negative cosine function is defined as

$$f(x) = -\frac{y_u}{y_{\max}} \cos\left(\frac{\pi x}{2\alpha}\right) \quad (41)$$

where $f(x)$ is a negative cosine function with the same amplitude and period of the vibration waveform, x is an independent variable, which is defined as $x = \omega t$.

The Fourier coefficient for Eq. (41) can be solved as

$$b_{nc} = \frac{1}{2\alpha} \int_0^{4\alpha} f(x) \sin \frac{n\pi\theta}{4\alpha} d\theta \quad (n=1, 2, 3, \dots) \quad (42)$$

$$b_{nc} = \frac{-y_u}{2\alpha y_{\max}} \left[-\frac{\cos(n\pi)}{\frac{\pi}{\alpha} + \frac{n\pi}{2\alpha}} - \frac{\cos(n\pi)}{\frac{n\pi}{2\alpha} - \frac{\pi}{\alpha}} + \frac{1}{\frac{\pi}{\alpha} + \frac{n\pi}{2\alpha}} + \frac{1}{\frac{n\pi}{2\alpha} - \frac{\pi}{\alpha}} \right] \quad (43)$$

The main parameters of the excitation structure and the hydraulic system are selected to be the same as its physical counterpart used in the experimental test. The numerical values of the parameters are listed in Table 1. According to Table 1, Eqs. (39) and (43) are solved simultaneously to obtain the Fourier coefficients, as shown in Table 2.

Table 1 Main parameters of the excitation system

Parameters	Value
Rod side area of piston A_r / m ²	1.43×10^{-2}
Head side area of piston A_h / m ²	2.86×10^{-2}
Discharge coefficient C_d	0.62
Diameter of the spool d / m	2.07×10^{-2}
Effective bulk modulus E_h / Pa	7.0×10^8
Load spring gradient K_L / N/m	2.74×10^7
Length of cylinder L / m	0.2
Mass m / kg	26.5
System pressure p_s / Pa	2.1×10^7
Axial displacement of spool x_v / m	9×10^{-3}
Number of grooves z	8
Density of oil ρ / kg / m ³	900
Working frequency f / Hz	250

Table 2 Fourier coefficients

Harmonics	Coefficients		
	$b_n (\times 10^{-4})$	$b_{nc} (\times 10^{-4})$	$ b_n - b_{nc} / b_{nc} \times 100\%$
1	14	14	0
3	-25	-24	4.2
5	-11	-9.7	13
7	-5.6	-6.3	11
9	-4.4	-4.7	6

Referring to Table 2, only the odd parts are in the Fourier coefficients, and for two waveforms (vibration

waveform and negative cosine waveform) the harmonic components are similar. Hence, the vibration waveform can be approximated by the standard waveform even though a small distortion is inevitable.

As discussed in Sections 3.1 and 4.2, the output waveform will not have a great difference, regardless of whether the orifice area varies with a sinusoidal or triangular wave. However, the vibration output is easier to be realized for a triangular input because it is obtained just by the structure of the 2D valve or its rotary motion.

5. Experimental System and Results

The experimental system of the metal material fatigue test is designed to test this new electro-hydraulic excitation type, as shown in Fig. 12(a), and a photograph of the physical system is shown in Fig. 12(b). The excitation wave is output by a single-rod piston actuator controlled by a 2D rotary valve. The rotary of the valve spool is driven by a hydraulic motor and a single mesh gear train with a gear ratio of four, while the linear motion of the valve spool is actuated by a combination of the eccentric mechanism and servomotor. Therefore, the frequency and amplitude of the output vibration could be controlled independently via two independent input signals to the 2D valve controller. In the open loop control for the rotary of the valve spool, the working frequency for the experimental system could attain resonant frequency of the system.

A pressure transducer is selected to measure the pressure in the control chamber of the cylinder. A force cell is placed between the end of the cylinder rod and the rigidly connected frame. A displacement transducer is mounted inside the cylinder rod to measure the displacement of the piston with respect to

the frame in the axial direction of the cylinder. By calibrating the hydrostatic force vs. the deformation, a fairly linear relationship is obtained. The system is therefore a rigid system with a very high spring constant, which establishes that the deformation of the frame is in the elastic range. Because the dominant load is coming from the spring force, the building of the theoretical model can be reasonably simplified, as shown in sections 2 and 4.

The natural frequency of the system could be adjusted by changing the position of the baffle plate connected with the single-rod piston. Usually, the accumulator is necessary in an experimental system, which functions to reduce the vibratory shock to the inlet and return lines. The whole test-bed is fixed on the cast iron platform, the weight of which is large enough that the effect of shock is weakened on the vibration output.

(a) Drawing of the electro-hydraulic excitation test-bed

(b) Photograph of the physical system

Fig. 12 Basic structure of experimental system

1—foundation; 2—plank; 3—anchor bolt; 4—baffle; 5—force

cell; 6—fitting; 7—pressure transducer; 8—three-way 2D valve; 9—displacement transducer; 10—cylinder with a single rod

The waveforms in the low-frequency domain range (below the natural frequency of the system) are given in Fig. 13. The vibration wave will be saturated if the control pressure increases to the system pressure. However, as illustrated in Fig. 13(a), the measured output during pressure saturation does not remain constant but shows some fluctuations. This result is attributed to such things as a water hammer phenomenon caused by fluid flow being suddenly stopped because of a rapid valve closure, and a very large pressure transient may result. Figs. 13(b) and (c) show the vibration waveforms at different frequencies. It can be seen that the experimental waveforms coincide well with the analytical results at low frequencies and large linear valve openings.

(a) $f=5\text{Hz}$

(b) $f=50\text{Hz}$

(c) $f=300\text{Hz}$

Fig. 13 Output waveforms of the experimental system

As illustrated in Fig. 13, a reduction in the amplitude of the vibration will reduce the effect of the elastic load on the excitation output. If the amplitude of the vibration is small enough, the system can be treated as linear. The vibration waveform is completely approximated as a sinusoidal waveform, especially at high frequencies. However, as the working frequency or its odd multiples equal the hydraulic resonant frequency, the corresponding harmonics of the output will be amplified. As a result, the output waveform displays more or less distortion, as shown in Fig. 14. This result becomes more apparent as the amplified harmonic approaches the

basic frequency of the output.

(a) $f=140\text{Hz}$

(b) frequency composition

Fig. 14 Vibration waveform and its frequency composition

As the working frequency is increased, the amplitude of the vibration decreases drastically. Although the amplitude can be increased by adjusting the axial opening of the valve spool, this compensation is limited considerably. This is because the axial displacement of the valve spool is impossible to increase indefinitely while taking into account the proper size of the 2D valve for the system or difficult manufacture in practice. However, when the working frequency reaches the system's natural frequency, the amplitude of the vibration is suddenly enlarged. This peculiar phenomenon is caused by hydraulic resonance. Unlike the mechanical resonance

phenomenon, the amplitude of the vibration does not increase to destroy all systems at the resonant frequency. This is because the hydraulic resonance elements composed of the spring, damping and flow forces are almost all made with flow power transmission mediums except for the mass. The hydraulic medium that is the working fluid not only acts as a force to drive the cycle excitation of the piston, but is also a centring spring of the stiffness about a function of the piston displacement, and takes into account effects such as viscous damping because of a leakage path acting to increase damping. This relevance among them inhibits the hydraulic resonance from destroying the system. Consequently, the amplitude of the vibration output is a constant, which is completely approximate to the sinusoidal waveform shown in Fig. 15.

Fig. 15 Resonant waveform

6. Conclusions

(1) A new structural rotary valve (2D valve) instead of a servo valve is used in an electro-hydraulic excitation system. Consequently, the working frequency of the system is extended largely. High-frequency excitation, even resonance excitation, can be obtained by this type of electro-hydraulic excitation.

(2) Analysis of the vibration output to the typical wave input yields an analytical solution of the vibration waveform excited by this electro-hydraulic system with a 2D rotary valve. The mathematical formulation of the harmonics is also given.

(3) It is too difficult to take into account both the frequency and the amplitude of electro-hydraulic excitation. However, the amplitude of a vibration can be enlarged greatly at a resonant frequency point if only the condition of hydraulic resonance is satisfied.

(4) Although this 2D valve employing rotary spool type construction can increase the upper frequency of the excitation system, it does not control the wave shape of the output like the servo valve used in the closed-loop system. However, parallel 2D valve group configuration can be developed in the future for controlling multi-flow to synthesize a random periodic waveform in an electro-hydraulic excitation system.

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Appendix A

Flows through the valve orifices are described as

$$Q_1 = C_d A_1 \sqrt{\frac{2(p_s - p_c)}{\rho}} \quad x \geq 0 \quad (\text{A.1})$$

$$Q_2 = -C_d A_2 \sqrt{\frac{2p_c}{\rho}} \quad x < 0 \quad (\text{A.2})$$

The critical **centre** type is considered first, **and** the equations for the pressure-flow curves can be written as

$$Q_L = Q_1 + Q_2 = \begin{cases} C_d A_1 \sqrt{\frac{2(p_s - p_c)}{\rho}} & x \geq 0 \\ -C_d A_2 \sqrt{\frac{2p_c}{\rho}} & x < 0 \end{cases} \quad (\text{A.3})$$

The head area is twice as the rod area, that is

$$A_h = 2A_r \quad (\text{A.4})$$

Applying the continuity equation to **the** control chamber yields

$$Q_L + C_f(p_s - p_c) = \frac{dV_h}{dt} + \frac{V_h}{E_h} \frac{dp_c}{dt} \quad (\text{A.5})$$

By the assumption of piston displacement and cross-port leakage in Section 2.2, Eq. (A.5) is simplified as

$$Q_L = \frac{A_h dy}{dt} + \frac{V_0}{E_h} \frac{dp_c}{dt} \quad (\text{A.6})$$

The force balance equation for the piston is

$$p_c A_h - p_s A_r = K_L y \quad (\text{A.7})$$

$$p_c = \frac{K_L y + p_s A_r}{A_h} \quad (\text{A.8})$$

The above equations may be solved simultaneously to obtain

$$\frac{C_d A_1}{\sqrt{\rho}} \sqrt{2(p_s - \frac{K_L y + p_s A_r}{A_h})} = \left(2A_r + \frac{V_0 K_L}{2A_r E_h} \right) \frac{dy}{dt} \quad (\text{A.9})$$

$$-\frac{C_d A_2}{\sqrt{\rho}} \sqrt{2 \frac{K_L y + p_s A_r}{A_h}} = \left(2A_r + \frac{V_0 K_L}{2A_r E_h} \right) \frac{dy}{dt} \quad (\text{A.10})$$

With a parameter C_0 defined, **Eqs.** (A.9) and (A.10) **become**

$$C_0 A_1 dt = \frac{1}{\sqrt{p_s - K_L y / A_r}} dy \quad x \geq 0 \quad (\text{A.11})$$

$$-C_0 A_2 dt = \frac{1}{\sqrt{p_s + K_L y / A_r}} dy \quad x < 0 \quad (\text{A.12})$$

where

$$C_0 = C_d \left/ \left[\sqrt{\rho} \left(A_h + \frac{V_0 K_L}{E_h A_h} \right) \right] \right. \quad (\text{A.13})$$

Appendix B

(B.11)

$$A_v = A_1 - A_2 = \begin{cases} \frac{4W}{T}t & 0 \leq t < \frac{T}{4} \\ 2W - \frac{4W}{T}t & \frac{T}{4} \leq t < \frac{T}{2} \\ 2W - \frac{4W}{T}t & \frac{T}{2} \leq t < \frac{3T}{4} \\ -4W + \frac{4W}{T}t & \frac{3T}{4} \leq t < T \end{cases} \quad (B.1)$$

Substituting Eq. (B.1) into Eqs. (A.11) and (A.12), the time integrals in the four-part form are given by

$$\int_0^t \frac{C_0 4Wt}{T} dt = \int_{-y_u}^y \frac{1}{\sqrt{p_s - K_L y / A_r}} dy \quad (B.2)$$

$$y = \frac{A_r}{K_L} \left[p_s - \left(\sqrt{p_s + \frac{K_L y_u}{A_r}} - \frac{K_L C_0 W t^2}{A_r T} \right)^2 \right] \quad (B.3)$$

When $t = \frac{T}{4}$,

$$y_1 = \frac{A_r}{K_L} \left\{ p_s - \left[\sqrt{p_s + \frac{K_L y_u}{A_r}} - \frac{K_L C_0 W T}{16 A_r} \right]^2 \right\} \quad (B.4)$$

$$\int_{\frac{T}{4}}^t C_0 \left(2W - \frac{4Wt}{T} \right) dt = \int_{y_1}^y \frac{1}{\sqrt{p_s - K_L y / A_r}} dy \quad (B.5)$$

$$y = \frac{A_r}{K_L} \left\{ p_s - \left[\sqrt{p_s + \frac{K_L y_u}{A_r}} - \frac{K_L C_0 W}{A_r T} \left(Tt - t^2 - \frac{T^2}{8} \right) \right]^2 \right\} \quad (B.6)$$

$$\int_{\frac{T}{2}}^t C_0 \left(2W - \frac{4Wt}{T} \right) dt = \int_{y_u}^y \frac{1}{\sqrt{p_s + K_L y / A_r}} dy \quad (B.7)$$

$$y = -\frac{A_r}{K_L} \left\{ p_s - \left[\sqrt{p_s + \frac{K_L y_u}{A_r}} + \frac{K_L C_0 W}{A_r T} \left(Tt - t^2 - \frac{T^2}{4} \right) \right]^2 \right\} \quad (B.8)$$

When $t = \frac{3T}{4}$,

$$y_2 = -\frac{A_r}{K_L} \left[p_s - \left(\sqrt{p_s + \frac{K_L y_u}{A_r}} - \frac{K_L C_0 W T}{16 A_r} \right)^2 \right] \quad (B.9)$$

$$\int_{\frac{3T}{4}}^t C_0 \left(-4W + \frac{4Wt}{T} \right) dt = \int_{y_2}^y \frac{1}{\sqrt{p_s + K_L y / A_r}} dy \quad (B.10)$$

$$y = -\frac{A_r}{K_L} \left\{ p_s - \left[\sqrt{p_s + \frac{K_L y_u}{A_r}} + \frac{K_L C_0 W}{A_r T} \left(t^2 - 2Tt + \frac{7T^2}{8} \right) \right]^2 \right\}$$

When $t = \frac{T}{2}$, Eq. (B.6) is solved to obtain

$$y_u = \frac{A_r}{K_L} \sqrt{p_s^2 - \left[p_s - \frac{1}{2} \left(\frac{K_L C_0 W T}{8 A_r} \right)^2 \right]^2} \quad (B.12)$$

Appendix C

$$b_n = \frac{1}{2\alpha} \int_0^{4\alpha} \bar{Y} \sin \frac{n\pi\theta}{4\alpha} d\theta \quad (n=1,2,3...) \quad (C.1)$$

where $\sqrt{p_s + \frac{K_L y_u}{A_r}} = A$, $\frac{K_L C_0 W T}{\omega_v A_r} = B$

$$b_n = \frac{1}{2\alpha p_s} \left\{ \int_0^\alpha p_s \sin \frac{n\pi\theta}{4\alpha} d\theta - \int_0^\alpha \left(A - B \frac{\theta^2}{4} \right)^2 \sin \frac{n\pi\theta}{4\alpha} d\theta \right. \\ \left. + \int_\alpha^{2\alpha} p_s \sin \frac{n\pi\theta}{4\alpha} d\theta - \int_\alpha^{2\alpha} \left[A - B \left(\alpha\theta - \frac{\theta^2}{4} - \frac{\alpha^2}{2} \right) \right]^2 \sin \frac{n\pi\theta}{4\alpha} d\theta \right. \\ \left. - \int_{2\alpha}^{3\alpha} p_s \sin \frac{n\pi\theta}{4\alpha} d\theta + \int_{2\alpha}^{3\alpha} \left[A + B \left(\alpha\theta - \frac{\theta^2}{4} - \alpha^2 \right) \right]^2 \sin \frac{n\pi\theta}{4\alpha} d\theta \right. \\ \left. - \int_{3\alpha}^{4\alpha} p_s \sin \frac{n\pi\theta}{4\alpha} d\theta + \int_{3\alpha}^{4\alpha} \left[A + B \left(\frac{\theta^2}{4} - 2\alpha\theta + \frac{7\alpha^2}{2} \right) \right]^2 \sin \frac{n\pi\theta}{4\alpha} d\theta \right\} \quad (C.2)$$

$$b_n = \frac{2K_L y_u}{p_s n \pi A_r} \left[-1 + 2 \cos \left(\frac{n\pi}{2} \right) - \cos(n\pi) \right] \\ + \frac{B^2 \alpha^4}{2p_s} \left[\frac{-1}{n\pi} \cos(n\pi) - \frac{32}{(n\pi)^3} \cos(n\pi) - \frac{1536}{(n\pi)^5} \cos(n\pi) + \right. \\ \left. \frac{1}{n\pi} \cos \left(\frac{n\pi}{2} \right) + \frac{32}{(n\pi)^3} \cos \left(\frac{n\pi}{2} \right) + \frac{3072}{(n\pi)^5} \cos \left(\frac{n\pi}{2} \right) \right. \\ \left. - \frac{1536}{(n\pi)^5} - \frac{32}{(n\pi)^3} \cos \left(\frac{n\pi}{4} \right) + \frac{768}{(n\pi)^4} \sin \left(\frac{n\pi}{4} \right) + \right. \\ \left. \frac{32}{(n\pi)^3} \cos \left(\frac{3n\pi}{4} \right) - \frac{768}{(n\pi)^4} \sin \left(\frac{3n\pi}{4} \right) \right] \\ + \frac{AB \alpha^2}{2p_s} \left[\frac{128}{(n\pi)^3} \cos \left(\frac{n\pi}{4} \right) - \frac{64}{(n\pi)^3} + \frac{4}{n\pi} \cos(n\pi) - \right. \\ \left. \frac{128}{(n\pi)^3} \cos \left(\frac{3n\pi}{4} \right) + \frac{64}{(n\pi)^3} \cos(n\pi) - \frac{4}{n\pi} \cos \left(\frac{n\pi}{2} \right) \right] \quad (C.3)$$

$$f(x) = -\frac{y_u}{y_{\max}} \cos \left(\frac{\pi x}{2\alpha} \right) \quad (C.4)$$

$$b_{nc} = \frac{1}{2\alpha} \int_0^{4\alpha} f(x) \sin \frac{n\pi\theta}{4\alpha} d\theta \quad (n=1,2,3...) \quad (C.5)$$

$$b_{nc} = \frac{-y_u}{2\alpha y_{\max}} \left[-\frac{\cos(n\pi)}{\frac{\pi}{\alpha} + \frac{n\pi}{2\alpha}} - \frac{\cos(n\pi)}{\frac{n\pi}{2\alpha} - \frac{\pi}{\alpha}} + \frac{1}{\frac{\pi}{\alpha} + \frac{n\pi}{2\alpha}} + \frac{1}{\frac{n\pi}{2\alpha} - \frac{\pi}{\alpha}} \right] \quad (\text{C.6})$$

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