Theory and Structure of a modified Optical Fiber Turbine Flowmeter

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Abstract

The measurement lower limit of traditional turbine flowmeter is too large to satisfy the measurement requirements of small flow rate. A modified optical fiber turbine flowmeter based on the traditional turbine flowmeter principle was purposed. The front guide vane was specially designed with a helix angle to reduce the measurement lower limit and improve the measurement sensitivity. The theoretical model for the modified flowmeter was established based on planar vector triangle of flow velocities. From theoretical analysis, the conclusion that if the helix angle of front guide vane plus the helix angle of turbine equals to 90° , the starting volume flow rate of turbine can get minimum value and the sensitivity of the flowmeter can get maximum value was found. The correctness of the conclusion was verified by the Computational Fluid Dynamics (CFD) simulation.

Index Terms: Turbine Flowmeter; Optical Fiber Technology; Front Guide Vane; Computational Fluid Dynamics

1. Introduction

Accurate flow rate measurement is the important precondition of many industry fields, such as medical, energy, petrochemical engineering, etc(1). Turbine flowmeter has become an important part of flow measurement instruments with high accuracy, good reproducibility and stability(2, 3). In addition, turbine flowmeter also can be used as a standard instrument to calibrate other flow measurement instruments.

The original helical turbine flowmeter was designed and primarily used in jet aircraft in the 1950s(4). After many years of research and exploration, its basic theory and the corresponding model has been nearly complete(5). However, the recent research of turbine flowmeter mainly focused on the response characteristics but not low flow rate. It was unable to satisfy the requirements of low flow rate measurement in actual production.

With the advantages of small size, anti-electromagnetic interference immunity, capability of multiplexing and anti-corrosion, optical fiber sensors have been widely used in flow measurement fields(6). Early optical fiber turbine flowmeter belongs to reflection-impulse-type flowmeter(7). The optical fiber sensor was applied to measure the rotating speed of turbine. Despite optical fiber flow measurement has developed rapidly in laboratory recently with the employment of Fiber Bragg Grating (FBG) and Fabry-Perot (FP) cavity(8, 9), the reflection-impulse-type optical fiber flowmeter is still the most applied in actual production.

In addition, Computational Fluid Dynamics (CFD) simulation has become an effective method to study flow distribution with the development of computational fluid mechanics theory and computer technology. Zhen and Zhang did research on tangential type turbine flowmeter with CFD simulation in 2007(10). In 2011, Zhao and Sun used CFD to study the viscosity impact of turbine flowmeter's performance(11). In 2015, Z. Saboohi developed a model for prediction of helical turbine flowmeter with CFD(12).

The paper purposed a modified optical fiber flowmeter based on traditional turbine flowmeter principle. Through the special design of front guide vane, the turbine can obtain maximum driving torque. Therefore, the minimum measurable flow limit of the traditional turbine flowmeter can be decreased and the sensitivity can be increased. The conclusion was discovered from theoretical analysis. In addition, the CFD simulation was carried out to verify the conclusion.

Nomenclature			
T_d	driving torque [N·m]	T_b	bearing drag torque [N·m]
$\sum f$	sum of all drag torque [N·m]	K	The sensitivity of the flowmeter [1/m ³]
β	helix angle of turbine [°]	u_i	the mean velocity in the i -direction [m/s]
ω	rotation speed of turbine [rad/s]	α_2, β_2	empirical coefficients
F	force on turbine blades [N]	g_i	i -component of gravitational acceleration [m/s ²]
θ	helixangle of front guide vane [°]	f_i	the i -component of the drag force [N]
r_1	wheel shell radius [m]	T_t	tip clearance drag torque [N·m]
r_2	top blade radius [m]	T_h	hub fluid drag torque [N·m]
f	light impulse frequency [Hz]	T_w	hub disc drag torque [N·m]
Z	number of turbine blades	μ_t	turbulence dynamic viscosity [pa·s]
A	sectional area of pipeline [m ²]	σ_k	empirical constant
Q_v	volume flow rate [m/s]	$\sigma_{\omega 2}$	empirical constant
Q_{Vs}	starting volume flow rate [m ³ /h]	P	perimeter of the opening [m]
ρ	density of fluid [kg/m ³]	R	radius of pipeline [m]
P_k	turbulence diffusion term	V	velocity of fluid [m/s]
P_ω	turbulence diffusion term	F_1, F_2	blending function

2. Structure Design and Theoretical Analysis

2.1 Basic Structure and Working Principles

The structure of the modified flowmeter is consisted of metal shell, front guide vane, back guide vane, turbine, Y-type optical fiber, illuminant and photoelectric detector, as shown in Fig. 1.

Fig. 1 structure of the modified flowmeter

As shown in Fig. 2, the front guide vane was specially designed with a helix angle which traditional front guide vane didn't have. In addition, the front guide vane provides structural support. After the process of polishing, the blades edge of turbine can reflect light. The shape of turbine and the back guide vane are shown in Fig. 3. The helix angle of front guide vane plus the helix angle of turbine equals to 90° . The back guide vane has the effect of the rectifier and provides structural support, too.

Fig. 2 diagram of front guide vane

Fig. 3 diagram of turbine and back guide vane

The working principle of primary sensor is as follows. Under the condition of axial flow direction, fluid flow through the front guide vane firstly. The velocity direction of fluid changes because of the front guide vane. Then, turbine rotates with the impact of fluid. Rotation speed is proportional to the average flow velocity. Finally, fluid flow through the back guide vane. After a long enough distance, the direction of flow restore to the original axial direction.

Fig. 4 schematic diagram of optical fiber sensor

The role of secondary sensor is played by the optical fiber sensor, as shown in Fig. 4. The light which comes from the illuminant pass through the branch of Y-type optical fiber to optical fiber flange. When turbine blades come to the underneath of the optical fiber flange, photoelectric detector receives a light impulse. **Light impulse frequency divided by the number of blades is turbine speed. As a result, the value of flow rate can be obtained with the turbine speed.**

2.2 Theoretical analysis

With the fluid flowing through the turbine, driving torque T_d will be produced by the impact of fluid. The turbine will begin to rotate when driving torque overcomes fluid viscous drag torque, bearing drag torque, etc(13). When the turbine flowmeter is operating under steady flow conditions, all torques acting on the rotor must be balanced. Let the sum of all drag torque as $\sum T_f$. The torque balance equation of the turbine flowmeter can be described as:

$$T_d = \sum T_f \quad (1)$$

Fig. 5 planar vector triangle of flow velocities

According to the structure of flowmeter, the planar vector triangle of flow velocities was established, as shown in Fig. 5. Along the blade height direction, the torque that blade surface element bears at radius r can be expressed as:

$$dT_d = r dF \quad (2)$$

Where dF is the force on turbine blades. According to the theorem of momentum, dF is given by:

$$dF = \rho V (2\pi r dr) [V(\cos \theta \tan \beta + \sin \theta) - r\omega] \quad (3)$$

Where ρ is fluid density. The integral of right side of equation (3) from wheel shell radius r_1 to top blade radius r_2 equals to the driving torque of turbine. Thus, driving torque T_d can be expressed as:

$$T_d = \int_{r_1}^{r_2} \rho V [V(\cos \theta \tan \beta + \sin \theta) - r\omega] (2\pi r) dr = \frac{2}{3} \rho V^2 \pi (\cos \theta \tan \beta + \sin \theta) (r_2^3 - r_1^3) - \frac{1}{2} \pi \rho V \omega (r_2^4 - r_1^4) \quad (4)$$

According to equations (1) and (4), turbine speed ω can be described as:

$$\omega = \left[\frac{4}{3} (\cos \theta \tan \beta + \sin \theta) \frac{r_2^3 - r_1^3}{r_2^4 - r_1^4} - \frac{\sum T_f}{\frac{1}{2} \pi \rho V^2 (r_2^4 - r_1^4)} \right] V \quad (5)$$

In addition, the light impulse frequency f is given by:

$$f = \frac{Z}{2\pi} \omega \quad (6)$$

Where Z is the number of turbine blades. The volume flow rate Q_v and flow rate V are linked through:

$$Q_v = A \cdot V \quad (7)$$

Where A is the cross sectional area of the pipe. According to equations (5), (6) and (7), Q_v can be expressed as follows.

$$Q_v = \frac{1}{\frac{4}{3}(\cos \theta \tan \beta + \sin \theta) \frac{r_2^3 - r_1^3}{r_2^4 - r_1^4} - \frac{\sum T_f}{\frac{1}{2}\pi\rho V^2(r_2^4 - r_1^4)}} \cdot \frac{2\pi f A}{Z} \quad (8)$$

The above equation is the theoretical relation between the volume flow rate and the light frequency.

2.3 Characteristic Analysis

2.3.1 Start characteristic

Turbine will start to rotate when the volume flow rate gets close to starting flow. At the same time, bearing drag torque T_b is much larger than others. Therefore, other drag torque can be ignored. At the moment, ω equals to zero. According to equation (1), (5) and (8), the starting flow of turbine is as follows:

$$Q_{vs} = A \cdot \sqrt{\frac{T_b}{\frac{2}{3}\pi\rho(\cos \theta \tan \beta + \sin \theta)(r_2^3 - r_1^3)}} \quad (9)$$

Where Q_{vs} is the starting volume flow rate of flowmeter. In order to decrease the starting flow of turbine, the way that increasing value of $\cos\theta \cdot \tan\beta + \sin\theta$ can be adopted. Defining $G(\theta)$ as $\cos\theta \cdot \tan\beta + \sin\theta$, that is:

$$G(\theta) = \cos \theta \tan \beta + \sin \theta \quad (10)$$

The maximum of $G(\theta)$ can be calculated by the derivation of equation (10). At the moment, the helix angle of front guide vane θ is:

$$\theta = 90^\circ - \beta \quad (11)$$

Therefore, if the helix angle of front guide vane plus the helix angle of turbine equals to 90° , the starting volume flow rate of turbine can get minimum value in the case of other parameters are fixed.

2.3.2 Sensitivity characteristic

According to equation (8), the sensitivity K of the flowmeter can be expressed as follows.

$$K = \left[\frac{4}{3}(\cos \theta \tan \beta + \sin \theta) \frac{r_2^3 - r_1^3}{r_2^4 - r_1^4} - \frac{\sum T_f}{\frac{1}{2}\pi\rho V^2(r_2^4 - r_1^4)} \right] \cdot \frac{Z}{2\pi A} \quad (12)$$

In a similar way of section 2.3.1, if the helix angle of front guide vane plus the helix angle of turbine equals to 90° , the sensitivity of the flowmeter can get maximum value

in the case of other parameters are fixed.

3. CFD Simulation

With the development of computational fluid dynamics (CFD) theory and computer technology, CFD has become an effective method to research flow distribution instead of experiment. Therefore, the CFD simulation was undertaken by the software (COMSOL) to verify the correctness of the above conclusion.

3.1 Governing equations

The basic equations for viscous incompressible and steady flow in Cartesian coordinates is solved by the software. The basic equations are well-known, such as continuity equation, momentum equation. For the sake of simplicity, the basic equations are described briefly as follows.

The continuity equation is mass conservation equation. The mass conservation equation and momentum equation can be described as(14):

$$\frac{\partial u_i}{\partial x_j} = 0 \quad (13)$$

$$u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\nu \frac{\partial u_i}{\partial x_j} \right) - \frac{\partial (u_i' u_j')}{\partial x_j} - f_i + g_i \quad (14)$$

Where u_i is the mean velocity in the i -direction, $u_i' u_j'$ is the Reynolds stress, g_i is the i -component of gravitational acceleration, and f_i is the i -component of the drag force.

3.1.1 Torque balance equation

When the flowmeter is working under steady flow conditions, all torques acting on the rotor must be balanced. The torque balance equation can be described as(13):

$$T_d = T_t + T_b + T_h + T_w \quad (15)$$

Where T_d , T_t , T_b , T_h , T_w are driving torque, tip clearance drag torque, bearing drag torque, hub fluid drag torque and hub disc drag torque. When the flowmeter is working normally, the T_h and T_w compared to other drag torque are small. Therefore, the T_h and T_w can be ignored.

According to torque balance equation, there must a certain rotational speed ω of turbine for specified inlet velocity. If rotational speed ω is not suitable, the resultant moment can not equal to zero. In simulation, the rotational speed is adjusted by monitoring the value of resultant moment. If the value of resultant moment is less than 10^{-3} , turbine can be regarded as balanced. The corresponding rotational speed ω is reasonable speed.

3.1.2 Turbulence model

k- ϵ and k- ω model are two of the most widely used turbulence model. k- ϵ model is used for the fluid with high Reynolds number. The flow in the near wall region is not fully developed with low Reynolds number. Therefore, k- ϵ model can not guarantee calculation accuracy in the near wall region. With the vane-blade arrangement proposed, flow separation and re-attachment will be increased in the near wall region.

In order to make simulation results close to actual conditions, the SST k- ω model of Menter was used to simulate inner flow field of the purposed flowmeter(15). The SST k- ω model not only captures the core flow and the boundary anisotropy well, but also captures the distribution of the pressure loss better. The equation of the model are described as follows.

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho u_i k)}{\partial t} = P_k - \frac{\rho k^{3/2}}{l_{k-\omega}} + \frac{\partial}{\partial x_i} \left[\left(\mu + \frac{\mu_t}{\delta_k} \right) \frac{\partial k}{\partial x_i} \right] \quad (16)$$

$$\begin{aligned} \frac{\partial(\rho \epsilon)}{\partial t} + \frac{\partial(\rho u_i \epsilon)}{\partial t} = & \alpha_2 \frac{\omega}{k} P_\omega - \beta_2 \rho \omega^2 + \frac{\partial}{\partial x_i} \left[\left(\mu + \frac{\mu_t}{\delta_{\omega 2}} \right) \frac{\partial k}{\partial x_i} \right] + \\ & 2\rho(1-F_1)\delta_{\omega 2} \frac{1}{\omega} \frac{\partial k}{\partial x_i} \frac{\partial \omega}{\partial x_i} \end{aligned} \quad (17)$$

$$\mu_t = \frac{\rho_m \alpha_1 k}{\max(\alpha_1 \omega, SF_2)} \quad (18)$$

Where P_k , P_ω are turbulence diffusion terms, F_1 , F_2 are blending functions, S is shear stress tensor constant term, δ_k , α_2 , β_2 , $\delta_{\omega 2}$ are empirical coefficients, ρ is the density of fluid and μ_t is turbulence dynamic viscosity.

3.2 Mesh and boundary conditions

In order to make the fluid state in flowmeter inlet and outlet close to full development, the upstream and downstream pipe lengths are 10 times of the inlet diameter. As shown in Fig. 6, the computational domain is divided into five parts: the front straight pipe, the front guide vane, the rotor, the back guide vane and the back straight pipe. The region of turbine was set to rotating region and other region which included front guide vane, back guide vane and pipe was set to be static area.

Firstly, a grid-independent numerical simulation is needed to analyze the simulation results. The study used five different grid densities to find the best matched grid which can make the simulation results independent of the computational grid. Table 1 shows the results of the grid independence study in terms of inlet velocity is 1m/s, helix angle of front guide vane is 0° and helix angle of turbine is 30°. For the purpose of saving calculation time with high accuracy, the total number of 2.2 million cells was chosen for the CFD simulation. The grid interval of upstream and downstream was relatively larger than grid interval of middle terms. The meshed computational grid of turbine is

shown in Fig. 7 .

Because the region of turbine was rotary, other machine parts were set to be stator and real walls. The helix angle β of turbine was set to 30° , 45° and 60° . When the helix angle β of turbine was set to 30° , the helix angle θ of front guide vane was set to 0° , 30° , 45° , 60° , 75° . When the helix angle β of turbine was set to 45° and 60° , the helix angle θ of front guide vane was set to 0° , 30° , 45° , 60° .

Table 1 Grid independence results in terms of light impulse frequency for 1(m/s) flow rate.

Grid cells number (million)	0.8	1.2	1.8	2.2	2.6
Calculated frequency (Hz)	62.74	63.42	66.85	68.96	69.12

Fig. 6 the total structural model

Fig. 7 mesh in region of turbine

The inlet flow rate was set to 1 m/s, 2 m/s, 3 m/s, 4 m/s, 5 m/s. According to turbulence balance equation, the rotational speed ω of turbine could be found. Using the rotational speed ω of turbine times the number of turbine blades, the light impulse frequency f could be got.

3.3 Results and discussion

The simulation results of light impulse frequency in different conditions of inlet flow rate and helix angle of front guide vane and turbine was shown in table 2, 3 and 4.

From table 2, 3 and 4, the relation curves of light impulse frequency and inlet flow rate in different conditions can be obtained, as shown in Fig. 8.

Table 2 light impulse frequency in terms of helix angle β of turbine was set to 30°

θ	1 m/s	2 m/s	3 m/s	4 m/s	5 m/s
0°	68.96 Hz	137.12 Hz	205.68 Hz	274.52 Hz	343.75 Hz
30°	121.58 Hz	243.16 Hz	363.96 Hz	486.54 Hz	606.89 Hz
45°	134.42 Hz	268.56 Hz	402.88 Hz	535.64 Hz	671.52 Hz
60°	141.62 Hz	283.18 Hz	423.86 Hz	566.08 Hz	707.86 Hz
75°	132.64 Hz	265.14 Hz	397.42 Hz	529.84 Hz	663.04 Hz

Table 3 light impulse frequency in terms of helix angle β of turbine was set to 45°

θ	1 m/s	2 m/s	3 m/s	4 m/s	5 m/s
0°	123.53 Hz	247.21 Hz	369.86 Hz	494.14 Hz	617.54 Hz
30°	169.45 Hz	338.52 Hz	508.42 Hz	676.92 Hz	847.06 Hz
45°	175.85 Hz	351.36 Hz	527.75 Hz	703.68 Hz	879.48 Hz
60°	167.34 Hz	333.58 Hz	502.45 Hz	669.38 Hz	836.62 Hz

Table 4 light impulse frequency in terms of helix angle β of turbine was set to 60°

θ	1 m/s	2 m/s	3 m/s	4 m/s	5 m/s
0°	218.64 Hz	437.14 Hz	655.52 Hz	874.64 Hz	1093.05 Hz
30°	252.58 Hz	505.62 Hz	757.24 Hz	1010.74 Hz	1262.58 Hz
45°	238.84 Hz	477.54 Hz	716.18 Hz	955.36 Hz	1194.72 Hz
60°	215.65 Hz	431.50 Hz	646.86 Hz	862.18 Hz	1078.48 Hz

As shown in Fig. 8, when the helix angle of front guide vane equals to 0° , the front guide vane is the traditional straight guide vane. In addition, the light impulse frequency got maximum value in different situation of inlet flow rate when the helix angle of front guide vane plus the helix angle of turbine equals to 90° .

The simulation results of sensitivity can be got with different value of helix angle of front guide vane and turbine from Fig. 8. At the same time, the sensitivity K of the flowmeter got maximum value when the helix angle of front guide vane plus the helix angle of turbine equals to 90° . From the theoretical analysis of section 2.2, the fundamental cause of the maximum value of sensitivity and light impulse frequency was the increase of driving torque. With the maximum value of driving torque, the starting volume flow rate of turbine can also get minimum value in same conditions.

(a) $\beta=30^\circ$

(b) $\beta=45^\circ$

(c) $\beta = 60^\circ$

Fig. 8 Relation curves of light impulse frequency and inlet flow rate

In addition, the pressure loss across the meter was obtained from the CFD simulation. The results was shown in Fig. 9.

Fig. 9 Relation curve of pressure loss and helix angle of front guide vane

As shown in Fig 9, if the helix angle of turbine blades is fixed, the pressure loss would increase rapidly with the increase of angle of front guide vane. Similarly, if the helix angle of front guide vane is fixed, the pressure loss would increase rapidly with the increase of angle of turbine blades. One cause of the phenomenon is the flow area decreases with the increase of angle of front guide vane and turbine blades. Meanwhile, some pressure was consumed on the momentum of turbine. So the pressure loss increased with the increase of turbine rotational speed. Thus it can be seen that the pressure loss would increase with the the increase of sensitivity and this is the negative effect of the angle increase.

4. Conclusion

In this paper, a modified structure of optical fiber turbine flowmeter has been designed. In addition, the theoretical model has been established based on the planar vector triangle of flow velocities. Through the theoretical analysis, the conclusion that if the helix angle of front guide vane plus the helix angle of turbine equals to 90° , the starting volume flow rate of turbine can get minimum value and the sensitivity of the flowmeter can get maximum value was discovered. In order to verify the conclusion, the CFD simulation has been carried out. From the results of simulation, the conclusion

is correct.

The pressure loss would increase with the improvement of sensitivity and this is the negative effect of the angle increase. A turbine meter is valued because of the low pressure loss and high accuracy. For practical application, sensitivity and pressure loss must be overall considered and the helix angle of front guide vane can be increased appropriately in the case of pressure loss is allowed.

The paper focused on the effects of helix angle of front guide vane and turbine on flowmeter characteristic. In the CFD simulation, the bearing drag torque was neglected. In actual application, the bearing drag torque is an important factor which affects flowmeter characteristic and it needs further research.

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