A dynamic reward-based incentive mechanism: reducing the cost of P2P systems

Kun Lu^a, Shiyu Wang^a, Ling Xie^a, Zhen Wang^{a,b}, Mingchu Li^{a,*}

^a116620, Software School, Dalian University of Technology, Liaoning Province, China

^bSchool of Computer Engineering, Nanyang Technological University, Singapore 639798

Abstract

Reward-based incentive mechanisms are used most commonly to promote cooperation in peer-to-peer (P2P) systems. Specifically, contributing agents are rewarded by the system. For a centralized P2P system, the central server is responsible for implementing the mechanism and rewarding those cooperative agents. Thus, providing incentives could be costly for the system server. In this paper, we propose a dynamic reward mechanism in which the reward changes accordingly as the system evolves, i.e., when there are more cooperative agents, the reward for each agent decreases, and when there are no free-riders, rewards are no longer needed. Through theoretical analysis using replication dynamics, we determined the evolution stable strategy (ESS) for different scenarios and qualitatively proved that our dynamic reward mechanism can reduce the overall cost of the system. The simulation proved that this dynamic reward mechanism can promote cooperation and reduce the cost of the system.

Keywords: incentive mechanism, dynamic reward, replicator dynamics, evolutionary game

1. Introduction

Peer-to-peer (P2P) communication technologies have been used extensively in file sharing [1] and video streaming applications [2, 3] due to their low bandwidth requirement and high flexibility. By definition, the stability

*Corresponding author.

Email address: mingchul@dlut.edu.cn (Mingchu Li)

5 of such systems relies on the voluntary contributions of agents. However, the
6 fundamental principle for these systems is to accommodate participants ra-
7 tional behaviors and design for choice [4]. These rational and strategic agents
8 will choose to share only if they have better long-term benefits. This leads
9 to the well-known free-riding problem [5], which degrades the performance
10 of a P2P system tremendously.

11 Incentive mechanisms are used to solve the lack of cooperation problem
12 in P2P systems [6]. Among these mechanisms, reward-based incentives are
13 used most commonly [7, 8, 9, 10, 11, 12, 13, 14, 15]. Generally, when reward
14 mechanisms are used, contributing agents may get additional rewards. Typ-
15 ically, reward mechanisms can be divided into two types, i.e., peer rewards
16 and institution rewards [16]. For peer rewards, agents exist who voluntarily
17 help to reward cooperative peers. But this method requires that agents have
18 great power to recognize and reward others, which means it is not realistic
19 in a real system. However, in the case of institution rewards, cooperative
20 agents are rewarded by a central server. The server tracks peers behaviors
21 and rewards them accordingly. However, since the system gives the rewards
22 in the institution reward mechanism, doing so can be costly to the server.
23 Thus, saving costs in the system under the premise of enhancing cooperation
24 is challenging.

25 In this paper, we propose a dynamic reward incentive mechanism. In
26 this mechanism, the reward is determined by the system according to the
27 proportion of agents with each strategy, i.e., when there are few cooperative
28 agents, each cooperative agent receives a greater reward, but when there are
29 more cooperative peers, each agent's reward is reduced. Thus, when all of
30 the agents in the system are cooperative, no one receives a reward. The
31 contributions of this work are as follows:

- 32 (1) A dynamic reward method is proposed that has a corrected reward
33 function, and the proportion of each strategy is considered to determine
34 the reward;
- 35 (2) Through our extensive theoretical analysis, we derive the evolutionary
36 stable strategy under various scenarios;
- 37 (3) The qualitative analysis and the simulation prove that our proposed
38 mechanism can promote cooperation effectively and reduce the cost of
39 the system.

The rest of the paper is structured as follows. In Section. 2, we briefly review related work on reward-based incentive mechanisms. In Section. 3, we introduce our transaction game model and dynamic reward function. In Section. 4, we use mathematical methods to predict the stable state of evolution. In Section. 5, the results of the simulation and performance analysis are presented. In Section. 6, our conclusions are presented.

2. Related Work

Providing incentives is one of the challenging issues in P2P systems. The concept of incentive mechanisms is borrowed from economics and management and applied in computer science. The core thoughts of incentive mechanisms always target a specific behavior that is desirable in the system [7]. The essence is to help cooperative agents gain more benefit in order to encourage cooperation.

One of the most commonly-used approaches is to provide rewards and punishment. Agents who are willing to share can get a reward, while free-riders may be penalized. Although punishment is more effective than reward [10], sanctioning a penalty may result in "anti-social problems" [17]. Vice versa, rewards may lead to cooperation without harming agents' reputations or provoking any retaliation [18]. Thus a reward mechanism is healthier to achieve cooperation with less probability of any destructive results [19].

Generally, the source of rewards can be divided into two categories, i.e., peer rewards and institution rewards. For peer reward mechanisms, some volunteer peers act as "police" or "managers". They share their own resources and voluntarily reward contributing agents. However, such mechanisms require agents that are able to recognize and understand other agents' behaviors and that have the authority to reward (or punish) these agents [20, 21]. It is difficult to achieve in a real system. Different from the peer reward approach, the institution reward mechanism uses a central server to track agents' behaviors and implement the reward mechanism. As in the institution reward mechanism, the rewards are given and managed by the central server, a mechanism that can be costly to the server. Especially, when a large number of cooperators must be rewarded, it can be either far too costly or ineffective. Thus, saving costs in the system under the premise of promoting cooperation is difficult to achieve. Ref [22] provides a novel insight in dynamic reward and punish mechanism by proposed "First carrot, then stick." The main idea is to reward or punish the minority peers in the

76 system. However, this method must switch from reward to punishment, and
77 properly setting the threshold switching mechanism is a great challenge. In
78 addition, the settings for reward and punishment must be decided before
79 implementing the mechanisms, and it is difficult to ensure the reward and
80 punishment approach will always work well as the system evolves.

81 Peers in P2P systems are rational and strategic [23]. They participate in
82 the system to maximize their benefits. Classic game theory, as a effective tool
83 to model and explain rational behaviors under conflicts between individual
84 and public interests, has been widely applied to the study and formal analysis
85 of cooperation in P2P networks [24, 25, 26]. In the classical game theoretic
86 method, peers can get their best strategies by global knowledge, which
87 requires great cognitive abilities (e.g., abilities of getting information of all
88 global transactions and strong abilities of analyzing and recognizing). Such
89 requirement is unfeasible in most cases. Due to the limited time for making
90 decisions, lack of information or other unknown reasons, peers may have
91 the probabilities making non-optimal decisions [27, 28]. This phenomena is
92 called bound-rational. Moreover, classic game theory cannot characterize the
93 evolutionary dynamics of peers strategies and how the strategies spread in
94 the system. Recently, evolutionary game theory is used to analyze incentive
95 mechanisms to overcome above drawbacks [29]. By leveraging evolutionary
96 game theory, peers only need to learn and imitate more successful strategies
97 of their neighbors. Such a simple imitative mechanism is easy to implement
98 by peers limited knowledge to optimize their strategies, and more detailed
99 insights into how the strategies spread in the network can be obtained. Generally,
100 EGT [30] approaches consist of three major phases, i.e., interaction,
101 evolution, and mutation [31]. First, agents play games according to the rules
102 specified by the system, and then each agent changes its strategy with the
103 same rule. However, due to errors in the system or propagation issues, agents
104 may change their strategies inappropriately, causing a small probability
105 of mutation occurring. A general platform is required to evaluate incentive
106 mechanisms. Zhao, et al. [32] provided a general framework for analyzing
107 and evaluating three classic incentive mechanisms in well-mixed networks.
108 In this paper, we focused on designing a dynamic reward mechanism based
109 on EGT and test the effectiveness using the evaluation framework proposed
110 by Zhao [32]. With this mechanism, a system effectively can encourage agents
111 to cooperate, thereby reducing the cost associated with the system because
112 no rewards are required when there are no free-riders to be eliminated.

113 3. System Model

114 3.1. System Description

115 In this paper, we consider a mixed structure P2P system with a special
 116 peer which is used to maintain the system resource, user information and
 117 performance, as shown in Fig. 1. After a period of transactions, the server
 118 checks the transaction records of each agent and rewards those who are willing
 119 to share resources.

Figure 1: Mixed P2P structure with reward mechanism

120 A real autonomous system consists of N different agents. Each agent
 121 i is equally regarded as a service provider and a service requester in one
 122 transaction. A transaction may occur when two agents interact with each
 123 other, i.e., one agent requests a service and its opponent responds. Each time
 124 an agent provides a service, it suffers a cost, β . On the contrary, a requester
 125 gets a benefit, α , when receiving a service. Participating in such systems
 126 may have other costs, such as the cost associated with acquiring a requester's
 127 transaction history. Therefore, the general average payoff of each agent, P_i ,
 128 is related to the probability of its serving other agents ($Prob_{serve}$), getting
 129 service ($Prob_{get}$), and other additional costs (extra cost). The equation used
 130 to calculate payoff is shown as Eq. 1. For analysis simplicity, we set $\beta = 1$.

$$P_i = \alpha * Prob_{get} - \beta * Prob_{serve} - extra_cost \quad (1)$$

131 Decentralized systems, such as the structure we use, can be classified
 132 into two scenarios: well-mixed and structured systems [34]. In a well-mixed
 133 population, an agent can interact with any other agent with equal probability
 134 in the system. Thus, we assume agents with same strategy share a common

average payoff, and P_i denotes the average payoff of every agent with strategy i . However, in a structured network, an agent can only interact with its direct neighbors, and the probability of an agent meeting another one agent is influenced by network structure. Therefore, even if agents have the same strategy, they may have different average payoffs. Thus, in a structured system, the parameter P_i stands for the average payoff of agent i .

3.2. Transaction Game

A transaction game is described as a symmetric game between two agents. In brief, we consider the three most common strategies, i.e., C, R, and D. The behaviors of those who use these strategies are depicted as follows (Fig.2): C users (cooperators) always grant a service without considering the history of transactions; D users (defectors) always deny a service; R users always are called reciprocators, because they grant services to cooperators and reciprocators and deny defectors requests [35]. According to the definition of the game model, we write the payoff matrix A as Eq. 2. Each item a_{ij} of this matrix symbolizes the payoff of one agent with strategy S_i when it meets another agent with strategy S_j . Take the item a_{21} for instance: the value is the payoff of a reciprocator when it meets a cooperator. If a reciprocator meets a cooperator, it will always grant a service to the cooperator and get a service from the cooperator. Thus it can obtain a benefit $\alpha - 1$. However, to know the opponent's strategy, the reciprocator has to search the server for information. So it has to bear an extra cost C_R . Therefore, the total payoff of a reciprocator in this transaction is $\alpha - 1 - C_R$. Since both cooperators and reciprocators provide services, we call them cooperative agents, and defectors are called defective agents.

Figure 2: Description of the transaction game

$$A = \begin{matrix} & C & R & D \\ \begin{matrix} C \\ R \\ D \end{matrix} & \begin{pmatrix} \alpha - 1 & \alpha - 1 & -1 \\ \alpha - 1 - C_R & \alpha - 1 - C_R & -C_R \\ \alpha & 0 & 0 \end{pmatrix} \end{matrix} \quad (2)$$

160 3.3. Reward Mechanism

161 The existence of P2P systems strongly relies on the voluntary of agents.
 162 However, sharing is costly. To encourage sharing behaviors, a reward mechanism is needed to help cooperative agents cover the cost of sharing resources[14].
 163 Rewards can be modeled as quantitative changes in parameters associated
 164 with an agent[7]. In this paper, we adopt an institutional reward mechanism: the reward is given by the central server to those cooperative agents (C
 165 and R) for their willingness to help other agents. Each time an agent shares
 166 resources with another agent, the central server gives the agent a reward in
 167 praise of voluntary behavior and help covering sharing costs. As the reward
 168 is given by the server, the server has to bear the cost. Thus, to alleviate
 169 the cost of giving rewards, we designed a dynamic reward-based incentive.
 170 Bearing this reward mechanism, the payoff matrix (A) can be modified as
 171 Eq. 3. As mentioned earlier, the system must bear a cost, C_S , the value
 172 of which can be calculated as Eq. 4, where N_C and N_R are the number of
 173 cooperators and reciprocators in the system, respectively.
 174
 175

$$A = \begin{matrix} & C & R & D \\ \begin{matrix} C \\ R \\ D \end{matrix} & \begin{pmatrix} \alpha - 1 + rd & \alpha - 1 + rd & -1 + rd \\ \alpha - 1 + rd - C_R & \alpha - 1 + rd - C_R & rd - C_R \\ \alpha & 0 & 0 \end{pmatrix} \end{matrix} \quad (3)$$

$$C_{S_F} = rd * (N_C + N_R) \quad (4)$$

176 3.4. Strategy Dynamics

177 Updating the strategy is a critical process in the evolution of the population.
 178 Those agents who have better payoff are favored by the system and have
 179 higher probability of surviving in the following transactions. In this paper,
 180 we used the most common pairwise comparison update rule, i.e., the Fermi
 181 update [36, 37, 38, 39, 40]. After each round, an agent i randomly selects
 182 another agent j in the system (a neighbor in a structured network). Player

183 i will learn to change its strategy to player j 's strategy with and calculate
184 the probability of changing its strategy to that of agent j by using the Fermi
185 function (Eq. 5). The parameter w is the selection intensity. The larger w
186 becomes, the faster the evolution proceeds. To make sure the system evolves
187 smoothly, we set $w = 10$.

$$p_{i \rightarrow j} = \frac{1}{1 + \exp[w(p_i - p_j)]} \quad (5)$$

188 4. Theoretical Analysis

189 Replicator dynamics [41] are the most common mathematical tools that
190 are used to analyze the evolution of the population. A core concept is fitness,
191 which represents the capability an agent to survive in the system. The evo-
192 lution of the population can be expressed by a differential equation (Eq. 6):

$$\dot{x} = x_i(f_i - \phi), \text{ where } \sum_i x_i = 1 \quad (6)$$

193 In this equation, f_i is the fitness of strategy, S_i , and $f_i = \sum_i x_i a_{ij}$. The
194 parameter ϕ is the average fitness, and $\phi = \sum_i x_i f_i$. If the fitness of strategy,
195 S_i , f_i , is larger than the system average fitness, ϕ ($f_i > \phi$), the population of
196 S_i will grow and survive in the long run. On the contrary, the worse strategy
197 will decrease and finally disappear. Thus, to analyze the evolutionary stable
198 strategy (ESS), we need only to compare the fitness of each strategy and find
199 the threshold of each result. The notations used in this paper are listed in
200 Table. 1.

Table 1: Definition of notations

Symbol	Definition
N	number of agents in the system
α	benefit when getting a service
β	cost when providing a service
C_R	cost for reciprocators to seek for information
rd	reward given to cooperative agents
i	an agent in the system
S_i	strategy i
x_i	the proportion of agents with strategy S_i
P_i	the payoff/fitness of strategy S_i
w	selection intensity
$p_{i \rightarrow j}$	the probability that agent i will use j 's strategy

201 4.1. Well-mixed Population

202 In a well-mixed population, each agent can interact with any other agent
 203 with equal probability. So, the average payoff is directly related to the pro-
 204 portion of each strategy. Let x_0 , x_1 , and x_2 denote the proportion of C, R,
 205 and D agents, respectively. Then, the average payoff of each strategy can be
 206 written as Eq. 7:

$$P_0 = \alpha(x_0 + x_1) - 1 \quad (7a)$$

$$P_1 = \alpha(x_0 + x_1) - (x_0 + x_1) - C_R \quad (7b)$$

$$P_2 = \alpha x_0 \quad (7c)$$

207 4.1.1. No Reward

208 Without considering the cost reciprocators seeking information ($C_R = 0$),
 209 we can derive the respective payoff as follows:

$$P_0 = \alpha(x_0 + x_1) - 1$$

$$P_1 = \alpha(x_0 + x_1) - (x_0 + x_1)$$

$$P_2 = \alpha x_0$$

210 Based on the above expressions, we have the following important obser-
 211 vations:

- 212 (1) As the summation of $x_0 + x_1$ is always less than or equal to 1, there
 213 will always be $P_0 \leq P_1$. Thus, to eliminate defectors, we need $P_1 > P_2$
 214 or $P_0 > P_2$. If $P_1 > P_2$, we have $\alpha > \frac{x_0+x_1}{x_0}$; if $P_0 > P_2$, we have $\alpha > \frac{1}{x_1}$.
- 215 (2) When $\alpha < \frac{x_0+x_1}{x_1}$, we have $P_2 > P_1 > P_0$. In other words, defectors have
 216 the best fitness, therefore defecting is the best strategy until $\alpha = \frac{x_0+x_1}{x_1}$.
- 217 (3) When $\frac{x_0+x_1}{x_1} < \alpha < \frac{1}{x_1}$, we have $P_1 > P_2 > P_0$, cooperators will evolve
 218 to defectors and reciprocators. When there are only defectors and
 219 reciprocators, if $\alpha > 1$, the ESS is R; When $\alpha = 1$, neutral drift occurs
 220 between reciprocators and defectors.
- 221 (4) If $\alpha > \frac{1}{x_1}$, we have $P_1 > P_2 > P_0$, therefore cooperators and reciproca-
 222 tors co-exist at stable status.

223 In general, when $C_R = 0$, for any $\alpha > 1$, R is the only ESS.

224 However, in a real system, to get information, an agent must incur a
 225 certain cost. When $C_R > 0$, $p_1 = \alpha(x_0 + x_1) - (x_0 + x_1) - C_R$, and defectors
 226 can be wiped out only if $C_R < (\alpha - 1)x_1 - x_0$. When there are only D and
 227 R, $x_0 = 0$, the inequation becomes $C_R < (\alpha - 1)x_1$.

228 4.1.2. Fixed and dynamic reward

229 By involving the reward mechanism, cooperator and reciprocators can
 230 get an additional benefit from the system. Thus, the payoff equations can be
 231 modified as follows:

$$\begin{aligned} P_0 &= \alpha(x_0 + x_1) - 1 + rd \\ P_1 &= \alpha(x_0 + x_1) - (x_0 + x_1) - C_R + rd \\ P_2 &= \alpha x_0 \end{aligned}$$

232 In this case rd is the reward value set by the system administrator before
 233 the system starts. To eliminate side effect of defectors, we should ensure coop-
 234 erative agents get best payoff. Thus, using the same method as in Sec. 4.1.1,
 235 we can get the following results:

- 236 (1) If $P_0 > P_1 > P_2$, we have $C_R > x_2$ and $rd > C_R + x_0 + (1 - \alpha)x_1$.
 237 Therefore, defectors will learn to behave cooperatively. When $rd =$
 238 $C_R + x_0 + (1 - \alpha)x_1$, neutral drift occurs between reciprocators and

defectors. However, these two strategies both evolve to cooperating strategy. Thus cooperators will dominate until the inequation isn't satisfied.

(2) If $P_1 > P_0 > P_2$, we have $C_R < x_2$ and $rd > 1 - \alpha x_1$. When $rd = 1 - \alpha x_1$, neutral drift occurs between cooperators and defectors. However, these two strategies both evolve to the reciprocative strategy. As x_1 increases, we'll always have $rd > 1 - \alpha x_1$. Thus, R strategy will be the ESS.

(3) When neither of the former two cases are fulfilled, D will dominate until one of the first two cases is satisfied and it returns to one of the first cases.

From the aforementioned fixed reward, we know that, if rd always satisfies the first two cases, we can make sure the defectors are defeated. However, no matter $P_0 > P_1$ or $P_0 < P_1$, if we can ensure that $P_1 > P_2$, the defectors will involve to reciprocators, then we can get full cooperative agents. Thus, to reduce the cost of the system and ensure the effectiveness of the incentive, we use a dynamic reward function. Each time the system incurs a reward, it calculates the real-time reward, as shown in Eq. 8. So the calculation of the systems cost can be modified as Eq. 9, where N_0 and N_1 are the number of cooperators and reciprocators, respectively, and $\delta > 0$.

$$rd(x_0, x_1) = \max(1 - \alpha x_1 + \delta, 0) \quad (8)$$

$$C_{S_D} = rd(x_0, x_1) * (N_0 + N_1) \quad (9)$$

In the scenario with dynamic reward, the system cost becomes a function of the proportion of defectors (Eq. 9). Thus, the cost difference between a dynamic and a fixed reward ($\Delta = C_{S_D} - C_{S_F}$) can be expressed as Eq. 10.

$$\Delta = [(x_0 - x_0^0) + (1 - \alpha)(x_1 - x_1^0)](N_0 + N_1) \quad (10)$$

As analyzed before, when $C_R > x_2$, C agents evolve to R and D agents, and D agents evolve to R agents. So, x_0 decreases, and x_1 increases. So the inequation is $\Delta < 0$, which means that using a dynamic reward can reduce the cost of the system. When $C_R \leq x_2$, the proportion of reciprocators keeps increasing, thus Δ will always be less than 0. Therefore, in general, with

Figure 3: Example of a defector's location and payoff

dynamic reward, the cost of the system for each agent can be reduced, and the the total payment for rewards also can be reduced in the long run.

4.2. Structured Networks

Different from a well-mixed population, in a structured network, agents can only interact with their neighbors. Thus, the location of an agent can influence the interaction among agents. For instance, a defector that is surrounded by four defectors can get no payoff (Fig. 3(a)). However, a defector surrounded by one cooperator and three agents with other strategies may get an $\alpha/4$ average payoff (Fig. 3(b)). Therefore, the correlation of two adjacent agents strategies should be taken into account.

To analyze the evolution on a structured network, we used replicator dynamics on graph [42] and considered the pairwise comparison update, such as the Fermi update rule. With the influence of the networks structure, for a network with average degree $k > 2$, the dynamic equation becomes $\dot{x}_i = x_i(f_i + g_i - \phi)$. For pairwise comparison updating, such as the Fermi update rule, the parameters in this replicator equation are as follows:

$$\begin{aligned}
 f_i &= \sum_j x_j a_{ij} \\
 b_{ij} &= \frac{a_{ii} + a_{ij} - a_{ji} - a_{jj}}{k - 2} \\
 g_i &= \sum_j x_j b_{ij} \\
 \phi &= \sum_i x_i (f_i + g_i) = \sum_i x_i f_i
 \end{aligned}$$

283 The term of b_{ij} describes local competition between strategy S_i and S_j ,
 284 and g_i characterizes the local competition among strategies. Note that the
 285 calculation of the value of b_{ij} involves the four items from the symmetric
 286 payoff matrix, so we can derive $b_{ij} = b_{ji}$. Thus $\sum_{i=1}^n g_i = 0$ and $\phi = \sum_i x_i(f_i +$
 287 $g_i) = \sum_i x_i f_i$, where $\sum x_i = 1$. Absorbing this method, we get a new matrix
 288 B (Eq. 11). Then, the new payoff matrix C can be modified as each item
 289 $c_{ij} = a_{ij} + b_{ij}$ (Eq. 12).

$$B = \begin{matrix} & \begin{matrix} C & R & D \end{matrix} \\ \begin{matrix} C \\ R \\ D \end{matrix} & \begin{pmatrix} 0 & \frac{2C_R}{k-2} & \frac{2rd-2}{k-2} \\ -\frac{2C_R}{k-2} & 0 & \frac{\alpha-1+2rd-2C_R}{k-2} \\ -\frac{2rd-2}{k-2} & -\frac{\alpha-1+2rd-2C_R}{k-2} & 0 \end{pmatrix} \end{matrix} \quad (11)$$

$$C = \begin{matrix} & \begin{matrix} C & R & D \end{matrix} \\ \begin{matrix} C \\ R \\ D \end{matrix} & \begin{pmatrix} \alpha-1+rd & \alpha-1+rd+\frac{2C_R}{k-2} & \frac{k(rd-1)}{k-2} \\ \alpha-1+rd-\frac{kC_R}{k-2} & \alpha-1+rd-C_R & \frac{\alpha-1+k(rd-C_R)}{k-2} \\ \alpha-\frac{2rd-2}{k-2} & -\frac{\alpha-1+2rd-2C_R}{k-2} & 0 \end{pmatrix} \end{matrix} \quad (12)$$

290 For the convenience of further analysis, we define a new variable $h_i = f_i + g_i$,
 291 and then we have the average fitness of three strategies (Eq. 13):

$$h_0 = \frac{(k-2+x_2)rd+2C_R-kx_2}{k-2} + (1-x_2)(\alpha-1) \quad (13a)$$

$$h_1 = (1+\frac{2x_2}{k-2})(\alpha-1+rd) - \frac{k-2x_1}{k-2}C_R \quad (13b)$$

$$h_2 = x_0(\alpha-\frac{2rd-2}{k-2}) - x_1\frac{\alpha-1+2rd-2C_R}{k-2} \quad (13c)$$

292 4.2.1. No reward

First, we consider the condition with no information cost or reward. If we let $C_R = 0$ and $rd = 0$, then we have:

$$\begin{aligned} h_0 &= (1 - x_2)(\alpha - 1) - \frac{kx_2}{k - 2} \\ h_1 &= (\alpha - 1)\left(1 + \frac{2x_2}{k - 2}\right) \\ h_2 &= x_0\left(\alpha + \frac{2}{k - 2}\right) - x_1\frac{\alpha - 1}{k - 2} \end{aligned}$$

293 Based on the above expressions, we have the following important observa-
294 tions:

- 295 (1) When $h_0 - h_1 = -x_2\frac{k\alpha}{k-2} \leq 0$, the reciprocators will always have better
296 performance than the cooperators. When $x_2 = 0$, neutral drift occurs
297 between the cooperators and reciprocators, and they will co-exist.
- 298 (2) When $\alpha > \frac{k+(k-4)x_2-x_1}{(k-1)x_1}$, we have $h_1 > h_0 > h_2$. Thus, defectors will
299 learn to behave cooperatively. As x_1 increases and x_2 decreases, the
300 value of $\frac{k+(k-4)x_2-x_1}{(k-1)x_1}$ decreases, the inequation will always be satisfied.
301 Therefore, defectors will be conquered in the end.
- 302 (3) When $\alpha < \frac{1-x_1}{1-x_0-\frac{3}{k}x_1}$, we have $h_0 < h_1 < h_2$. Thus, defecting is the best
303 strategy and dominate the system.
- 304 (4) When $\alpha > \frac{1-x_1}{1-x_0-\frac{3}{k}x_1}$, we have $h_1 > h_2$. Thus defectors will evolve to
305 reciprocators. However, if we cannot fulfil the inequation in Case (2),
306 cooperators will evolve and become defectors and reciprocators. In this
307 case, when there are only defectors and reciprocators, R adopters will
308 dominate until $\alpha = \frac{1-x_1}{1-\frac{3}{k}x_1}$ and then neutral drift will occur between
309 them and they will co-exist continuously.

When we consider the cost seeking information, $C_R > 0$ and $rd = 0$, we have:

$$\begin{aligned} h_0 &= \frac{2C_R - kx_2}{k-2} + (1-x_2)(\alpha-1) \\ h_1 &= \frac{2x_2 + k-2}{k-2}(\alpha-1) - \frac{k-2x_1}{k-2}C_R \\ h_2 &= x_0(\alpha + \frac{2}{k-2}) - x_1\frac{\alpha-1-2C_R}{k-2} \end{aligned}$$

310 In this case, we get results as follows:

311 A When $C_R > \frac{kx_2}{k-2-2x_1}\alpha$, $h_0 > h_1$;

312 B when $C_R > \frac{x_1+(k-1)(2x_2-\alpha)+k-2}{2(1+x_1)}$, $h_0 > h_2$;

313 C when $C_R < \frac{(k-4)x_0-x_1}{k+2x_1}\alpha - \frac{k+x_1+2x_2-2}{k+2x_1}$, $h_1 > h_2$.

314 From the above cases, we can derive the following: when the status of the
315 system satisfies case A and B at the same time, we can be sure that the system
316 will evolve to the status of being fully comprised of cooperative agents.

317 4.2.2. Fixed and dynamic rewards

318 Under the influence of reward mechanism, we used the fitness functions
319 as shown in Eq. 13 and we solved the similar inequations. The results showed
320 that:

321 (1) When $rd > C_R + \frac{(k-1)x_0-(k-1)(\alpha-1)x_1-(\alpha-1)x_2}{k}$, R agents will rule the
322 system

323 (2) When there are only R and D users, if $rd > C_R - \frac{(k-1)(\alpha-1)x_1+(\alpha-1)x_2}{k}$,
324 R strategy is the ESS.

325 (3) When only R and C agents in the system. $h_0 - h_1 = \frac{k}{k-2}C_R > 0$. Thus,
326 when these two strategies co-exist, C strategy is the only ESS.

327 Similarly, when the reward is dynamic, if the value of the reward is slightly
328 larger than the threshold, the system effectively can become cooperative, and
329 the cost of the system will be reduced. In this scenario, we set the dynamic
330 reward function as Eq. 14, $\vec{x}$ symbolizes the fraction of each strategy.

$$rd(\vec{x}, C_R) = C_R - \frac{(k-1)(\alpha-1)x_1 + (\alpha-1)x_2}{k} + \delta \quad (14)$$

331 5. Performance Evaluation

332 The performance evaluation results are presented in this section. Initially,
 333 we investigated and compared the performance of the proposed incentive
 334 mechanism with dynamic rewards and fixed rewards. Unless there was a
 335 special need, we used the parameters in Table. 2.

Table 2: Parameters

Parameter	Definition	value
N	number of agents in the system	10,000
β	cost when providing a service	1
(x_0, x_1, x_2)	initial proportion of each strategy	(0.3,0.2,0.5)
w	selection intensity	10
$< k >$	average degree of structured networks	4

336 Our simulation was divided into Monte Carlo steps. As described in Alg.
 337 1, in each step, an agent plays the transaction game with its all neighbors,
 338 excluding itself (with all agents in a well-mixed population). Then, each
 339 agent simultaneously calculates its payoff and adopts a new strategy accord-
 340 ing to the Fermi update. In addition, a real system architecture is always
 341 structured, and we also simulated a structured network by using a square
 342 lattice and captured the snapshots along with the evolution of the system.

343 5.1. Effectiveness of incentives in a well-mixed population

344 In a well-mixed population, each agent can interact with any other agent
 345 in the system. Figs. 4-6 show the evolution of strategies in well-mixed pop-
 346 ulation under different scenarios.

347 As we analyzed in Sec. 4.1.1, when there is no reward, if α is smaller
 348 than the threshold, the system will be filled with defectors, but, if there are
 349 rewards, the system will be filled with reciprocators. However, cooperators
 350 always have less payoff than reciprocators and defectors, so they converge
 351 to D and R and disappear in the system. When there are only D and R,
 352 $\alpha = 1$ is the systems threshold. When $\alpha < 1$, defectors rule; when $\alpha > 1$,
 353 reciprocators rule; and, when $\alpha = 1$, neutral drift occurs between defectors
 354 and reciprocators (Fig. 4).

355 In a real system, when an agent needs some information, it must incur a
 356 cost (e.g., bandwidth and CPU cycle [43]). Thus, it is necessary to consider

Algorithm 1 Evolution framework

Require: Network structure, network size, strategy distribution

```
1: Initialization;
2: while  $step < MC\_STEPS$  do
3:   for each agent  $i$  do
4:     for  $j = 1$  to  $k$  do
5:       Play game with neighbor  $j$ ;
6:       Update agent  $i$ 's payoff
7:     end for
8:   end for
9:   for each agent  $i$  do
10:    Strategy update;
11:   end for
12:   Clear payoff information;
13: end while
```

357 the cost of information. First we consider the influence of cost for seeking
358 information ($C_R > 0$). The results were exactly the same with our analysis.
359 When $C_R < (\alpha - 1)x_1$, reciprocators can easily eliminate the defectors. How-
360 ever, when $C_R > (\alpha - 1)x_1$, the defectors conquer the reciprocators. Thus, to
361 have an effective incentive mechanism, the cost of seeking information must
be sufficiently small.

Figure 4: Evolution with different α and 0 cost (a) $\alpha = 0.5$ (b) $\alpha = 1$ (c) $\alpha = 1.5$

362

Figure 5: Evolution with different cost, $\alpha = 1.5$
Initial proportion of each strategy $(x_0, x_1, x_2) = (0.3, 0.2, 0.5)$ (a) $C_R = 0.075$ (b) $C_R = 0.15$

363 However, in a real system, the system cannot control the cost for each
364 agent. So, to help those cooperative agents, the system must reward them
365 for their contributions. Figs.6(a) and 6(b) show that, only if the reward is
366 large enough, the system can effectively help cooperative agents survive .
367 However, if the system continues to change, the threshold of reward is also
368 changing, and we must consider dynamic rewards. As a result, the system
369 keeps controlling the value of the reward and it can still effectively incentivize
370 cooperative agents.

Figure 6: Evolution with different reward, $\alpha = 1.5$, $C_R = 0.3$ Initial proportion of each strategy $(x_0, x_1, x_2) = (0.3, 0.2, 0.5)$

371 Even though the reward mechanism can promote cooperation, the system
372 will incur extra costs. As shown in Fig. 7, for a fixed reward system, when
373 the system is full of cooperative agents, all of them must be paid. The
374 accumulative cost keeps increasing as more agents evolve and become more
375 cooperative. However, when we consider dynamic reward, since there are

fewer and fewer defectors, the reward for each agent decreases. Thus, the cumulative cost will decrease in the long run.

Figure 7: System cost with reward ($\delta = 0.05$)

5.2. Effectiveness of incentives in a structured network

In a real system, a well-mixed population rarely exists. To make the model more realistic, we tested our model on a square lattice, a commonly used network structure when analyzing the evolution of strategies. Initially, each agent processes a vertex of a 100×100 square lattice and assigned a strategy with the distribution $(x_0, x_1, x_2) = (0.3, 0.2, 0.5)$, which was the same as the distribution in a well-mixed population. In this circumstance, each agent plays a transaction game with each neighbor once (excluding itself). To understand the influence of the networks structure, we captured the curves of evolution and the snapshots of each step.

First, we investigated the simplest scenario, i.e., without information cost and reward (Fig. 8). Similar to the results in a well-mixed population, $\alpha = 1$ is the evolution threshold. But, due to the correlations of agents, the speed of evolution is faster.

Figure 8: Evolution with different α and 0 cost on a square lattice (a) $\alpha = 0.5$ (b) $\alpha = 1$ (c) $\alpha = 1.5$

392 To see more clearly how the correlation of strategies influences the systems
 393 dynamics, we captured the evolution snapshot (Fig. 9). In the first few
 394 steps, agents tend to form small clusters with the same strategy. Without
 395 considering C_R and reward, under three different values of α , cooperators
 396 always have the worst payoff. So, unless they are surrounded by agents
 397 with the same strategy, they will change their strategy, C, to the other two
 398 strategies [Figs.9(b), 9(f),9(j)]. However, when there are only reciprocators
 399 and defectors, the results differ. When $\alpha < 1$, a reciprocator, at most,
 400 can get *zero* payoff when surrounded by four defectors. Even if theres only
 401 one reciprocator links with it, it gets a negative payoff. So, the cluster of
 402 reciprocators narrows until it disappears [Figs.9(g), 9(h)]. Opposite from this
 403 scenario, when $\alpha > 1$, payoff of a reciprocator is greater than or equal to 0,
 404 but defectors can only get 0 payoff, as they cannot get service from defectors
 405 or reciprocators [Figs.9(k), 9(l)]. When $\alpha = 1$, they all get 0 payoff, so
 406 neutral drift occurs, and the system will maintain its stable state.

Figure 9: Snapshot of the evolution strategies
 top to bottom: $\alpha = 0.5, 1, 1.5$
 red nodes: cooperators, green nodes: defectors, blue nodes: reciprocators

407 When we consider the cost of seeking information ($C_R > 0$), the payoff of
 408 reciprocators changes. If C_R is small, reciprocators can still conquer defectors
 409 as they may still get a positive payoff when there are only R and D agents
 410 (Fig.10). But when the cost is larger than the threshold we analyzed in
 411 Sec. 4.2, reciprocators can no longer exist.

Figure 10: Evolution with different cost, $\alpha = 1.5$
 Initial proportion of each strategy $(x_0, x_1, x_2) = (0.3, 0.2, 0.5)$
 (a) $C_R = 0.1$ (b) $C_R = 0.3$

412 A cost for knowing more information is mandatory in most systems. To
 413 encourage agents to be more cooperative, we still consider the reward mech-
 414 anism. Similar with well-mixed population, when the value of the reward
 415 was less than the threshold, the reward provides very little incentive, and it
 416 can slightly decrease the speed of the defectors invasion (Fig.11(a)). When
 417 the reward is larger than the threshold, the system will converge to full co-
 418 operative agents (Fig.11(b)). As the system is dynamic, a proportion of each
 419 strategy changes all the time, and we consider the dynamic reward. The
 420 rewards value is slightly larger than the threshold, and, when the system is
 421 full of cooperative agents, we set $rd = 0$. Thus, the dynamic reward can
 422 effectively promote cooperation as well.

Figure 11: Evolution with different reward (rd) on a square lattice
(a) $rd = 0.08$ (b) $rd = 0.3$ (c) Dynamic reward

423 The main purpose of using a dynamic reward is to reduce the cost of the
424 system. So, we compare the overall system cost in each step (Fig. 12). In
425 a fixed reward system, as the number of cooperative agents keeps increasing,
426 the systems cost also increases. When the system is full of cooperative agents,
427 the cost will stay at the peak in all following rounds. But when we consider
428 dynamic reward, the system cost will decrease as with less defectors, a small
429 reward will help defeat defectors. When defectors disappear, the system no
430 longer needs rewarding agents, and the cost of the system can be reduced.

Figure 12: System cost with reward on a square lattice

431 6. Conclusions

432 To solve the free-riding problem in P2P systems, we introduced a dynamic
433 reward mechanism into classical incentive mechanisms. Through theoretical
434 analysis, we found the optimal system settings to encourage agents to coop-
435 erate, and we compared the effectiveness of each scenario. In addition, we
436 theoretically and qualitatively proved that the cost of the system can be re-
437 duced with a dynamic reward. Through numerous simulations from different
438 aspects, we tested and confirmed the results that were obtained from the

analysis. A relatively large reward may lead the system to be fully cooperative. However, using dynamic reward can get the same effect, but dynamic reward may reduce the cost of the system in the long run. With the growing use of new, decentralized, and self-organized methods of computing and sensing, these systems are facing similar problems concerning cooperation. Our paper provides a simple and effective way to implement rewards in a real system.

However, for further studies, we may consider the dynamic of networks as well, when agents join and leave the system at various times. The co-evolution of strategies and network topology will be our research direction in the following studies.

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