A relevant Subspace based Contextual Outlier Mining Algorithm

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ABSTRACT:

For high-dimensional and massive data sets, a relevant subspace based contextual outlier detection algorithm is proposed. Firstly, the relevant subspace, which can effectively describe the local distribution of the various data sets, is redefined by using local sparseness of attribute dimensions. Secondly, a local outlier factor calculation formula in the relevant subspace is defined with probability density of local data sets, and the formula can effectively reflect the outlier degree of data object that does not obey the distribution of the local data set in the relevant subspace. Thirdly, attribute dimensions of constituting the relevant subspace and local outlier factor are defined as the contextual information, which can improve the interpretability and comprehensibility of outlier. Fourthly, the selection of N data objects with the greatest local outlier factor value is defined as contextual outliers. In the end, experimental results validate the effectiveness of the algorithm by using UCI data sets.

Keywords:

Contextual outlier; Relevant subspace; Interpretability and comprehensibility; Local sparsity; Probability density.

1. INTRODUCTION

Outlier is a general pattern or behavior, which significantly deviates from the other data. This type of data are inconsistent with other existing data[1], and often contain a large number of information which is not easily to be found but offers a very high level of research and application value. Outlier mining, an important branch of data mining and data analysis, has been widely used in astronomical spectroscopy[2], credit card fraud[3], network intrusion mining[4,5], data cleaning[6] and so on. At present, outlier mining algorithms can be broadly classified into such methods as statistics-based [7], distance-based[8], density-based[9,10], deviation-based [11], angle-based methods[3, 12] and so on. However these methods assume that each dimension plays a uniform contribution for detecting outliers [13]. The existence of some irrelevant dimensions will increase effects of "disaster dimension" on mining results, and lead to unable to find some outliers hidden in the subspace.

For the high-dimensional and massive data, the effectiveness and efficiency of outlier mining have been seriously influenced and some outliers hidden in the subspace may not be found. In most cases, the outliers are such data objects which are apparently inconsistent with the distribution features of the local dataset. But the inconsistent valuable information can be given in some attribute dimensions, while nothing important information is afforded on the other attribute dimensions and these dimensions are nearly irrelevant to detect the outliers[14]. Therefore, it can effectively reduce the disturbance of "disaster dimension" and find the hidden outliers by searching and deleting the attribute

dimensions that cannot provide some valuable information in the high dimensional datasets. Aiming at the problem of high-dimensional and massive data, a new contextual outlier mining algorithm based on relevant subspace is proposed in this paper. From the perspective of the local sparsity, the relevant subspace is redefined by making use of the local sparsity factor in the local dataset, and thus effectively described the distribution characteristics of all the local datasets. For the data objects in the relevant subspace, the calculation formula of the local outlier factor is given by using the probability density and Gaussian error function of the local dataset. Then the data attributes constituting the relevant subspace are regarded as the data objects' contextual information, and a contextual outlier mining algorithm is proposed, and accordingly it effectively reflects the inconsistency degree of the distribution in the local dataset and data objects in the relevant subspace, and provides the origin, meaning and features of outliers. The selection of n data objects which are of the most outlier, are identified as the contextual outlier. Finally, the interpretability and performance of the algorithms are validated by taking UCI data as experimental datasets. In this paper, the main highlights and contributions are summarized as follows:

- The relevant subspace, which can effectively describe the local distribution of the various data sets, is redefined by using the local sparseness of attribute dimensions.
- We proposed that the attribute dimensions and outlier factor of data object in the relevant subspace are regarded as the contextual information, which can effectively explain the outlier's meaning and cause, and is benefit for the understanding of outlier.
- The computing formula of outlier factor in data object's relevant subspace is presented to define the object's outlier degree.

2. RELATED WORK AND ANALYSIS

Since mostly traditional outlier mining methods [7,8,9,10,11,12], detect outliers from all the dimensions of the data space, some irrelevant dimensions would have an effect on the efficiency and precision of outlier mining. It is one of the hot spots of high-dimensional outlier mining to project the data onto a subspace for outliers mining [14, 15, 16], however, it is more difficult to search for the meaningful subspace [15], and has the dimension-exponential complexity [14]. Aiming at outlier mining tasks, the typical methods of selecting meaningful subspace in the high-dimensional data set mainly include two categories: one is sparsity subspace method[2,17,18] and the other is relevant subspace method [13,19,20]. The sparse subspace method projects all overall the data onto the sparse subspace according to sparse coefficient threshold given by the user, and the data objects which are contained in the subspace are determined as outliers. Therefore, the sparse subspace method is a type of global subspace method. Typical work are: c c.Agarwal et al used genetic algorithm to search for sparse subspace [17], but the algorithm performance

was affected by the initial population, the completeness and accuracy of outlier mining's results cannot be guaranteed. Aiming at the deficiencies in [17], Zhang Jifu et al, took concept lattice as a description tool for the subspace [2,18], and determined the sparse subspace through introducing the density coefficient. The method further improves the accuracy and completeness of the mining results, and has obtained better application in the astronomical spectrum data. However, the mining efficiency is lower because the construction of concept lattice is complex.

The basic idea of the relevant subspace method [13,14,16,19,20] is to look for the relevant subspace composed of some meaningful attribute dimensions in the datasets, from which the outliers are determined. The mainly used method is based on linear correlation of local reference datasets [13,19,20], as well as statistical models of local reference datasets [14, 16] and so on. Typical work includes: H.-P.Kriegel et al [19] proposed an outlier detection method in axis-parallel subspaces of high dimensional data (SOD). By sharing the nearest neighbor (SNN), the method looks for a similar subset for each data object, then determines the axis-parallel linear correlation subspace in the similar subset based on the theory of attribute-based dimensions with low variance in the relevant subspace [19]. In the SOD, since the outlier degree of data objects is depicted with only one relevant subspace, this method will not be able to distinguish their outlier degree when the data objects are located in two or more than two subspace [14]. Clearly, it has some obvious deficiencies to detect outliers in the relevant subspace by using the mean-dimension of the Euclidean distance method. E.Muller et al [14] proposed a method to select the meaningful subspace from datasets by using Kolmogorov-Smirnov test statistics method, and search for non-uniform distribution subspace (relevant subspace) from low-to high-dimensional in recursive ways, then regard the multiplicative of local outlier in each relevant subspace of data objects, as the ultimate outlier degree of the data objects, and accordingly solved the problem of outlier coefficient comparability in different subspaces. However, the complexity of determining all the relevant subspace in this algorithm is exponential time of the dimension [3], and the efficiency is low. Furthermore, the scalability of dimension for high dimensional datasets is poor, and unable to be adapted to the high-dimensional and massive data. F.Keller et al [16] using Monte Carlo methods to find relevant subspace set, and this method measures the outlier degree of objects from the local subset which the data objects correspond to each relevant subspace, but the relevant subspace are actually achieved from the point of global view. Hans-Peter Kriegel et al [13], on the basis of the theory of principal component analysis, detected outlier by using of Mahalanobis distance with gamma distribution methods. Relevant subspace achieved by this method is arbitrary subspace of linear correlation, and the relevant subspace has good adaptability to linear data. However, this method is based on the deviation from the local data distribution, and it is liable to make mistakes in the relevant subspace obtained from maximum likelihood estimation when outliers was in the regional which not obviously reflects of correlation. Therefore, the method needs enough local data to reflect the obviously deviation trend, and the complexity of this algorithm is the cube of the dimension and difficult to fit for the high dimensional data. In addition, Mohamed et al [20] find the relevant subspace by using of sparse density matrix, and make clustering analysis in the relevant subspace.

Most of outlier mining algorithms only to focus on the mining methods, and the reasons with the interpretation of outlier's generation are relatively few. So, the origins, meaning and characteristics of the outlier's generation have not been able to effectively explained. Guanting Tang et al proposed an outlier model with contextual information in [21]. On this basis, an outlier mining algorithms with the contextual information is presented, but the mining efficiency is low because the sphere of searching outlier is first narrowed, then outlier is searched with enumerating method. Wang and Davidson proposed a probabilistic approach based on random walks graph, which is essentially a homogeneous Markov chain as characterized by a transition matrix [22]. The principal eigenvector of the transition matrix gives the stationary distribution of the nodes being visited in the graph under a global random walk, so the method can simultaneously explore meaningful contexts and score contextual outliers.

3. RELEVANT SUBSPACE AND LOCAL OUTLIER PROBABILITY

According to the mining tasks on the different data sets, the data objects in the datasets only reflect the valuable information in some specific attribute dimensions, while there may little or no valuable information in other attribute dimensions [14]. Suppose DS be an arbitrary d-dimensional dataset, the attribute set F , $FS = \{A_1, A_2, \dots, A_d\}$, and $x_{ij} (i=1,2,\dots,n, j=1,2,\dots,d)$ denote the j-th attribute value of the i-th data object. Referring to the reference [14] and [23], the basic concepts of the relevant subspace and local outlier probability are described as follows:

For an arbitrary subspace S , $S \subseteq FS$, data object o , $o \in DS$, and $N(o, S)$ is the nearest neighbor of o . If S is a relevant subspace, then $N(o, S)$ is non-uniform distribution. If S is not a relevant subspace, then $N(o, S)$ is uniform distribution.

In [14], the non-uniform distribution of the subspace can effectively embody the valuable information of "outlier", while the uniform distribution of attribute dimensions cannot reflect the valuable information of "outlier". Therefore, the measurement and searching for the relevant subspace has become the key to detect outliers since the relevant subspace can effectively reflect the valuable information of "outlier".

In [23], for an arbitrary data object o , $o \in DS$, suppose S be the correlation dataset, $S \subseteq DS$ and $o \in S$, then the probability distance $pdist$ of o in s , satisfy the condition that $\forall s \in S: [d(o, s) \leq pdist(o, S)] \geq \phi$.

Intuitively, $pdist(o, S)$ is the probability of the data objects' number contained in S , which is in the spherical with the center of o and the radius of $pdist$. $Pdist(o, S)$ can be indirectly used to estimate the density of s , that is, the density of S is defined as the formula (1):

$$pdens(o, S) = \frac{1}{pdist(o, S)} \quad (1)$$

Suppose o be the center of s , $s \in S$, and the set of distance between s and o approximately follows the half Gaussian distribution, which is similar to the standard deviation. The standard distance of s and o is computed as the formula (2):

$$\sigma(o, S) = \sqrt{\frac{\sum_{s \in S} d(o, s)^2}{|S|}} \quad (2)$$

In Formula (2), because the distance $d(o, o) = 0$ replaces the average $E[d(o, S)]$, the standard distance of o and its standard deviation $\text{Stddev}(d(o, S))$ is different. Therefore, we cannot assume that the set of distance values between $s(s \in S)$ and o follows normal distribution, but suppose that S follow normal distribution with the centre o . So, we determine the correlation dataset S by taking advantage of this hypothesis, namely $S=o$, and S is the nearest neighbors of K . The distance between o and λ corresponding probability set of S is defined as the formula (3):

$$\text{pdist}(\lambda, o, S) = \lambda \cdot \sigma(o, S) \quad (3)$$

Where, the parameter λ is used for controlling the approximate value of the density.

In Formula (3), $\text{pdist}(\lambda, o, S)$ is used to estimate the density of the correlation set s . For arbitrary data object o , and $o \in DS$, suppose $S (S \subseteq DS)$ be the o 's correlation dataset, then the related probability local outlier factor (PLOF) of λ is defined as formula (4):

$$\text{PLOF}_\lambda(o) = \frac{\text{pdist}(\lambda, o, S(o))}{E_{s \in S(o)}[\text{pdist}(\lambda, s, S(s))]} - 1 \quad (4)$$

Where E denotes the mean value, and $S(o)$ denotes o 's correlation dataset.

By using Gaussian error function, the probability value that the data objects $o(o \in DS)$ is outlier(local outlier probability (LoOP)) is defined as formula (5)

$$\text{LoOP}_S(o) = \max \left\{ 0, \text{erf} \left(\frac{\text{PLOF}_{\lambda, S}(o)}{\sqrt{2 \cdot n \text{PLOF}}} \right) \right\} \quad (5)$$

Where $n \text{PLOF} = \lambda \cdot E[(\text{PLOF})^2]$.

Remark. The value of λ does not affect the sort of the probability values but only the outlier's probability values.

4. RELEVANT SUBSPACE, SUBSPACE-DEPENDENT VECTOR, AND CONTEXTUAL OUTLIERS

4.1 Relevant Subspace

Since the real datasets often contain the cluster (Cluster), which is generated from a variety of different mechanisms, different mechanisms may act on the different subsets of attribute dimensions[13]. So, the subset of attribute dimensions reflects that the appropriate mechanism can generate the corresponding subsets of data. Furthermore, it embodies a kind of similar local distribution characteristics, namely for the data subset generated from the specific mechanism, the subset of attribute dimensions has the characteristic of partial correlation: the common characteristic of local sparse distribution, namely some attribute dimensions are non-uniform distribution, while others are uniform distribution. Generally, only does the data subset generated from the specific mechanism reach a certain number, the common

characteristics of local distributions can become apparent. Therefore, outlier can be regarded as the data objects which are not generated from such mechanism, namely, outlier is one type of data objects which are clearly inconsistent with the partial correlativity. For arbitrary data objects obj , attribute dimensions which are inconsistent with the common characteristics of the local sparse distribution features, can embody a kind of valuable information. They can also be described by a non-uniform distribution shown from the value of the local data attribute dimensions. However attribute dimensions, consistent with the common characteristics of the local sparse distribution features, cannot reflect any valuable information, but can be depicted by an uniform distribution on the value of the local data attribute dimensions. Referring to [14], the relevant subspace is redefined as follows:

Definition 1. For the detection of outlier, suppose DS be the d -dimensions dataset, the global space FS , $FS=A$ (the attribute set A , and $A=\{A_1, A_2, \dots, A_d\}$, and the nearest neighbors of data object obj are $N(obj, FS)$ (namely the local dataset LDS). If the value of $N(obj, FS)$ conforms to the uniform distribution in the A_i 's attribute dimension, then A_i does not provide any valuable information and belongs to the attribute dimensions in the irrelevant subspace. Conversely, if the value of $N(obj, FS)$ conforms to the non-uniform distribution in A_i attribute dimensions, then A_i can provide some valuable information and belong to the attribute dimensions in the relevant subspace.

In [14], whether each subspace S is a relevant subspace, is determined by using the nearest-neighbor $N(o, S)$, so for the datasets DS , the distribution characteristics of containing datasets in S will also change with the adjustment of S ' dimension. Therefore, if S is not FS , then $N(o, S)$ will lose the meaning of local dataset, and cannot effectively reflect the common features of the DS 's local distribution characteristic in FS . However, according to definition 1, the relevant subspace is determined only by using $N(obj, FS)$ in FS , and the above deficiencies in [14] can be effectively avoided.

4.2 Subspace-dependent Vector

In the local dataset LDS , since the value of uniform distribution attribute dimensions is more uniform, the sparsity (density) are similar or small differences, the uniform distribution attributes can be measured by taking the sparsity of the local attribute dimensions. Thus, the relevant subspace of data objects can be effectively identified by removing attribute dimensions of the uniform distribution and retaining non-uniform distribution attribute dimensions.

Suppose obj be the i -th data object in datasets DS , and $LDS(obj)$ be the local dataset constructed from obj with K -NN methods. For $LDS(obj)$, the sparsity (density) of obj in each attribute dimension can be measured by the following local sparse factor λ_{ij} :

$$\lambda_{ij} = \frac{\sum_{y \in p(x_{ij})} (y - x_{ij})^2}{K + 1} \quad (6)$$

Where $p(x_{ij})$ is a sequence which is composed of $LDS(obj)$'s values on the attribute dimensions A_j , and K is the number of nearest neighbors.

From formula (6), we can know that the local sparse factor λ_{ij} is the mean square value of Euclidean distance from each attribute dimension values of A_j in the local dataset $LDS(obj)$ and x_{ij} . Similar to formula (2), the larger the λ_{ij} is, and $LDS(obj)$ on the attribute dimensions A_j is more sparse. Conversely, the smaller the λ_{ij} is, the more dense of $LDS(obj)$ on the attribute dimensions A_j is. In short, λ_{ij} can reflect the local sparsity (density) of local dataset LDS on its attribute dimension A_j .

Obviously, from formula (6), we can determine the local sparse factor of each attribute dimension's of all the data objects, and accordingly generate the sparsity factor matrix $[Z_\lambda]_{n \times d}$ of the datasets DS . Furthermore, $[Z_\lambda]_{n \times d} = [\lambda_{ij}]$, where $n = |DS|$, d is the degree of DS 's dimension. From $[Z_\lambda]_{n \times d}$, we can construct the obj's local sparse factor matrix $[Z_{L\lambda}]_{k \times d}(obj)$ on $LDS(obj)$, namely the local sparse factor matrix $[Z_{L\lambda}]_{k \times d}(obj)$ of obj , where $k = |LDS(obj)|$.

To describe the difference of the attribute dimensions' local sparsity, we take the following local sparse difference factor d_{ij} to depict the sparsity difference of the obj's j -th attribute dimensions on $LDS(obj)$.

$$d_{ij} = \sqrt{\left| \frac{(\lambda_{ij} - C_{\lambda ij})}{C_{\lambda ij}} \right|} \quad (7)$$

Where: $C_{\lambda ij}$ is the mean square of $[Z_{L\lambda}]_{k \times d}(obj)$ ' values on the attribute dimensions A_j .

From Formula (7), d_{ij} depicts the difference degree of the local sparse factor λ relative to the mean values. If d_{ij} is greater, then the local sparse factor differences on the j -dimensional of the data objects obj 's local sparse factor matrix $[Z_{L\lambda}]_{k \times d}(obj)$ is larger, namely, if the local density value of the obj's local dataset LDS on the attribute dimensions A_j is more uneven, then the value of obj's local dataset LDS on the attribute dimensions is not uniform. Conversely, If d_{ij} is less, then the local sparse factor differences on the j -dimensional of the data objects obj 's local sparse factor matrix $[Z_{L\lambda}]_{k \times d}(obj)$ is smaller, namely, if the local density value of the obj's local dataset LDS on the attribute dimensions A_j is more even, then the value of obj's local dataset LDS on the attribute dimensions is more uniform. In conclusion, whether the value of obj's local dataset LDS on the attribute dimensions is uniform can be measured by using of d_{ij} .

Definition 2. Let ε be the threshold of local sparse difference factor, d_{ij} be the local sparse difference factor of the i -th data objects obj 's j -th dimension. If $d_{ij} < \varepsilon$, then $v_{ij} = 0$, or else, $v_{ij} = 1$, then $v_i = \{v_{ij}\}$, and v_i is a vector defined by the obj's subspace, among which $i = 1, 2, \dots, |DS|$, $j = 1, 2, \dots, d$, and d is the dimension of DS . In v_i , the subspace constructed from the attribute dimensions with v_{ij} value being 1 is called as the obj's relevant subspace, and the subspace constructed from the attribute dimensions with v_{ij} value being 0 is called as the obj's irrelevant subspace.

When the dataset's dimension increases, the distance difference is not obvious, it will have no meaning by using distance to measure

differences among data objects, but be meaningful for the nearest-neighbor sort [24]. From formula(7) and definition 2, we can know that in $LDS(obj)$, the local sparse factor of each neighbor $loc_obj (loc_obj \in LDS(obj))$ is obtained from $LDS(loc_obj)$, that is, the vector defined by a data object obj 's subspace is determined from the attribute dimensions of $LDS(obj)$ and $LDS_{loc_obj} \in LDS(obj)(loc_obj)$. Therefore, this subspace vector can accurately reflect the distribution characteristic of the local data, and ensure the correctness of the vector defined by the subspace. In addition, since the value of each dimension in the vector defined by the subspace is determined from the distribution condition of their corresponding attribute values, the determination of subspace vector has nothing to do with the dimension of DS .

In [13], the relevant subspace which is obtained from the correlated Pearson or Spearman methods is linear, and this method reflects the linear distribution characteristics of the local dataset. But by using the relevant subspace, we only discover that the outlier is obviously deviate from the linear distribution. The relevant subspace defined by definition 2 is determined from the local sparse difference factor of the local dataset, and constructed from the attribute dimensions whose density differences are more obvious. Similar to the analysis of [16], the vector defined by the subspace can determine arbitrarily nonlinear subspace, and be more accurate to describe the data objects with sparse density.

4.3 Contextual Outlier

From the above definition 2, if the relevant subspace does not exist in the i -th object obj , namely each dimension of the obj's subspace vector v is 0, then it indicates that obj is consistent with local distribution features. If the relevant subspace exists in the i -th object obj , then it indicates that obj is inconsistent with the local distribution characteristics. In order to describe the degree of inconsistency (outlier), referring to [23], the outlier degree of obj (Factor (obj)) in the subspace is redefined as formula (10) by combining with definition 2.

Let v_i be a vector defined from the obj's subspace, d be the v_i 's dimensions, and λ_{ij} be the corresponding local sparse factor of j 's attribute dimensions (see: Formula(6)), $d_{RS}(o, s)$ represents the Euclidean distance of data objects s and o in the relevant subspace RS . Referring to Formula (2) to (5), relative to the local dataset $LDS(obj)$, the standard distance of obj and its relevant subspace RS is defined as the formula (8):

$$\sigma_{RS}(obj, LDS(obj)) = \sqrt{\frac{\sum_{o \in LDS(obj)} d_{RS}(obj, o)^2}{|LDS(obj)|}} = \sqrt{\sum_{j=1}^d v_i \cdot \lambda_{ij}} \quad (8)$$

In the obj's relevant subspace RS , loc_obj is the obj's arbitrary nearest neighbor, and the standard distance of loc_obj ($loc_obj \in LDS(obj)$) (let loc_obj be the m -th data objects) relative to $LDS(loc_obj)$ is defined as the formula (9).

$$\sigma_{RS}(loc_obj, LDS(loc_obj)) = \sqrt{\frac{\sum_{o \in LDS(loc_obj)} d_{RS}(loc_obj, o)^2}{|LDS(loc_obj)|}} = \sqrt{\sum_{j=1}^d v_i \cdot \lambda_{mj}} \quad (9)$$

According to formula (4), the probabilistic local outlier factor PLOF of obj in its relevant subspace RS is defined as the formula(10).

$$PLOF_{RS}(obj) = \frac{\sigma_{RS}(obj, LDS(obj))}{E_{o \in LDS(obj)}(\sigma_{RS}(o))} - 1 \quad (10)$$

In the relevant subspace RS of obj, the probabilistic local outlier factor PLOF of loc_obj (loc_obj $\in$ LDS(obj)) is defined as the formula (11).

$$PLOF_{RS}(\text{loc_obj}) = \frac{\sigma_{RS}(\text{loc_obj}, LDS(\text{loc_obj}))}{E_{o \in LDS(\text{loc_obj})}[\sigma_{RS}(o, LDS(o))]} - 1 \quad (11)$$

In formula(10) and (11), $PLOF_{RS}(obj)$ and $PLOF_{RS}(\text{loc_obj})$ are similar to the local outlier factor LOF in the obj's relevant subspace RS, in that they both depict the local outlier degree of loc_obj in the obj's relevant subspace RS. It can be easily seen that, if we do not adopt relevant subspace RS, the definition of $PLOF_{RS}(obj)$ and $PLOF_{RS}(\text{loc_obj})$ is no different in the space of FS composed of all the dimensions in DS (that is, RS=FS).

PLOF is not a probability value, nor a standardized value. If the value of PLOF is less than 0, obj can be described as the normal data object, or else, if the value of PLOF is greater than 0, the larger value PLOF shows the higher outlier degree of obj. But it is not comparable by adopting PLOF to measure different distribution of datasets [23]. Since there exists different dimensions of relevant subspace in datasets DS, the data distribution characteristic of different relevant subspace is different. Therefore, PLOF in the different relevant subspace is not comparable.

To convert PLOF value to be a probability value, all dimensions are assumed to be independent each other, furthermore, each dimension has impact on the PLOF value [23]. According to the central limit theorem, we can assume that the space composed of all the dimensions obeys the normal distribution of $E[PLOF^2]$ (the variance with the center of 0)[23]. According to definition 2, if the value is less than ϵ , it is unable to provide valuable information, namely, attribute dimensions in the relevant subspace determined by definition 2 can have some impact on PLOF. Therefore, it can be assumed that $PLOF_{o \in LDS(obj)}(o)$ in the relevant subspace (obj) (o) obeys the normal distribution of $E_{o \in LDS(obj)}[(PLOF_{RS}(o))^2]$ (the variance with the center of 0), and also represents the probability of data objects being outlier with Gaussian error function. Referring to the Formula (5), the outlier degree(outlier factor) of obj in its relevant subspace RS is redefined as follows:

Factor(obj) =

$$\max \left\{ 0, \text{erf} \left(\frac{PLOF_{RS}(obj)}{\sqrt{2} \cdot \sqrt{E_{o \in LDS(obj) \cup obj}[(PLOF_{RS}(o))^2]}} \right) \right\} \quad (12)$$

Similar to [23], erf denotes the Gaussian error function. Because the parameter λ in [23] does not affect the sort of outlier degree,

but essentially divides the variable value of Gaussian error function to λ . Therefore we do not consider the selection of erf's value, similar to the condition of λ being 1 in [23], if necessary, we can deal with in similar.

From Formula (10), (11) and (12), $PLOF_{RS}(\text{loc_obj})$ is not determined from the relevant subspace of loc_obj, but from the relevant subspace RS of obj since the inconsistency degree of Obj and its local distribution characteristics can be only embodied in the obj's relevant subspace RS when to determine Factor(obj).

In [23], each dimension is supposed to have the same effect on measuring outliers and calculating the data objects' PLOF in the space which is composed of all the dimensions. However, the properties of the uniform distribution which is also involved in calculating PLOF and many uniform distribution of attribute dimensions, are likely to make the PLOF values in formula (10) and (11) close to 0. In short, attribute dimensions in the uniform distribution will not provide any valuable information for distinguishing between normal data and outliers, instead, their existence can probably change the original outlier into a normal data. In Formula (12), the outlier degree of data objects is determined in the relevant subspace RS. Since RS contains only attribute dimensions which follow the non-uniform distribution, and effectively avoids attribute dimensions in the uniform distribution to participating the calculation process of outlier degree. Meanwhile, formula (12) can be adapted to different distributions of data characteristics, as well as a variety of subspaces, namely it can be regarded as the unified measurement pattern in determining the outlier in the relevant subspace, and effectively improving the incomparable problem of measurement pattern in multiple subspaces in [19]. Especially, the normalization of PLOF is from the view of data sets in [23], and the PLOF in all over the datasets is supposed to obey the normal distribution. But, it is from a general perspective in formula (12), and the normalization of different clusters in datasets is also considered, furthermore, it was assumed that PLOF of the local dataset in the relevant subspace follows the normal distribution.

Definition 3 Let $v_i = \{v_{ij}\}$ be a vector defined from the subspace of data objects obj, where $i=1,2,\dots, |DS|$, $j=1, 2, \dots, d$, and d is the DS's dimension degree. In v_i , the corresponding Factor and attribute dimensions (obj) of the v_{ij} value being 1 is called as the obj's contextual information, and the outlier with contextual information is called as contextual outlier.

From definition 3, we can know that the contextual information describes and depicts the attribute dimension information of the outlier's non-uniform distribution. Since the subspace of non-uniform distribution can effectively embody the outlier's valuable information, this type of information can effectively explain the characteristic, implication and the cause of outlier, and contribute to the understanding of outlier.

5. THE RELEVANT SUBSPACE-BASED CONTEXTUAL OUTLIER MINING ALGORITHM

The basic idea of outliers mining in the high-dimensional datasets DS is described as follows. Firstly, the global sparsity factor matrix is generated, and the obj's local sparse factor matrix which LDS corresponds to is constructed according to formula (6). Then, the local sparse difference factor which each attribute dimensions

in obj corresponds to is computed by formula(7), and the obj's subspace vector is determined from definition 2 because the subspace vector can embody that obj is a relevant subspace or irrelevant subspace. If the relevant subspace exists in obj, then the obj's subspace is computed according to formula (12), otherwise, it shows that obj is a normal data object, and the outlier degree of obj is assigned with 0. Finally, some data objects with the greatest outlier degree are selected as the outlier. The mainly algorithms are described as follows.

Algorithm COAS(Contextual Outlier Detection in Arbitrarily relevant subspaces)

Input: datasets DS, the sparsity threshold ϵ

Output: NN number of contextual outlier

```

    /* the default value  $\epsilon$  is 0.1
1) L=DS; /* DS is a dataset and is stored to L
2) d=DS's dimension number;
    /* the number of DS's attributes
3) n=|DS|; /*the number of data objects in DS
4)  $K=\sqrt{n}$ ;
    /* the number of neighborhood is generally set with  $\sqrt{n}$ 
5) For ( i=1;i<= n;i++){
6) L[i]=L /* L is the i-th data object;
7) L[i].KNN=ComputerKNN(L[i],L,K);
    /* referring to [13], compute the i-th data object's KNN
8) L[i].LDS=LKNN  $\cup$  L[i];
    /* determine the i-th data object's local dataset
9) L[i]. $\lambda$ =ComputerL $\lambda$ ( L[i].LDS,K);
    /* according to definition 2, compute each attribute's sparsity
factor of the i-th data object(see function 1)
10) L. $\lambda$  Matrix.add(L[i]. $\lambda$ );
    /* the sparsity factor of the i-th data object was added to the line
of sparsity factor matrix which the datasets DS is corresponding to.
11) }
12) For ( i=1;i<= n;i++){
13) L[i].LDS $\lambda$ Matrix=Compute L $\lambda$ Matrix(L. $\lambda$  Matrix, L[i].LDS);
    /* select the i-data object's local sparse factor matrix according
to the DS's sparsity factor matrix.
14) Factor(L[i])=Computer Factor(L[i].LDS $\lambda$ Matrix,d,K);
    /* compute the outlier degree of the i-th data object(see
function 2).
15) If (Factor(L[i])>0){ OutlierMap.add(Factor, L[i]);}
    /* save the value of the i-th data object's outlier degree
16) }
17) Sort OutlierMap by Factor;
    /* the data object is sorted from big to small according to the
outlier degree of the data objects.
18) Print Top(NN);
    /* Select NN number of data objects with the greatest outlier
degree, and their corresponding contextual information(attribute

```

dimensions which construct the relevant subspace) as contextual outlier.

END COAS

Function1 ComputerL λ (L[i].LDS,K){

```

    /*compute each attribute's sparsity factor of the i-th
data
object
1) For ( m=1; m<=d; m++) {
2) For( j=1; i<=K+1; j++){
3) mSet[j]=(L[i].LDS[j][m]);
    /* Add each attribute values of the L[i]'s local data in
the m-th column to an array
4) }
5)  $\lambda_{im}$ =computer $\lambda_{im}$ (Set);
    /* according to formula (6), we compute the sparse
factor of the attribute in the L[i]'s m-th dimension.
6)  $\lambda_i[m]=\lambda_{im}$ ;
    /* Store the sparsity factor of the attribute in the L[i]'s
m-th dimension to an array.
7) }
8) Return  $\lambda_i$ ;
END ComputerL $\lambda$ 

```

Function2 ComputerFactor(L[i].LDS λ Matrix, d, K)

```

    /* compute the outlier degree of the i-th data object.
1) For ( j=1; j<=d; j++) {
2) x=0;
3) For ( m=1; m<=K+1; m++){
4) x+=L[i].LDS $\lambda$ [m][j];
5) }
6) Compute  $d_{ij}$ (L[i]) according to formula(7);
    /*the local sparse difference factor of the i-th data
object's j-th column attribute
7) If ( $d_{ij}$ (L[i])< $\epsilon$ ){  $v_i[j]=0$ ; } else {  $v_i[j]=1$ ; }
8) }
9) If ( $\|v_i\|=1$ ){ Factor(L[i]) =0; } else{ Factor(L[i]); }
    /*computing the outlier degree of the i-th data objects
according to formula (10).
10) Return Factor (L[i]);
END ComputerFactor

```

In COAS algorithm, the data object obj's subspace vector is determined through the local sparse difference factor, and the outlier degree is determined in the relevant subspace. If there exists irrelevant subspace, namely the subspace vector is total 0, this indicates that the difference of the local density is close to 0, obj is a normal data, and the outlier degree is assigned with 0. The

calculating of the data objects' KNN is realized by sequentially traverse, and the time complexity is $O(n \cdot n)$, but the average time complexity can be reduced to $O(n \cdot \log n)$ with the aid of the index structure[13]. For each local data object, each attribute's local sparse factor is calculated, the sparsity factor matrix of the original data is also generated, and the time complexity is $O(n \cdot k \cdot d)$. In addition, the time complexity of determining each data object's subspace vector and outlier degree is $O(n \cdot k \cdot d)$. Therefore, the complexity of COAS algorithm is $O(n \cdot \log n + n \cdot k \cdot d + n \cdot k \cdot d)$.

6. EXPERIMENTAL ANALYSIS

We use java to implement the COAS and HiCS algorithms [16], COPAlgorithm[13] algorithm and OUTRES algorithm[14], and conduct experiments by using a server that contains a Pentium D925 CPU, 256M memory, Windows XP operating system, ORACLE 9i DBMS development platform.

6.1 Accuracy Analysis of The Algorithm

To be convenient for outlier mining, 4 datasets from UCI are preprocessed according to the methods in [3,14]. As can be seen from table 1, "the outlier point category and its number" represents the type of outlier points and the retained data objects'

number, and "the normal point category and its number" represents the type of normal points and the retained data objects' number.

Fig.1 impact of ϵ on the mining accuracy

Table 1. Preprocessing of UCI data

UCI data sets	number of Data dimension	normal point category and its number	outlier point category and its number	total number of data	number of outlier
Yeast	8	ME3 400 CYT 400 MIT 200 NUC 100	ERL 5	1105	5
Ionosphere	34	Good 200	Bad 5	235	5
mfeat	649	0-7 category, 150 of each category	8-9, 5 of each category	1210	10
Isolet	617	1-20 category, 200 of each category	21-26 category, 8 of each category	4048	48

It can be seen from Fig. 1, when the local sparse difference factor threshold value ϵ is bigger or smaller, the mining efficiency is lower. The main reason is that if ϵ is set with the smaller value, most attribute dimensions are considered to be with significant differences in density according to definition 2, and should situate in the relevant subspace, and thus cause the relevant subspace dimension to be expanded. For the high dimensional datasets, it will make results severely affected by "the curse of dimensionality", and the accuracy being decreased. Moreover, the outlier in the subspace could be hidden. For the low-dimensional datasets, the influence of "curse of dimensionality" is smaller, but it still hides the outlier in subspace. If ϵ is set with the bigger value, most attribute dimensions are considered that the density difference is not obvious, and should situate in the irrelevant subspace, and thus cause to determine the outlier with significant difference in density as the normal data, and the accuracy will decrease. Therefore, the ϵ curve in Fig. 1 shows "protruding" shape for high-dimensional data, while presents "near linear" shape for low-dimensional data. By definition 2, the local sparse difference factor of each attribute dimensions in unrelated is close to 0, and the difference was not obvious. For the high dimensional datasets, once the threshold ϵ becomes slightly smaller, a large number of attribute dimensions in the irrelevant subspace could be divided into the relevant subspace, and increases the "the curse of

dimensionality" influence on the results. Therefore, the ϵ curve which high dimensional of datasets correspond to is steeper on the left. As the threshold ϵ increases to a certain degree, the outlier's attribute dimensions in the relevant subspace could be probably divided into the irrelevant subspace, and the outliers with significant differences in density are regarded as the normal data. Due to the significant differences exist in the density of the outlier, namely the difference of each attribute dimension's local sparse difference factor in the relevant subspace is always larger, the greater change of ϵ can divide the outlier's attribute dimension in the relevant subspace to the irrelevant subspace, and thus the outlier may be considered to be a normal data. Therefore the curve of the threshold value ϵ is generally flatter in the right. When the outlier's density in the datasets exist significantly difference, only the threshold value ϵ become to larger value, can the outlier be regarded as a normal data. The mining accuracy is higher if ϵ is within the larger range. Conversely, when the outlier's density difference is not obvious, once the threshold value ϵ become to a smaller value, the outlier can be regarded as a normal data. The mining accuracy is higher if ϵ is within the smaller range. Since the influence of "the curse of dimensionality" in the Yeast and Ionosphere datasets is small, the corresponding ϵ curve is relatively flatter on the left, and the outlier's density exists significantly difference, so the high accuracy of gentle transition

phase is wider. In short, the threshold ϵ has a certain impact on the mining accuracy.

As can be seen from table 2, the accuracy of COAS algorithm is higher than that of the COP algorithm. The main reason is that COP algorithm is based on the distribution pattern, and the relevant subspace is obtained from all over the attribute dimensions by maximum likelihood estimation, and the accuracy is greatly influenced by the data distribution and the curse of dimensionality. In COAS algorithm, the dimensional differences of various properties is used to determine the vectors of subspace, and the accuracy is less affected by the data distribution and the curse of dimensionality, so the accuracy of COAS algorithm is higher than that of the COP algorithm. Since

the determination of relevant subspace is based on a global approach in HiCS algorithm, and the relevant subspace candidates are searched through the similar Apriori algorithm, it may lead to the relevant subspace discovered from the datasets is all incomplete, and some outlier cannot be found. But the COAS algorithm can find the relevant subspace in the data, and also effectively discover the outliers hidden in any relevant subspace. Similar to HiCS algorithm, the relevant subspace in OUTRES algorithm is also based on a global, and thus the accuracy of OUTRES algorithm is lower than COAS algorithm. In short, the accuracy of COAS is relatively higher on the different datasets, especially with pretty obvious advantages in the high-dimensional data.

Table 2. Accuracy comparisons on UCI data sets

UCI data sets	Number of outlier	COP algorithm		COAS algorithm ($\epsilon=0.1$)		HiCS algorithm ($\alpha=0.1, M=50$, candidate_cutoff=50)		OUTRES algorithm ($\epsilon=15, \alpha=0.01$)	
		Outlier	Accuracy	Outlier	Accuracy	Outlier	Accuracy	Outlier	Accuracy
Yeast	5	5	100%	5	100%	5	100%	5	100%
Ionosphere	5	4	80%	5	100%	3	60%	3	60%
mfeat	10	4	40%	9	80%	5	50%	6	60%
Isolet	48	18	37.5%	41	85.4%	24	50%	31	64.6%

6.2 Efficiency Analysis of The Algorithm

The Experimental datasets are discrete Star spectrum datasets SDSS provided by China National Solar Observatory [2, 18, 25], and the experimental results are shown as figures 2, 3 and 4.

As can be seen from Fig. 2, the influence of the local sparse difference factor ϵ is smaller on the mining efficiency of COAS algorithm. The main reason is that most operations of COAS

algorithm focus on searching for the k-nearest neighbor, and it has nothing to do with ϵ . The value of ϵ only affects the value of each dimension in the subspace vector. The subspace vector with totally 0 can be possibly increased with the increasing of ϵ . Due to it need not be computed according to formula (14) when the space calculation vector is totally 0, so with the increasing of ϵ , the overall amount of computation can be reduced. In short, the value of ϵ has less impact on the mining efficiency.

Fig. 2 Mining Efficiency On different ϵ (Spectral data Records=10000, dimensions = 200)

Fig. 3 Mining Efficiency On attribute dimension (spectral data Records=10000, $\epsilon = 0.1$)

It can be seen from Fig. 3, when the number of data sets are the same, the pruning effects are very limited while the relative dimension of the subspace is higher because the complexity of COP algorithm is the cube of the dimension, and the complexity of OUTRES algorithm is exponential with the dimension in spite of the fact that the pruning strategies can be taken to reduce the time when determining the relative subspace in [18]. Similar to Apriori algorithm, two dimension subspaces of all the dimensions are first generated in HiCS algorithm, and the candidate set of two dimension subspace are selected, then three dimension candidate set are generated from the two dimension of candidate set, until $d+1$ dimension subspace cannot be generated from the candidate set $d+1$ dimension candidate subspace, in the end the candidate set of d dimension subspace is the related subspace set. The

candidate_cutoff is retained in each candidate set (namely the maximum number of candidate set's items, and the suitable candidate item is more or less than 500 which is obtained from many times of experiments in [19]). The pruning is constantly processed in the generation process in spite of that the exponent of dimension is effectively done with, the candidate set is acquired through the sorting of the difference value of subspaces. The difference value of each subspace is obtained through M iterations process, and the impact of the dimension degree is larger and the computation amount is also quite large. The computation complexity of the COAS Algorithm is relatively small since it is nearly linear relationship with the dimension. Thus, the efficiency of COAS algorithm is higher than that of COP algorithm OUTRES algorithm and HiCS algorithm.

Fig. 4 Mining Efficiency On Different Data Sets (Data dimension=200, $\epsilon = 0.1$)

As can be seen from Fig. 4, the time complexity of three algorithms is nearly linear, but the complexity of COP algorithm is the cube of the dimension, while the complexity of COAS algorithm is nearly to the linear relationship with the dimension. Therefore, the tilt degree of COP algorithm is greater than COAS algorithm, and its efficiency is lower than COAS algorithm. Similar to the algorithm analysis of HiCS algorithm in Fig. 4, the impact of dimension is still high and the tilt degree of HiCS algorithm is greater than that of COAS algorithm and the efficiency is lower than the COAS algorithm although the Apriori approach and the pruning strategies are taken by it.

6.3 Understandability of The Algorithm

To evaluate the understandability of the algorithm, we made experiments on the Wholesale customers Data Set from UCI

datasets (<http://archive.ics.uci.edu/ml/datasets/Wholesale+customers>). The data set refers to clients of a wholesale distributor. It includes the annual spending in monetary units (m.u.) on diverse product categories in different sales channels and regions. This data set contains a total of 440 pieces of data, among which every piece of data denotes some information on this wholesale distributors, and is composed of 8 dimensions: CHANNEL, REGION, Fresh, Milk, Grocery, Frozen, Detergents_Paper and Delicatessen. In this analysis, we first make preprocess with the data. Since CHANNEL and REGION are qualitative data, these dimensions of data are ignored during the calculation process. In the case of local sparse difference factor thresholds $\epsilon=0.1$, two outliers whose outlier factors are the largest, are obtained by COAS algorithm and shown in Table 3. Table 4 and Table 5 are the two outliers' reference or local data set, respectively.

Table 3. Outliers

No.	Oulier factor	Relevant subspace
obj183	0.886	{ Fresh, Milk, Frozen, Detergents_Paper, Delicatessen }
obj86	0.886	{ Fresh, Milk, Frozen, Detergents_Paper, Delicatessen }

As can be seen from Table 3, Table 4 and Table 5, the outlier data obj183 of {36847, 43950, 20170, 36534, 239, 47943} contains the following context information: the reference data set of {183, 23, 325, 427, 435, 87, 196, 202, 265, 384, 71, 24, 12, 92, 40, 47, 103, 165, 45, 251, 254}, the relevant subspace {Fresh, Milk, Frozen, Detergents_Paper, Delicatessen} and the outlier Factor=0.886. The outlier data obj86 of {22925, 73498, 32114, 987, 20070, 903} contains the following context information: the reference data set

of {86, 47, 23, 61, 211, 56, 92, 319, 251, 49, 171, 45, 28, 437, 65, 384, 265, 163, 331, 216, 309}, the relevant subspace {Fresh, Milk, Frozen, Detergents_Paper, Delicatessen} and the outlier Factor=0.886. The outlier data obj183 can be explained as follows: the annual spending on Fresh, Milk, Frozen, Detergents_Paper and Delicatessen are 36847, 43950, 36534, 239, and 47943 respectively. The difference between these data and the reference data set is very large, i.e. the values on Fresh, Milk, Frozen and Delicatessen are much larger, while the value on Detergents_Paper is much smaller. In addition, the outlier data obj86 can be explained as follows: the annual spending on Fresh, Milk, Frozen, Detergents_Paper, and Delicatessen are 22925, 73498, 987, 20070, and 903 respectively. The difference between these data and the reference data set is very large, i.e. the values on Fresh, Milk and Detergents_Paper are much larger, while the values on Frozen and Delicatessen are much smaller.

Table 4. Reference or Local data set of outlier object 183

No.	Fresh	Milk	Grocery	Frozen	Detergents_Paper	Delicatessen
obj183	36847	43950	20170	36534	239	47943
obj12	31714	12319	11757	287	3881	2931
obj23	26373	36423	22019	5154	4337	16523
obj24	22647	9776	13792	2915	4482	5778
obj40	24025	4332	4757	9510	1145	5864
obj45	5181	22044	21531	1740	7353	4985
obj47	44466	54259	55571	7782	24171	6465
obj71	18291	1266	21042	5373	4173	14472
obj87	43265	5025	8117	6312	1579	14351
obj92	9198	27472	32034	3232	18906	5130
obj103	56082	3504	8906	18028	1480	2498
obj165	15615	12653	19858	4425	7108	2379
obj196	30624	7209	4897	18711	763	2876
obj202	25203	11487	9490	5065	284	6854
obj251	6134	23133	33586	6746	18594	5121
obj254	10379	17972	4748	4686	1547	3265
obj265	5909	23527	13699	10155	830	3636
obj325	32717	16784	13626	60869	1272	5609
obj384	10683	21858	15400	3635	282	5120
obj427	31012	16687	5429	15082	439	1163
obj435	29703	12051	16027	13135	182	2204

Table 5. Reference or Local data set of outlier object 86

No.	Fresh	Milk	Grocery	Frozen	Detergents_Paper	Delicatessen
obj86	22925	73498	32114	987	20070	903
obj23	26373	36423	22019	5154	4337	16523
obj28	4113	20484	25957	1158	8604	5206
obj45	5181	22044	21531	1740	7353	4985
obj47	44466	54259	55571	7782	24171	6465
obj49	4967	21412	28921	1798	13583	1163
obj56	4098	29892	26866	2616	17740	1340
obj61	35942	38369	59598	3254	26701	2017
obj65	85	20959	45828	36	24231	1423
obj92	9198	27472	32034	3232	18906	5130
obj163	5531	15726	26870	2367	13726	446
obj171	200	25862	19816	651	8773	6250
obj211	12119	28326	39694	4736	19410	2870
obj216	2532	16599	36486	179	13308	674
obj251	6134	23133	33586	6746	18594	5121
obj265	5909	23527	13699	10155	830	3636
obj309	918	20655	13567	1465	6846	806
obj319	9759	25071	17645	1128	12408	1625
obj331	11223	14881	26839	1234	9606	1102
obj384	10683	21858	15400	3635	282	5120
obj437	14531	15488	30243	437	14841	1867

7. CONCLUSION

In the high-dimensional and massive datasets, the implantation of data mining task from the relevant subspace, constructed from the meaningful dimension, is one of the hot spots and difficult problems. In this paper, a relevant subspace based contextual outlier mining algorithm is proposed. In the algorithm, the relevant subspace is redefined by using the local sparse degree of attribute dimensions in the local data set, the attribute dimensions constituting the relevant subspace is regarded as the data objects' contextual information, and accordingly the influence of irrelevant attribute dimensions on the outlier can be effectively avoided and the context-sensitive outliers hidden in the subspace can be detected. Furthermore, the algorithm has the better mining effects. The future work would focus on the degree of outlier values are smaller when the local dataset are difficult to reflect the distribution characteristics of clusters.

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