Prestressed glulam beams reinforced with CFRP bars

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ABSTRACT

This paper describes an experimental test program and theoretical analysis which examines the reinforcing in flexure of glued laminated timber (glulam) beams using bonded-in carbon fiber reinforced polymer (CFRP) bars. A series of four-point bending tests were conducted till failure on unreinforced, passively reinforced and prestressed Douglas fir glulam beams in a simply-supported scheme. The focus of this research was to evaluate the reinforcing efficiency of both passively reinforced and prestressed beams. Test results showed that the flexural capacity of the reinforced, prestressed, prestressed & reinforced (bottom prestressed and top reinforced) beams greatly increased by 64.8%, 93.3% and 131%, respectively. While the maximum improvement of the bending stiffness reached 42.0%. Another important finding was that the extreme fiber tensile strain of timber beams at failure could be remarkably increased due to the presence of the tension reinforcement, which indicated it overcomes the effects of local defects and therefore the failure mode was changed from brittle tension failure to ductile compression failure. Based on the experimental results, a theoretical model was proposed to predict the flexural capacity of unreinforced, reinforced and prestressed timber beams, which was validated by the test data.

Key words: Glulam beams; Prestressed; Timber structures; CFRP bars; Four-point bending.

1. Introduction

Timber is renewable and sustainable, and it also has the lowest energy consumption and the lowest carbon dioxide emission among many building materials. Thus it is one of the world's most environmentally friendly building materials. Glued laminated timber (glulam), as an engineered wood product, was developed during the 19th century

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in Europe and is widely used nowadays in buildings and bridges [1]. Glulam has an excellent strength-to-weight ratio, shape and size flexibility, as well as high strength and dimensional stability.

However, despite all of these benefits, glulam beams are usually underused due to the naturally defects such as knots and cross grain [2]. Another problem is the relative low stiffness as a result of which the design of glulam beams is often controlled by deflection limits [3]. For these reasons, many attempts have been made to reinforce or strengthen glulam or solid timber beams by using high tensile strength materials. In the earlier decades, the majority of this work focused on the use of metallic reinforcement [4-7]. More recently fiber reinforced polymers (FRP) was used as structural reinforcement for timber beams, which in the form of sheets, plates and bars [8-19]. It showed from these researches that the reinforcement in the tension zone would improve the strength, stiffness and ductility. Furthermore, both short-term and long-term deflections of the reinforced timber beams were decreased [3,20].

But the reinforcing materials usually has a notable higher ultimate tensile strain than that of wood, which means it was not effectively used while the failure occurs in this kind of timber members with passive reinforcement. Thus its economic efficiency was argued by some researchers [21,22]. Attempt then was made by introducing prestress in reinforcing materials [2,23-27]. As a result, the flexural strength is further increased due to the full use of both FRP and wood, while bending stiffness is greatly improved because of the pre-camber produced in the flexural members. Also the introduction of prestress may provide an extra strength at a small additional cost [27].

Since there was less research upon prestressed timber beams especially using FRP bars. In this paper, we focus on the improved performance of flexural behavior of glulam beams reinforced with prestressed CFRP bars. Four-point bending tests were conducted on the unreinforced control, passively reinforced and prestressed glulam beams with longitudinally bonded and end anchored CFRP bars. Subsequently a theoretical model was developed, in which the increased tensile strain of glulam timber resulted from reinforcement was taken into consideration. A calibration of the theoretical model was then undertaken based on the experimental results.

2. Experimental program

2.1. Materials

The glulam beams, CFRP bars and adhesive used in this investigation were purchased from several manufacturers, as detailed below, with the mechanical characteristics either obtained from test results in the university laboratory or furnished by the manufacturers.

2.1.1. Glulams

The Douglas fir homogeneous glulam was visually classified by the manufacturer and then tested in the university laboratory according to BS EN13183-1:2002 [28] and BS EN 408:2010 [29]. The characteristic strength properties

(5% value) directly from test results were shown in Table 1. And the moisture content of the glulam ranged from 12.9% to 14.6% with the mean value of 13.8%, while the density ranged from 461 kg/m³ to 583 kg/m³ with the mean value of 531 kg/m³. The experimental characteristic strength and stiffness properties were compared to those presented by BS EN 1194:1999 [30], which including the grade of GL 28h and GL 36h, as shown in Table 1.

Table 1

Characteristic strength and stiffness properties of homogeneous glulam.

Material property	Test results (N/mm ²)	GL 28h [30] (N/mm ²)	GL 36h [30] (N/mm ²)
Tension strength parallel to grain	32.8	19.5	26.0
Compression strength parallel to grain	37.0	26.5	31.0
Modulus of elasticity parallel to grain, mean value	12,500	12600	14700
Modulus of elasticity parallel to grain, 5% value	11,400	10200	11900

2.1.2. CFRP bars

The CFRP bars had diameters of 11.0 mm and 16.0 mm to obtain two different reinforcement ratios in the tension zone of the glulam. The bars were composed of 65% unidirectional carbon fiber by volume and 35% thermoset epoxy resin. The tensile strength of the bars was 2,300 N/mm² and modulus of elasticity in tension was 165,000 N/mm², as was provided by its manufacturer.

2.1.3. Adhesive

A two-component epoxy resin named XK390, with the density of about 1,430 kg/m³, was used between CFRP bars and glulam beams in this research. The mechanical properties of the adhesive, provided by the manufacturer, are listed in Table 2.

Table 2

Mechanical properties of the epoxy resin provided by the manufacturer.

Specification	Value (N/mm ²)
Compressive strength	70.0
Bending strength	65.0
Splitting strength	9.2
Modulus of elastic in tension	3320

2.2. Specimens preparation

Twelve glulam beam specimens were tested to failure under monotonic load in four-point bending configuration. The specimens were 6.0 m long with a cross section of 75 mm × 300 mm. The reinforced and prestressed specimens were longitudinally slotted at the corresponding face of the CFRP bars (see Fig. 1). The slots were 30 mm deep and 20 mm wide for bonded bottom CFRP bars, while 30 mm deep and 16 mm wide for bonded top reinforced CFRP bars. The

slots were filled with a small timber lamina for all of the specimens with bonded bars after the gluing process. The beam specimens were then divided into four groups with three replicates for each (see Fig. 1 and Table 3).

Fig. 1. Details of cross section types.

Table 3

Series and specimen information.

Series reference	No. of replicates	Reinforcement		Diameter (mm)	Anchorage type	Cross-section ratio (%)	Prestress force ^c (kN)
		Type	Location				
C	3	-	-	-	-	0.00	-
R11	3	passive reinforcement	Bottom	11	End anchor and bonded	0.42	0.0
P16	3	prestressed	Bottom	16	End anchor and bonded	0.89	50.8
P16R11	3	prestressed and reinforcement	Bottom and top	16 and 11 ^a	End anchor and bonded	0.89 and 0.42 ^b	50.8

^a Bottom prestressed bar was 16 mm in diameter and top reinforced bar was 11 mm in diameter.

^b Cross-section ratio for bottom prestressed bar was 0.89 and for top reinforced bar was 0.42.

^c Prestress force was average strain values of the CFRP bars at mid-point of the three glulam beams in each series.

Prior to the bonding operation, the glulam slots were cleaned with high-pressure air while the CFRP bars were cleaned with acetone. The prestress was produced by the Preflex process [31], as shown in Fig. 2, through the symmetrical four-point bending loads. The end anchorages were tightened then as the glulam beam deformation came to the predicted value. Furthermore, the CFRP bar could be re-tightened if necessary by the nuts of the straight sleeve bonded-type anchorages (see Fig. 3). And a detailed description and comprehensive review of these kind of anchorages without nuts can also be found in the literature [32]. Subsequently, the slots were filled with the adhesive for the reinforcement/prestressed glulam specimens. The preflexion loads remained constant for about 72 hours at the environmental temperature of $20 \pm 2^\circ\text{C}$. And then the applied Preflex load were slowly released while the concerned

98 strains and deflections, including pre-camber, were recorded. The prestress force was then calculated from the
 99 average strain values of the CFRP bars at mid-point of the glulam beams and presented in Table 3. The glulam beam
 100 was immediately overturned and tested to failure, which presents a detailed report in the following part of this paper.

101

102 **Fig. 2.** Phases of construction of prestressed beam: (a) assemblage of glulam beam, (b) pre-deflection of glulam
 103 beam and filling the slots with adhesive, (c) release of glulam beam after the adhesive is well cured and (d) Overturn
 104 of glulam beam and prepare to test.

105

106 **Fig. 3.** Straight sleeve anchorages with nuts.

107 According to Negrão [27], during specimen preparation and test procedure, the possible prestress loss mainly caused
 108 by the contact condition of the anchor nuts and the elastic recovery of the specimens. However, the tensile strain of
 109 the prestressing FRP bars has been recording during this stage so that the prestress force is known to us. Also Negrão
 110 [27] discussed the prestress loss under long-term condition and pointed out that creep and reinforcement relaxation is
 111 one of the most concerning aspects. However, the issue on prestress loss is not the focus of this research.

112 2.3. Flexural test procedure and beam instrumentation

113 All glulam beam specimens were tested using the four-point bending method in accordance to BS EN 408:2010 [29],
 114 with the test setup as shown in Fig. 4. The loading rate was set to 5.0 mm/min and was held constant until failure.
 115 Lateral roller-type restraint was provided to prevent buckling or lateral torsional effects. Since the mid-span
 116 deflection is a relative value about that of supports, linear variable differential transformers (LVDTs) were located at
 117 both midpoint and end supports of the glulam beam, while strains were monitored by paper based strain gauge at
 118 mid-span of the beam both throughout the depth with a space of 75 mm and in CFRP bars. It should be noted that
 119 strain gauges just included on one side of glulam beams, so the twist of beams and deviations of wood quality were
 120 not accounted for.

122 **Fig. 4.** Test set-up and arrangement.

123 2.4. Experimental results & discussion

124 2.4.1. Load-deflection behavior

125 Figs. 5-8 represents the load-deflection behaviour of the four series of glulam beams including unreinforced,
 126 reinforced and prestressed:

- 127 - The load-deflection behaviour of the unreinforced control beams (C-1, C-2 and C-3) are shown in Fig. 5. All the
 128 glulam beams exhibited linear elastic behaviour till the failure at tension face due to the presence of either knots or
 129 cross grain. No compression yielding occurred in the compression zone of the glulam beams.
- 130 - As can be seen from the load-deflection curves of the bottom reinforced beams (R11-1, R11-2 and R11-3) shown in
 131 Fig. 6, non-linear behaviour is introduced into the beams before the maximum load was reached. This means that
 132 the yielding of the wood in the compression zone occurred before the wood of tension face reached the ultimate
 133 tensile strain.
- 134 - Fig. 7 plots the load-deflection curves of the two bottom prestressed beams (P16-1 and P16-2). The bottom
 135 prestressed beams shows almost the same load-deflection behaviour as that of bottom reinforced beams.
 136 Furthermore, the deflection of the prestressed beam decreased, when compared to the reinforced beams, due to the

137 pre-camber produce by the prestress. The load-deflection data of the specimen P16-3 was not recorded because of
 138 the faulty LVDT.

139 - Fig. 8 illustrates the load-deflection curves of the bottom prestressed & top reinforced beams (P16R11-1, P16R11-2
 140 and P16R11-3). Due to the presence of the top reinforcement in the compression zone, as can be seen from Fig. 8, it
 141 shows less non-linear behaviour than that of just prestressed beams. It indicates that the compression reinforcement
 142 suspend or prevent the wood yielding in the compression zone. And as a result, the ultimate load of this series is
 143 even higher.

144 It should be noted that the post failure behaviour of most reinforced or prestressed beams was not recorded during the
 145 test. So it is not possible to predict the ductility of the tested beams.

146
 147 **Fig. 5.** Load-deflection curve for the unreinforced
 148 control beams (C series).

Fig. 6. Load-deflection curve for the bottom reinforced
 beams (R11 series).

149
 150 **Fig. 7.** Load-deflection curve for the bottom prestressed
 151 beams (P16 series).

Fig. 8. Load-deflection curve for the bottom prestressed
 & top reinforced beams (P16R11 series).

152 The test results are also summarized in Table 4. It can be seen from Table 4 that the failure load was significantly
 153 improved for either reinforced or prestressed glulam beams. These improvements reached 64.8%, 93.3% and 131%

for the passive reinforced, prestressed and prestressed & reinforced glulam beams, respectively. The average failure load of the prestressed beams was increased from 61.3 kN to 71.9 kN as compared to the passively reinforced beams, which represents an improvement of 17.3% while increasing the CFRP reinforcement ratio from 0.42% to 0.89%. As to the load while the beam deflection reached 1/250 of the beam span, the enhancement was even more evident for the prestressed glulam beams. This improvement in load-carrying capacity reached a maximum of 211%, which is considered of a benefit from both the reinforcement itself and the pre-camber.

Table 4

Comparison of experimentally obtained results.

Specimen type	Beam code	Failure load F_u (kN)	Failure moment M_u (kNm)	Load at the mid-span deflection reached $L/250 F_{L/250}$ (kN)	Stiffness $E_{mg}I$ (10^{12} Nmm ²)	Failure modes
Unreinforced control beam (C series)	C-1	33.5	31.8	13.7	2.03	Tension
	C-2	39.0	37.1	12.7	1.91	Tension
	C-3	39.3	37.1	13.1	1.94	Tension
	Average	37.3	35.3	13.2	1.96	—
	Standard deviation	3.3	3.0	0.50	0.06	—
Reinforced beam (R11 series)	R11-1	59.1	56.1	17.0	2.52	Compression
	R11-2	57.3	54.4	16.1	2.13	Compression
	R11-3	67.7	64.3	16.7	2.33	Compression
	Average	61.3	58.3	16.6	2.33	—
	Standard deviation	5.6	5.3	0.46	0.18	—
	% increase ^a	64.8	64.8	26.1	18.9	—
Prestressed beam (P16 series)	P16-1	65.6	62.3	39.7	2.62	Compression
	P16-2	70.9	67.4	38.0	2.61	Compression
	P16-3 ^b	79.2	75.2	—	—	Compression
	Average	71.9	68.3	38.9	2.61	—
	Standard deviation	6.9	6.5	1.20	0.01	—
	% increase ^a	93.3	93.3	195.1	33.2	—
Prestressed & reinforced beam (P16R11 series)	P16R11-1	90.6	86.1	41.8	2.91	Flexure-shear
	P16R11-2	82.7	78.6	41.2	2.84	Flexure-shear
	P16R11-3	84.2	80.0	40.0	2.61	Flexure-shear
	Average	85.9	81.5	41.0	2.79	—
	Standard deviation	4.2	4.0	0.92	0.14	—
	% increase ^a	131	131	211.4	42.3	—

^aImprovement compared to the unreinforced control beam in average value.

^bThe deflection and stiffness values are not available due to the faulty LVDT.

165 It also can be seen from Table 4 that, as compared to the unreinforced control beams, the beam stiffness ($E_{mg}I$,
 166 calculated using Eq. (1) shown below, which was derived from BS EN 408: 2010 [29]) increased by 18.9%, 33.2%
 167 and 42.3% for the passive reinforced, prestressed, and prestressed & reinforced glulam beams, respectively.

$$168 \quad E_{mg}I = \frac{3al^2 - 4a^3}{24\left(\frac{2(w_2 - w_1)}{F_2 - F_1} - \frac{6a}{5Gb h}\right)} \quad (1)$$

169 Where E_{mg} (MPa): global modulus of elasticity; I (mm⁴): the section moment of inertia; a (mm): distance to loading
 170 point (see Fig. 4); l (mm): beam span (see Fig. 4); w_1 (mm): deformation at F_1 ; w_2 (mm): deformation at F_2 ; F_1 (N):
 171 applied load at 10% of estimated maximum load; F_2 (N): applied load at 40% of estimated maximum load; G (MPa):
 172 shear modulus ($G = 850$ MPa for glulam with the grade of GL28h according to BS EN 1194:1999 [30]); b (mm):
 173 beam width; h (mm): beam depth.

174 2.4.2. Failure modes

177 **Fig. 9.** Typical failure modes of the beams: (a) tension failure, (b) compression failure and (c) flexure-shear failure.

178 Out of the 4 series of beams tested, the unreinforced control beams all failed in tension mode at the location of
 179 natural defects (C series, see Fig. 9(a)), 2 series failed in compression mode (R11 series and P16 series, see Fig. 9(b)),
 180 while the series of prestressed & reinforced beams all failed in combined flexure and shear mode (P16R11 series, see
 181 Fig. 9(c)), as summarized in Table 4. The introduction of the CFRP bars caused a change in the failure mode from
 182 brittle tension in the unreinforced beams to a ductile compression failure, which is a common phenomenon for the
 183 reinforced/prestressed timber beams reported by many authors [11,13,16,17,19,36]. There was no rupture of the

CFRP bars due to their high strength. As a result of the addition of the end anchorage as well as the longitudinal bonding of CFRP bars, none of the failures were due to debonding.

2.4.3. Strain distribution

Typical load-strain curves are illustrated in Figs. 10-13. It can be seen from the figures that the load-strain curve for the unreinforced beams (see Fig. 10) remained nearly linear, while the reinforced and prestressed ones showed a non-linear behaviour at the late stage of the curves, as a result of the wood crushing in the compression zone (see Fig. 11 and Fig. 12). For prestressed & reinforced glulam beams, there was less non-linear behaviour due to the reinforcement in compression zone, which could reduce the compressive strain of the wood (see Fig. 13).

Fig. 10. Typical load-strain curve for an unreinforced reinforced control beam (C-3).

Fig. 11. Typical load-strain curve for a bottom beam (R11-3).

Fig. 12. Typical load-strain curve for a bottom prestressed beam (P16-2).

Fig. 13. Typical load-strain curve for a bottom prestressed & top reinforced beam (P16R11-2).

Figs. 14-17 plot the strain profiles at mid-span for typical specimens of the four series under different load levels till failure. As the load increases to failure, the mechanical neutral axis position moves from -4.4 mm to -8.7 mm against the geometric neutral axis for unreinforced beams (see Fig. 14), while from -4.4 mm to -12.1 mm for passively

reinforced beams (see Fig. 15). As to the prestressed beams, it was from -15.4 mm to -29.6 mm (see Fig. 16). While for prestressed & reinforced beams, it became -13.3 mm to -19.4 mm (see Fig. 17).

Fig. 14. Typical strain profile for an unreinforced control beam (C-3).

Fig. 15. Typical strain profile for a bottom reinforced beam (R11-3).

Fig. 16. Typical strain profile for a bottom prestressed beam (P16-2).

Fig. 17. Typical strain profile for a bottom prestressed & top reinforced beam (P16R11-3).

The average extreme fibre strain for both compressive and tensile at failure is summarized in Table 5. As can be seen from Table 5, both the average extreme fiber tensile and compressive strain at failure were all about 0.25% for the unreinforced beams. The introduction of the reinforcement or prestress could maximally increase the average extreme fibre tensile and compressive strain by 77.6% (with a tensile strain value of about 0.44%) and 88.2% (with a compressive strain value of about 0.47%), respectively. This finding is consistent with many studies [11,33-36]. Since the strain gauges could not record the exact global compressive strain of wood in the late loading stage, the extreme fiber compressive strain (as listed in Table 5) was then estimated from the strain values at the beam height of 0 mm and 75 mm (or 0 mm and -75 mm for specimen P16-2), by extending the straight line between these two points of strain values to the compression face.

218 **Table 5**

219 Comparison of extreme fibre strain at failure of the specimens.

Specimen type	Beam code	Extreme fiber compressive strain $\epsilon_{c,max}$ (microstrain)	Extreme fiber tensile strain $\epsilon_{t,max}$ (microstrain)
Unreinforced control beam (C series)	C-1	-2758	2288
	C-2	-1724	2572
	C-3	-3025	2638
	Average	-2502	2499
	Standard deviation	687	186
Reinforced beam (R11 series)	R11-1	-4110	4450
	R11-2	-4120	4550
	R11-3	-5500	4317
	Average	-4577	4439
	Standard deviation	800	117
	% increase ^a	82.9	77.6
Prestressed beam (P16 series)	P16-1	-5625	3300
	P16-2	-3810	2820
	P16-3	-4690	3200
	Average	-4708	3107
	Standard deviation	908	253
	% increase ^a	88.2	24.3
Prestressed & reinforced beam (P16R11 series)	P16R11-1	-3541	3871
	P16R11-2	-3199	3143
	P16R11-3	-3800	3320
	Average	-3513	3445
	Standard deviation	301	380
	% increase ^a	40.4	37.8

220 ^aImprovement compare to the unreinforced control beam in average value.221 **3. Theoretical model**222 *3.1. Proposed methodology*

223 A general theoretical model for all reinforced or/and prestressed glulam beam in bending was used to predict the
 224 resistant moment after the compressive strain of wood reaches elastic strain limits, as shown in Fig. 18. The
 225 analytical values were then compared with experimental ones. This type of model has been used by most researchers
 226 dealing with reinforced glulam [10,11,24,35-40].

227 The model adopted here assumed the following:

228 - plane sections remain plane;

- 229 - plane sections remain plane;
- 230 - bonding between CFRP bars and glulam was assumed as perfect;
- 231 - the stress-strain behaviour of timber was based on the model proposed by Bazan [39] and modified by Buchanan
- 232 [40]: tension behavior was assumed to be linear-elastic for the glulam; a bilinear relationship was assumed for the
- 233 compression behavior, with a linear part up to compressive strength f_{wcu} and a corresponding strain ϵ_{wcy} ; the slope
- 234 of the descending branch was a constant fraction, m ($m \leq 0$), of the modulus of elasticity of the timber, E_w (see
- 235 Fig. 19), $m = -0.25$ herein proposed by Lindyberg [41].
- 236 - the stress-strain relationship for the CFRP bar in tension and in compression was linear-elastic until failure (Fig.
- 237 20).

Fig. 18. Cross section of the glulam beam with the notation of the theoretical model.

Fig. 19. Stress-strain relationship for glulam.

Fig. 20. Stress-strain relationship for CFRP bar.

- 242 As can be seen from Fig. 18, it is a general model for all kinds of beams. As to a passively reinforced glulam beams,
- 243 the pre-existing strain (ϵ_{p0}) in the prestressing CFRP bar is assumed to equal zero. While for the glulam beams
- 244 reinforced only in the tension zone, the cross section of the CFRP bar (A_{rc}) and the internal force in the compression
- 245 zone (F_{rc}) are all assumed to equal zero then.
- 246 As a widely accepted fact, due to the stress distribution effect in a timber flexural members, the extreme fiber tensile

stress at failure in bending, f_{wm} , is greater than the axial tensile strength f_{wtu} . And the relationship between these two parameters as indicated in Eq. (2), proposed by Madsen and Buchanan [42], was widely accepted worldwide and therefore adopted in this paper.

$$f_{wm} = \left(\frac{k_3 + 1}{c} \right)^{1/k_3} f_{wtu} \quad (2)$$

Where k_3 (dimensionless): the stress distribution parameter; c (dimensionless): the ratio of the depth of the neutral axis from the tension face with respect to the depth of the section.

In this investigation, however, it was not possible to obtain f_{wtu} directly because of the limited samples of the axial strength test specimens. For this reason, the extreme fiber tensile stress at failure in bending, f_{wm} , was obtained directly from the experimental mean strain values of the unreinforced glulam beams.

Since the presence of the reinforcement can remarkably improve the extreme fiber tensile strain of glulam at failure [11,33-36], a modification factor α_m was introduced in the theoretical analysis to account for this enhancement of the tensile strength in bending, as proposed by Gentile et al. [11] and by Gentile [36]. Also it is considered that $\alpha_m = 1.3$, which was proposed by the authors [11,36], is reasonable when taking into account of these all experimental results including this research.

Thus the tensile strength in bending of reinforced glulam is:

$$f_{wmr} = \alpha_m f_{wm} = \alpha_m E_w \varepsilon_{wmu} \quad (3)$$

Where f_{wmr} (MPa): the tensile strength in bending of reinforced timber; α_m (dimensionless): a constant modification factor ($\alpha_m = 1.3$, as proposed by the literature [11,36]); E_w (MPa): Modulus of elasticity of timber; ε_{wmu} (dimensionless): the ultimate tensile strain of timber in bending.

Generally, since there was no yielding of the reinforcement, there are two typical flexural failure modes for reinforced or prestressed glulam beams:

- reinforced timber tensile failure as the extreme fiber tensile strain reaches ultimate tensile strain ($\varepsilon_{wt} = \alpha_m \varepsilon_{wmu}$);
- timber compressive failure as the extreme fiber compressive strain reaches ultimate compressive strain ($\varepsilon_{wc} = 1.2\%$, as assumed from Brunner [38]).

Then based on the theoretical model and the stress-strain relationships of the materials presented above, the neutral axis location and then the ultimate bending moment could be evaluated using the equilibrium condition on the cross section of the beam, as discussed in the following section.

3.1.1. Flexural capacity for timber tensile failure

According to the theoretical model shown in Fig. 18, the internal forces were defined by the following:

$$F_{rc} = A_{rc} \sigma_{rc} = A_{rc} E_r \varepsilon_{rc} \quad (4)$$

$$F_{wcp} = A_{wcp} \cdot (f_{wcu} + \sigma_{wc}) / 2 = E_w b h_p [(2-m)\varepsilon_{wcy} + m\varepsilon_{wc}] / 2 \quad (5)$$

$$F_{wce} = A_{wce} \cdot f_{wcu} / 2 = E_w b (h_c - h_p) \varepsilon_{wcy} / 2 \quad (6)$$

$$F_{wt} = A_{wt} \cdot \sigma_{wt} / 2 = E_w b (h - h_c) \varepsilon_{wt} / 2 \quad (7)$$

$$F_{rt} = A_{rt} \sigma_{rt} + F_{p0} = A_{rt} E_r \varepsilon_{rt} + F_{p0} \quad (8)$$

Where h_c (mm): the depth of neutral axis; h_p (mm): the depth of plastic compression zone; F_{p0} (N): the prestress load of the CFRP bar in tension zone ($F_{p0} = A_{rt} E_r \varepsilon_{p0}$); the other parameters are all indicated in Fig. 18 to Fig. 20.

Since the failure is due to the timber tension failure, ε_{wt} was then given as $\alpha_m \varepsilon_{wmu}$. The other strain quantities can be calculated according to the assumption that the sections remain planar, as shown in Fig. 18:

$$\varepsilon_{wc} = \frac{h_c}{h - h_c} \varepsilon_{wt} \quad (9)$$

$$\varepsilon_{wcy} = \frac{h - h_p}{h - h_c} \varepsilon_{wt}, \Rightarrow h_p = (1 + \frac{\varepsilon_{wcy}}{\varepsilon_{wt}}) h_c - \frac{\varepsilon_{wcy}}{\varepsilon_{wt}} h \quad (10)$$

$$\varepsilon_{rc} = \frac{h - h_{ac}}{h - h_c} \varepsilon_{wt} \quad (11)$$

$$\varepsilon_{rt} = \frac{h - h_c - h_{at}}{h - h_c} \varepsilon_{wt} \quad (12)$$

The internal force equilibrium requires that:

$$F_{wt} + F_{rt} = F_{rc} + F_{wce} + F_{wcp} \quad (13)$$

Then the depth of neutral axis h_c can now be calculated:

$$h_{c,tf} = \frac{-b_1 \pm \sqrt{b_1^2 - 4a_1c_1}}{2a_1} \quad (14)$$

where $h_{c,tf}$ (mm): the depth of neutral axis corresponding to timber tensile failure;

$$a_1 = (m-1)E_w b (\alpha_m \varepsilon_{wmu} + \varepsilon_{wcy})^2 / (2\alpha_m \varepsilon_{wmu});$$

$$b_1 = E_r \alpha_m \varepsilon_{wmu} (A_{rc} + A_{rt}) + F_{p0} + \frac{E_w b h}{\alpha_m \varepsilon_{wmu}} [\alpha_m^2 \varepsilon_{wmu}^2 + (1-m)\varepsilon_{wcy} (\varepsilon_{wcy} + \alpha_m \varepsilon_{wmu})];$$

$$c_1 = \frac{E_w b h^2}{2\alpha_m \varepsilon_{wmu}} [(m-1)\varepsilon_{wcy}^2 - \alpha_m^2 \varepsilon_{wmu}^2] - E_r \alpha_m \varepsilon_{wmu} (A_{rc} h_{ac} + A_{rt} h - A_{rt} h_{at}) - F_{p0} h.$$

The resultant resisting moment can then be predicted as follows:

$$M_{tf} = F_{rc}d_{rc} + F_{wcp}d_{wcp} + F_{wce}d_{wce} + F_{wt}d_{wt} + F_{rt}d_{rt} \quad (15)$$

where M_{tf} (Nmm): the resisting moment of glulam beams at timber tensile failure; the corresponding lever arms of the internal forces are:

$$d_{rc} = (h_c - h_{ac}) \quad (16)$$

$$d_{wcp} = h_c - \frac{2(h - h_c)\varepsilon_{wcy} + h_c\alpha_m\varepsilon_{wmu}}{3[(h - h_c)\varepsilon_{wcy} + h_c\alpha_m\varepsilon_{wmu}]} h_p \quad (17)$$

$$d_{wce} = 2(h_c - h_p)/3 \quad (18)$$

$$d_{wt} = 2(h - h_c)/3 \quad (19)$$

$$d_{rt} = h - h_c - h_{at} \quad (20)$$

3.1.2. Flexural capacity for timber compressive failure

In this phase, only the timber in compression has exceeded its plastic limit and the given condition is $\varepsilon_{wc} = 1.2\%$, as assumed from Brunner [38]. Then the depth of neutral axis h_c can now be found out as following:

$$h_{c,cf} = \frac{-b_2 \pm \sqrt{b_2^2 - 4a_2c_2}}{2a_2} \quad (21)$$

where $h_{c,cf}$ (mm): the depth of neutral axis corresponding to timber compressive failure;

$$a_2 = (m - 1)E_w b(\varepsilon_{wcu} - \varepsilon_{wcy})^2 / (2\varepsilon_{wcu});$$

$$b_2 = E_r \varepsilon_{wcu} (A_{rc} + A_{rt}) - F_{p0} + E_w b h \varepsilon_{wcu};$$

$$c_2 = \varepsilon_{wcu} [E_r A_{rt} (h_{at} - h) - E_r A_{rc} h_{ac} - E_w b h^2 / 2].$$

The resultant resisting moment is then found out to be:

$$M_{cf} = F_{rc}d_{rc} + F_{wcp}d_{wcp} + F_{wce}d_{wce} + F_{wt}d_{wt} + F_{rt}d_{rt} \quad (22)$$

where M_{cf} (Nmm): the resisting moment of glulam beams at timber compressive failure.

And the corresponding lever arms are the same as above except that:

$$d_{wcp} = h_c - \frac{2\varepsilon_{wcy} + \varepsilon_{wcu}}{3(\varepsilon_{wcy} + \varepsilon_{wcu})} h_p \quad (23)$$

Then it can be determined by comparing the calculating results from Eq. (15) and Eq. (22), the ultimate bending moment could be:

$$M_u = \min \{ M_{tf}, M_{cf} \} \quad (24)$$

3.1.3. Bending stiffness

The bending stiffness of glulam beams reinforced or prestressed with CFRP bars was computed based on the transformed section method using Eq. (25).

$$EI = E_w \left[I_w + (\alpha_E - 1) A_{rt} (h/2 - h_{at})^2 + (\alpha_E - 1) A_{rc} (h/2 - h_{ac})^2 \right] \quad (25)$$

where I_w (mm^4): the section moment of inertia of unreinforced glulam beam; α_E (dimensionless): the modulus of elasticity ratio of CFRP bar to glulam beam.

3.2. Comparison of theoretical analysis to test data

Table 6

Comparison between theoretical and experimental results.

Series reference	Bending stiffness $EI(10^{12}\text{Nmm}^2)$			Flexural capacity M_u (kNm)		
	Experimental results	Analytical results	Difference (%)	Experimental results	Analytical results	Difference (%)
C	1.96	2.11	+7.7	35.3	30.9	-12.4
R11	2.33	2.35	+0.8	58.3	50.5	-13.4
P16	2.61	2.63	+0.8	68.3	66.1	-3.2
P16R11 ^a	2.79	2.87	+2.9	81.5	72.1	-11.6

^a It showed experimental flexure-shear failure modes for this series.

Using the assumptions and procedure that were previously described, the flexural capacity and bending stiffness were calculated and compared with the corresponding experimental results (Table 6). The predicted flexural capacity M_u and bending stiffness EI were calculated using Eq. (15,22,24) and Eq. (25), respectively.

As can be seen from Table 6, the analytical values agreed well with the experimental mean values. The predicted bending stiffness showed an even greater accuracy. While the predicted flexural capacity was lower (from 3.2% to 13.4%) than that of experimental results, indicating that the theoretical model was much more conservative for the flexural capacity prediction.

Through the theoretical model of this research, it can also predict the influence of both the prestressing and the increase area of reinforcement. For example, given a passive reinforcement with one 16 mm diameter CFRP bars (reinforcement ratio is 0.89%), the flexural capacity will then be calculated to be 58.5 kN, which indicates an improvement of 15.8% when compared to series R11 (with passive reinforcement ratio of 0.42%). Since the overall improvement of the flexural capacity is 30.9% by comparing series P16 with R11 based on the theoretical analysis (see Table 6), an increase of the flexural capacity can be then obtained to be 15.1% due to the prestressing. Thus it can be seen that both the prestressing and the increase area of reinforcement has notable influence on the flexural capacity.

Furthermore, the correlation of the calculated and experimental values of the flexural capacity is presented in Fig. 21,

which includes the test data from the literature [36,37,41]. It should be noted that the specimens of “Beam 1” and “Beam 2” of the literature [37] were prestressed timber members, while “C series” of this work, HS0 of the literature [36], 0% WH and 0% DF of the literature [41] were unreinforced timber beams. All of the others were passively reinforced timber beams. It shows a good agreement between the analytical results and the experimental ones from different research and different reinforcing types, with the R^2 value was 0.89. However it still need further verification due to the limited sample size.

Fig. 21. Correlation of the calculated and experimental values of flexural capacity

4. Conclusions

The following conclusions may be drawn from the research regarding the reinforcement and prestressing of the Douglas fir glulam beams with CFRP bars.

- Unreinforced control glulam beams failed with a brittle failure due to the flexural tension failure of timber in tension zone.
- Both passively reinforced and prestressed glulam beams show a ductile behaviour with timber compressive failure.
- For prestressed & reinforced glulam beams, it failed in a flexural-shear failure mode, which can be prevented by either decreasing the reinforcement ratio of the CFRP bar or increasing the cross section of glulam beam.
- Passively reinforced glulam beams using CFRP bar with a 0.42% reinforcement ratio can significantly improve the flexural capacity from 35.3 kNm to 58.3 kNm, which represent an increase of 64.8%. While the bending stiffness increases about 18.7%.
- Prestressed glulam beams using CFRP bar with a 0.89% reinforcement ratio results in an enhancement of the ultimate capacity by 93.3% and a notable increase of bending stiffness by 33.2%.
- Glulam beams both prestressed in tension zone and reinforced in compression zone increases the flexural capacity and bending stiffness by 131% and 42%, respectively.

371 - As a result from the prestress and pre-camber in the prestressed glulam beams, the load-carrying capacity at the
 372 beam deflection of 1/250 of the beam span can be greatly improved. This improvement increases from 13.2 kN to
 373 41.0 kN, which indicates an increase of 211%.

374 - Based on the theoretical model, both the prestressing and the increase area of reinforcement has a notable influence
 375 on the flexural capacity.

376 - Combined longitudinally bonding with end anchorages of the CFRP bars resulted into a good compatibility of
 377 deformation and therefore showed an obvious integrity.

378 - The flexural capacity of the unreinforced glulam beams was well predicted using a theoretical model based on work
 379 by Buchanan [40]. The model was modified to account for the presence of CFRP bars.

380 - The addition of the reinforcement, either passively reinforced or prestressed, could increase the tensile strain or
 381 strength of the timber beams in tension zone (increased by 24.3% to 77.6% from the experimental values). A
 382 constant modification factor $\alpha_m = 1.3$, as proposed by Gentile et al. [11,36], was considered reasonable.

383 - Correlation of the calculated and experimental values of several different research works showed satisfactory
 384 agreement, therefore the theoretical method presented a practical approach to discriminate the flexural behaviour of
 385 the FRP reinforced timber beams with different reinforcement ratios and reinforcing types.

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Table 1

Characteristic strength and stiffness properties of homogeneous glulam.

Material property	Test results (N/mm ²)	GL 28h [30] (N/mm ²)	GL 36h [30] (N/mm ²)
Tension strength parallel to grain	32.8	19.5	26.0
Compression strength parallel to grain	37.0	26.5	31.0
Modulus of elasticity parallel to grain, mean value	12,500	12600	14700
Modulus of elasticity parallel to grain, 5% value	11,400	10200	11900

Table 2

Mechanical properties of the epoxy resin provided by the manufacturer.

Specification	Value (N/mm ²)
Compressive strength	70.0
Bending strength	65.0
Splitting strength	9.2
Modulus of elastic in tension	3320

Table 3

Series and specimen information.

Series reference	No. of replicates	Reinforcement					
		Type	Location	Diameter (mm)	Anchorage type	Cross-section ratio (%)	Prestress force ^c (kN)
C	3	-	-	-	-	0.00	-
R11	3	passive reinforcement	Bottom	11	End anchor and bonded	0.42	0.0
P16	3	prestressed	Bottom	16	End anchor and bonded	0.89	50.8
P16R11	3	prestressed and reinforcement	Bottom and top	16 and 11 ^a	End anchor and bonded	0.89 and 0.42 ^b	50.8

^a Bottom prestressed bar was 16 mm in diameter and top reinforced bar was 11 mm in diameter.

^b Cross-section ratio for bottom prestressed bar was 0.89 and for top reinforced bar was 0.42.

^c Prestress force was average strain values of the CFRP bars at mid-point of the three glulam beams in each series.

Table 4

Comparison of experimentally obtained results.

Specimen type	Beam code	Failure load F_u (kN)	Failure moment M_u (kNm)	Load at the mid-span deflection reached $L/250 F_{L/250}$ (kN)	Stiffness $E_{mg}I$ (10^{12} Nmm ²)	Failure modes
Unreinforced control beam (C series)	C-1	33.5	31.8	13.7	2.03	Tension
	C-2	39.0	37.1	12.7	1.91	Tension
	C-3	39.3	37.1	13.1	1.94	Tension
	Average	37.3	35.3	13.2	1.96	—
	Standard deviation	3.3	3.0	0.50	0.06	—
Reinforced beam (R11 series)	R11-1	59.1	56.1	17.0	2.52	Compression
	R11-2	57.3	54.4	16.1	2.13	Compression
	R11-3	67.7	64.3	16.7	2.33	Compression
	Average	61.3	58.3	16.6	2.33	—
	Standard deviation	5.6	5.3	0.46	0.18	—
	% increase ^a	64.8	64.8	26.1	18.9	—
Prestressed beam (P16 series)	P16-1	65.6	62.3	39.7	2.62	Compression
	P16-2	70.9	67.4	38.0	2.61	Compression
	P16-3 ^b	79.2	75.2	—	—	Compression
	Average	71.9	68.3	38.9	2.61	—
	Standard deviation	6.9	6.5	1.20	0.01	—
	% increase ^a	93.3	93.3	195.1	33.2	—
Prestressed & reinforced beam (P16R11 series)	P16R11-1	90.6	86.1	41.8	2.91	Flexure-shear
	P16R11-2	82.7	78.6	41.2	2.84	Flexure-shear
	P16R11-3	84.2	80.0	40.0	2.61	Flexure-shear
	Average	85.9	81.5	41.0	2.79	—
	Standard deviation	4.2	4.0	0.92	0.14	—
	% increase ^a	131	131	211.4	42.3	—

^aImprovement compare to the unreinforced control beam in average value.^bThe deflection and stiffness values are not available due to the faulty LVDT.

Table 5

Comparison of extreme fiber strain at failure of the specimens.

Specimen type	Beam code	Extreme fiber compressive strain $\epsilon_{c,max}$ (microstrain)	Extreme fiber tensile strain $\epsilon_{t,max}$ (microstrain)
Unreinforced control beam (C series)	C-1	-2758	2288
	C-2	-1724	2572
	C-3	-3025	2638
	Average	-2502	2499
	Standard deviation	687	186
	% increase ^a	82.9	77.6
Reinforced beam (R11 series)	R11-1	-4110	4450
	R11-2	-4120	4550
	R11-3	-5500	4317
	Average	-4577	4439
	Standard deviation	800	117
	% increase ^a	88.2	24.3
Prestressed beam (P16 series)	P16-1	-5625	3300
	P16-2	-3810	2820
	P16-3	-4690	3200
	Average	-4708	3107
	Standard deviation	908	253
	% increase ^a	88.2	24.3
Prestressed & reinforced beam (P16R11 series)	P16R11-1	-3541	3871
	P16R11-2	-3199	3143
	P16R11-3	-3800	3320
	Average	-3513	3445
	Standard deviation	301	380
	% increase ^a	40.4	37.8

^aImprovement compare to the unreinforced control beam in average value.**Table 6**

Comparison between theoretical and experimental results.

Series reference	Bending stiffness $EI(10^{12}Nmm^2)$			Flexural capacity M_u (kNm)		
	Experimental results	Analytical results	Difference (%)	Experimental results	Analytical results	Difference (%)
C	1.96	2.11	+7.7	35.3	30.9	-12.4
R11	2.33	2.35	+0.8	58.3	50.5	-13.4
P16	2.61	2.63	+0.8	68.3	66.1	-3.2
P16R11 ^a	2.79	2.87	+2.9	81.5	72.1	-11.6

^a It showed experimental flexure-shear failure modes for this series.

Fig. 1. Details of cross section types.

Fig. 2. Phases of construction of prestressed beam: (a) assemblage of glulam beam, (b) pre-deflection of glulam beam and filling the slots with adhesive, (c) release of glulam beam after the adhesive is well cured and (d) Overturn of glulam beam and prepare to test.

Fig. 3. Straight sleeve anchorages with nuts.

Fig. 4. Test set-up and arrangement.

Fig. 5. Load-deflection curve for the unreinforced control beams (C series).

Fig. 6. Load-deflection curve for the bottom reinforced beams (R11 series).

Fig. 7. Load-deflection curve for the bottom prestressed beams (P16 series).

Fig. 8. Load-deflection curve for the bottom prestressed & top reinforced beams (P16R11 series).

Fig. 9. Typical failure modes of the beams: (a) tension failure, (b) compression failure and (c) flexure-shear failure.

Fig. 10. Typical load-strain curve for an unreinforced control beam (C-3).

Fig. 11. Typical load-strain curve for a bottom reinforced beam (R11-3).

Fig. 12. Typical load-strain curve for a bottom prestressed beam (P16-2).

Fig. 13. Typical load-strain curve for a bottom prestressed & top reinforced beam (P16R11-2).

Fig. 14. Typical strain profile for an unreinforced control beam (C-3).

Fig. 15. Typical strain profile for a bottom reinforced beam (R11-3).

Fig. 16. Typical strain profile for a bottom prestressed beam (P16-2).

Fig. 17. Typical strain profile for a bottom prestressed & top reinforced beam (P16R11-3).

Fig. 18. Cross section of the glulam beam with the notation of the theoretical model.

Fig. 19. Stress-strain relationship for glulam.

Fig. 20. Stress-strain relationship for CFRP bar.

Fig. 21. Correlation of the calculated and experimental values of flexural capacity