A generalized rate-dependent constitutive law for elastomeric bearings

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ABSTRACT

Elastomeric bearings have been a mature and efficient technique to mitigate structural damage caused by earthquakes. This paper proposed a generalized mathematical model for accurately evaluating the rate-dependent stress-strain response of natural rubber bearing (NRB), high damping rubber bearing (HDRB) and super high damping rubber bearing (SHDRB). A novel strain energy function including an additional stiffness correction factor α is proposed to describe the Fletcher-Gent effect of elastomeric materials, which featured by high initial stiffness at small strain ranges. The proposed constitutive model is composed of two parts, the first part consisting of a hyperelastic spring represents the rate-independent equilibrium stress, while the second part consisting of a Maxwell element expresses the rate-dependent overstress. Parameter identification scheme is implemented based on the results of the multi-step relaxation tests and monotonic shear tests. Numerical simulations for monotonic shear

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tests were conducted to demonstrate the capacity of the proposed model in predicting the stress-strain relationship of elastomeric bearings at different strain rates. Finally, in order to investigate the accuracy and feasibility of the proposed model on the application to the seismic response assessment of isolated bridges, real-time hybrid simulation (RTHS) test was performed and favorable results were obtained.

Keywords: Elastomeric bearing; Constitutive law; Rate-dependency; Seismic response

1. Introduction

Elastomeric bearings have been widely used as seismic isolators to protect buildings and bridges from earthquakes for both new constructions and retrofit projects, and have proven to be effective and reliable. Among various types of elastomeric bearings, natural rubber bearing (NRB) and lead rubber bearing (LRB) have been well known and the application of them in civil structures has increased substantially during the last decades, especially in New Zealand, Japan and United States [1]. The NRB uses alternate layers of natural rubbers and steel plates, it has small damping and often be used to accommodate the shrinkage of the deck of isolated bridges. In recent years, two evolving types of elastomeric bearings were invented, named as high damping rubber bearing (HDRB) and super high damping rubber bearing (SHDRB). The rubber material of HDRB and SHDRB possesses high damping due to the add of specific chemical fillers like carbon black, plasticizer and oil during the vulcanization process. With these additives, HDRB and SHDRB provide structural systems additional damping to dissipate seismic energy and

lengthen their fundamental period resulting in the decrease of the seismic response [2-3].

Since the acceptance of the performance-based design philosophy [4], nonlinear time history analysis is recommended by some design specifications [5-6] for the seismic response assessment of isolated bridges [7]. Hence, accurate modeling of the mechanical behavior of the elastomeric bearings is of great importance to a rational design of isolated bridges. In these specifications, the hysteretic behavior of NRB is approximated by equivalent linear method, and that of HDRB and SHDRB are represented by bilinear elasto-plastic model. However, the chemical fillers in rubber make the hysteretic behavior of the elastomeric bearings, in particular the HDRB and SHDRB, considerably complicated. Therefore, any existing linear models cannot reproduce their mechanical characteristics with satisfactory accuracy, especially the rate-dependent property as found by many researchers [8-9]. In the past studies, a few rate-dependent constitutive models for NRB and HDRB were proposed to ameliorate the aforementioned limitations [8-12], while the analytical models available to SHDRB are rare [3, 13]. Moreover, most of these theoretical models are based on the experimental hysteretic loops of the elastomeric bearings, this indicates the necessity for the further study of elastomeric bearings in terms of rubber material. On the other hand, NRB, HDRB and SHDRB are often used together in engineering practices, nowadays the design and seismic analysis of isolated bridges with elastomeric bearings are extremely complex because different hysteretic models should be employed, in this regard, a major need exists for a simplified analytical and design

approach. Development of a generalized constitutive law to describe the mechanical behaviors of these three types of elastomeric bearings is motivated by these reasons.

In this paper, a generalized rate-dependent constitutive law for three types of elastomeric bearings (i.e., NRB, HDRB, and SHDRB) under the horizontal shear deformation is presented from the viewpoint of rubber material. A modified version of classical Zener model is established to express the stress-strain function of rubber material and the total stress is decomposed to rate-independent equilibrium stress and rate-dependent overstress to describe the fundamental viscoelastic behavior of the rubber. A nonlinear viscous damping coefficient is introduced into the mathematical formulation to characterize the rate-dependent effect. Furthermore, multi-step relaxation tests and monotonic shear tests are performed to identify the material parameters. The numerical and experimental results are compared to verify the accuracy of the proposed model. Finally, in order to investigate the accuracy and feasibility of the proposed model on the application to the seismic response assessment, real-time hybrid simulation (RTHS) test was performed and favorable results were obtained.

2. Strain energy function

According to the phenomenological theory, rubber is a hyperelastic material and the mechanical properties of rubber can be characterized by its strain energy function W [14], which can be represented in terms of deformation tensor invariants (I_1, I_2, I_3) ,

$$W = W(I_1, I_2, I_3) \quad (1)$$

where

$$\begin{cases} I_1 = \text{tr} \mathbf{B} = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \\ I_2 = \frac{1}{2} \left[(\text{tr} \mathbf{B})^2 - (\text{tr} \mathbf{B}^2) \right] = \lambda_1^2 \lambda_2^2 + \lambda_2^2 \lambda_3^2 + \lambda_3^2 \lambda_1^2 \\ I_3 = \det \mathbf{B} = \lambda_1^2 \lambda_2^2 \lambda_3^2 \end{cases} \quad (2)$$

tr and det represent trace and determinant of left Cauchy-Green deformation tensor $\mathbf{B}$, λ_i ($i=1, 2, 3$) denotes principal stretches. Moreover, considering that rubber is incompressible, the third invariant $I_3=1$, thus W is expressed as a function of I_1 and I_2 only, i.e., $W=W(I_1, I_2)$.

Since an adequate strain energy function is of great importance in establishing a reliable constitutive model, there are many strain energy function expressions in the literature [15], the most general one is proposed by Rivlin [16] as follows:

$$W_{Rivlin} = \sum_{i=0, j=0}^{\infty} C_{ij} (I_1 - 3)^i (I_2 - 3)^j \quad (3)$$

where C_{ij} is the material parameter.

The Mooney-Rivlin model [17] is a first-order polynomial form of the Rivlin's expression and has been widely used for its simple form and sufficient accuracy to predict the response of rubber material from small to moderate strain amplitudes.

$$W_{Mooney-Rivlin} = C_{10} (I_1 - 3) + C_{01} (I_2 - 3) \quad (4)$$

where C_{01} and C_{10} are the material parameters.

However, strain energy functions available in published researches are mainly focusing on rubber-like materials, the investigations on the behavior of innovative rubber materials aforementioned are found only in a few works [12]. Furthermore, previous researches mainly deals with the uni-axial stress-strain relationship of rubber material, hence, these studies fail to investigate the shear regime, which is essential

for elastomeric bearings under the earthquake exaction. Therefore, development of an adequate strain energy function applicable to shear deformation for seismic application is aroused by these reasons.

Past investigations on the performances of existing strain energy functions showed their incapability to represent the complex mechanical behaviors of hyperelastic materials, especially the high stiffness at small strains [12]. This feature is referred to as the Fletcher-Gent effect on the effect of fillers in NRB [18], it should be noted that this effect seems more significant in HDRB and SHDRB due to the higher fillers contents. In this study, in order to not only reproduce the rubber response accurately but also minimize the material parameters and ensure the simplicity of mathematical formulation, a strain energy function of I_1 and I_2 with an additional stiffness correction factor α is proposed as follows:

$$W(I_1, I_2) = C_1(I_1 - 3) + \frac{2}{3}C_2(I_1 - 3)^{\frac{3}{2}} + \frac{1}{2}C_3(I_1 - 3)^2 + \frac{2}{5}C_4(I_1 - 3)^{\frac{5}{2}} + \frac{C_5}{\alpha + 1}(I_2 - 3)^{\alpha + 1} \quad (5)$$

where C_i ($i=1\sim5$) and α are the material parameters.

3. Constitutive equation for rate-dependent property

To model the rate-dependent phenomenon of elastomeric bearings, a phenomenological Zener model shown in Fig. 1 is considered. The hyperelastic spring A represents the rate-independent equilibrium stress, while the Maxwell element consisting of a hyperelastic spring B and a nonlinear dashpot C expresses the rate-dependent overstress.

Fig. 1. Schematic of modified hyperelastic Zener model.

The total stress $\mathbf{S}$ is given by

$$\mathbf{S} = \mathbf{S}_{eq} + \mathbf{S}_{ov} \quad (6)$$

where $\mathbf{S}$ symbolizes the Cauchy stress, the subscript eq and ov denote equilibrium stress and overstress, respectively. Furthermore, the Cauchy stress is expressed as

$$\mathbf{S} = 2 \left(\mathbf{B} \frac{\partial W}{\partial I_1} - \mathbf{B}^{-1} \frac{\partial W}{\partial I_2} \right) - p \delta \quad (7)$$

where δ is the Kronecker symbol, p denotes the hydrostatic pressure determined by the boundary condition, $\mathbf{B}$ is the left Cauchy-Green deformation tensor.

For shear deformation, we get

$$\begin{cases} \lambda_2 = 1 \\ \lambda_3 = \lambda_1^{-1} \\ \gamma = \lambda_1 - \lambda_3 \end{cases} \quad (8)$$

where γ is the simple shear strain.

substituting Eq. (8) into Eq. (2), I_1 and I_2 can be expressed as

$$I_1 = I_2 = 3 + \gamma^2 \quad (9)$$

the left Cauchy-Green deformation tensor $\mathbf{B}$ and $\mathbf{B}^{-1}$ can be calculated as

$$\mathbf{B} = \mathbf{F} \cdot \mathbf{F}^T = \begin{bmatrix} 1 + \gamma^2 & \gamma & 0 \\ \gamma & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B}^{-1} = \begin{bmatrix} 1 & -\gamma & 0 \\ -\gamma & 1 + \gamma^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (10)$$

where $\mathbf{F}$ is the deformation gradient tensor defined by

$$\mathbf{F} = \begin{bmatrix} 1 & \gamma & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (11)$$

substituting Eqs. (9)-(11) into Eq. (7), the shear stress component is given by

$$\mathbf{S}_{12} = 2\gamma \left(\frac{\partial W}{\partial I_1} + \frac{\partial W}{\partial I_2} \right) \quad (12)$$

combining Eq. (5) and Eq. (12), one gets,

$$\tau_{eq} = 2 \left(C_{1eq} \gamma_{eq} + C_{2eq} \gamma_{eq}^2 + C_{3eq} \gamma_{eq}^3 + C_{4eq} \gamma_{eq}^4 + C_{5eq} \gamma_{eq}^{2\alpha_{eq}+1} \right) \quad (13)$$

$$\tau_{ov} = 2 \left(C_{1ov} \gamma_{ov} + C_{2ov} \gamma_{ov}^2 + C_{3ov} \gamma_{ov}^3 + C_{4ov} \gamma_{ov}^4 + C_{5ov} \gamma_{ov}^{2\alpha_{ov}+1} \right) \quad (14)$$

where C_{ieq} , C_{iov} ($i=1\sim 5$), α_{eq} and α_{ov} are the material parameters determined from the equilibrium response and instantaneous response, which corresponds to infinitely slow and fast strain rate, respectively.

Based on a work by Huber and Tsakmakis [19], the rate of Left Cauchy-Green deformation tensor $\mathbf{B}$ is defined by

$$\dot{\mathbf{B}}_B = \mathbf{B}_B \mathbf{L}_A^T + \mathbf{L}_A \mathbf{B}_B - \frac{2}{\eta} \mathbf{B}_B \mathbf{S}_B^D \quad (15)$$

where η is the viscosity parameter represented by the nonlinear dashpot element C.

The $(\cdot)$ indicates material time derivative. Superscript D denotes the deviatoric component. $\mathbf{L}$ is the velocity gradient tensor given by

$$\mathbf{L} = \dot{\mathbf{F}} \cdot \mathbf{F}^{-1} \quad (16)$$

combining Eqs. (15)-(16), the expression of rate of $\mathbf{B}_B$ is obtained.

$$\dot{\gamma}_{ov} = \dot{\gamma}_{eq} - \frac{4 \left(C_{1ov} \gamma_{ov} + C_{2ov} \gamma_{ov}^2 + C_{3ov} \gamma_{ov}^3 + C_{4ov} \gamma_{ov}^4 + C_{5ov} \gamma_{ov}^{2\alpha_{ov}+1} \right)}{\eta} \left(\frac{2}{3} \gamma_{ov}^2 + 1 \right) \quad (17)$$

when all the parameters are identified, the constitutive equation is developed.

4. Material tests and parameter identification

4.1 Test specimen and instrument

The test specimen comprises two rubber layers and three steel blocks, it has a net shear area of $25 \text{ mm} \times 20 \text{ mm}$, detailed geometric size is illustrated in Fig. 2 (a). All tests were carried out using a computer-controlled servo hydraulic loading machine with a 10 kN load cell. The maximum stroke capacity of the load cell was 50 mm. Specified displacements were exerted on the one end of the specimen (the other end was fixed on the machine) and the corresponding restoring force was measured by the load cell.

(a) Specimen (unit: mm)

(b) Instrument

Fig. 2. Test specimen and instrument.

4.2 Multi-step relaxation test and monotonic shear test

A multi-step relaxation (MSR) tests was employed in the experimental scheme to obtain the equilibrium stress. In the tests, each specimen was tested up to the specified shear strains and held constant with a time interval of 20 minutes, Fig. 3 presents the applied strain history utilized in the test.

Fig. 3. Applied strain histories utilized in MSR tests.

Fig. 4 shows the resultant stress histories obtained by MSR tests. It is clearly seen that at the end of each relaxation interval of 20 minutes, each stress history converges to an almost constant value in all three types of rubber specimens, a similar characteristic of carbon black filled rubber was also observed by Lion [20]. By connecting all the minimum stress values at each corresponding strain level, the equilibrium response can be achieved as shown in Fig. 5.

Fig. 4. Stress time history obtained by MSR tests.

Fig. 5. Equilibrium response obtained by MSR tests.

Theoretically, the instantaneous response can be ideally obtained when the rubber material is loaded at an infinitely fast rate. However, the maximum stroke velocity of the displacement controlled device inherently limits the loading rate of the specimen. To overcome this limitation, a series of monotonic shear tests were conducted. The tests were carried out at four different loading rates up to a maximum shear strain range of 2.00. The four strain rates of 0.08, 0.8, 1.6 and 4.0 1/s were utilized in the tests. Fig. 6 shows the stress-strain response that were observed in four rate cases, in general, the curves contained a threefold feature corresponding to small, moderate and large strains, among which the extreme high initial stiffness at the small strain level is the most prominent feature. Comparing the experimental data at different strain rates, it was seen that all three types of elastomeric bearings possess a pronounced rate-dependent behavior, especially in the case of HDRB and SHDRB. It should be mentioned that the stress at the strain rate of 1.6 1/s is smaller than that of 0.8 1/s for shear strain beyond about 1.25 for the HDRB, this abnormal phenomenon can be mainly attributes to the experimental error in the tests and manufacturing error of HDRB. A further comparison of the curves at different strain rates indicates a diminishing trend, that at higher strain rate, the increase of the stress response was declining. Similar phenomenon of HDRB was also observed in the cyclic shear tests [10], in which the fastest strain rate was used to approach the neighborhood state of the instantaneous response. For a similar consideration, the stress response obtained at 4.0 1/s is regarded as an acceptable instantaneous response in this study.

Fig. 6. Stress-strain relationships obtained by monotonic shear tests.

4.3 Material parameter identification

The parameters for equilibrium stress can be determined by applying the least-square method to Eq. (13) with sets of the equilibrium stress τ_{eq} and corresponding shear strain γ_{eq} obtained by the multi-step relaxation test. It should be mentioned that since the equilibrium response does not possess the high stiffness which shown in instantaneous response, set $C_{5eq}=0$ for the purpose of reducing material parameters. The results of parameters are shown in Table 1.

Table 1 Material parameters for equilibrium stress.

Specimen	C_{1eq} (MPa)	C_{2eq} (MPa)	C_{3eq} (MPa)	C_{4eq} (MPa)	C_{5eq} (MPa)
NRB	0.7642	-0.7091	0.3883	-0.0619	0
HDRB	1.0615	-1.5318	0.9779	-0.1994	0
SHDRB	2.0347	-3.0981	1.9878	-0.4239	0

The overstress τ_{ov} is obtained by subtracting the equilibrium stress τ_{eq} from the total stress obtained by the monotonic shear tests using the case of 4.0 1/s. Firstly, the parameter α_{ov} in Eq. (14) is determined by the shear stress-strain curve fitting technique in the small strain range wherein the high initial stiffness is dominant. Then, other parameters C_{iov} ($i=1\sim5$) are identified by the least-square fitting method using

the sets of the overstress τ_{ov} and the corresponding shear strain γ_{ov} obtained from the case of 4.0 1/s. The values of the overstress parameters are summarized in Table 2.

Table 2 Material parameters for overstress.

Specimen	C_{1ov} (MPa)	C_{2ov} (MPa)	C_{3ov} (MPa)	C_{4ov} (MPa)	C_{5ov} (MPa)	α_{ov}
NRB	0.3561	-0.4315	0.4666	-0.1399	0.0141	-0.35
HDRB	1.1306	-1.0488	0.7869	-0.2551	0.0264	-0.35
SHDRB	1.7699	-2.0761	1.7718	-0.7801	0.1346	-0.35

The viscosity coefficient η obtained from the tests results is found not to be constant, showing variation within the progress of deformation of the specimen. In this study, it is assumed that the viscosity coefficient has a functional relationship with strain and strain rate, therefore, Gaussian function and a 3rd order polynomial function were utilized to calculate viscosity coefficient, which provides a convenient way to describe the strain and strain rate dependence of η .

$$\eta = \{Z_0 + A \exp[-0.5(\frac{\gamma - x_c}{w_1})^2 - 0.5(\frac{v - y_c}{w_2})^2]\} \times (B + Cv + Dv^2 + Ev^3) \quad (18)$$

where Z_0 , A , x_c , w_1 , y_c , w_2 , B , C , D and E are material parameters, and v is the strain rate. The material parameters are determined by the nonlinear multivariate fitting procedure, parameter results are listed in Table 3.

Table 3 Material parameters for dashpot element C.

Z_0	A	x_c	w_1	y_c	w_2	B	C	D	E
1.076	3.159	1.779	0.978	12.989	9.632	14.291	-22.096	5.778	2.191

5. Model validation

5.1 Comparison of simulation results with monotonic shear tests

The proposed model with the identified parameters were used to simulate the monotonic shear tests at varied strain rate cases. Fig. 7 shows the numerical simulation in comparison with test data for all three types of specimens, it indicates that all the three different strain rate cases are found to be well described, including the high stiffness feature at small strains.

Fig. 7. Numerical simulation and monotonic shear tests of NRB.

Fig. 8. Numerical simulation and monotonic shear tests of HDRB.

Fig. 9. Numerical simulation of monotonic shear tests of SHDRB.

5.2 Comparison of simulation results with real-time hybrid simulation tests

In this section, the performance of seismic response assessment of the proposed model is verified in comparison with the results of real-time hybrid simulation (RTHS) test. In this test, an isolated single-column bridge as illustrated in Fig. 10 was exerted by an accelerogram with a peak ground acceleration (PGA) of 207.8 gal. Using the hybrid simulation technique, the bridge is split into numerical and experimental parts. In this study, elastomeric bearings were tested physically to take advantage of RTHS test in evaluating the dynamic response of rate-dependent devices and the remaining components of the bridge was numerically simulated with sufficient accuracy. The major challenging issue for RTHS test is to impose command displacement in real-time, to overcome this problem, an innovative approach characterized by the velocity-based method was adopted in this work, which improved the accuracy of RTHS test results and simplified the test control algorithm [21].

Fig. 10. Schematic view of a single-column bridge.

Comparisons of numerical simulation and RTHS test results are illustrated in Fig. 11. Although the difference in the peak amplitude is visible with a maximum error of nearly 10% and more distinct variations with a maximum error of almost 14% can be

seen in the case of HDRB, coherent results were observed over the entire duration of the earthquake excitations. It can be concluded that the displacement time histories in three types of elastomeric bearings can be predicted accurately by the proposed model and the model is accurate in the seismic response assessment of isolated bridge.

(a) NRB

(b) HDRB

(a) SHDRB

Fig. 11. Comparisons of numerical simulation and RTHS test results.

6. Conclusions

This paper proposed a generalized mathematical expression to accurately represent the rate-dependent stress-strain relationship of three types of elastomeric bearings (i.e., NRB, HDRB and SHDRB). The conclusions are summarized as follows:

1. The newly developed model is a thermodynamically compatible rheological model including three elements to express the rate-dependent property of elastomeric

bearings. Comparing the simulations using the proposed model with monotonic shear tests and RTHS tests, it is concluded that the proposed model can reproduce the rate-dependent behavior of elastomeric bearings with a good accuracy.

2. A novel strain energy function containing an additional stiffness correction factor is proposed and it contributes to improve the description of high stiffness of elastomeric bearings at small strains.

3. The proposed generalized model will be greatly facilitated and accelerated the design and analysis process of isolated bridges. It can be utilized practically instead of rigorous equivalent linear and bilinear models in predicting the stress-strain behavior of elastomeric bearings, especially the HDRB and SHDRB which have prominent nonlinear rate-dependent property.

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