Controlled Manipulation of Floating Objects on Deformed Fluid Interfaces and Conditions for Stable Equilibria

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Keywords

Capillarity, electrowetting, interfacial assembly, capillary charge, semi-immersed rod

Abstract

At the millimeter scale, interactions between floating and semi-immersed objects are significant. The local curvature of the interface is modified by the weight/buoyancy forces of floating objects, and by the surface properties of semi-immersed objects. The curvature changes generate attractive (or repulsive) interactions between floating parts, and semi-immersed objects. This work demonstrates how electrowetting can manipulate these interactions in order to position, align, assemble and transport parts attached to the fluid interface. This demonstrates one way in which fluid interfaces can provide an alternative to standard pick and place technology for part positioning/assembly. Typically, the part/rod forces are purely attractive or repulsive, but under some conditions, floating objects reach a stable equilibrium with a finite gap between the floating and semi-immersed bodies. Stable equilibrium positions were measured for rectangular prisms suspended on a water/oil interface and a fixed cylindrical rod. Measurements showed that the equilibrium position depends on the ratio of $\frac{\Delta\rho t}{w}$ where t is the part thickness; w is its width, and $\Delta\rho$ the part/fluid density difference. The stable equilibrium position provides for repeatable positioning without risk of parts sticking to the semi-immersed bodies.

1. Introduction

Particles at interfaces play a key role in many applications including cleaning, separations (Lopetinsky, Masliyah, & Xu, 2006; Menon & Wasan, 1988), colloid stabilization

(Lopetinsky et al., 2006), and inter-particle assembly (Bowden, Terfort, Carbeck, & Whitesides, 1997; Kuncicky & Velez, 2008). Over a broad range of wetting conditions, particles remain stable on interfaces (McHale & Newton, 2011). These particles can deform the interface and interact with other semi-immersed and floating objects that also deform the interface.

Microscale interactions at fluid-fluid interfaces have proven useful for facilitating particle manipulation through different methods. For instance, photo-induced effects have been used for manipulating floating particles. The flow of a photo-responsive surfactant (and consequently, the fluid flow) around a floating particle was controlled by shining light to the interface (Varanakkottu et al., 2013). In another work, holographic optical tweezers were used to manipulate floating dielectric particles (Jesacher, Furhapter, Maurer, Bernet, & Ritsch-Marte, 2006).

Microscale manipulation by traditional grasp and release processes proves challenging due to surface forces (Morris, Stauth, & Parviz, 2005). Surface forces that are negligible at larger scales including capillary and Van der Waals forces are larger than gravity at these sizes. In traditional manufacturing environments they are often poorly controlled—making it difficult to release objects consistently. However, developing systems that harness surface forces could improve microscale manipulation to simplify the challenges of sorting, positioning and aligning microscale components. In the Knuesel & Jacobs work (Knuesel & Jacobs, 2010), a water-oil interface served to pre-align microscale components for subsequently self-assembling onto a semi-immersed substrate without a tool or any serial manipulation. The authors demonstrated a high-yield, high-rate microscale assembly process. Sowerby, et al used the axisymmetric meniscus of a fluid-air interface around a cylindrical rod for concentrating buoyant particles at the three-phase contact line (Sowerby, Mirams, Hill, & Paulin, 2011) to efficiently collect

biological particles as samples for detection/screening (White, Cooke, Wakes, & Sowerby, 2013).

Fluid interfaces are of particular interest for manipulating floating particles with both attractive and repulsive forces possible. The weight/buoyancy force on a floating particle (i.e. a “floatation force” as illustrated in Figure 1-c,d) or the wetting properties of a solid surface in contact with the interface (i.e. an “immersion force” as illustrated in Figure 1-a,b) deform the fluid interface. Kralchevsky and Nagayama concluded that energy minimization drives an attractive force between two components if both components generate a similar interface curvature (Figure 1-b, d) while opposite interface curvatures generates a repulsive force (Figure 1a,c) (Deryagin & Starov, 1977; Kralchevsky & Nagayama, 2001). The concept of “capillary charges” allowed the authors to calculate the inter-particle force, F , with a formula that is analog to Coulomb’s law (Kralchevsky & Nagayama, 2000) :

$$F = -2\pi \cdot \gamma \cdot \frac{Q_1 \cdot Q_2}{L} \quad \text{Equation 1}$$

where Q_1 and Q_2 are the capillary charges of each interacting particle (Kralchevsky & Nagayama, 2001)), γ is the surface tension and L is the inter-particle distance. The capillary charge of a floating spherical particle on a flat interface is given by

$$Q = r \sin \varphi \quad \text{Equation 2}$$

where r is the particle diameter and φ is the angle of the meniscus relative to the horizontal. See (Crane, Onen, Carballo, Ni, & Guldiken, 2013) for a more comprehensive review.

Figure 1 Capillary interaction force between semi-immersed rods that create different (a) and same curvatures (b). Capillary floatation force between floating components that create different (c) and similar curvatures (d)

The afore-mentioned works have added valuable insight for understanding interactions of spherical particles at fluid interfaces. This work focuses on floating microscale components with a prismatic geometry. In order to use the interfacial forces to manipulate components at the fluid interface, a method is required for changing the fluid interface with a practical control input. Electrowetting is a promising approach. In electrowetting, the apparent contact angle of a fluid changes under an applied electric field. It is reversible and applicable to a wide range of fluids and configurations. (Mugele & Baret, 2005; Nelson & Kim, 2012). Using EW, the apparent contact angle of a sessile drop can be modulated by over 100 degrees.

Recent work has shown that electrowetting of flat surfaces can be used to manipulate objects (Yuan, Feng, & Cho, 2015). This work complements this work by showing how electrowetting can provide a reversible control method around partially-wetted rods. Rods allow for additional flexibility in the path of floating objects compared to flat plates.

Typically, the interface curvature around the rods is determined by the wetting properties of the rod alone and cannot be readily modulated. Sowerby demonstrated an alternative interface control method in which mechanical motion of a rod with a hemispherical tip deformed the

interface without penetrating it (Sowerby et al., 2011). Similar results might be obtained using a semi-immersed rod with high contact angle hysteresis. These mechanical methods may be able to maintain a surface deformation for longer periods and do not require advanced manufacturing techniques to create thin dielectric layers required in electrowetting. However, mechanical methods of controlling the interface are limited in their speed and are difficult to implement with high spatial density. Direct electrical actuation via electrowetting facilitates tighter packing of the actuated rods. Recent publications have shown promising advances in actuation reliability (Dhindsa, Heikenfeld, Weekamp, & Kuiper, 2011; Schultz, Chevalliot, Kuiper, & Heikenfeld, 2013) and development of low cost electrowetting material systems (Khodayari, Carballo, & Crane, 2012). Additionally, the Sowerby method is not capable of fully reversing the interface curvature from positive to negative curvatures. However, curvature reversal is not needed to concentrate floating parts near the deformed interface to facilitate imaging of dilute particle solutions.

The paper will illustrate part motion control using an array of electrowetted rods then address a method for avoiding irreversible bonding of the parts to the semi-immersed objects by preventing contact with the rod due to the dual curvature of the fluid interface around the partially wetted rod. Eliminating part contact could improve the reliability of positioning and aligning floating parts. This non-contact manipulation method prevents damaging the component, and eliminates the need for expensive grippers and actuators. The present work also explains an experimental correlation between the existence of a stable non-contact equilibrium and a combination of part geometry and density parameters.

2. Electrowetting Modulated Surface Forces

A cylindrical rod that is semi-immersed through an oil-water interface produces an axisymmetric meniscus. Wetting contact angles ($\varphi < 90^\circ$) produce a positive slope in the radial direction (see Figure 2-a) and non-wetting ($\varphi > 90^\circ$) produce negative slopes (see Figure 2-b). A floating part in the region of curved interface around the rod experiences gravitational and capillary forces due to the curved interface profile. At large distances, gravitational forces drive the part motion. If the interface curves downward (hydrophobic rod), then parts that deform the interface downward approach the rod while lower density parts move away from the rod. The curvature of the interface due to the “floating” parts depends on how the parts are wet by the interface. For a prismatic part with only the bottom surface wet by the lower fluid, then the curvature is determined by the relative density of the parts and the top fluid. However, if the sides of the part are wet by the lower fluid so that only the part top is wet by the top fluid, then the interface curvature will be determined by the relative density of the parts and the lower fluid.

Figure 2 Part-rod interaction on an oil-water fluid interface. The floating prismatic part interacts with a semi-immersed, cylindrical rod. The interface curves downwards (a) or upwards (b) if the rod surface is rendered hydrophobic, or hydrophilic, respectively.

Electrowetting (EW) provides an effective control strategy with simple electrical inputs and high actuation speed. Contact angle is given by the Young-Lippman equation (Mugele & Baret, 2005)

$$\cos \theta - \cos \theta_0 = \frac{\epsilon_0 \epsilon_r}{2d\gamma} V^2$$

where θ is the apparent contact angle and θ_0 is the zero voltage contact angle, and $\epsilon_0 \epsilon_r$ is the dielectric constant of the dielectric layer. The voltage across the dielectric is V , d is the dielectric thickness and γ is the fluid surface energy. By applying the EWOD principle to the semi-immersed rod configuration, the local curvature of the fluid interface around the rod wall can be manipulated (see Figure 3).

Figure 3 Illustration of the EWOD configuration for semi-immersed rods. The rod creates a negative curvature when is inactive (a), and a positive curvature when activated (b).

Aluminum rods (diameter ~ 0.5 mm) were dip-coated with a commercially available fluoropolymer (Cytop CTL-809M, Asahi Glass Co. Chemicals). The thickness of the resulting coating was about $1 \mu\text{m}$. The aluminum rod was then partially submerged into a bath of 0.1M citric acid solution to create a self-healing environment that improves the EWOD system

reliability (Dhindsa, Heikenfeld, et al., 2011; Khodayari et al., 2012). Next, a layer of hexadecane oil was added over the water. An immersed grounding electrode and a positive voltage applied to the semi-immersed rod provided an electrical circuit through the citric acid solution (as illustrated in Figure 3). At zero applied voltage, the contact angle at the three-phase contact line on the fluoropolymer-coated-rod, φ is approximately 145° (Figure 3-a) based on measurements of similarly prepared flat surfaces. A voltage potential of 90 V between the rod and the citric acid solution switched the interface slope positive, decreasing φ to approximately 70° (Figure 3-b).

When a part with density greater than water is suspended at the solution-oil interface of the EWOD configuration in Figure 3 while at zero potential, the part falls along the downwards slope meniscus towards the rod. However, upon applying a voltage, the gravitational force draws the parts away from the rod due to the slope reversal. Electrowetting modulation can occur on millisecond timescales and is proven to function consistently for many thousands of cycles (Dhindsa, Heikenfeld, et al., 2011; Khodayari et al., 2012). Therefore, this hydrophobic rod creates a simple, high-speed modulation system for varying the local fluid interface around the rods.

In order to demonstrate this manipulation concept, EWOD driven rods were spaced 10 mm apart from each other, in a 2x2 array (see Figure 4). Square dies of plastic shim stock (300 μm thick and 3 mm in length) were suspended on the fluid interface. Upon applying (90 V) voltage to three rods to create a repelling force, the part moved toward the inactive rod. The part approached the inactive rod but appeared to reach an equilibrium separation (Figure 4a,e,g) and did not stick to the rod. At some points (Figure 4 c), it is unclear from the image whether the part touched the rod or not. This uncertainty could make reliable actuation more difficult. To

Figure 4 Sequential activation of a EWOD-driven rods in a 2x2 array for the purpose of programmable part positioning (Crane et al., 2013). The pattern of electrode actuation illustrated below each image.

address, the conditions for the stable equilibrium are tested in a later section. Repeated actuations of a combination of four energized rods moved the part between the rods (see Figure 4).

In a subsequent experiment, parts were transported long distances by creating an array of EWOD-driven rods. One $5 \times 5 \times 0.3 \text{ mm}^3$ part (approximate dimensions) was manually positioned inside the array and a second one was placed on the edge of the array (see Figure 5). Sequential on/off switching of the EW-actuated rods brought the second part to the first. As the distance narrowed, the capillary interactions between the two parts attracted them to each other and aligned their edges. Additional switching of the rod voltages transferred the assembly out of the array. This simple demonstration illustrates how modulating the wetting of the rods can manipulate parts on the interface for transport and assembly. The success of this approach requires that the attractive interactions between the floating parts and the rods be reversible. However, depending on the wetting state and relative size of the parts, they have the potential to be trapped at the interface if they touch the rod— hindering further part manipulation. In these

initial demonstrations, the parts closely approached and may have touched the rods in at least some cases (Figure 4b, Figure 5). While the floating parts described above did not stick to the rods, other test parts (such as 500 μm silicon cubes) did stick to flat surfaces. The rest of the paper considers the conditions for maintaining a stable gap between a rod and part, so that the attraction can always be fully reversed.

The basic response of the parts attracting to or being repelled from the rods was very repeatable. As some combinations of activated electrodes produce multiple attraction points with unstable equilibria in between, feedback control may be required to achieve reliable positioning. It is also difficult to control the orientation of the square parts since each edge interacts with the forces identically. The force interactions between parts can be helpful as illustrated in the joining of two adjacent parts along their edges. If further orientation control were required, additional electrodes or other features might be required to break the part symmetry.

While no degradation in electrowetting response was observed in these tests, the reliability of the actuation system is a practical concern that would need to be addressed in implementing this actuation system. The configuration demonstrated here requires energizing the electrodes in order to repel parts denser than the fluid medium. As motion typically requires more repelling electrodes than energized electrodes, the voltages may be maintained for long periods. While this can be challenging for reliable actuation, very promising results have been shown for electrode longevity (Chevalliot, Heikenfeld, Clapp, Milarcik, & Vilner, 2011; Dhindsa, Kuiper, & Heikenfeld, 2011; Khodayari et al., 2012). AC excitation could extend reliability further. Finally, inverting the system wetting so that only the attractive electrode is energized would reduce the electrowetting demands. This could be accomplished by using the

Figure 5 Sequential activation of EW-driven rods in a 4x4 array transports and then assembles the floating parts. The outer part is first brought to proximity with the inside part (a-e). The parts self-align by capillary interactions (f). The assembled parts are then transferred out of the array by sequential rod-activation.

top fluid as the electrowetting fluid or by using parts that are lower in density than the fluid medium.

3. Predicting equilibrium state of part-rod interaction

Previous work has shown that some part-rod interactions reach equilibrium with a repeatable gap between the part and a cylindrical rod (Carballo, Ni, Vasquez, Chatzisavas, & Crane, 2012; Sowerby et al., 2011). Equilibrium requires a balance between opposing forces. The attractive force is from the gravitational (buoyancy) force and it is opposed by the deformation of the curved interface around the planar parts (Kralchevsky & Nagayama, 2001). At the equilibrium point, these forces balance. In high curvature regions close to the rod, the interfacial energy increase required to conform the interface to the rigid part is larger than the decrease in gravitational potential energy. The wetting of thin silicon parts floating at a water/hexadecane interface had negligible impact on their location (Carballo et al., 2012; Caroline E. Liberti & Crane, 2011).

A simple system model can explain the part-rod equilibrium point. The long-range attractive force is due to gravity. The suspended parts are trapped on the interface by the interfacial energy, but they can respond to the net vertical force (buoyancy force plus gravity) by moving along the sloped liquid interface. For small floating parts, the net vertical force is small compared to the interfacial tension. Hence, the force can be estimated from the slope of the interface around the rod. For a square prism with thickness ' t ' and planar dimension ' w '

$$F_G = \Delta\rho_p \cdot g \cdot t \cdot w^2 \sin \theta \quad \text{Equation 3}$$

where $\Delta\rho_p$ is the density difference between the part and the covering (top) medium. The covering medium is used as a reference on the assumption that only the bottom surface of the part is wet by the water. The angle θ is the slope of the tangent to the surface in the direction of travel measured from the horizontal to the interface. The part slope is approximated as the slope of the undeformed surface, at the location of the part's edge nearest to the rod. See Figure 6 for an illustration of each parameter composing Equation 3.

Figure 6 Illustration of part parameters in Equation 3. The forces acting along the interface are illustrated as well.

The opposing force is generated by the deformation of the fluid interface to wet the flat part. This adds energy to the system either through the bending of the parts or the deformation of the fluid surface. This work considers the case in which the surface tension is insufficient to bend the parts significantly which holds for all the parts considered here (see Section 4 for part geometry and material details). Hence, the energy increase is dominated by the additional fluid

surface areas created by the presence of the part. This energy increase is assumed proportional to the sum of the magnitude of the principal curvatures k_1 and k_2 . For the present case (fluid interface around a cylindrical rod), the principal curvatures k_1 and k_2 have opposite signs and similar magnitudes. Figure 7 displays the principal curvature components. Accordingly,

$$\Delta E_K \approx \beta \cdot \gamma \cdot A \cdot (w|k_1| + w|k_2|) = \beta \cdot \gamma \cdot w^3 |K| \quad \text{Equation 4}$$

where β is geometry-dependent scaling factor, A is the area of the part on the interface, and $|K|$ is the sum $|k_1| + |k_2|$. Equation 4 uses the sum of the magnitude of the curvatures ($|K|$) because the surface energy increases from conforming both positive and negative curvatures to the flat plate. The resulting radial force can be found by differentiating the energy ΔE_K with respect to the radial position r away from the rod axis,

$$F_K = \beta \cdot \gamma \cdot w^3 \frac{d|K|}{dr} \quad \text{Equation 5}$$

where

$$\frac{d|K|}{dr} = \frac{d|k_1|}{dr} + \frac{d|k_2|}{dr} \quad \text{Equation 6}$$

Figure 7 Slope and curvature of oil-water interface profile, around a semi-immersed, hydrophobic rod.

As a part interacting with a semi-immersed rod achieves equilibrium, the gravitational and curvature forces must be in balance. Setting Equation 3 equal to Equation 5 and rearranging, we have,

$$\left. \frac{d|K|}{dr} \right|_{r=r_{eq}} \cdot \frac{\beta\gamma}{g \sin \theta} = \frac{\Delta\rho_p \cdot t}{w} \quad \text{Equation 7}$$

The radial location r_{eq} that satisfies Equation 7 is the point where the two competing forces are balanced and the parts should reach equilibrium. In other words, the geometry and density of the part (summarized by the right hand side of Equation 7) determine the interface geometry conditions (left hand side of Equation 7) at which the part achieves equilibrium. This relationship predicts that parts with the same value of $\Delta\rho_p \cdot t/w$ will have the same equilibrium position.

4. Experimental Test Method

To test this prediction, an axisymmetric fluid interface was created by placing a hydrophobic glass rod of known diameter ($d = 1, 1.5, 2, 2.5, 3, 4$ mm) in the center of a cylindrical container as seen in Figure 8. The rod was dip-coated with a hydrophobic coating (Cytop CTL-809M diluted 2:1 with solvent CT-solv 180, Asahi Glass Co. Chemicals) and cured at 200°C for 1 hour. The water-oil interface was created by adding deionized water and then slowly adding hexadecane oil on top of the water, to reach a oil-layer thickness between 3 mm and 5 mm.

The water-oil interface profile was imaged as illustrated in Figure 8. Using a pipette as a capillary gripper, parts were released within the hexadecane layer and allowed to fall onto the water interface with minimum disruptions to the fluid-fluid interface. The dropping location was selected to be in the attractive zone of the meniscus region so that the parts would approach the rod until reaching an equilibrium. After reaching the equilibrium point, the acrylic container was rotated until the radial line from the part to the rod was perpendicular to the optical axis. An

Figure 8 Experimental Set-up for imaging the oil-water interface around a cylindrical, semi-immersed rod. The set-up is illustrated from a top view (a) and the container from a side-view (b). Images of the actual set-up (c) and of the fluid interface profile (d) are also shown.

image was recorded for measurement of the equilibrium distance r_{eq} as observed from above. The container axis is aligned over the axis of rotation of a rotary stage so that the center rod does not move during rotation. This set-up is different than (Caroline Elizabeth Liberti, 2011) in that only one rod was used and the position was observed horizontally rather than from above. Table 1 summarizes the ranges of rod diameter (which determines interface curvature) and parts that were tests. Images of the fluid interface (without parts) were also made for each rod diameter.

The cylindrical container had a 3 inch outer diameter and 0.14 inch thick wall. The contact angle of the water-oil interface was nearly 90° . The large diameter of the container provided nearly normal viewing angles to minimized image distortion over the region of interest near the rod. The distortion was measured (<0.03 mm over the region of interest). The images were compensated using an image of a rectangular grid as a reference. Image resolution was $5.5 \mu\text{m}$ and a position measurement repeatability of $8 \mu\text{m}$ was demonstrated.

5. Results

Figure 9 shows representative equilibrium position data from all the parts tested on the 3mm rod. Each part density is distinct, as it is seen for the equilibrium distance vs. width plot in Figure 9a. However, when plotted against the parameter identified in Equation 7 ($\Delta\rho_p \cdot t/w$), the change of equilibrium distance follows a clear trend (see Figure 9b). Figure 9 depicts results for all rod diameters, in the form of equilibrium distance vs. $\Delta\rho_p \cdot t/w$ plots. This indicates that $\Delta\rho_p \cdot t/w$ could be used as a metric for summarizing the effects that part design have on the outcome of interactions at fluid interfaces.

Table 1 Experimental parameters for equilibrium distance measurements.

Rod Diameter (mm)	Width, w (mm)	Density (g/cm ³)	Thickness, t (mm)	Resultant*, $\Delta\rho_p \cdot t/w$
2.0	0.5, 1.0, 1.5, 2, 2.8, 3.3	2.2	0.150	0.08 – 0.84
	0.8, 1.2, 1.7, 2.8, 3.3		0.250	
	1.0, 2.0, 4.0, 5.0, 5.9	7.9	0.075	0.08 – 1.54
	1.0, 2.0, 3.0		0.254	
2.5	0.5, 1.0, 1.5, 2.0, 2.8, 3.9, 4.5, 5.0	2.2	0.150	0.05 – 0.85
	0.8, 1.3, 1.8, 2.8, 3.3		0.250	
	1.0, 2.0, 4.0	7.9	0.075	0.11 – 1.54
	1.0, 2.0		0.254	
3.0	0.5, 1.0, 1.5, 2.0, 2.8, 3.3, 4.5, 5.0	2.2	0.150	0.05 – 0.85
	0.8, 1.3, 1.8, 2.8, 3.3, 3.9, 4.3		0.250	
	1.0, 2.0, 4.0, 5.0, 5.9, 6.9	7.9	0.075	0.07 – 1.54
	1.0, 2.0, 3.0, 4.0		0.254	
4.0	0.5, 1.0, 1.5, 1.9, 2.0, 2.8, 3.3, 4.5, 5.0	2.2	0.150	0.05 – 0.85
	0.8, 1.3, 1.8, 2.8, 3.3, 3.9, 4.3		0.250	
	1.0, 2.0, 4.0, 5.0, 5.9, 6.9	7.9	0.075	0.07 – 1.54
	1.0, 2.0		0.254	

* The quantity $\Delta\rho_p$ corresponds to the difference between the density of the part, and that of the covering medium, which in this case is 0.77 g/cm³ for hexadecane oil.

Figure 9 plots of equilibrium distance with respect to width $-w$ (a), thickness $-t$ (b), part density $-\rho$ (c) and the parameter combination $\Delta\rho_p \cdot t/w$ (d). Data was acquired using a 3 mm rod

Each rod diameter showed a range of values for which $\Delta\rho_p \cdot t/w$ yielded stable positions at a certain distance r_{eq} . In Figure 10, data points at distances above the rod radius (greater than 1 in the vertical axis) correspond to such stable states, away from the rods. Interface curvature would also be affected by the interfacial energy and the relative density differences of the two fluids. However, the consistency across part sizes confirms that the predicted ratio is a good indication of part stability positions for a given interface profile. As the part parameter $\Delta\rho_p \cdot t/w$ decreases, the equilibrium distance decreases. Below a value of approximately 0.07 g/cm³, all rods maintained equilibrium distances. However, smaller rods could maintain equilibrium distance at $\Delta\rho_p \cdot t/w < 0.2$ g/cm³.

Figure 10 Plot of equilibrium distance measurements normalized by the diameter for the rod with respect to $\Delta\rho_p \cdot t/w$. Parts with a distance ratio above one are at stable equilibrium. Error bars represent two standard deviation (95% confidence interval). Samples in contact with the rod have zero variation.

In order to further test Equation 7, the values of $\Delta\rho_p \cdot t/w$ for parts that achieved equilibrium away from the rod were compared to the rate of change of interface curvature $d|K|/dr$ at the part equilibrium position r_{eq} (see Figure 11). The curvature was calculated from a surface evolver model of the interface at two values of contact angle (ϕ_{lower} , ϕ_{upper}) with the rod that bounded the measured profile. The supplementary material explains how this relationship was calculated from the profile data. Since Equation 7 predicts that the data should have a linear relationship between $\Delta\rho_p \cdot t/w$ and $\frac{d|K|/dr}{\sin \theta}$, the linear fits of the data were extracted, and summarized in Table 2. Rod diameters below 2 mm were not plotted, as these parts did not align an edge tangent to a circle concentric to the rod so that the distance could not be reliably measured. For each rod size, dK/dr was calculated using both the ϕ_{lower} and ϕ_{upper} surface

Figure 11 Plots of $(d|K|/dr)/\sin(\tan^{-1}(y'))$ vs $\Delta\rho_p \cdot t/w$ for each rod diameter. Error bars show two standard deviations (95% confidence interval).

profiles. Since the results show a negligible difference, only the lower bound values are reported.

The slopes of the linear fit with a zero intercept correspond to $g/(\gamma \cdot \beta)$, providing an experimentally derived estimate of the constant β . Table 2 summarizes the average value of β as derived from the linear fit slopes with $g = 9.81 \text{ m/s}^2$ and $\gamma = 0.05 \text{ N/m}$. An average β value of $1.63 \cdot 10^{-5} \pm 3.11 \cdot 10^{-6}$ was obtained from the linear fitting results. The variation in the measurements is significant with standard deviations of β from all rod diameters falling roughly within 20% of the average values. This likely reflects some limits in the use of curvature and slope and curvature of the non-deformed surface at a single point when many of the parts are

large relative to the diameter of the rod. Additional error may arise due to the possibility of multiple wetting states for the parts at the interface. Depending on the wetting state, the reference density in the $\Delta\rho_p \cdot t/w$ term could vary between the density of the covering fluid and the lower fluid. This could induce an uncertainty in $\Delta\rho_p \cdot t/w$ of 4% in high-density parts but up to 16% in the low-density test parts. Nevertheless, the similarity in the calculated β values supports the proposed model.

Table 2 Summary of linear fit results

Rod Diameter	Line Fit Slope $g/(\gamma\beta)$	β^*
2mm	$1.59 \cdot 10^7$	$1.23 \cdot 10^{-5}$
2.5mm	$1.24 \cdot 10^7$	$1.62 \cdot 10^{-5}$
3mm	$1.15 \cdot 10^7$	$1.66 \cdot 10^{-5}$
4mm	$9.84 \cdot 10^6$	$1.99 \cdot 10^{-5}$
Mean (μ)	$1.24 \cdot 10^4$	$1.63 \cdot 10^{-5}$
Standard Deviation (σ)	$2.55 \cdot 10^3$	$3.11 \cdot 10^{-3}$
R.S.D.* ($100 \cdot \sigma / \mu$)	20.5%	19.11%

* Value calculated after implementing $\gamma = 0.05$ N/m

** Residual standard deviation

Results indicate that part-rod interactions are governed by the surface tension of the fluid, part geometry, part density, and the radial curvature-change of the fluid interface. This empirically confirmed relation between forces (gravitational and interfacial) provides critical insight into the interactions between floating parts and semi-immersed cylindrical bodies. We have identified a parameter ($\Delta\rho_p \cdot t/w$) that permits understanding in the tradeoffs between part geometry and material. Additionally, we have proposed a relationship that could support comparison between surfaces with different curvature such as due to different rod diameters.

These findings provide valuable guidance in developing fluidic systems that rely on these interactions such as the electrowetting actuation system presented above. The most important characteristic of these systems is the presence or absence of a stable equilibrium point that would

assure that parts would not touch the rods. Secondly, the actual equilibrium distance might be useful in differentiating between parts with different geometries and/or densities based on their varying equilibrium distances. While such a distinction in equilibrium distance could be used for sorting, the uncertainty in the experimental results suggests the effect may be too small to serve as an effective sorting criterion under the conditions studied in this work.

While Velev et al. previously demonstrated stable equilibria of floating spheres near a planar surface (Velev, Denkov, Paunov, Kralchevsky, & Nagayama, 1994), no stable equilibria were observed between the parts tested in this work and a vertical glass plate. The lack of a stable equilibrium can be explained by the large difference in $d|K|/dr$ around a flat and a cylindrical object. As observed above, the rod creates a surface with two non-zero curvatures of opposing sign and nearly equal magnitude. Given that

$$\Delta p = \gamma \cdot (k_1 + k_2) = \rho g \Delta h$$

The opposing curvatures can have much higher curvatures for the same hydrostatic pressure. Thus, $d|K|/dr$ is much higher for a rod than for an interface near a flat wall which has only one curvature component. Figure 12 shows the calculated $d|K|/dr$ values for rods and plates with the

Figure 12 Comparison of $d|K|/dr$ for a rod and a flat plate. The rod has much higher value that enables stable equilibria for floating parts.

same wetting properties. To find the interface curvature for a flat plate meniscus, the height profile $y(r)$ of such meniscus was found from Kralchevsky and Nagayama's analytical solution (Kralchevsky & Nagayama, 2000). The change of curvature profile was calculated through numerical differentiation. This means that for many parts near a partially submerged planar surface, the repulsive surface curvature force is insufficient to resist the gravitational force. This explains why stable equilibria are not commonly observed near flat plates for the parts studied in this work.

6. Conclusions

A simple method for positioning and aligning floating parts of sub-millimeter dimensions is presented. Electrowetting is used to modulate the wetting state of a semi-immersed rod. By modulating the wetting around a 2D array of rods, floating parts are transported through the array. Therefore, the results from this work aid future efforts for aligning component design parameters with the process requirements for manipulation and assembly. This approach could be extended to create a manipulation system capable of positioning, aligning and assembling components at micrometer and millimeter scales.

This work identifies a simple parameter that determines how a part will interact with a curved interface and verifies that it effectively correlates with the presence of an equilibrium gap for a wide variety of part/rod interactions. The primary part factors for a square prismatic part are the density, thickness, and part width. This equilibrium state assures that floating parts will not bind to the rod. From a manipulation system perspective, this is very useful for trapping, releasing, and transporting floating parts without risk of the parts sticking to the manipulation tool.

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1.1. Calculation of Interface Curvature and its Derivative

The components of curvature and the rate of change of (absolute) curvature, $d|K|/dr$, were calculated for each measured equilibrium location using the procedures summarized below. The radial interface curve profile was measured from the digital image (Figure 8-d), but errors in these measurements did not enable calculation of $d|K|/dr$ (a 3rd order derivative in radius) through numerical differentiation. Hence, smooth profiles were necessary.

Figure 13 Image of fluid interface as modeled by Surface Evolver software package. Side view image (a). Top view image (b).

Surface Evolver was used to model the interface geometry around a semi-immersed rod (Brakke, 1992). Surface Evolver solves for the equilibrium surface by energy-minimization. The boundary conditions were the contact angle ϕ between the interface and the rod and a zero-slope

at approximately 35mm away from the rod to approximate an undisturbed interface). See Figure 13 for an illustration of the interface generated by Surface Evolver.

Table 3. Input Parameters for Surface Evolver model of water-oil interface around a cylindrical hydrophobic rod

ρ_{oil}	770 Kg/m ³			
ρ_{water}	1,000 Kg/m ³			
$\gamma_{water-oil}$	0.05 N/m			
d_{rod}	2 mm	2.5 mm	3.0 mm	4.0 mm
$\phi_{lower}, \phi_{upper}$	140°, 150°			150°, 160°

Table 3 shows the entire range of model input parameters. The model output is the interface profile when in equilibrium, represented by a $y(r)$ curve. The coordinates y and r indicate the vertical and radial distances away from the rod axis, respectively. Figure 14 plots both model and experimental surfaces, for the case for a 3 mm rod diameter and two values of contact angle (i.e. ϕ_{lower} and ϕ_{upper}). These two contact angles are shown to bracket the actual curves acquired from experiments (see Figure 14). Moreover, the equilibrium distance data of the floating parts and $d|K|/dr$ calculations showed little sensitivity to the contact angle between the lower and upper bound values. Hence, for clarity, only the lower contact angle calculations are shown in the Results sections above. Section 4 describes the procedure for measuring the equilibrium distance. The rest of this appendix describes the approach for calculating $d|K|/dr$.

In order to calculate $d|K|/dr$, the curvature (K) must first be found. Based on the symmetry, the planes of the principle radii are known. The plane of the first principal curvature (k_I) of the interface is the r - z plane as shown in Figure 7. Therefore, the SE model interface profile was used to calculate k_I . The first and second derivatives were calculated using numerical differentiation. The principal curvature k_I was then calculated, using the definition for a plane curve. Hence,

Figure 14 Comparison of profiles from menisci around a 3mm rod. Dotted profiles correspond to images acquired experimentally. Solid lines correspond to menisci simulated by Surface Evolver models with different contact angles (140° and 150°) between rod and interface.

$$k_1(r) = \frac{|y''|}{(1+y'^2)^{3/2}} \quad \text{Equation 8}$$

For convenience in the later calculations, the curvature was regression-fitted to the power equation:

$$k_1(r) = a \cdot (r - d_{rod}/2)^b \quad \text{Equation 9}$$

The power equation allowed analyzing the rate of change of curvature dk_1/dr at any point of interest. Accordingly,

$$\frac{dk_1}{dr} = a \cdot b \cdot (r - d_{rod}/2)^{(b-1)} \quad \text{Equation 10}$$

The second principal curvature was calculated from the Laplace equation that relates the fluid pressure to the curvature:

$$\Delta p = \gamma \cdot (k_1 + k_2) \quad \text{Equation 11}$$

The pressure difference across the fluid is driven by the gravitational potential so that $\Delta p = \Delta \rho_{fluid} \cdot g \cdot h$, where $\Delta \rho_{fluid} = \rho_{water} - \rho_{oil}$ and h is the height of the fluid interface at the part edge closest to the rod relative to the height of the planar interface (at $r = \infty$). Hence,

$$\gamma \cdot (k_1 + k_2) = \Delta \rho_{fluid} \cdot g \cdot h \quad \text{Equation 12}$$

Next, Equation 9 and Equation 12 were combined and rearranged in order to solve for k_2 .

$$k_2(r) = \frac{\Delta\rho_{fluid} \cdot g \cdot h}{\gamma} - a \cdot (r - d_{rod}/2)^b \quad \text{Equation 13}$$

Equation 13 was then differentiated with respect to x , and rearranged:

$$\frac{dk_2}{dr} = \frac{\Delta\rho_{fluid} \cdot g}{\gamma} \cdot y' - \frac{dk_1}{dr} \quad \text{Equation 14}$$

Finally, $d|K|/dr$ was calculated by combining Equation 6, Equation 10 and Equation 14,

$$\begin{aligned} \frac{d|K|}{dr} = & \left| a \cdot b \cdot \left(r - \frac{d_{rod}}{2} \right)^{(b-1)} \right| + \\ & \left| \frac{\Delta\rho_{fluid} \cdot g}{\gamma} \cdot y' - a \cdot b \cdot (r - d_{rod}/2)^{(b-1)} \right| \end{aligned} \quad \text{Equation 15}$$

For each point, $d|K|/dr$ was calculated from each r_{eq} measurement with a rewritten version of Equation 7, where the angle θ was substituted with $\tan^{-1}(y'(r))$,

$$\frac{d|K|}{dr} \cdot \frac{1}{\sin(\tan^{-1}(y'))} = \frac{g}{\beta \cdot \gamma} \cdot \frac{\Delta\rho_p \cdot t}{w} \quad \text{Equation 16}$$

The force balance relation was evaluated for parts of different design parameters ($\Delta\rho_p$, t , and w) and the left side of Equation 16 vs. $\Delta\rho_p \cdot t/w$. The slope of the linear fit provides an estimate of $g/\beta\gamma$.

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