Improved modeling of photoluminescent and electroluminescent coupling in multijunction solar cells

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Abstract

The performance of tandem stacks of Group III-V multijunction solar cells continues to improve rapidly, both through improved performance of the individual cells in the stack and through increase in the number of stacked cells. As the radiative efficiency of these individual cells increases, radiative coupling between the stacked cells becomes an increasingly important factor not only in cell design, but also in accurate efficiency measurement and in determining performance of cells and systems under varying spectral conditions in the field. Past modeling has concentrated on electroluminescent coupling between the cells, although photoluminescent coupling is shown to be important for cells operating near their maximum power point voltage or below or when junction defect recombination is significant. Extension of earlier models is proposed to allow this non-negligible component of luminescent coupling to be included. The refined model is validated by measurement of the closely related external emission from both single and double junction cells.

Keywords: radiative coupling, photoluminescent coupling, electroluminescent coupling, tandem solar cells, III-V materials

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Limiting solar cell performance is obtained when cells operate at the radiative limit, with all recombination in the cell occurring radiatively. This situation was first analyzed by Shockley and Queisser [1] for individual cells, with the analysis subsequently extended to multiple junction cell stacks by several authors [2-5]. Marti et al. [5] in particular studied the consequences of radiative coupling between the cells in this limit. Brown and Green [6] showed how radiative coupling could improve the balance between cell currents in blue rich spectra. The performance of experimental cells has since improved sufficiently that radiative coupling now needs to be taken into account in cell design [7], measurement [8] and in predicting cell and system performance under varying spectral conditions [6].

Published analyses to date include electroluminescent (EL) coupling between the cells but neglect the photoluminescent (PL) coupling due to carrier generation from light absorption within each cell. While both refer to reabsorption of higher bandgap cells' rear emission by lower bandgap cells, the former is a component due to EL emission and the latter is a component due to PL emission at short circuit. The neglect of this PL coupling component is appropriate for limiting efficiency calculation due to the often-implicit assumption of infinite carrier mobilities [2-6]. However, for practical cells, PL coupling can be an important coupling component near the maximum power point voltage where cells would ideally operate or when junction depletion region recombination is the dominant component of total device recombination.

Low injection is assumed throughout the bulk regions of individual cells. The current-voltage characteristics of each junction i in a series-connected multijunction solar cell can be formulated generally as the difference between a recombination component J_i^{rec} and a light generated component J_i^L :

$$J(V) = J_i^{rec}(V) - J_i^L \quad (1)$$

where J is negative in the power producing quadrant. The light generated component J_i^L is limited by the ideal current J_i^{Lmax} that would correspond to all carriers photogenerated in the cell of interest being collected and therefore contributing to current output. The actual current collected will be a fraction of this ideal current with this fraction approaching unity for good quality cells. Following the approach described elsewhere [9], J_i^{rec} can be expressed

generally as the sum of two components:

$$J_i^{rec}(V_i) = J_i^{01}(e^{qV_i/kT} - 1) + J_i^{0m}(e^{qV_i/mkT} - 1) \quad (2)$$

where J_i^{01} and J_i^{0m} are the reverse saturation components of the two current components and V_i is the voltage across the cell corrected for series resistance effects. The first term describes recombination in the bulk regions of the device under low injection conditions [10, 11], including band-to-band radiative processes, non-radiative recombination through defect levels and Auger recombination [12]. The second term with $m > 1$ describes recombination via defects in junction depletion regions [13] and other regions where electron and hole concentrations are comparable [The second term can also include recombination through defect levels in bulk regions under high level injection; under these conditions, a third term of the same form as the second but with $m = 2/3$ could be added to describe Auger recombination [12, 14] although accommodated by the first term where $m = 1$ under low level injection conditions; radiative recombination remains described by an $m = 1$ term under both low and high level injection conditions [12, 14]].

These equations apply for each cell in the stack but with different values of the parameters for each cell. The EL current in cell j due to the EL flux emitted by a higher bandgap overlying cell i can be expressed as [9]:

$$J_{ij}^{EL} = \eta_{ij} J_i^{01}(e^{qV_i/kT} - 1) \quad (3)$$

where η_{ij} is a coupling efficiency, restricted to a value less than unity since not all recombination in cell i is radiative, some light is emitted in directions where it will not reach the underlying cell, some will be reabsorbed in the emitting cell. As elsewhere [9], this leads to the following expression relating J_i^{rec} and J_{ij}^{EL} if the minus 1 term is neglected:

$$J_i^{rec} = \frac{J_{ij}^{EL}}{\eta_{ij}} + J_i^{0m} \left(\frac{J_{ij}^{EL}}{\eta_{ij} J_i^{01}} \right)^{1/m} \quad (4)$$

This allows the EL component to be determined. For the particular but common case where $m = 2$, the following simple analytical solution can be found:

$$J_{ij}^{EL} = \eta_{ij} \left[\sqrt{\varphi_i^2 + J_i^{rec}} - \varphi_i \right]^2 \quad (5)$$

where $\varphi_i = \frac{J_i^{02}}{2\sqrt{J_i^{01}}}$ is a previously [9] defined parameter associated with the light-emitting upper cell i .

Here the formulation differs from that given earlier [9]. Since this EL component is zero when the voltage across cell i is zero, it follows that this model predicts no luminescent coupling from cell i to cell j when cell i is short-circuited. In actual cells, photoluminescence does occur at short circuit [15], being zero only in the ideal case of infinite carrier mobility as noted elsewhere [16]. In cells under illumination, carriers build up across the base region on short circuit, with these giving rise to what is referred to in this work as a PL contribution. Given the linearity of the equations governing the carrier build-up, this contribution would be expected to be proportional to the short-circuit current J_i^L but would be bounded by the amount of recombination occurring on short-circuit, which can be expressed as $(J_i^{Lmax} - J_i^L)$. Here, J_i^{Lmax} is the maximum possible value for J_i^L when all photogenerated carriers in cell i contribute to its light generated current. Hence, the total luminescent coupling current in cell j becomes:

$$J_{ij}^{LC} = J_{ij}^{EL} + J_{ij}^{PL} = \eta_{ij} \left[\sqrt{\varphi_i^2 + J_i^{rec}} - \varphi_i \right]^2 + \kappa_{ij} J_i^L \quad (6)$$

where $\kappa_{ij} \approx \eta_{ij}(J_i^{Lmax}/J_i^L - 1) \ll \eta_{ij} < 1$. Note that this approximation would slightly underestimate κ_{ij} for a usual cell i but moderately overestimate κ_{ij} for a rear-junction cell i . The additional term will be large compared to the first when $J_i^{rec} \ll \kappa_{ij} J_i^L / \eta_{ij}$, as is the case for voltage in cell i below the maximum power point voltage, or when $J_i^{rec} \ll \varphi_i^2$, as is the case when junction defect recombination dominates in this cell, as likely at low illumination levels. The light-generated current term, J_i^L , on the far right of Eq. (6) is given by:

$$J_i^L = J_i^{LExt} + \sum_{h=1}^{i-1} J_{hi}^{LC} \cong J_i^{LExt} + J_{(i-1)i}^{LC} \quad (7)$$

where J_i^{LExt} is the current generated by the externally incident light source. The approximation follows if each cell is opaque to its overlying cell's band edge luminescence.

With this modification to the definition of the luminescent coupling current, the analysis can proceed by expressing the total recombination current in cell i in terms of light generated currents as in earlier [9] work. One further refinement that could be worth considering if sufficient information is available is that, as apparent from the approximate expression above, κ_{ij} can depend upon the spectral composition of J_i^L . Noting that J_i^L can be expressed as the sum of an external J_i^{LExt} and internal luminescent coupling component $\sum_{h=1}^{i-1} J_{hi}^{LC}$ with the latter generated by photons of energy at the upper end of the energy range incident on cell i and hence absorbed closer to its junction, $\kappa_{ij} J_i^L$ in Eq. (6) could be replaced by the more complex expression:

$$\kappa_{ij} J_i^L = \kappa_{ij}^{Ext} J_i^{LExt} + \kappa_{ij}^{LC} \sum_{h=1}^{i-1} J_{hi}^{LC} \quad (8)$$

where κ_{ij}^{Ext} would be larger than κ_{ij}^{LC} , for the normal cell structures. The term in κ_{ij}^{LC} disappears if i is the uppermost cell.

For a tandem stack of arbitrary number of cells in the order of descending bandgaps, only cells above the limiting junction have an impact on the overall current output. This simple principle suggests a sequential algorithm for extracting κ_{ij} and other fitting parameters of each cell i . In brief, second-junction limited cases should be analyzed first to extract the luminescent coupling parameters of the top cell; then, third-junction limited cases can be investigated with the parameters of the top cell inserted to facilitate the extraction of the corresponding parameters of the second cell on top, and so on. The consequences of neglecting the PL coupling can be severe in some cases, particularly the extreme cases that are conducive to extracting the coupling parameters.

From Eq. (6), the PL coupling current J_{ij}^{PL} becomes significant when either $\kappa_{ij} J_i^L / \eta_{ij}$ or ϕ_i^2 is comparable with or greater than J_i^{rec} , where $\kappa_{ij} J_i^L / \eta_{ij} \approx (J_i^{Lmax} - J_i^L)$. Taking the positive correlation between J_i^{rec} and J_i^L into account, the former condition can be regarded as a restriction on the position of the operating point (lower than maximum power point

voltage), with no restriction on the absolute magnitude of the voltage, it can refer to a wide range of illumination intensities. The later condition, however, requires $J_i^{rec} \ll \varphi_i^2$ with φ_i being a constant, which more often refers to low illumination levels where J_i^{rec} is always small.

Measuring a closely related emission component, the light emitted from the top surface of the cell stack, validates the presence of the additional PL term and its dependencies. In general, we can express the top surface emission current for each cell J_i^{em} in terms of the luminescence coupling components by introducing empirical coupling constants β that connects the external emission arising from cell i to the luminescent currents induced in the immediately underlying cell j [17]:

$$J_i^{em} = \frac{J_{ij}^{PL}}{\beta_{ij}^{PL}} + \frac{J_{ij}^{EL}}{\beta_{ij}^{EL}} \quad (9)$$

where the β terms approximately equals n^2 where n is the index of refraction for a solar cell of constant index, but this can be dramatically different when layers of significantly differing index are included [18]. Additionally $\beta_{ij}^{PL} > \beta_{ij}^{EL}$ for the standard cell structure, since the carrier distributions at short circuit are more concentrated towards the cell rear compared with those responsible for the EL emission, enhancing the relative PL emission towards the rear. The opposite would be the case for rear-junction devices.

We demonstrate the importance of the additional PL term for explaining luminescent coupling and quantify the empirical parameters introduced above through experimental measurements of single and two-junction III-V solar cells. As in earlier work, we measure and quantify external emission [19] and J_{sc} as we vary the calibrated illumination [9] on these cells. The external illumination is quantified, as in Ref. [9], by the number of suns relative to the one-sun photocurrent under a specific reference spectrum (typically AM1.5d for concentrator cells or AM1.5g for non-concentrator cells).

In the case of single-junction devices no luminescent coupling is possible, but we can investigate relationships between the parameters appearing in Eqs. (6) and (9) by measuring the external emission as a function of illumination at short-circuit. At short-circuit (SC),

$V_1 = 0$ and $J_1^{rec} = 0$, and $J_1^L = J_{sc}$. Even though there is no cell to couple to in this case, only the substrate and rear contact, this allows the ratio $\kappa_{1r}/\beta_{1r}^{PL}$ that would apply to such coupling to be determined since inserting these values into Eqs. (6) and (9) gives $J_1^{em}|_{sc} = (\kappa_{1r}/\beta_{1r}^{PL}) J_{sc}$. (Here we define J_{sc} as a positive quantity and the subscript r denotes emission towards the rear.) At open-circuit (OC), $J_1^{rec} = J_1^L = J_{sc}$, allowing $J_1^{em}|_{oc}$ to be similarly related to J_{sc} . In Fig. 1, we plot both $J_i^{em}|_{sc}$ and $J_i^{em}|_{oc}$ as a function of J_{sc} for three separate cells and fit to the relevant equations, with fitting parameters shown in Table. I.

FIG. 1. External emission current density at short-circuit (SC) and open-circuit (OC) and their ratios as a function of the short-circuit current density for three different single-junction III-V solar cells (“RHJ” and “trad” are abbreviations for rear-heterojunction and traditional structures respectively).

TABLE I. The fit parameters for the three single-junction III-V solar cells shown in Fig.1.

Sample ID	$\beta_{1r}^{EL}/\eta_{1r}$	ϕ_1 ($\sqrt{\text{mA}}/\text{cm}$)	$\kappa_{1r}/\beta_{1r}^{PL}$
MM937	17.4	0.42	0.0062
MM989	74	3.37	0.0002
MM958	5.8	1.22	0.0036

As discussed in earlier work, $J_1^{em}|_{oc}$ is a non-linear function of J_{sc} while the weaker $J_1^{em}|_{sc}$ can be well fitted by a linear function. The latter is strongest relative to the former when J_{sc} is small (devices MM958 and MM989 in particular), φ_1 is large (device MM989) or κ_1 is large (device MM937), as previously discussed.

Fig. 2 shows the emission spectra along with the EQE of a two-junction GaInP/GaAs tandem solar cell in which both junctions are rear-heterojunction devices which have been shown to have high radiative emission [19, 20]. In the bottom-junction limited case (top cell over-illuminated), the bottom junction is reverse biased and gives no EL emission, but there is still measurable luminescence around the bandgap wavelength which is hence identified as the short-circuit PL contribution. Fig. 3 shows how the tandem J_{sc} and the emission J_{em} from each cell varies as the illumination on each cell is varied with the tandem at short circuit. The data for the tandem J_{sc} by itself can be fit well both with and without using the additional PL term but the data for the emission J_{em} from the top and bottom cells clearly show the necessity for the PL term. From Table II, the inclusion of this PL contribution results in significantly different conclusions for the one-sun current density J_2^{1sun} and the optical geometric coupling factor β_{12} (β_{12}^{PL} assumed equal to β_{12}^{EL} in this case to simplify data extraction) and the correlated coupling efficiency η_{12} . [The φ_i and β_{ij}/η_{ij} terms can be difficult to decouple with this data, but this becomes much clearer by fitting to dark EL data, although not discussed further here.] Besides the above analyzed situations, the overall PL coupling for a tandem stack is expected to become more significant as the number of cells increases, especially in a bottom-junction limited case where the effect of non-zero κ_{ij} for each cell above accumulates.

FIG. 2. The PL emission spectra (line) and EQE (markers) for a two-junction GaInP/GaAs tandem solar cell with high radiative efficiency. The units of the PL emission have been calibrated for the emission peaks (but are not appropriate for the reflected probe light). The shaded regions are integrated to determine J_i^{em} in Fig. 3.

FIG. 3. The current density and the photon emission flux density from individual cells where the series-connected tandem is held at short-circuit while a) varying 470nm illumination as 740nm illumination is held constant at 0.54 suns; b) varying 740nm illumination as 470nm illumination is held constant at 0.55 suns; c) varying 470nm illumination with no 740nm illumination.

TABLE II. The fit parameters for the GaInP/GaAs tandem solar cell shown in Fig. 3.

Model	$J_1^{1\text{sun}}$ (mA/cm ²)	$J_2^{1\text{sun}}$ (mA/cm ²)	$\beta_{12}^{EL}/\eta_{12}$	$\beta_{2r}^{EL}/\eta_{2r}$	φ_1 ($\sqrt{\text{mA/cm}}$)	φ_2 ($\sqrt{\text{mA/cm}}$)	$\kappa_{12}/\beta_{12}^{PL}$	$\kappa_{2r}/\beta_{2r}^{PL}$	β_{12}
Old	13.48	13.73	25	3.4	0.22	6.7	0	0	14.6
Present	13.48	12.63	25	3.4	0.22	6.7	0.0066	0.0057	11.6

In summary, an improved model for luminescent coupling in monolithic tandem cells is developed that accounts for not only EL but PL coupling that previous work neglects. Such PL coupling is important when the corresponding cell is operating below the maximum power point voltage or junction defect recombination dominates in that cell as more likely at low illumination levels, and the overall effect of PL coupling on the performance of a

tandem stack is expected to be more important as the number of cells increases. The refined model is validated by measurement of the related external emission in single- and double-junction devices.

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