IMPACT OF DENSITY INFORMATION ON RAYLEIGH SURFACE WAVE INVERSION RESULTS

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Abstract

We assessed the impact of density on the estimation of inverted shear-wave velocity (V_s) using the multi-channel analysis of surface waves (MASW) method. We considered the forward modeling theory, evaluated model sensitivity, and tested the effect of density information on the inversion of seismic data acquired in the arctic. Theoretical review, numerical modeling and inversion of modeled and real data indicated that the density ratios between layers, not the actual density values, impact the determination of surface-wave phase velocities. Application on real data compared surface-wave inversion results using: a) constant density, the most common approach in practice b) indirect density estimates derived from refraction compressional-wave velocity observations, and c) from direct density measurements in a borehole. The use of indirect density estimates reduced the final shear-wave velocity (V_s) results typically by 6-7 % and the use of densities from a borehole reduced the final V_s estimates by 10-11% compared to those from assumed constant density. In addition to the improved absolute V_s accuracy, the resulting overall V_s changes were unevenly distributed laterally when viewed on a 2-D section leading to an overall V_s model structure that was more representative of the subsurface environment. It was observed that the use of constant density instead of increasing density with depth not only can lead to V_s overestimation but it can also create inaccurate model structures, such as a low-

velocity layer. Thus, optimal V_s estimations can be best achieved using field estimates of subsurface density ratios.

Keywords: Rayleigh surface wave; surface-wave inversion; density; density sensitivity; shear-wave velocity; V_s

1. Introduction

Estimation of shear-wave velocity (V_s) is important for the evaluation of the stiffness properties of the near-surface materials; V_s increases as material shear strength (rigidity) increases. The multichannel-analysis of surface-wave (MASW) method was developed to estimate near-surface S-wave velocity from high-frequency (≥ 2 Hz) Rayleigh-wave data (Miller et al., 1999; Park et al., 1999; Xia et al., 1999). Shear-wave velocities estimated using MASW have been reliably and consistently correlated with shallow-well data. Using the MASW method, Xia et al. (2000) noninvasively measured V_s within 15% of downhole measurements in wells located along the seismic profile. As confirmed by numerous borings, Miller et al. (1999) mapped limestone bedrock with 0.3-m accuracy at overburden depths of about 4.5-9 m. Similar successful results have been reported in the literature, e.g. Ismail and Anderson (2007), including good agreement between surface-wave estimated V_s for the upper 30 m (a.k.a. V_{s30}) and invasive-methods (Moss, 2008; Comina et al., 2011). Anbazhagan and Sitharam (2008) estimated a relationship between MASW V_s and standard penetration test (SPT) N values that correlates well with the Japan Road Association equations. The MASW method has been used to characterize pavements (Ryden and Park, 2004), investigate sea-bottom sediment stiffness (Kaufmann et al., 2005; Park et al., 2005), map fault zones (Ivanov et al., 2006a), study Arctic

ice sheets (Tsoflias et al., 2008) and subglacial sediments (Tsuji et al., 2012), obtain 3D V_s estimations in 3D reflection data (Strobbia et al., 2011), and map build up stress above voids (Ivanov et al., 2013). Studies using the MASW method have evaluated approaches for determining quality factor (Q) of the near-surface (Xia et al., 2002), and constraining seismic refraction inversion models (Ivanov et al., 2006b; Dal Moro et al., 2007; Ivanov et al., 2010; Socco et al., 2010a; Piatti et al., 2013). A review of established approaches of surface wave methods (SWM) can be found in Socco et al. (2010b).

The MASW method requires several unique steps (Miller et al., 1999). First, a single seismic-data record is acquired using a line of equally spaced (usually, for convenience) low-frequency (e.g., 4.5 Hz) vertical geophones. The seismic wavefield from such a shot record is transformed into a phase-velocity – frequency domain image (i.e., dispersion-curve image) (Park et al., 1998). This image is used to evaluate the dispersion-curve characteristics of the fundamental-mode Rayleigh wave. Next, the estimated dispersion curve is inverted (Xia et al., 1999) to produce a 1-D V_s model of the subsurface. This 1-D V_s function is assigned to a point in the middle of the geophone line (a.k.a., spread). A pseudo-2-D V_s model is then built by assembling numerous 1-D V_s models each derived from seismic shot gathers recorded after each incremental advancement of shot and receiver locations (a.k.a., roll).

Most recent developments of the SWM include the definition of a suitable spatial window (e.g., MOPA method) to control near-field effect and lateral variations (Strobbia and Foti, 2006) and one based on wavelengths (Lu, 2015), the use of lateral constraints (Boiero and Socco, 2010), and space-varying special windowing (Bergamo et al., 2012) to handle 2-D

effects. Recent research also includes the expansion with the use of the horizontal component of the Rayleigh wave (Boaga et al., 2013) and the Love-waves (Dal Moro et al., 2015), evaluation at landfill sites (Suto, 2013), understanding fault geometry (Ikeda et al., 2013), development of V_s profiles to depths of 100m or deeper for earthquake-response microzonation (Murvosh et al., 2013), and the simultaneous use of guided-waves with multi-mode surface waves in land and shallow marine environments (Boiero et al., 2013).

MASW dispersion-curve inversion for V_s is commonly performed using predefined values for compressional-wave velocity (V_p) and density. It is preferred that such a-priori information is available from other measurements. However, practical MASW applications often lack this information and as a result the V_p and density estimates are determined based on assumptions for each specific site. It has been postulated that the resulting error from using parameter assumptions is insignificant. Xia et al. (1999) showed that a 25 % increase in V_p resulted in less than 3 % average change in the dispersion-curve phase-velocity values from forward modeling. Furthermore, forward modeling showed that decreasing the density in the top 2 layers by 25 % and increasing it in the rest of the layers by 25 % resulted in average V_s change of less than 10 %.

In this study, we expand the above work reviewing the numerical surface-wave formulations and applying three types of model sensitivity studies to better understand the impact of density change within a single layer, a single density contrast change within a layered section, and overall density trend ratios. We also experimentally evaluated the effect of density information on the estimation of V_s . We employed the MASW method in the arctic over snow

and ice where density and V_s were well constrained, compared to typical near-surface geologic environments. Using constant density as a function of depth resulted in unrealistic V_s estimates and velocity inversion structures that were unlikely in an environment dominated by gradual compaction and densification of snow with depth. We observed the most realistic V_s estimates when we employed a density profile measured at a nearby ice-core.

2. Background

2.1 Theoretical Background

The impact of density on surface wave inversion can be investigated by examining any of the numerical algorithms for Rayleigh-wave dispersion-curve estimation. Knopoff's method (Schwab and Knopoff, 1972), which is an improved version of the Thomson-Haskell technique (Thomson, 1950; Haskell, 1953) can be used for a layered earth model (Xia et al., 1999). This transfer matrix method is a preferred algorithm because of its simplicity and ease of computer implementation (Foti et al., 2015). However, other approaches, such as the dynamic stiffness matrix method (Kausel and Roesset, 1981) or the reflection and transmission coefficients method (Kennett and Kerry, 1979) can also be used. For brevity we list only a few parts of the implementations outlined by Schwab and Knopoff (1972) required to demonstrate the impact of density on the forward modeling computations (i.e., formulas are incomplete).

The Rayleigh-wave dispersion curve calculation function $F_R(\omega, c)$ can be written as:

$$F_R(\omega, c) = T^{(0)} \bar{F}^{(1)} F^{(2)} \bar{F}^{(e)} \dots F^{(n-2)} \bar{F}^{(n-1)} T_{solid}^{(n)} \quad (1)$$

if n is even (when n is odd the formula is similar), where n is the total number of layers and $T^{(0)}$ and $F^{(m)}$ are matrixes and m is layer number.

The $T^{(0)}$ component (not shown for brevity) in equation (1) contains the density of the first layer, only (i.e., ρ_1). The $F^{(m)}$ matrix elements contain variables that include density only in the form of ratios in various relationships, e.g.:

$$\begin{aligned} \varepsilon_0^{(m)} &= \rho_{m+1}/\rho_m & \varepsilon_7 &= \varepsilon_1 \varepsilon_3 \\ \dots & & \varepsilon_8^{(m)} &= \varepsilon_1^{(m)} \varepsilon_4^{(m)} \\ \varepsilon_2 &= \varepsilon_1 - 1 & \varepsilon_{10} &= \varepsilon_2 \varepsilon_3 \\ \varepsilon_3 &= \varepsilon_1 + \varepsilon_0 & \varepsilon_{11} &= \varepsilon_2 \varepsilon_4 \\ \varepsilon_4 &= \varepsilon_2 + \varepsilon_0 \end{aligned} \quad (2)$$

It can be noticed in equations (2) that density ratios can be observed in the expression of $\varepsilon_0^{(m)}$, which is part of the expressions for ε_2 , ε_3 , and ε_4 which in turn are components of the expressions for other ε values. In these numerous interdependent expressions it is difficult to observe direct relationships and/or dependencies between variables. However, it can be noticed that in all expressions densities are presented in pairs and as ratios, i.e., their absolute values appear irrelevant to the computation process, and it is their ratios that affect the computations.

2.2 Previous Sensitivity Analysis

Xia et al. (1999) used a 6-layer model (Table 1) to calculate surface-wave dispersion curves, observe their changes due to changes in the model parameters, such as density, V_s , V_p , and thickness, and concluded that V_s is the dominant parameter influencing the Rayleigh-wave velocity. Furthermore, they measured V_s inversion errors when using inaccurate V_p and density values up to 25%, separately and in combination, and reported 8% V_s deviation from the true model with as much as 23% for some layers. In order to maximize the effects of density variability on the dispersion curve, the first and second layers were decreased by 25 % and the

rest of the layers were increased by 25%. The resulting average relative change in phase velocity between the calculated dispersion curves before and after density changes were estimated to be <10% (Xia et al., 1999).

3. Methods and Results

3.1 Model Sensitivity Analysis

3.1.1 Absolute Density Value Changes

In this study the 6-layered model (Table 1) proposed by Xia et al. (1999) was used for a broader density sensitivity assessment. Figure 1 shows calculated dispersion curve values for the fundamental mode and the next five higher modes of the Rayleigh wave for the original 6-layered model. To test the proposition that the absolute density-value changes do not affect the dispersion- curve estimations we used densities 75% from their original values for all the layers to again calculate the fundamental-mode and the first five higher modes of the Rayleigh wave (Figure 1). All calculated dispersion-curves before density changes were identical to those from the model after all densities were reduced to 75%. For simplicity, further density sensitivity analysis will focus on the fundamental-mode, only.

3.1.2 Single-Layer Sensitivity Estimations

A common approach to sensitivity studies is to change the model parameter of one layer (e.g. V_s , thickness), calculate the corresponding data (i.e. dispersion-curve values) and compare the results to observations from the original model (Liang et al., 2008; Calderón-Macías and Luke, 2010). We applied this analysis to the 6-layer model but with constant densities across all layers and changed in sequence the density of each single layer by +66% (Figure 2). Changing

the density of one layer effectively changed two parameters in the calculation formula because it changed the density ratios with respect to two neighboring layers and thus changed two parameters in the dispersion-curve sets of equations. Increasing density by 66% from layer to layer shows different phase-velocity change patterns. When the density of the first layer increases phase-velocities decrease by at least 7% in the ~23-50 Hz range and by 20-25% in the 15-23 Hz range (Figure 2). Inverting with constant density (e.g., as result of lack of density information) the layers related to the low-frequencies at ~20 Hz during the inversion would have notably underestimated V_s . Other shallower layers will be affected as well but to a lesser degree. For this model inversion, calculations indicated V_s underestimations for the second, third, fourth, fifth, and sixth layers to be ~15%, 12%, 8%, 5%, and 1% respectively (Figure 3). However, the first layer had no V_s estimation errors. This was an unexpected observation that using lower-than-true densities of the top layer during the inversion can lead to V_s underestimations not for the top layer, but to the layers below, with the largest error for the nearest layer and decaying with distance (Figure 3).

When the density of the second layer increased 66%, phase-velocities still decreased but to a lesser degree ~5-15% in the ~15-22 Hz range, however above 25 Hz the phase-velocities increased reaching ~10% change at ~39 Hz. Such a pattern suggested that if constant densities were used during the inversion, some parts (relatively shallower) of the V_s section would be overestimated and others (relatively deeper) underestimated, which could have potential for creating a false structure. When the density of each of the rest of the layers increased by 66%,

phase-velocities increased but at lower and narrower frequency range as the layers got deeper (Figure 2).

We repeated the analysis, this time decreasing the density of each layer by 40% so that the density ratios between the neighboring layers would be identical to those of the increasing density tests (e.g., 1.66). When the density of the first layer decreased phase-velocities increased by at least 10% in the ~17-41 Hz range and by 20% in the ~19-30 Hz range (Figure 4). Thus, inverting with constant density (e.g., as result of lack of density information) resulted in overestimated V_s for the second, third, fourth, fifth, and sixth layers by ~19%, 7%, 8%, 20%, and 1% respectively (figure not shown for brevity). Thus, using higher-than-true density of the top layer can lead to noticeable overestimations (~19-20 %) of the second and fifth layers. It is intriguing to observe a different error distribution throughout layers in comparison to the underestimated case (i.e., ~15%, 12%, 8%, 5%, and 1%).

When the density of the second layer decreased by 40%, phase-velocities decreased by at least ~10% in the ~20-35 Hz range (Figure 4). It is interesting to observe that regardless of whether the density of the second layer decreased (Figure 4) or increased (Figure 2), there is a frequency range (e.g., 5-15% and ~20-35 Hz, accordingly) within which the dispersion-curve values decreased. With the density decreasing for the deeper layer the range of the affected dispersion curve values narrowed.

First-layer dispersion-curve changes trends are opposite to those from the tests of the rest of the layers. I.e., values decrease/increase when density values for the first layer increase/decrease. For the rest of the layers dispersion-curve values increase/decrease with the

densities of those layers (i.e., when they increase/decrease, accordingly). For all top layers changes occur above ~17 Hz, while for half space, density changes occur below ~17 Hz. Thus, this ~17 Hz appears as a pivotal (reference) frequency, with respect to which sensitivity values change. At this frequency the fundamental mode wavelength is ~30.5 m, ~520 m/s divided by 17 Hz (Figure 1), ~42% of which corresponds to the depth to half space (12.8 m; Table 1). Consequently, we can consider this pattern-changing-frequency at ~17 Hz to be indicative of the wavelength vs depth-to-half-space relationship.

The different dispersion-curve sensitivity responses from the first-layer density changes can be attributed to the $T(0)$ component of equation (1), which contains the density of the first layer, only.

3.1.3 Single Density Contrast Sensitivity Estimations

To change just one parameter for sensitivity calculations, another approach was adopted that uses a single density contrast. The density of the target layer was changed by 66% and the densities of all the layers below were changed accordingly to preserve the existing density ratios between them (Figure 5). By changing the density of only one layer at a time and then changing all the layers below it by the same percent simultaneously, the first layer change is equivalent to increasing all layers by the same rate (i.e., 66%), i.e., rescaling them. In such a manner only one density ratio was changed and only one parameter was modified during forward modeling. As expected, in the case of the first layer there were no phase-velocity changes at all frequencies, which is consistent with the formula observations and initial tests above when all density ratios in the model were preserved. When the second layer and the ones below were changed by 66%,

the resulting model was similar (identical density ratios) to the sensitivity test by Xia et al. (1999). The density ratio from decreasing densities of the first two layers by 25% and increasing the rest by 25% can also be obtained by keeping the densities for the first two layers unchanged and only increasing the densities in the remaining layers (i.e., 3-6) by 66%. Compatibility and extension of the work by Xia et al. (1999) was the motivation for selecting the value of 66% in our studies. Significant phase-velocity changes 8-25% could be observed in the 18-28 Hz range (Figure 5), which is consistent with Xia et al. (1999), who reported that the average relative phase-velocity changes were <10% in the 5-30 Hz range.

3.1.4 Increasing Density Trend Sensitivity Estimations

The density-change patterns were extended to better understand real-world situations (density distributions). The next density-changes pattern included density ratios that changed across the whole 1-D vertical model. It was estimated that a linear density-trend change between the first and last layer (a.k.a. half space) provided nearly identical density-ratio changes across the whole model. Six models with linear density trends with last-layer/first-layer ratio starting from 1 and ending at 1.66 (Table 2) were used to calculate phase-velocity sensitivity (Figure 7) with respect to those from the original six-layer model (Table 1). The density ratios between the neighboring layers were nearly identical within each trend (Table 3) being 1 (i.e., constant densities), ~1.025 for the second trend, and ~1.105 for the last trend. The densities from the second model (column 1.13; Table 2) were practically identical to the original 6-layer model from Xia et al. (1999). Consequently, the sensitivity test for the second density trend at $\rho_{hs}/\rho_1=1.13$ is at ~0 (Figure 7). The same result can be reproduced with different density values that may appear

more realistic and are, for example, 75% lower from the densities initially used (Figure 6). Since all values were rescaled the corresponding density ratio table did not change (Table 3), therefore the resulting forward modeling calculations were identical. The maximum extent of the used density trends (and even greater) can be considered reasonable at some sites as a result of natural geologic processes including compaction, desiccation, cementation, etc. Phase-velocity changes 4-12% could be observed in the ~15-33 Hz range at $\rho_{hs}/\rho_1 = 1.66$. When the dispersion curve from this trend was inverted with constant densities V_s was overestimated by 3%, 11%, 4%, 12%, and 3% for the second, third, fourth, fifth, and sixth layers, respectively (Figure 8) with the average overestimation of ~7%. Again there was no error for the first layer. It can be noticed that the distribution of the overestimations through the layers are irregular, i.e., for every other layer (2nd, 4th, and 6th layers) it was relatively small, ~3-4%, while relatively large for the rest, 11-12%. Observations of such irregularities (most likely due to the non-linear nature of the problem) make it difficult to predict how inaccurate density information might affect the final V_s estimates.

The numerical-model sensitivity tests above can be considered representative for the particular six-layer model, which we selected to use for research continuity purposes. Sensitivity results may change with different layer models (Liang et al., 2008; Calderón-Macías and Luke, 2010) and it may be appropriate to perform density sensitivity studies on a case-by-case basis.

Density trends decreasing with depth can imply similar V_s trends that may lead to complexities, such as Lamb-wave patterns (Ryden et al., 2004; Pan et al., 2013), etc., which may

not be strictly approached as Rayleigh waves (Thrower, 1965) and may be addressed by future research.

3.2 Field Seismic Data

3.2.1 Field Site and Data Acquisition

The practical density impact evaluations on the MASW inversion can be performed on seismic data that are collected at a site, for which compressional- wave velocity, shear-wave velocity and density information is available from laboratory or field measurements, or such information can be reliably predicted from analogue sites. We scrutinized the MASW method using a seismic data set acquired at Jakobshavn Glacier in western Greenland. The subsurface consisted of snow and ice with density and seismic p- and s-wave velocities increased exponentially with depth. This setting offered well-constrained density and velocity information compared to typical near-surface geologic environments. The Arctic ice-sheet site appeared to be suitable for studying density impacts on surface waves because for similar sites V_p , V_s and density information for snow and ice are available from laboratory measurements (Kohnen, 1974; Patterson, 1994; Waite et al., 2000) and indirect estimates from field observations can be found in the literature (King and Jarvis, 2007). Density measurements are available from Crawford Point, a relatively near-by site (~ 55 km north of the seismic data site) where a shallow ice core was obtained for seasonal accumulation measurements in west central Greenland (Porter and Mosley-Thompson, 2014)

From the perspective of the practical application of the surface-wave inversion the aforementioned a-priori data types can be summarized into three approaches for using density

information in view of the amount and likelihood of its availability. The first type is using constant density from a single sparse estimate, which is the most common approach in practice due to the often lack of density information. Next is using indirect density estimates with depth (e.g., derived from refraction compressional -wave velocity evaluations), and the third is using direct subsurface density measurements with depth from boreholes.

The MASW data analysis was applied using SurfSeis software developed by the Kansas Geological Survey to data originally acquired for a reflection survey at Jakobshavn Glacier, Greenland (Horgan et al., 2008). The goal of the reflection survey was to evaluate the thickness of the ice sheet to about 2000 m. Data were collected along a 10-km profile using a 0.5 kg charge at 10 m depth as a source and twenty-four 28 Hz p-wave vertical phones on 20 m spacing resulting in a 460 m spread. The spread incremental movement (roll) was 160 m. Our research effort was to evaluate the seismic data for possible MASW Vs estimations. The phase-velocity – frequency image of data showed the high-resolution fundamental-mode dispersion-curve trend within a wide frequency range, 10-100 Hz (Figure 9). The lowest observable phase velocity was at ~10 Hz, which was lower than the 28 Hz resonant geophone frequency. However, such observation was consistent with our previous research (Ivanov et al., 2008), which showed that surface-wave phase velocity can be imaged with the phase-shift method (Park et al., 1998) at frequencies as low as about 1.5 octave lower than the resonant geophone frequency. With a phase velocity of 1800 m/s at 10 Hz and 1000 m/s at 100 Hz with corresponding wavelengths between 180 m and 10 m, the expected approximate depth estimates (based on 50% of the wavelengths assumption) were from 90 m to 5 m.

3.2.2 Field Data V_s Estimation

Phase-velocity vs frequency images from 62 records along the 10 km line were obtained, fundamental-mode dispersion-curves were estimated, and inverted one by one with the inversion algorithm implemented in SurfSeis (Xia et al., 1999) using a 10-layer model. Root-mean-square (RMS) error between the modeled and observed dispersion-curves ranged between 5 and 22 m/s and was practically unchanged when using different models discussed below.

The obtained initial inversion V_s estimates (Figure 10a) were higher than expected at about 50 m and deeper, exceeding the maximum V_s velocity values for ice found in the literature (2000 m/s). In an effort to achieve more realistic results we incrementally used reference values for ice: a constant V_p/V_s ratio of 2.0 (Waite et al., 2000) and density of 0.8 g/cm³ (Patterson, 1994), every time obtaining identical (to the ones from the initial inversion) final V_s estimates, which is consistent with the numerical formulas (2). Other constant density values were tested, for thoroughness, without having any impact on the final results (Figure 10a). Additional inversion tests were performed in an attempt to identify other types of inversion parameters, such as various V_p model assumptions [including King and Jarvis (2007) from refraction tests which can be found in the fifth column in Table 4], number of layers, initial model, and depth conversion ratio. It was examined if any of these would make the inversion V_s match better or get closer to the referenced maximum values for ice (i.e., below 2000 m/s), however, the tests were unsuccessful.

The sought dispersion-curve inversion results modification took place when variable density values, namely increasing with depth, were used for the inversion. We applied an

indirectly estimated density/depth function (Figure 11) obtained by King and Jarvis (2007) after analyzing p- and s-wave seismic refraction data acquired in the Antarctic (Adelaide Island). Not only did the final Vs results change (Figure 10b) but most of the absolute values decreased and matched better to those reported in the literature values, while still maintaining some extremes at few locations. In addition, most of the velocity inversions disappeared and different structural features could be observed. A couple of velocity inversions at ~70 m depth centered at ~1050 m and ~5200 m became even more pronounced. However, it can be seen that density values used for the inversion were constant below 50 m (Figure 11). Although the King and Jarvis (2007) density/depth function came from the opposite hemisphere than our Greenland seismic data, it provided an improvement over the constant density results by eliminating many of the unrealistically high Vs estimates.

Inversion results proved even more realistic (in terms of better matching expected maximum values and gradual Vs vertical trend) after using densities measured in a relatively near-by shallow ice core in Greenland (Crawford Point ice core density data provided by Porter and Mosley-Thompson (2014)) (Figure 11) with Vs estimates decreasing further (Figure 10c) resulting in values consistent with laboratory measurements of ice and snow Vs (i.e., below 2000 m/s). Furthermore, the final pseudo-2-D Vs section did not have any velocity inversions that were consistent with glaciologists' expectations of an environment dominated by snow and firn gradual compaction and densification with depth (Porter and Mosley-Thompson, 2014). The ice core density trend confirmed the resulting gradual increase in Vs with depth (Figure 11). However, based on observation of Vs lateral variability along the profile (Figure 10c) we

speculate that obtaining more accurate density information from wells positioned along the profile could lead to more accurate Vs results.

3.2.3 Field Data Sensitivity Analysis

Motivated by the formula observations, earlier model sensitivity tests, and lack of changes in field data inversion results when using different constant densities we tested the inversion process using rescaled density functions (e.g., 10 times greater than the measured density in the ice core). We did not observe any changes in the inversion solutions, which is consistent with our expectations.

Similarly to the six-layer model sensitivity analysis applied earlier, we ran sensitivity estimations on our data focusing on the density-trend approach, which seemed most appropriate for this arctic site. Using record 601 dispersion-curve (Figure 9) inversion results with a ten-layer model (Table 4), ten models with linear density trends with last-layer/first-layer ratio starting from 1 and ending at 2 (Table 5) were used to calculate phase-velocity sensitivity with respect to those from the ten-layer model with constant densities (Figure 12). The densities from the tenth model ($\rho_{10}/\rho_1=2$; Table 5) were almost identical to those from ten-layer model with the borehole densities (Table 4).

Phase-velocity changes 4-9% could be observed in the ~8-60 Hz range at $\rho_{hs}/\rho_1 = 2$ with the biggest changes 7-9% between ~11-26 Hz range. When the dispersion curve from this trend was inverted with constant densities Vs was overestimated by ~3%, 4%, 7%, 9%, 9%, 9%, 8%, 5%, and 3% for the second, third, fourth, fifth, sixth, seventh, eighth, ninth, and tenth layers, respectively (Figure 13) with the average overestimation of ~6%. Again there was no error for

the first layer. It can be noticed that for this model the distribution of the overestimations through the layers are gradually increasing and then decreasing. This smooth pattern is different in comparison to the more irregular distribution of the analogous sixth-layer model sensitivity discussed earlier (Figure 8).

We calculated the percent changes between the obtained solutions at all locations to study the impact of each type of density information (Figure 14). The use of direct density measurements from the ice core reduced almost half of the Vs values (Figure 14a) by approximately 10-11% (mostly at mid-depths) in comparison to using assumed constant density. Another portion of Vs values, most of which distributed at the top or bottom of the section, were reduced by 4-5 % and very few by more than 17 %. Closer examination of Vs modifications indicated that the biggest decreases occurred for the middle 5 -6 layers of the 10-layer model, while there were no changes for the first layer. At very few locations (e.g., 100 m and 900-1500 m) at depth (70 m) Vs estimates increased by about 5-6 %.

There are several subtle vertical sequences of bulls-eye features (e.g., at horizontal locations 100 m, 1050 m, 1400 m, and 4300 m) that can be observed on the inversion results when using constant density (Figure 10a). These features disappeared when using accurate density information from the well (Figure 10c). Interestingly, while the use of indirect density estimates produced a section with Vs values closer to the expected (below 2000 m/s) reducing most of the section by 4-10 % (Figure 14b) in comparison to when using a constant density, some of the vertical sequences of anomalous velocity areas (bulls-eye features) become even stronger (Figure 10b) (e.g., at horizontal locations 1050 m and 4300 m), some new features

appeared (e.g., at horizontal locations 3450 m and 4900-5450 m), and some disappeared (e.g., at horizontal locations 100 m, and 1400 m).

The most significant change between solutions from using direct ice core measurements and indirect estimates (Figure 14c) are several columns having consecutive bulls-eye shapes of increased and decreased V_s values. Few areas of the section decreased by 4-5 % (e.g., below 30 m at horizontal locations 2100-3600 m). However, most of the changes were unevenly distributed throughout the section contributing to the modification of the overall V_s model structure that more accurately coincided with the geologic expectations.

The purpose of the above numerical studies was to obtain insights on the impact of using different patterns of inaccurate versus accurate densities during the inversion on both modeled and real data. The purpose of this research was not the interpretation of the final V_s results from the Jakobshavn Glacier data; this will be addressed by another manuscript.

4. Discussion

All numerical and real data from the research presented in this paper indicated that V_s of the first top layer is not affected by density inaccuracies. The observation that underestimated top-layer densities can lead to V_s underestimation of the layers below (e.g., 12-15%; Figure 3) appears new and unique. We speculate that underestimated densities could contribute reduced V_s for models with high-velocity half-space (Casto et al., 2009). The same is valid for the vise-versa case of overestimated top-layer densities.

Using constant density of 2 g/cm^3 for all layers, Xia et al. (2000) noninvasively measured V_s within 15% of V_s measured in wells. Based on the observations that use of constant density can lead to V_s overestimation of about 10-11% and the production of inaccurate model structures we hypothesize that the utilization of correct density depth function has the potential of improving further the match between the MASW and well measurements V_s to as low as 4-5 % mismatch.

Even with a general expectation about the density depth function from the characteristics of the Arctic field location, the final V_s result exhibited velocity inversions, which were unrealistic in view of the gradual ice compaction with depth. These results come as a warning that similar features in final V_s sections can be due to using inadequate density-depth function and not related to the true geology. Vice-versa, extreme, unrealistic values may be indicative of an inaccurate density function. In addition, closer study of the inversion process showed that several subtle vertical sequences of bulls-eye features are a result of inversion instability. When accurate density measurements from well were used, these patterns were not observed on the inversion results. These observations suggest that models with increasing density with depth are less prone to instability.

As a result of the density-impact observations we speculate that the use of different density-depth functions may affect the comparison between different surface-wave inversion algorithms (e.g., benchmark tests, etc.).

These results suggest the use of as much a-priori information as possible concerning the density distribution with depth when absolute V_s evaluations are important for a specific site

investigation. When such a-priori information is not available, the assumption that a density increasing with depth trend appears more appropriate in view of the geologic processes taking place at the very near surface; this could lead to more accurate final results. Nevertheless, this can also lead to unrealistic vertically expressed bulls-eye sequences, as previously mentioned.

We consider our efforts to be a step toward a better understanding of the density impact on the Rayleigh-wave inversion V_s results using only the fundamental-mode. However, further research can incorporate more complicated density models (e.g., densities decreasing with depth) and the involvement of higher modes.

5. Conclusions

The main contribution of this research demonstrates that the benefit of using accurate increasing-with-depth density information is two-fold, it decreased the absolute V_s values and it also improved the overall structural appearance of the pseudo 2-D MASW V_s sections. The V_s results reduction can be of significant value to the engineering community because it can prevent site overestimation, which can have various safety aspects (e.g., earthquake hazard estimations). The observation of more realistic (e.g., there were no velocity inversions) and consistent with site location expectations inversion results from using accurate density information can be of significance for the general understanding of the subsurface and improving surface-wave method accuracy and reliability.

One of the contributions of this work is the observation that varying a single constant density value for the whole model or rescaling by a constant density-depth function does not affect the final V_s results but it is the vertical density-ratio function ($\sim$ density gradient) that does.

The observation of the counter-intuitive impact of underestimated/overestimated top-layer density on underestimating/overestimating layers beneath can also be considered as a contribution.

Another contribution of our efforts is the expansion of the single-layer-parameter variations for Rayleigh-wave density-sensitivity studies with the single-parameter contrast and vertical-trend-parameter variations, which can provide additional perspective to evaluate sensitivity.

This manuscript provides formulas, numerical, and practical examples for the utilization of density values at a given site. One of the goals of our work is to demonstrate the use of assumed (based on general lab measurements), approximate (from indirect measurements), or relatively accurate a-priori density data and its impact on the final surface-wave inversion results.

From practical point of view we can generalize, that from the perspective of minimizing V_s inversion errors due to the use of different types of density information, field measurements have the highest ranking, followed by indirect measurements, then by assumptions about the vertical density trend consistent with geological expectations, and, finally, a constant value for the entire section.

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Layer	Depth (m)	Thickness (m)	Vs (m/s)	Vp (m/s)	Density (g/cm ³)
1	2	2	194	650	1.82
2	4.3	2.3	270	750	1.86
3	6.8	2.5	367	1400	1.91
4	9.6	2.8	485	1800	1.96
5	12.8	3.2	603	2150	2.02
6	HalfSpace	N/A	740	2800	2.09

Table 1. Six-layer model used by Xia et al. (1999).

Layer	$\rho_6/\rho_1=1$	$\rho_6/\rho_1=1.13$	$\rho_6/\rho_1=1.26$	$\rho_6/\rho_1=1.40$	$\rho_6/\rho_1=1.53$	$\rho_6/\rho_1=1.66$
1	1.82	1.82	1.82	1.82	1.82	1.82
2	1.82	1.87	1.92	1.96	2.01	2.06
3	1.82	1.92	2.01	2.11	2.20	2.30
4	1.82	1.96	2.11	2.25	2.40	2.54
5	1.82	2.01	2.20	2.40	2.59	2.78
6	1.82	2.06	2.30	2.54	2.78	3.02

Table 2. Density values for six models each using different increasing-density trends.

Layer	$\rho_6/\rho_1=1$	$\rho_6/\rho_1=1.13$	$\rho_6/\rho_1=1.26$	$\rho_6/\rho_1=1.40$	$\rho_6/\rho_1=1.53$	$\rho_6/\rho_1=1.66$
1						
2	1.000	1.026	1.053	1.079	1.105	1.132
3	1.000	1.026	1.050	1.073	1.095	1.117
4	1.000	1.025	1.048	1.068	1.088	1.105
5	1.000	1.024	1.046	1.064	1.080	1.094
6	1.000	1.024	1.044	1.060	1.074	1.086

Table 3. Density-ratio values, $\frac{\rho_{i+1}}{\rho_i}$, between the next and previous layers for the six increasing-density trends in Table 2.

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Layer	Depth (m)	Thickness (m)	Vs (m/s)	Vp (m/s)	Density (g/cm ³)
1	2.60	2.60	1120.00	1869	0.45
2	5.84	3.24	1041.42	1965	0.51
3	9.90	4.06	1170.15	2263	0.55
4	14.97	5.07	1287.17	2599	0.61
5	21.31	6.34	1386.56	2934	0.66
6	29.24	7.93	1492.35	3225	0.70
7	39.14	9.90	1615.20	3495	0.75
8	51.52	12.38	1765.29	3762	0.80
9	67.00	15.48	1796.34	3762	0.84
10	HalfSpace	N/A	1988.21	3721	0.88

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Table 4. Ten-layer model from horizontal location 22800 m obtained from the MASW analysis of record 601 (Figure 9) using borehole densities.

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Layer	$\rho_{10}/\rho_1=1$	$\rho_{10}/\rho_1=1.11$	$\rho_{10}/\rho_1=1.22$	$\rho_{10}/\rho_1=1.33$	$\rho_{10}/\rho_1=1.44$	$\rho_{10}/\rho_1=1.55$
1	0.45	0.45	0.45	0.45	0.45	0.45
2	0.45	0.456	0.461	0.467	0.472	0.478
3	0.45	0.461	0.472	0.483	0.494	0.506
4	0.45	0.467	0.483	0.500	0.517	0.533
5	0.45	0.472	0.494	0.517	0.539	0.561
6	0.45	0.478	0.506	0.533	0.561	0.589
7	0.45	0.483	0.517	0.550	0.583	0.617
8	0.45	0.489	0.528	0.567	0.606	0.644
9	0.45	0.494	0.539	0.583	0.628	0.672
10	0.45	0.50	0.55	0.60	0.65	0.70

Layer	$\rho_{10}/\rho_1=1.66$	$\rho_{10}/\rho_1=1.77$	$\rho_{10}/\rho_1=1.88$	$\rho_{10}/\rho_1=2$
1	0.45	0.45	0.45	0.45
2	0.483	0.489	0.494	0.50
3	0.517	0.528	0.539	0.55
4	0.550	0.567	0.583	0.60
5	0.583	0.606	0.628	0.65
6	0.617	0.644	0.672	0.70
7	0.650	0.683	0.717	0.75
8	0.683	0.722	0.761	0.80
9	0.717	0.761	0.806	0.85
10	0.75	0.80	0.85	0.90

Table 5. Density values for ten models each using different increasing-density trends based on the ten-layer model in Table 4.

Figure 1. Calculated Rayleigh-wave fundamental- and higher-mode dispersion curves for the six-layer model using original densities shown in Table 1 (symbols only), which are identical to a six-layer model using density values 75% of the original (lines only).

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Figure 6. Six layer models with the six density trends.

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Figure 14. MASW V_s changes along the seismic line due to using varying density with depth. a) Ice core direct density versus constant density, b) indirect estimates versus constant density, and c) Ice core direct density versus indirect estimates density.

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