1
2 **On the reliability and limitations of the SPAC method with a**
3 **directional wavefield**

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Abstract

The spatial autocorrelation (SPAC) method is one of the most efficient ways to extract phase velocities of surface waves from ambient seismic noise. Most studies apply the method based on the assumption that the wavefield of ambient noise is diffuse. However, the actual distribution of sources is neither diffuse nor stationary. In this study, we examined the reliability and limitations of the SPAC method with a directional wavefield. We calculated the SPAC coefficients and phase velocities from a directional wavefield for a four-layer model and characterized the limitations of the SPAC. We then applied the SPAC method to real data in Karamay, China. Our results show that, 1) the SPAC method can accurately measure surface wave phase velocities from a square array with a directional wavefield down to a wavelength of twice the shortest interstation distance; and 2) phase velocities obtained from real data by the SPAC method are stable and reliable, which demonstrates that this method can be applied to measure phase velocities in a square array with a directional wavefield.

Keywords: SPAC; directional wavefield; phase velocity; wavelength limitations.

1. Introduction

The dominant energy of ambient seismic noise recorded by receivers near the Earth's surface is surface waves (Toksöz and Lacoss, 1968). The spatial autocorrelation (SPAC) method is one of the most efficient ways to retrieve surface waves from ambient seismic noise (Aki, 1957, 1965). Recent studies of the SPAC method include an extension of the SPAC method to mixed-component correlations (Haney et al., 2012), a generic theory using three-component data (Cho et al., 2006), applications using different array configurations (Ling and Okada, 1993; Bettig et al., 2001; Ohori et al., 2002; Cho et al., 2004; Chavez-Garcia et al., 2005; Xu et al., 2012), direct inversion of SPAC coefficients (Wathelet, 2005), discussion of the relationships between the SPAC method and the traditional cross-correlation technique (Chávez-García and Luzón, 2005; Yokoi and Margaryan, 2008; Ekström et al., 2009; Tsai and Moschetti, 2010), different ways to calculate SPAC coefficients (Weemstra et al., 2013; Weemstra et al., 2014), multimode inversion of surface waves (Ikeda et al., 2012), and extraction of attenuation coefficients (Prieto et al., 2009; Lawrence and Prieto, 2011; Tsai, 2011; Nakahara, 2012; Lawrence et al., 2013; Weemstra et al., 2013; Menon et al., 2014).

So far, most studies were based on the assumption that the wavefield of ambient noise is diffuse. However, the actual distribution of noise sources is neither diffuse (Mulargia, 2012) nor stationary, especially in areas which are far away from the coast. A directional wavefield will lead to biases in phase velocity measurement in the traditional ambient noise cross-correlation method (Shapiro et al., 2005; Bensen et al., 2007). Generally, The SPAC method can remedy the bias by azimuthally averaging the SPAC coefficients over the receiver array. Okada (2006) considered a directional wavefield and investigated the minimum receiver number required for a circular array (i.e., one receiver at the center of a circle of uniformly distributed receivers), and recommended that a 3-station circular array (i.e., three receivers in the circle) to be

the most efficient array in the SPAC method. [Cho et al. \(2008\)](#) considered a directional wavefield and provided a more rigorous theoretical framework to assess the applicability of the SPAC method. These two studies, however, were restricted to circular arrays, which hardly represent arrays of distributed receivers. [Gouédard et al. \(2008\)](#) proposed to use the noise correlation slant stack technique to extract phase velocities from a directional wavefield. But it requires that the wavefield comes from only a known direction.

The most commonly used arrays consist of scattered distributed receivers. The reliability and limitations of the SPAC method with a directional wavefield need to be analyzed for such arrays. For example, whether the SPAC method can correctly measure phase velocities from a directional wavefield, and if it does, what are the wavelength limits of the measured phase velocities. In this paper, after briefly summarizing the basics of the SPAC method, we calculate the SPAC coefficients and phase velocities for one velocity model. We then use synthetic data to analyze dispersion measurement limits by the SPAC method with a known directional wavefield. Finally, we apply the SPAC method to real data in Karamay, China.

2. Theory and method

[Aki \(1957\)](#) first introduced the SPAC method based on a statistical investigation of seismic waves. [Henstridge \(1979\)](#) obtained similar results based on spectral representations of stationary random signals. [Tsai and Moschetti \(2010\)](#) derived the same theory in a more explicit way. Here, we briefly summarize the basics of the SPAC method.

2.1 SPAC theory

In the one-dimensional case, the SPAC coefficients $\rho(r, \omega)$ of two receivers separated by distance r is expressed as,

$$\rho(r, \omega) = \cos(r\omega / c(\omega)) \quad (1)$$

where ω is the angular frequency and the $c(\omega)$ is the frequency dependent phase velocity. Eq. (1) also holds in the two-dimensional case if the wavefield is unidirectional; the distance r should be replaced by the apparent interstation distance along the direction of the incident wave, i.e., $r|\cos(\varphi-\theta)|$ (Fig. 1), where φ is the azimuth of the incident wavefield, and θ is the direction of the receiver-receiver line.

In the two-dimensional case, for an isotropic wavefield or uncorrelated noise sources (with random phases), the SPAC coefficients are given by

$$\rho(r, \omega) = \frac{1}{2\pi} \int_0^{2\pi} \cos[\omega r \cos(\varphi - \theta) / c(\omega)] d\varphi = J_0(r\omega / c(\omega)), \quad (2)$$

where J_0 is the first kind Bessel function of order zero. Because of the $\varphi - \theta$ symmetry in Eq. (2), we can switch θ with φ and obtain the same result. This means that the requirement of an isotropic wavefield can be satisfied with a number of uniformly distributed receivers for an azimuthal average of the SPAC coefficients, which remedies the biases in phase velocity measurements caused by a nonisotropic or directional wavefield.

2.2 Computation of SPAC coefficients

The first step of the SPAC method is computation of SPAC coefficients. A common way is to ensemble-average the cross-spectra and normalize it by the averaged power-spectra (Ohori et al., 2002). Recently, some studies (Prieto et al., 2009; Lawrence and Prieto, 2011; Weemstra et al., 2013) normalize the cross-spectra before the ensemble averaging,

$$\rho(r, \omega) = \left\langle \frac{\Re \{C_{x_n x_m}(\omega)\}}{\sqrt{C_{x_n x_n}(\omega) C_{x_m x_m}(\omega)}} \right\rangle \quad (3)$$

where ρ is the SPAC coefficients, $\langle \dots \rangle$ is the ensemble averaging, $\Re \{ \dots \}$ is to take the real part of a complex argument, $C_{x_n x_m}(\omega) = u(\mathbf{x}_n, \omega) u^*(\mathbf{x}_m, \omega)$ is the cross-spectra (where $n \neq m$) or the power-spectra (where $n = m$), $u(\mathbf{x}_n, \omega)$ and $u(\mathbf{x}_m, \omega)$

are the Fourier spectra of seismic records $u(\mathbf{x}_n, t)$ and $u(\mathbf{x}_m, t)$, respectively, and the asterisk indicates complex conjugation.

2.3 Phase velocity measurement

The second step of the SPAC method is to estimate the phase velocities from the SPAC coefficients. The application of the SPAC method requires that the array should be regular, and the most frequently-used array is circular. [Ohori et al. \(2002\)](#) proposed to use the extended spatial autocorrelation (ESAC) method, which can be adapted to arbitrary arrays. The ESAC method treats the SPAC coefficients as a function of interstation distance r and extracts the phase velocities by finding the best fit between the observed and the theoretical SPAC coefficients. [Weemstra et al. \(2013\)](#) added a frequency dependant factor $A(\omega)$ in Eq. (4) to account for the cross-terms. As a result, phase velocity $c(\omega)$ and $A(\omega)$ can be estimated by a grid search to minimize,

$$\text{Misfit}(\omega_i, A_j, c_k) = \text{RMS} \left(\rho(r, \omega_i) - A_j J_0 \left(\frac{r \omega_i}{c_k} \right) \right) \quad (4)$$

where $\text{RMS}(\dots)$ denotes the root mean square.

3 Determination of phase velocity

Layered velocity models are widely used for shallow Earth structure ([Xia et al., 1999](#); [Beatty et al., 2002](#)). We used a four-layer model ([Table 1](#)) to characterize the SPAC method with a directional wavefield. The phase velocity dispersion curve of the model was calculated by the Knopoff method ([Schwab and Knopoff, 1972](#)) and is shown in [Fig. 2b](#). The dispersion curve exhibits a complex shape and indicates a low velocity layered model, which is commonly encountered in the near surface. Generally, a square array can represent a two-dimensional scattered array of distributed receivers. We used two square arrays, which contain $7 \times 7 = 49$ ($N=7$) and $3 \times 3 = 9$ ($N=3$) receivers, respectively, to characterize the SPAC method. An example of the square array (where $N=7$) is shown in [Fig. 2a](#). We set the directional wavefield to

come from the northeast at an azimuth $\varphi=60^\circ$.

We calculated the SPAC coefficients of the directional wavefield for two arrays. Comparing with the SPAC coefficients derived from a uniform wavefield (Fig. 3a, c), the SPAC coefficients of the directional wavefield (Fig. 3b, d) vary as a cosine function (Fig. 3f) instead of a Bessel function (Fig. 3e).

We conducted the grid search method proposed by Weemstra et al. (2013) to find the phase velocities from the SPAC coefficients. We restricted the values of the scale factor between 0 and 2 rather than between 0 and 1 used by Weemstra et al. (2013) for a better fit between the observed and theoretical SPAC coefficients. In the three-dimensional grid search, the frequency f is sampled from 0.1 to 40Hz at an increment of 0.1Hz, the phase velocity c is searched from 0.1 to 0.8km/s at an increment of 0.005km/s, and the scale factor A is searched at an increment of 0.01. Figure 4 shows the phase velocity dispersion energy images from the SPAC coefficients. For the uniform wavefield, phase velocities are accurately extracted over the whole frequency range of 0.1 to 40Hz, though resolution of the dispersion image for the array of $N=3$ (Fig. 4c) is lower than that of $N=7$ (Fig. 4a). For the directional wavefield, phase velocities can be correctly extracted for both arrays in a given wavelength range. The Nyquist wavelength (Okada, 2006) controls the range of $N=7$ (Fig. 4b) while it does not control $N=3$ (Fig. 4d). This suggests the wavelength limits in a two-dimensional array cannot be simply described by the Nyquist wavenumber but somehow does relate to the receiver numbers of an array. Generally, larger receiver numbers of a square array will have a better coverage of interstation azimuth. So the wavelength limits may also relate to interstation azimuth in some extents.

4 Wavelength limits

In addition to the Nyquist wavelength and receiver numbers, the maximum

interstation distance which controls the resolution of an array also plays an important role in the wavelength limits (Cornou et al., 2007; Gouédard et al., 2008). So, we investigated these three elements of an array, i.e., the receiver numbers (N^2), the minimum ($d_{\min}$) and maximum ($d_{\max}$) interstation distance.

We conducted four tests to study the limitations of the SPAC method in a directional wavefield. We used the same square arrays and the directional wavefield as in the preceding section (Fig. 2a). To simplify the analysis, we used a constant phase velocity of 2.5km/s as the input dispersion curve. 1) In the first two tests, we set $N=7$ and $N=3$ and let $d_{\min}$ increase from 0.01 to 0.4km at an increment of 0.002km, consequently $d_{\max}$ also increases. These two tests are to study the influence of $d_{\min}$ and $d_{\max}$ compared to the wavelength limits. 2) In the third test, we analyzed the influence of N and $d_{\min}$ compared to the wavelength limits. We fixed $d_{\max}$ to $\sqrt{2}$ km (one side of the square array is 1km) and let N increase from 2 to 20 consequently $d_{\min}$ decreases. 3) In the fourth test, we explored the influence of N and $d_{\max}$ compared to wavelength limits. We set $d_{\min}$ to 0.05km and again let the N increase from 2 to 20; $d_{\max}$ will increase accordingly. As a result, the first two tests have 196 cases for $N=7$ or $N=3$, the third and the fourth test have 19 cases.

We followed the processing steps described in the previous section and obtained the SPAC coefficients of directional wavefield for the square array by using Eq. (1). Then we used Eq. (4) as the misfit function and measured the phase velocities by grid search method. We searched the phase velocity c from 2 to 3km/s at an increment of 0.01km/s. Finally, we calculated the relative errors between the measured phase velocities ($V_{measured}$) and the theoretical ones (V_{th}),

$$Errors = \frac{|V_{measured} - V_{th}|}{V_{th}} \times 100 \quad (5)$$

Figure 5a, b show the error distributions for the first two tests. For $N=7$, the

boundaries of areas with few errors ($\leq 1\%$) follow hyperbolas of $\lambda=2d_{\min}$ and $\lambda=d_{\max}$. Large relative errors ($\approx 3\% \sim 5\%$) exist around $\lambda=d_{\max}$ at low frequencies. The wavelength range narrowed for $N=3$. Few errors ($\leq 1\%$) appear for $\lambda > d_{\max}$ for both arrays, suggesting that the lower frequency limits are not influenced by $d_{\max}$ nor receiver numbers at extremely low frequency ranges.

The error distribution of the third test is presented in Fig. 5c. Since we fix the $d_{\max}$ in this test, $\lambda=d_{\max}$ is a vertical line. The wavelength range with few errors ($\leq 1\%$) is roughly limited by $\lambda=2d_{\min}$ for all N 's and increases as N increases. A slight deviation from the Nyquist wavelength is due to incomplete azimuth sampling by the station pairs. For one-dimensional distributed arrays the Nyquist wavelength is in a strict sense (Cornou et al., 2007). We speculated that it is also applicable for two-dimensional cases but not in a strict sense. Figure 5d shows the error distribution of the fourth test. Due to the constant $d_{\min}$ in this test, $\lambda=2d_{\min}$ is a vertical line. The actual upper wavelength limit is less than $\lambda=2d_{\min}$ when N is small (e.g., $N < 7$) and exceeds $\lambda=2d_{\min}$ when N becomes large (e.g., $N > 16$), though the increases relative to the $\lambda=2d_{\min}$ are quite limited. This suggests that it is unnecessary to increase the receiver number to increase the frequency upper limit; rather it is more important to decrease the station spacing as observe in Fig. 5a, b.

Large relative errors ($\approx 3\% \sim 5\%$) emerge around $\lambda=d_{\max}$ for all four tests. These large errors, however, may hardly be detected from real data because other influences at low frequencies. Few errors ($\leq 1\%$) are observed at extremely low frequencies ($\lambda > d_{\max}$) for all four tests. This is a little surprising because we could believe that the $d_{\max}$ controls the lower frequency limit. Generally, the actual phase velocity dispersion at the extremely low frequencies can hardly be extracted, however, it is not hard to understand since we used SPAC coefficients over the whole frequency band in the tests. These results suggest the lower limits of frequency ranges are not influenced by

$d_{\max}$ nor the receiver numbers. If one wants to determine the low frequency limit, other influential factors should be considered, such as the incoherent noise, the power-spectra of noise sources and the dissipative medium.

To investigate how the results vary with azimuth, we analyzed the error distributions of another four source azimuths ($\varphi = 0^\circ$, $\varphi = 15^\circ$, $\varphi = 30^\circ$, $\varphi = 45^\circ$). We used a square array $N=7$ and let $d_{\min}$ increase from 0.01 to 0.4km at an increment of 0.002km. The results (Fig. 6) show that error distributions do not vary significantly with azimuth.

We then did the tests with three other kinds of arrays (first column in Fig. 7), which are triangular, circular and spiral arrays (e.g., Xu et al., 2012; Cho et al., 2013; Kennett et al., 2015), to investigate the effect of different geometries. Again, we set the directional wavefield to come from the northeast at an azimuth $\varphi=60^\circ$. We fixed the maximum radius of each array to 0.5km and let receiver number increase from 4 to 49 with an increment of 1 for the circle array and 3 for the other two arrays. Following the procedure for the square array, we obtained error distributions of each array (second column in Fig. 7). The errors do not follow the Nyquist wavelength except for the square array, which demonstrates that error distributions can depend on array configurations. To do a further investigation, we calculated interstation azimuths with receiver number equaling to 49 and obtained histogram of azimuths (third column in Fig. 7). It turns out that more uniformly distributed interstation azimuths will result in a better phase velocity measurements.

5. A real data application

To further test robustness, we use the SPAC method to measure phase velocities near Karamay, China (Fig. 8). Ninety-six hours of seismic noise data were recorded by a square array of 45 stations equipped with sensors of 2.5Hz corner frequency in

July 2013. The minimum interstation distance $d_{\min}$ is 0.2km and the maximum interstation distance $d_{\max}$ is 1.7km.

We used a beamforming algorithm described by [Gerstoft et al. \(2006\)](#) to assess the wavefield distribution. We first removed mean, trend, and instrument response in 1-hour-long waveforms. Moreover, signals from large events (e.g., earthquakes) were removed by truncating the amplitudes exceeding one standard deviation. The time series were then whitened in the frequency domain. Finally, the frequency domain beamforming was applied to get the beamformer of each hour, and the final beamformers were obtained by stacking the results of all 96 hours.

[Figure 9](#) shows the beamformer of four different frequency bands. We observed a strong and a relative weak energy from the northeast and southwest, respectively. The azimuths of these two energies are consistent with the city of Karamay and the village of Daoban ([Fig. 8](#)), which are about 30km and 8km from the array, respectively. In the low frequency band, the dominant energy came from Karamay ([Fig. 9b](#)), while in the high frequency band, it came from Daoban ([Fig. 9d](#)). In the medium frequency band, the energy came from both azimuths ([Fig. 9c](#)). This is reasonable because Karamay is further away our site than Daoban. We speculate that the energy from Karamay was dissipated at high frequency over the longer distance, while the high-frequency energy from Daoban was less dissipated over the shorter distance.

Phase velocities were measured by grid search of phase velocity from 1 to 5 km/s at an increment of 0.01 km/s and the scale factor A between 0 and 2 at an increment of 0.01. We note that the same result is obtained if we search A between 0 and 1. We used an outlier algorithm proposed by [Parolai et al. \(2006\)](#) where the observed data points outside 2 standard deviations were removed before the next iteration of the grid search. However, we used 2.8 standard deviations instead of 2 because the directional

293 wavefield generally leads to more scattered SPAC coefficients.

294
295 Figure 10a-c show the procedure of fitting the observed SPAC coefficients to find
296 the phase velocity. At each frequency, the observed SPAC coefficients (circles in Fig.
297 10a-c) were fit to the theoretical model (red line in Fig. 10a-c), and the corresponding
298 phase velocities were estimated. Figure 10d shows the final phase-velocity dispersion
299 from the ambient noise over the whole array ($N=7$). One can see that reliable phase
300 velocities were extracted in the range of $\lambda > 2d_{\min}$, which is consistent with the tests in
301 the previous section. However, it is hard to observe the small errors ($\approx 3\% \sim 5\%$)
302 around $\lambda = d_{\max}$, which were identified in the tests (Fig. 5). Those errors may be easily
303 influenced by other factors in real data, such as, the incoherent noise, the
304 power-spectra of noise sources and the dissipation in the medium.

305
306 To further assess the frequency range, we divided the $N=7$ square array (Fig. 8)
307 into subarrays. Figure 11a shows an example of a subarray containing $5 \times 5 = 25$
308 receivers. Thus, we obtained 25 subarrays for $N=3$, 16 subarrays for $N=4$, 9 subarrays
309 for $N=5$, and 4 subarrays for $N=6$. We averaged the phase velocities of all subarrays
310 for each N . Figure 11b shows phase velocity dispersion curves of different N 's. We
311 can see that all the reliable phase velocities are restricted in $\lambda > 2d_{\min}$. The widest
312 frequency range is for $N=7$ (red line in Fig. 9) and, as N decreases, the frequency
313 range becomes narrow. A larger N -array generally has a better sampling of the
314 interstation azimuth. Phase velocities around $\lambda = d_{\max}$ for all N 's can be measured,
315 which is consistent with numerical tests (Fig. 5). So one cannot deem $d_{\max}$ as a
316 definite limitation of the SPAC method with a directional wavefield.

317
318 In addition, we calculated the interstation azimuths relative to the north direction
319 for all subarrays to illustrate the influence of interstation azimuths. Azimuths larger
320 than 180° were changed into $(0^\circ, 180^\circ)$ by subtracting 180° . We then divided the

azimuths into twelve bins of 15° . Figure 12 shows the histograms of interstation azimuths. We can observe that there exist four dominant directions (i.e., 45° , 90° , 135° and 180°) in all N's, and the relative azimuth sample distribution becomes more even with increasing of N, resulting in a more uniformly sampling the azimuthal space. Consequently, a better phase velocity measurements would be obtained, which is consistent with the results we found in the synthetic examples (e.g., Fig. 7).

6. Conclusion

In this study, the main goal is to analyze the ability and the limitation of the SPAC method to extract phase velocities from a directional wavefield. We show here by a four-layer model and two square arrays that phase velocities can be accurately extracted from a directional wavefield. Results show that, the upper frequency limit of the square array is roughly restricted by $\lambda=2d_{\min}$. It is, however, also influenced by the receiver numbers and interstation azimuths. As the receiver number increases, the high frequency limit increases by a small amount. Phase velocities at the extremely low frequencies can also be accurately extracted and, the low frequency limit seems not controlled by the maximum interstation distance. The interstation azimuth also determines the detectable wavelength ranges and the accuracy of phase velocity measurements. A field data example shows that reliable phase velocities were extracted at $\lambda>2d_{\min}$ when $N=7$, and as N decreases the accuracy deteriorates which is consistent with the results of the numerical tests.

The frequency limits that we obtained here should serve as the most “conservative” limits of the SPAC method in a square array because the wavefield in the real-world is generally more isotropic than the directional wavefield. However, in the real-world, numerous other factors such as the incoherent noise, the power-spectra of noise sources and the dissipative medium, will also affect these limits. Further studies are needed to investigate those influential factors. In addition, real-world

arrays are generally irregular, and an appealing solution is to use average interstation spacing to replace the minimum distance in determining wavelength limits.

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Figure captions

Figure 1. Geometry of the wavefield and two receivers (black solid circle) separated by distance r .

Figure 2. (a) Example of a square array and a directional wavefield. The square array consists of $N \times N = N^2$ receivers (black triangle). (b) The theoretical dispersion curve of a four-layer velocity model (Table 1).

Figure 3. Theoretical SPAC coefficients as a function of interstation distance and frequency. (a) $N=7$, a uniform wavefield. (b) $N=7$, a directional wavefield. (c) $N=3$, a uniform wavefield. (d) $N=3$, a directional wavefield. (e) $N=7$, a uniform wavefield, with around 0.04km interstation distance. (f) $N=7$, a directional wavefield, with around 0.04km interstation distance.

Figure 4. Colors represent normalized misfits of the SPAC coefficients in Fig. 3. The black solid line is the theoretical dispersion curve, and the gray solid circles are the best estimated phase velocities.

Figure 5. Error distributions of the four tests. (a) and (b) are error distributions of the first two tests for arrays $N=7$ and $N=3$, respectively. (c) is the error distribution of the third test, and (d) is for the fourth test. The gray dashed lines correspond to two particular wavelengths denoted.

Figure 6. The same error distributions as Fig. 5a with different azimuths of directional wavefield. (a) $\varphi=0^\circ$. (b) $\varphi=15^\circ$. (c) $\varphi=30^\circ$. (d) $\varphi=45^\circ$.

Figure 7. Four kinds of array configurations (the first column) and the corresponding error distributions (the second column) and histogram of interstation azimuths

(the third column). (a) Triangular array. (b) Square array. (c) Circle array. (d) Spiral array.

Figure 8. Location of the seismic array (red reverse triangle) near Karamay (back star) and Daoban (back diamond) in western China. The square array consists of $7 \times 7 = 49$ receivers, however, only 45 receivers were used and displayed in the figure.

Figure 9. Beamformer results for different frequency bands normalized by the maximum in each panel, where the radial direction denotes the slowness and the circumference is the azimuth. (a) 2 to 5Hz. (b) 2 to 3Hz. (c) 3 to 4Hz. (d) 4 to 5Hz.

Figure 10. An example of using SPAC coefficients to estimate phase velocities. (a), (b) and (c) show the fitting process at 2.5, 3.5 and 4.5Hz, respectively. The solid circles denote the SPAC coefficients. The black ones were used, while the gray ones were discarded. The red solid line represents the corresponding SPAC coefficients of the estimated phase velocity. The scale factor and other parameters are showed in the box. (d) The final phase velocity dispersion (gray circles) of the square array $N=7$.

Figure 11. (a) An example of dividing the array into subarrays. The receivers into the blue rectangle form one subarray of $N=5$. 9 such subarrays can be formed by moving the rectangle. (b) Averaged phase velocity dispersion curves of the arrays with different N 's.

Figure 12. Interstation azimuths histogram of different N 's for the real data. (a) $N=3$. (b) $N=4$. (c) $N=5$. (d) $N=6$. (e) $N=7$.

Table1.

Parameters of a four layered model (modified from Luo et al. (2011))

Layer	$v_s(\text{m/s})$	$v_p(\text{m/s})$	$\rho(\text{g/cm}^3)$	$h(\text{m})$
1	400	800	2.0	10
2	200	400	2.0	10
3	600	1200	2.0	10
4	800	1600	2.0	Infinite

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

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542 Figure 6

Figure 7

Figure 8

Figure 9

Figure 10

Figure 11

Figure 12