First-principles study of coherent interfaces of Laves-phase MgZn_2 and stability of thin MgZn_2 layers in Mg-Zn alloys

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The β_1' and β_2' are important precipitates in Mg-Zn alloys. Two types of precipitates both are composed of MgZn_2 , but the orientation relationships of MgZn_2 with the α -Mg matrix in two precipitates are different. To understand the properties of β_1' and β_2' , we use the first-principles method to perform a systematical investigation of coherent interfaces of β_1'/α -Mg and β_2'/α -Mg. Here, we present exhaustive analysis of the structure and stability for these interfaces, and find that the interfacial energy is highly dependent on the average coordination of interfacial structures. Furthermore, we find that the MgZn_2 slab in matrix with $[11\bar{2}0]_{\text{MgZn}_2}/[0001]_{\alpha}$, $(0001)_{\text{MgZn}_2}/(11\bar{2}0)_{\alpha}$ orientation relationship (OR1) is less stable than that with $[11\bar{2}0]_{\text{MgZn}_2}/[10\bar{1}0]_{\alpha}$, $(0001)_{\text{MgZn}_2}/(0001)_{\alpha}$ orientation relationship (OR2), which can explain that the β_2' plate shape precipitates that are frequently observed in Mg-Zn alloys comply with OR2 instead of OR1.

Key words: Mg alloys; precipitates; Laves-phase MgZn_2 ; orientation relationships;

First-principles calculations

1. Introduction

Mg alloys, which offer a 30% weight reduction compared to aluminum alloys and a 75% weight reduction compared to steels, have received renewed attention due to the increasing demand to reduce carbon dioxide emissions in the transportation sector. One of drawbacks is that the strength of most commercial Mg alloys is relatively low. Precipitate

hardening is an important strengthening method. It is known that ZK series alloys (Mg-Zn-Zr alloy) possess the greatest precipitation hardening among Mg-based alloys, but they still need to increase strength for industrial application [1-3]. Therefore, understanding its mechanism of precipitation is critical to improve the mechanic properties of existing alloys and design new alloys.

Pure Mg-Zn alloys whose grain size is large have almost no application in industry [4]. Eighty years ago, Sauerwald found that addition of Zr into Mg-Zn can change the precipitate in the grain boundary and refine the grain of the alloy, which efficiently improves the strengthening and ductile properties [5]. Now, the Mg-Zn-Zr alloy is one of the widest application wrought Mg-based alloys, and Mg-2.4Zn-0.16Zr (at. %, ZK60) is probably the material which has the highest specific strength. Though the ZK series alloys have a good combination of strength and ductility, the strength is still relatively low. For example, the yield strength of ZK60 is only 240 MPa [6-7], which is much lower than that of medium-strength Al alloys, 300 MPa [6].

Because of the lack of polymorphic transformation of Mg matrix in heat treatment, the solid-to-solid transition cannot be used to strengthen Mg alloys. The strengthening methods of Mg alloys include solid solution strengthening, grain boundary strengthening and precipitation strengthening. For wrought Mg alloys, the precipitates actually tightly influence both fine grain strengthening and precipitation strengthening. First, at the stage of hot extrusion and dynamic recrystallization process, the precipitate will inhibit the grain growth by pinning the grain boundaries and refine the grain size, which results in

fine grain strengthening. Second, the precipitate can prevent the motion of dislocation and resist basal slip, which results in precipitation strengthening. Therefore, ZK series alloys have potential for heat-treatable precipitation hardening, but different ZK alloys hardening responses differ significantly [8]. For example, under the same heat treatment condition, the Mg–2.4Zn, Mg–2.4Zn–0.16Zr, Mg–2.4Zn–0.1Ca and Mg–2.4Zn–0.1Ag alloys have relative low hardness increment, 18, 15, 25 and 35 VH, while Mg–2.4Zn–0.1Ag–0.1Ca and Mg–2.4Zn–0.1Ag–0.1Ca–0.16Zr have relatively high hardness increment, 43 and 45 VH [3]. It indicates that the precipitation strengthening can be changed by micro alloying, and it also provides a promising method to enhance the strength of Mg-based alloys. Therefore, to reveal the mechanism of precipitation in ZK alloys is not only a foundational issue, but also an emergent task in engineering fields.

The precipitation sequence in Mg–Zn based alloys during aging has been generally considered to occur as follows:

SSSS (super-saturated solid solution) $\rightarrow$ pre- β_1' /Guinier Preston (GP) zones $\rightarrow \beta_1'$
(MgZn₂, Mg₄Zn₇) $\rightarrow \beta_2'$ (MgZn₂) $\rightarrow \beta$ equilibrium phase (MgZn, Mg₂Zn₃)

β_1' and β_2' are the main strengthening phases [9]. β_1' occurs at peak-aged condition, while β_2' occurs at overage condition. It is now generally accepted that β_1' phase has MgZn₂ or Mg₄Zn₇ structures, while β_2' has the MgZn₂ structure. It is interesting that MgZn₂ can form both β_1' and β_2' phases. The main differences of two precipitated phases that are composed of MgZn₂ are shapes and orientation relationships (ORs). β_1' is rod shape, also named needle shape or lath shape, while β_2' is plate shape. The orientation

relationships of two precipitated phases in alloys with different compositions are listed in Table 1.

In the theoretical side, because of the importance of strengthening effects of β_1' phase and the role of micro alloying on precipitation strengthening, the properties of MgZn_2 and Mg_4Zn_7 received extensive study. First, the basic properties such as crystal structure parameters, thermodynamic stability, elastic constants and electronic structures of MgZn_2 have been well studied [10]. Specially, many works compare the elastic properties of MgZn_2 with other similar intermediate phases, such as MgNi_2 , MgCu_2 , Mg_2Y , Mg_2Ca etc., to reveal the correlation of mechanical properties between the precipitation and matrix [11-15]. Second, to understand the effects of alloying elements on the microscopic structure, the effects of alloying elements and atomic vacancy on the stability of MgZn_2 and Mg_4Zn_7 also attract the attention [16]. Third, due to the importance of deformation properties of Mg alloys, the deformation mechanism of precipitates has been also investigated, and it was found that the deformation of MgZn_2 is performed by partial dislocations [17].

Most of studies above focus on the bulk properties of intermediate phase, but the precipitates embodied in the matrix inevitably have interfaces. In general, the phase structure, OR, density of precipitates, which are highly influenced by the precipitate interface, are predominant factors influencing the precipitation strengthening. For example, meta-stable precipitation phases, such as bcc Cu in Fe [18], $\beta''\text{-Mg}_5\text{Si}_6$ in Al [19], $\beta_1'\text{-MgZn}_2$ in Mg alloys[20, 21], that form coherent interfaces with the matrix have

the largest age-hardening responses. Finely dispersed precipitates with various habit planes usually have more effective precipitation hardening than precipitates with the same OR do. In addition, due to the importance of micro alloying, understanding the partition and segregation of alloy elements at the interface of precipitates is highly valuable [22-23], which also requests the knowledge of the interface structure. Therefore, a careful investigation of the interface between precipitates and matrix is very necessary.

Table 1 The ORs, shapes and names of precipitates which are composed of MgZn_2 in alloys with different compositions.

Ref.	ORs	Name and shape of precipitates	Alloys
OR1			
Gallot [24] (1965)	$[11\bar{2}0]_{\text{MgZn}_2} // [0001]_{\alpha}, (0001)_{\text{MgZn}_2} // (11\bar{2}0)_{\alpha}$	β_1' , rod-like	Mg-Zn
Singh [25] (2010)	$[11\bar{2}0]_{\text{MgZn}_2} // [0001]_{\alpha}, (0001)_{\text{MgZn}_2} // (11\bar{2}0)_{\alpha}$	β_2' , rod-like	Mg-Zn-Y
Kim [26] (2010)	$[11\bar{2}0]_{\text{MgZn}_2} // [0001]_{\alpha}, (0001)_{\text{MgZn}_2} // (11\bar{2}0)_{\alpha}$	β_1' , rod-like, lath	Mg-Zn-Y
Rosalie [27] (2010)	$[11\bar{2}0]_{\text{MgZn}_2} // [0001]_{\alpha}, (0001)_{\text{MgZn}_2} // (11\bar{2}0)_{\alpha}$	β_1' , rod-like	Mg-Zn-Y
Zeng [28] (2006)	$[10\bar{1}0]_{\text{MgZn}_2} // [1\bar{1}00]_{\alpha}, (0002)_{\text{MgZn}_2} // (11\bar{2}0)_{\alpha}$	β_1' , rod-like	Mg-Zn-Al-Y
Park [29] (2007)	$[01\bar{1}0]_{\text{MgZn}_2} // [01\bar{1}0]_{\alpha}, (2\bar{1}\bar{1}0)_{\text{MgZn}_2} // (0002)_{\alpha}, (0002)_{\text{MgZn}_2} // (2\bar{1}\bar{1}0)_{\alpha}$	β_1' , rod-like	Mg-Zn-Mn-Al
Zou [30] (2008)	$[10\bar{1}0]_{\text{MgZn}_2} // [1\bar{1}00]_{\alpha}, (0002)_{\text{MgZn}_2} // (11\bar{2}0)_{\alpha}, (1\bar{2}10)_{\text{MgZn}_2} // (0002)_{\alpha}$	β_1' , rod-like	Mg-Zn-Al-Y
Shi [31] (2013)	$[01\bar{1}0]_{\text{MgZn}_2} // [01\bar{1}0]_{\alpha}, (0001)_{\text{MgZn}_2} // (\bar{2}110)_{\alpha}$	β_1' , needle-shaped	Mg-Zn-Sn
OR2			
Gallot [24] (1965)	$[11\bar{2}0]_{\text{MgZn}_2} // [10\bar{1}0]_{\alpha}, (0001)_{\text{MgZn}_2} // (0001)_{\alpha}$	β_2' , plates	Mg-Zn
Komura [32] (1980)	$[11\bar{2}0]_{\text{MgZn}_2} // [10\bar{1}0]_{\alpha}, (0001)_{\text{MgZn}_2} // (0001)_{\alpha}$	β_2' , plates	Mg-Zn
Wei [21] (1995)	$[11\bar{2}0]_{\text{MgZn}_2} // [10\bar{1}0]_{\alpha}, (0001)_{\text{MgZn}_2} // (0001)_{\alpha}$	β_2' , plates	Mg-Zn-RE
Gao [9](2007)	$[11\bar{2}0]_{\text{MgZn}_2} // [10\bar{1}0]_{\alpha}, (0001)_{\text{MgZn}_2} // (0001)_{\alpha}$	β_2' , plates	Mg-Zn

To date, there are many TEM experiments revealing the interface structures between MgZn_2 and α -Mg matrix [9, 25-31]. Table 1 lists the shapes and names of precipitate with two most common ORs, i.e., $[11\bar{2}0]_{\text{MgZn}_2} // [0001]_{\alpha}, (0001)_{\text{MgZn}_2} // (11\bar{2}0)_{\alpha}$ (OR1) and $[11\bar{2}0]_{\text{MgZn}_2} // [10\bar{1}0]_{\alpha}, (0001)_{\text{MgZn}_2} // (0001)_{\alpha}$ (OR2), in alloys with different compositions. Most authors named rod-like precipitates with OR1 as β_1' , and named plate shaped with

OR2 as β_2' . The coherent interface of β_1' precipitate parallel to $(0001)_{\text{MgZn}_2} // (11\bar{2}0)_\alpha$, while the coherent interface of β_2' precipitate parallel to $(0001)_{\text{MgZn}_2} // (0001)_\alpha$. The first-principles method is a powerful tool to investigate precipitated phases and interfaces in alloys [33-34]. Since the properties of these two types of interfaces are critical to understand the β_1' and β_2' precipitates, in this paper, we use first-principles calculations performing a systematical investigation of the structural properties of coherent interfaces of β_1' and β_2' in α -Mg alloys. We first describe the calculation details in section 2. Second, we present the interfacial energies of 16 coherent interfaces complying with OR1 and OR2, and describe the structural features of each interface by using coordination. We find that the interfacial energy is tightly influenced by the average coordination number of the interface structure. Third, we present the strain energies and formation energies for the slabs with different interfaces. Using results of formation energies, we explain the relative stability of β_1' and β_2' . Finally, we discuss the atomic structures and electronic structures of two favorable interfaces.

2. Methodology

In this paper, first-principles calculations based on density functional theory (DFT), within the generalized gradient approximation (GGA) [35] as incorporated in the Vienna Ab Initio Simulation Package (VASP) [36], are employed. The ion–electron interaction is described using the projector augmented wave (PAW) method [37-38]. A kinetic energy cut-off value of 280 eV was used for plane wave expansions. The Brillouin zone (BZ) integrations for all calculations are performed by the special k-point sampling of the

Gamma centered grid type. For Mg and Zn, the valence electrons considered are $3s^2$ and $3d^{10}4s^2$, respectively. The internal atoms in supercells are allowed to relax until the maximum forces on unconstrained atoms converged to less than 0.01 eV/Å.

We first calculate the lattice constants of α -Mg and MgZn₂. For the BZs of Mg and MgZn₂, we use the k-point meshes of $(8 \times 8 \times 4)$ and $(8 \times 8 \times 6)$, respectively. For pure α -Mg, our calculated lattice constants are 3.22 Å and 10.42 Å, which are well consistent with the experimental values, 3.21 Å and 5.21 Å. For MgZn₂, our calculated lattice constants are 5.20 Å and 8.43 Å, which are also well consistent with the experimental value 5.23 Å and 8.58 Å [9].

To study the properties of the Laves-phase MgZn₂, which complies with α -Mg matrix in different ORs, we construct different supercells to descript the interfacial structures. For interfaces with OR1 (Fig. 2a), we set $\vec{a}$ parallel to $[1\bar{1}00]_\alpha$ and $[1\bar{1}00]_{\text{MgZn}_2}$, $\vec{b}$ parallel to $[0001]_\alpha$ and $[11\bar{2}0]_{\text{MgZn}_2}$, and $\vec{c}$ parallel to $[11\bar{2}0]_\alpha$ and $[0001]_{\text{MgZn}_2}$ for the supercells. For interfaces with OR2 (Fig. 2b), we set $\vec{a}$ parallel to $[21\bar{3}0]_\alpha$ and $[1000]_{\text{MgZn}_2}$, $\vec{b}$ parallel to $[1\bar{1}00]_\alpha$ and $[01\bar{1}0]_{\text{MgZn}_2}$, and $\vec{c}$ parallel to $[0001]_\alpha$ and $[0001]_{\text{MgZn}_2}$ for the hexagonal supercells. The lengths of the lattice of supercells are illustrated in Fig. 2. We use a $(8 \times 4 \times 1)$ k-point mesh for the interfacial structures that comply with OR1 and a $(8 \times 8 \times 1)$ k-point mesh for the interfacial structures that comply with OR2 respectively.

Fig. 1. The dependence of calculated in-plane lattice parameters of Mg(0001) plane on the thickness of Mg layers within supercells simulating interfaces.

Here , we use supercells with different Mg layers and the same MgZn₂ layers to test the influence of the thickness of Mg layers on the lattice constants parallel in the interfacial plane. Fig. 1 shows the dependence of calculated in-plane lattice constants with the number of Mg layers. We find that the in-plane lattice constants of the interface are strongly influenced by the number of Mg layers. With increasing Mg layers, the in-plane lattice constants approach the ideal lattice of bulk Mg (5.58 Å). We can infer that in-plane lattice constants of the interface between MgZn₂ and α -Mg matrix must be the same with that of bulk α -Mg. There are two strategies to simulate the interface: one is using a very large thickness of Mg layers to emulate the real α -Mg matrix, and it at least needs Mg layers with thickness larger than 50 layers, which needs a very huge computational expenditure. To avoid huge computational expenditure, another strategy that we adopt here is using a relatively thin matrix (4 layers of Mg) and directly set the in-plane lattice constants as the same with our calculated value of Mg bulk lattice; we implement a series of static total energy calculation to scan the energy profile along the $\vec{c}$

direction of supercells and determine the favorable interfacial structure. This method has been widely used to simulate the coherent interface of precipitates in alloys. For example, thin slabs are used to emulate the real alloy matrix in the models, which simulate the coherent interfaces of Al_2Cu in the Al-Cu alloys [39-40], γ' - Ni_3Al in γ -Ni [41], Al_2MgC_2 in Mg alloys [42] and Al_4C_3 in Mg alloys [43]. The interface properties, such as the interfacial energy and segregation behavior etc., are mainly influenced by chemical bonding at the interface, which is only determined by the nearest neighbor of the interfacial atoms, while the atoms far from the interface have no influence on the interfacial properties (We can also find that the coordination number, which reflects the number of nearest neighbor atoms, has the same variation trend as interfacial energy in section 3.3.). Therefore, a thin slab of matrix can be used to construct the structure of the coherent interface of precipitates in alloys.

The formation energy (E_f) reflects the stability of a given structure, which is given by:

$$E_f = E_{Int} - N1_{Mg}E_{Mg} - N_{MgZn2}E_{MgZn2} - N2_{Mg}E_{Mg} - N2_{Zn}E_{Zn} \quad (1)$$

where E_{Int} represents the total energy of the supercell, E_{Mg} and E_{Zn} represent the energy per atom of Mg and Zn in their equilibrium states respectively. E_{MgZn2} represents the energy of the stoichiometric part MgZn_2 in equilibrium state. $N1_{Mg}$ and N_{MgZn2} represent the number of Mg atoms in α -Mg layers and the number of atoms of stoichiometric part MgZn_2 in the supercell, respectively. $N2_{Mg}$ and $N2_{Zn}$ respectively represent number of Mg and Zn atoms in MgZn_2 except for the stoichiometric part. It is worth to mention that, in all simulations of the interfaces, we use symmetrical structures

in supercells that contain two equivalence interfaces to compute the interface formation energy. The slabs in these supercells simulating interfaces would have different thickness of MgZn₂ slabs (see Table 2).

The formation energy as defined in Eqn. (1) contains contributions from both the interfacial energy and the elastic strain energy. Therefore, the formation energy of Eqn. (1) can be also expressed as following equation:

$$E_f = \sigma + \zeta \quad (2)$$

where σ represents the interfacial energy, and ζ is the elastic strain energy.

The elastic strain energy of the interface is due to the lattice mismatch of MgZn₂ with α -Mg matrix. Since we use the thin slab of MgZn₂ in Mg layers to simulate precipitate in alloys, in which the thickness of MgZn₂ is much thinner than that of matrix, the influence of strain on the matrix is negligible. From Fig. 1, we also find that, for a thin slab of MgZn₂ model, the lattice constants of thick Mg layers almost are the same with those of its equilibrium state. Therefore, strain of the interface can be computed by only considering the strain of the MgZn₂ part. The strain energy ζ of the supercell can be calculated by following equation:

$$\zeta = E_{st} - E_{op} \quad (3)$$

where E_{st} represents the total energy of MgZn₂ slab with same distortion in the supercell simulating interface, and E_{op} is the total energy of MgZn₂ slab with same lattice as equilibrium state.

3. Results and discussion

3.1 Interface structures

Fig. 2(a) shows the $(11\bar{2}0)$ Mg plane and (0001) MgZn₂ plane (OR1 interface) and Fig. 2(b) shows the (0001) Mg plane and (0001) MgZn₂ plane (OR2 interface). Clearly, the (0001) MgZn₂ can easily form coherent interface with both $(11\bar{2}0)$ Mg and (0001) Mg planes. The mismatch of OR1 and OR2 interfaces are 19.29% and 13.16%, respectively. Due to the complicate structure of MgZn₂, the truncated surface of MgZn₂ has 8 different confirmations (see Fig. 3 and 4). Therefore, both OR1 and OR2 coherent interfaces have 8 different structures, which are denoted as I₁-I₈ in Fig. 3 and Fig. 4 respectively.

Fig. 2. The parallel planes from Mg and MgZn₂ for interfaces: (a) OR1 interface, (b) OR2 interface. The orange balls represent Mg atoms, the dark grey balls represent Zn atoms, the light grey sticks represent the Mg-Mg bonds, and the solid black lines are the boundaries of supercells. For clarity, we do not plot the bonds of Mg-Zn and Zn-Zn.

3.2 Stability of interfaces

Using Eqn. (1-3), we obtain the interfacial energies for OR1 and OR2 interfaces, which are shown in Fig. 5. The interfacial energies of OR1 interfaces are among $-4.9 \sim 34.2 \text{ meV/\AA}^2$, and the interfacial energies of OR2 interfaces are among $-2.7 \sim 49.7 \text{ meV/\AA}^2$. The OR1-I₄, OR1-I₆ and OR2-I₁ interfaces are three most stable energy

interfaces, whose interfacial energies are -2.4 , -4.9 and -2.7 $\text{meV}/\text{\AA}^2$, respectively. The OR2-I₃ and OR2-I₇ are two least stable interfaces, whose interface energies are 49.8 and 46.8 $\text{meV}/\text{\AA}^2$, respectively. Though the mismatch of OR2 is much little than that of OR1 interfaces, the interfacial energy of OR2 type is not always lower than that of OR1 type. The most stable OR1-I₆ interface is even lower than that of OR2-I₁. Therefore, the interface stability between MgZn_2 and $\alpha\text{-Mg}$ must relate to the bonding formation at the interface, due to the diversity of the structures truncated surface MgZn_2 .

Fig. 3. The atomic structure of coherent interface $(0001)_{\text{MgZn}_2} // (11\bar{2}0)_{\alpha}$ complying with OR1. The blue dashed lines marked by (I₁-I₈) are the truncated planes of MgZn_2 which appear in the slab used to construct interfaces in right and bottom panels.

Fig. 4. The atomic structure of coherent interface $(0001)_{\text{MgZn}_2}/(0001)_a$ complying with OR2. The blue dashed lines marked by (I₁-I₈) are the truncated planes of MgZn₂ which appear in the slab used to construct interfaces in right and bottom panels.

Fig. 5. Interfacial energies of interfaces that comply with OR1 and OR2.

3.3 Interface coordination

To reveal the bonding numbers in the interfacial structures, we compute the average coordination for all atoms in these interfaces, N_C^{all} . Also, we compute the average coordination for all Mg and Zn atoms in these interfaces, N_C^{Mg} and N_C^{Zn} , respectively. The coordination number N_C^A is defined as follow:

$$N_C^A = \frac{1}{N_0^A} \sum_{i=1}^{N_0^A} \sum_{j=1(i \neq j)}^N \lim_{k \rightarrow \infty} \frac{1}{1 + \exp[-2k(d_{i,j} - d_0)]} \quad (4)$$

N_0^A is the number of A (A=Mg, Zn, Mg+Zn) atoms in the supercell, N is the total number of atoms in the supercell, i and j are the atom labels, $d_{i,j}$ is the distance between the i atom and j atom, d_0 is a constant equal to 4.0 (Å). The N_C^A actually reflects the average number of nearest neighbor atoms around A atoms in the structure, therefore it can reflect the bonding number around A atom in the structure. We list the average coordination numbers, Zn concentration in MgZn₂ part, and the thickness of MgZn₂ slab in the interfaces in Table 2, and these data are useful to describe the feature of interfaces.

Table 2 The average coordination of Mg and Zn of the interface: N_C^{all} is the average coordination of all atoms in the cell, N_C^{Mg} is the average coordination of all Mg atoms in the

cell, N_C^{Zn} is the average coordination of all Zn atoms in the cell. C represents the Zn concentration in the $MgZn_2$ part, and L represents the thickness of $MgZn_2$ slab in the interfacial structure.

	N_C^{all}	N_C^{Mg}	N_C^{Zn}	C (at. %)	L (Å)
Mg	12.00	12.00	0	0	-
$MgZn_2$	13.33	16.00	12.00	66.67	-
OR1 interface					
I ₁	13.13	14.34	12.20	69.23	25.76
I ₂	12.83	13.38	12.31	63.64	22.19
I ₃	13.00	13.97	12.17	67.74	21.30
I ₄	12.87	13.50	12.26	65.52	19.18
I ₅	13.00	14.16	12.03	70.37	18.24
I ₆	13.21	13.72	12.58	61.90	13.85
I ₇	12.74	13.57	11.85	68.42	13.68
I ₈	12.24	12.54	11.86	64.70	11.76
OR2 interface					
I ₁	13.10	14.33	12.00	69.23	27.39
I ₂	12.89	13.67	12.00	63.64	23.48
I ₃	12.37	13.27	11.43	67.74	22.85
I ₄	13.07	14.00	12.00	65.52	20.53
I ₅	12.62	13.50	11.68	70.37	18.97
I ₆	12.73	13.20	12.00	61.90	15.26
I ₇	12.00	12.67	11.08	68.42	14.63
I ₈	12.97	13.56	12.00	64.71	12.21

The average coordination of all atoms in Mg and $MgZn_2$ are 12.00 and 13.33, respectively. Naturally, the average coordination of all atoms (N_C^{all}) in the structures for OR1 and OR2 interfaces are between 12.00 and 13.33. It is found that the concentration (C) of Zn in $MgZn_2$ part which used to construct the interface is different from that of stoichiometric $MgZn_2$. We notice that the thicknesses of slab of $MgZn_2$ part (L) for constructing different interfaces also have a relatively large difference (The thickness of Mg layers in all interfaces is the same, so the thickness of $MgZn_2$ can reflect the concentration of $MgZn_2$ in supercells). However, the average coordination for the interface is not a monotonic function of the concentration of Zn in $MgZn_2$ or thickness of

the MgZn₂ slab. Therefore, we can infer that the average coordination of all atoms in these structures should be influenced by the complicated structure of truncated surface, which introduces many atoms with different coordination numbers.

Because both components, Mg and MgZn₂, of the interface include Mg atoms, the average coordination of Mg atoms (N_C^{Mg}) of the interface depends on both the thickness of MgZn₂ and the structure of the truncated surface. In contrast, Zn atoms only exist in MgZn₂, therefore the average coordination of Zn atoms (N_C^{Zn}) just depends on the structure of the truncated surface. We stress the role of N_C^{Zn} to reveal the interfacial structure feature is more important than that of N_C^{Mg} . Interestingly, we find that the number of average coordination of Zn atoms of most OR1 interfaces (except I₇ and I₈) is even larger than that of MgZn₂. For OR2 interface, all the average coordination number of Zn atoms is not larger than that of MgZn₂. The average coordination of Zn atoms can actually reflect the atom density at the interface. For example, the atom density at OR1-I₆ interface, whose N_C^{Zn} is 12.58, is much greater than that at OR1-I₇ and OR1-I₈, whose N_C^{Zn} respectively are 11.85 and 11.86. (See Fig. 3)

3.4 Coordination vs. interfacial energy:

The average coordination can reflect the average number of bonding in structures. The larger the average coordination number of the structure is, the more bonds the structure has, and the lower total energy the structure is. Here N_C^{all} can mainly reflect the bonding around both Mg and Zn, which is influenced by the interface, and N_C^{Zn} exactly reflect the bonding around Zn at the interface. For example, the two most stable OR1-I₆

and OR2-I₁ interfaces have the two largest values of N_C^{all} , 13.21 and 13.10, while the two least stable OR1-I₈ and OR2-I₇ interfaces have the two smallest values of N_C^{all} , 12.24 and 12.00. Also, the most two stable OR1-I₆ and OR2-I₁ interfaces have the largest two values of N_C^{Zn} , 12.58 and 12.00, while the least stable OR2-I₃ and OR2-I₇ interfaces have the smallest two values of N_C^{Zn} , 11.43 and 11.08. However, though N_C^{Mg} can reflect the bonding number around Mg atoms, it is greatly influenced by the proportions of Mg and MgZn₂, causing N_C^{Mg} cannot directly reflect change of bonding number influenced by the interface. Therefore, N_C^{Mg} cannot reflect the change of interfacial energy. For example, the coordination value N_C^{Mg} of the most stable OR1-I₆ interface is 13.72, which is smaller than that of OR1-I₁ that is less stable, 14.34.

3.5 Strain energy and formation energy

Using Eqn. (3) we can obtain the strain energies of the slabs used to simulate interfaces. From Fig. 6, we find that the strain energy has a rough linear relation with atomic number in the slab of MgZn₂. The average strain energy per atom of OR1 and OR2 are 0.10 and 0.046 eV/atom, respectively. The reason why the strain energy of OR1 interface is larger than that of OR2 interface is the mismatch of OR1 interface is larger than that of OR2 interface.

The formation energy, which is the sum of the interfacial energy and strain energy, determines the stability of the precipitates. Fig. 7 presents the average formation energy per atom of MgZn₂ slabs with OR1 and OR2 interfaces. The formation energies of slabs with OR1 interfaces are among 89.41-157.49 eV/atom, while the formation energies of

slabs with OR2 interfaces are among 40.61-109.83 eV/atom. The most stable slab with the OR1 interface is the slab with interface OR1-I₆, while the most stable slab with OR2 is the slab with OR2-I₁. It is found that most of the MgZn₂ slabs with OR2 interfaces are much more stable than those slabs with OR1 interfaces when the slab thickness is from 11 to 28 Å. However, the formation energies of the slab with OR2-I₃ and OR2-I₇ are 87.73 meV/atom and 109.83 meV/atom, respectively, which are comparable to and even exceed that of the slab with OR1-I₆, 89.41 meV/atom, indicating the slab with OR2-I₃ and OR2-I₇ are unfavorable compared to other slabs with OR2 having thickness among 12~28 Å. This energetics can well explain that the β_2' plate shape precipitates that are frequently observed have OR2 relationship instead of OR1 [9, 21-22, 30]. It also indicates that β_2' is more stable than β_1' , which is consistent with experiments.

Fig. 6. Strain energies of interfaces that comply with OR1 and OR2.

Fig. 7. Formation energies of MgZn₂ slabs with interfaces that comply with OR1 and OR2.

3.6 The structures of OR1-I₆ and OR2-I₁ interfaces

Fig. 8. The structures of interfaces OR1-I₆ (a, b and c) and OR2-I₁ (d, e, and f): (a, d) show the atomic configuration of cross-section parallel to basal plane of MgZn₂ for unrelaxed interfacial structure, (b, e) show the atomic configuration of cross-section parallel to basal plane of MgZn₂ for relaxed interfacial structure, and (c, f) show the atomic configuration of cross-section parallel to prism plane of MgZn₂ for relaxed interfacial structure.

Fig. 8 presents the atomic structures of the most stable interfaces for OR1 and OR2, namely OR1-I₆ and OR2-I₁, respectively. Fig. 8(a, b) show the unrelaxed atomic position and optimized atomic position of cross-section along the basal plane of MgZn₂ for OR1-I₆ interface, and we find there is obvious reconstruction in this interface. In contrast, from Fig. 8(d, e), which show the unrelaxed atomic position and optimized atomic position in the cross-section along the basal plane of MgZn₂ for the OR2-I₁ interface, we find there is

almost no reconstruction. The atomic configurations in cross-sections along prism planes of MgZn_2 of unrelaxed and relaxed planes are also presented in Fig. 8(c, f), which also indicate that OR1-I₆ has obvious reconstruction but OR2-I₁ almost has no reconstruction.

Table 3 The coordination numbers (N_C^{all}) of interfacial atoms that marked in Fig. 8.

OR1-I ₆							
Mg1	Mg2	Mg3	Mg4	Mg5	Mg6	Mg7	Mg8
14	15	15	16	13	13	13	12
Zn1	Zn2	Zn3	Zn4	Zn5	Zn6	Zn7	Zn8
11	13	13	15	13	12	12	13
OR2-I ₁				MgZn ₂			
Mg1	Mg2	Mg3	Zn1	Zn2	Zn3	Mg	Zn
12	16	12	12	12	12	16	12

Table 3 lists the coordination numbers of interfacial atoms, which are marked in Fig. 8, of OR1-I₆ and OR2-I₁ interfaces. We describe the features of OR1-I₆ and OR2-I₁ respectively as following:

OR1-I₆: Compared to atoms in the corresponding single phase (α -Mg or MgZn_2), the coordination number of interfacial Mg atoms at α -Mg side all becomes larger, while that of interfacial Mg atoms at MgZn_2 side all becomes smaller. The total number of coordination of all these interfacial Mg atoms decreases by 1. Most of the coordination of interfacial Zn atoms becomes larger compared to that of the MgZn_2 phase. The total number of coordination of all these interfacial Zn atoms increases by 5. The total number of coordination of all atoms of this interface will increase by 4. Now, it is clear that the increase of total coordination number of this interface is because of the contribution of Zn atoms. Therefore, due to the atoms in this interface providing more chemical bonds

compared to the MgZn_2 phase, interfacial Zn atoms involve in the enhancement of interface cohesion, which also leads to a very low interfacial energy.

OR2-I₁: The coordination numbers of all interfacial atoms, except Mg2, are the same with the corresponding single phase. Since only the coordination number of Mg2 increase by 4 compared to single phase α -Mg, the total coordination number of this interface also increases by 4 compared to the corresponding single phase. Therefore, the increase of coordination of this interface is due to the contribution of the Mg2 atom, and the Mg2 atom is a strong joint point for OR2-I₁.

Fig. 9(a, b) show the local densities of states (LDOS) of the interfacial atoms. Here, we sum up the LDOS of all Mg atoms and Zn atoms at the interfaces, which are marked in Fig. 8. It is shown that the LDOS of Mg is delocalized because the valence electronic states of Mg atoms are all s and p states, while the LDOS of Zn is more localized because the valence electronic states of Zn atoms are mainly d states. The localized states usually are the main contributors to chemical bonds. Here, it is found that the peak of the d states of Zn atoms at the interface is closer to the Fermi level compared to that of Zn atoms in the equilibrium state of MgZn_2 . In addition, the states of Mg atoms are also closer to the Fermi level compared to that of Mg atoms in the equilibrium state of MgZn_2 . The shallower electronic bands of interfaces, especially for d-band of Zn atoms, compared to that of the equilibrium state, indicate the higher bonding energies and weaker bonding strength, which is induced by the stretch of the bond length. For example, the Zn-Zn bond lengths around Zn6 in OR1-I₆ are 2.65, 2.69, 2.72, 2.76, 2.80, 2.98 and 3.12 Å, which are

obviously larger than the Zn-Zn bond lengths (2.60, 2.61, 2.68, 2.68, 2.83 and 2.83 Å) in the equilibrium state of MgZn_2 .

Fig. 9. The local density of states (LDOS) of Mg and Zn atoms within interface and MgZn_2 with stretch in basal plane: (a, b) illustrate the LDOS of Mg and Zn atoms for the interface respectively, and (c, d) illustrate the LDOS of Mg and Zn atoms for MgZn_2 with stretch in basal plane respectively. The number (in eV) in each panel is the difference of energies between electronic bands.

As mentioned in the previous section, the in-plane stretch of OR1-I₆, 19.29%, is larger than that of OR2-I₁, 13.16%. To verify the reason of the formation of relatively high energy electronic bands of atoms at the interface, we present LDOS of the MgZn_2 phase with the same extent of stretch along the basal plane as the interface, which is shown in Fig. 9(c, d). We find that the peak of the d states of Zn atoms and band bottom of Mg atoms in the stretched phase are also closer to the Fermi level than the peak of the d

states of Zn atoms and band bottom of Mg atoms in the equilibrium state of MgZn_2 , respectively.

For a detail survey, we estimate the band centers of d-bands of Zn, which are marked in Fig. 9(b, d). We find that the d-band centers of bands of Zn for two interfaces are both higher than that of Zn in equilibrium phase. The center of d-band of Zn in interface OR1-I₆ that has relatively large in-plane stretch is higher than that of Zn in interface OR2-I₁ by 0.12 eV. Similarly, the d-band centers of bands of Zn for stretched phases are both higher than that of Zn in equilibrium phases. The center of d-band of the stretched phase that has relatively large stretch is higher than that of Zn in the stretched phase that has relatively small stretch by 0.08 eV, which approach the value of energy difference between d-band centers of Zn atoms in OR1-I₆ and OR2-I₁. Therefore, we can confirm the change of bands of Zn atoms of the interface is induced by the structure distortion and bonding stretch.

It is also found that the band bottom of Mg atoms at OR2-I₁ is higher than that of Mg atoms in equilibrium state by 0.77 eV, and band bottom of Mg atoms at OR1-I₆ is higher than that of Mg atoms in OR2-I₁ by only 0.21 eV. Similarly, the band bottom of Mg atoms in the 13.16% stretched phase is higher than that of Mg atoms in the equilibrium state by 0.87 eV, and band bottom of Mg atoms in the 19.29% stretched phase is higher than that of Mg atoms in the 13.16% stretched phase by only 0.09 eV. The similar changes of electronic states of Mg atoms in interfaces and stretched phases also reflect that the

change of LDOS energy is mainly contributed by the structure distortion and bonding stretch.

From the LDOS of interfaces, we can infer that the bonding energy within the interface is higher than that within equilibrium MgZn_2 , which has an effect on increasing the interfacial energy. However, from the coordination of OR1-I₆ and OR2-I₁, the bonding number for interfacial atoms increases, which has an effect on decreasing the interfacial energy. The effect of bonding number is much larger than the change of bonding energy induced by the distortion, and therefore the interfacial energies of these two interfaces are negative.

4. Summary

Using first-principles calculations to perform a systematic study on the coherent interfaces belong to β_1' and β_2' precipitates, $(0001)_{\text{MgZn}_2} // (11\bar{2}0)_\alpha$ and $(0001)_{\text{MgZn}_2} // (0001)_\alpha$. The conclusions can be summarized as following:

1. The most stable interface of $\beta_1'/\alpha\text{-Mg}$ is OR1-I₆, while the most stable interface of $\beta_2'/\alpha\text{-Mg}$ is OR2-I₁. The interfacial energy of OR1-I₆ is lower than that of OR2-I₁, because the OR1-I₆ structure has more average coordination than that of OR2-I₁.
2. The interfacial energy is highly influenced by the coordination of the interface structure, but is less influenced by the energy of bonding between atoms at the interface compared to coordination.

3. The strain energies of the slabs with interfaces that comply with given OR have a linear relationship with the atom numbers of MgZn_2 in slabs, and the values of the strain energy per atom in the slabs with given OR are almost constant.
4. Most of the MgZn_2 slabs with OR2 interfaces are more stable than that with OR1 interfaces, which explains that the β_2' plate shape precipitates that are frequently observed have OR2 relationship instead of OR1. It also indicates that β_2' is more stable than β_1' , which is consistent with experiments.
5. Electronic analysis shows that the energy of bonding between atoms in the OR1-I₆ interface is higher than that in OR2-I₁, and the reason is that OR1-I₆ has larger bond stretch than OR2-I₁ does.

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