Modeling Decision and Game Theory Based Pedestrian Velocity Vector Decisions with Interacting Individuals

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Abstract

This paper explores the incorporation of decision and game theory in the velocity vector decisions found in human movement. These decisions are made by individuals when maneuvering in an environment occupied by other people or obstacles. The model developed in this paper examines how human movement can be captured as a result of a decision-making process rather than modeling the behavior directly. In this paper, a novel utility function is formed from the biological-based preference of minimizing energy consumption. This energy function is partially fashioned from the energies due to moving, colliding, and waiting. Time and risk preferences are incorporated into the utility function to uniquely capture human desires. Decision and game theories are used to model the decision making process of individuals when alone and when interacting with a range of evacuee types (panicked, distracted and alert). Beliefs are formed and updated concerning both the environment and individuals in the environment, to truly capture the knowledge of individual evacuees. Assumptions that were made during the creation of this novel decision-focused movement model are discussed throughout the paper. This work expands upon previous research performed using the obstacle avoidance technique known as the velocity obstacle method. Common human traits and tendencies are proven to exist in the model through multiple examples. Future modifications of this alternative approach to traditional behavior-focused movement models are discussed.

Keywords: Evacuation, Game theory, Decision Analysis, Movement Decision, Velocity Selection

1. Introduction

In the United States in 2011, deaths from residential and non-residential building fires totalled 2,530, injuries totalled 15,000, and the total dollar loss was more than 9 billion dollars (2012). Due to the need to better understand the causes of these large losses, models have been created to predict the evacuation of humans in different types of environments (Santos and Aguirre, 2004). Through this understanding, it is possible for the designers to optimize the environment to maximize occupant safety as it relates to the evacuation process.

The evacuation process of an individual can be broken down into two stages - an exit or goal selection process (Mesmer and Bloebaum, 2014), which was addressed in previous work, and a movement or velocity vector selection process, addressed in this paper. This paper focuses on the determination of which velocity direction and magnitude an evacuee ought to make, and assumes that the goal that the evacuee is trying to reach is predetermined. While closely related, this paper and the past research (Mesmer and Bloebaum, 2014) offer necessary mathematical techniques to simulate two separate decision processes that are inherent to evacuations. It is the culmination of the individual decision making processes that forms the evacuation as a whole, which in turn results in deaths, injuries, and damage. Therefore, by modelling the individual decisions of the evacuees, an evacuation model can be formed that can be used to explore the safety of the environment's design. While the examples of this paper explore a constricted space, the method can be directly related to larger environments, since the same fundamental decision making processes are present (selection of an exit and selection of a velocity vector).

The model used for the movement of individuals is a vital component of an evacuation simulation. Methods to model movement range from modelling the evacuees as particles in a

flow problem to cells in a cellular automata problem, as reviewed in (Santos and Aguirre, 2004). Proposed methods have been both nodal based, such as Exodus (Owen et al., 1996) and SIMULEX (Thompson and Marchant, 1995), and non-nodal based such as Vacate (Mesmer and Bloebaum, 2012; Ries, 2006; Xue, 2009b). One method that has been used to describe motion planning of objects in dynamic environments is the velocity-obstacle (VO) method (Fiorini and Shiller, 1998).

The VO method determines which velocity vectors will result in a collision with obstacles of known velocity. VO and similar approaches have been used in many applications, such as robot motion planning (Fiorini and Shiller, 1993) and missile guidance (Chakravarthy and Ghose, 1998), as well as path finding (Chakravarthy and Ghose, 1998; Fiorini and Shiller, 1993; Kluge and Prassler, 2006). The basis of the VO method, the collision cone, is shown in Figure 1. In Figure 1 an agent (individual making a velocity decision), P_i , and an obstacle (other individual that the agent can see), P_j , are depicted. For the agent, any relative velocity that is chosen that falls within the collision cone will result in the agent with radius r_i and the obstacle with radius r_j colliding at some point in time. While non-linear movements can be incorporated into VO (Shiller et al., 2001), for this research combinations of linear movements are assumed. Previous research (Kluge and Prassler, 2006) explains the traditional velocity obstacle method in more detail.

Figure 1: Collision Cone Representation

The traditional VO method assumes the obstacles' characteristics are known by the agent. Previous research on the VO method has modified the method to allow for uncertainty in the obstacle's size and velocity, producing a probability of collision which is a function of an obstacle's characteristics (Kluge and Prassler, 2004). Kluge and Prassler (Kluge and Prassler, 2004) introduced individual evacuee utilities as a function of the individual's velocity, velocity reachability, and probability of collision. This utility function can be combined with optimization techniques to determine a 'best velocity', however there is not a clear human basis for this type of utility function. This paper forms a biological-based utility function applicable to a non-nodal environment that simulates human traits of discounting and risk preferences.

Interactions between individuals is at the core of movement decisions. Previous research created a methodology of recursive reflection in an effort to capture the interactions between intelligent objects (Kluge and Prassler, 2004) where objects would anticipate other objects'

velocity decisions. However, the recursive reflection methodology limits the number of recursions that occur which then limits the ability of the objects. This paper uses the mathematical method of game theory (Von Neumann and Morgenstern, 1944) to develop a novel model for human interactions in movement vector determination incorporated in the VO method and demonstrated through multiple examples.

The incorporation of game theory principles in evacuation simulators has previously been examined for evacuee movement in a limited context. Previous studies (Dogbé, 2010; Hoogendoorn and H.L. Bovy, 2003) explored the creation of a utility function and the use of differential games to approximate the flow of crowds. Another study (Tanimoto et al., 2010) has discussed the inherent properties of crowd maneuvering and their relationship to games. The incorporation of game theory principles, with a limited strategy set of three, in a cellular automata model has also been examined (Zheng and Cheng, 2011a). The human behaviors of cooperation and competitiveness in the context of evacuees has been examined using game theory (Zheng and Cheng, 2011b). (Lo et al., 2006) have examined the selection of an optimum exit alternative through a game theory model, where the players are the crowd and a capacity restricting entity.

This paper differentiates itself from previous game theory research by creating a biological-based utility function for velocity determination, formed from a meaningful measure of desire. This paper also uses traditional normative game theory to model the populations' interactions where the players are represented as individuals. Individual preferences and beliefs concerning other individuals are also accounted for in this work. Different types of individuals (such as panicked, distracted, and psychologically alert) are represented and beliefs are formed accordingly. Beliefs are represented through probability distributions, allowing for the use of Bayes theorem to integrate new information correctly. Unlike past research, individuals can have limitless strategy options partially due to the decision-making model being incorporated into a non-nodal based evacuation environment. Hence, the work in this paper is novel because it incorporates a meaningful utility function into the VO method, which captures human interactions through the mathematically rigorous means of utility and game theory, while accounting for individual preferences and beliefs. The utility function and game interactions created in this paper are not perfect representations of human choice modeling and appropriate assumptions are made to simplify the complex problem into a programmable and manageable model (addressed throughout the paper). However, the model developed in this paper offers an initial step based on rigorous mathematics in decision-focused modeling to determine human movements. The decision-focused movement model developed in this paper will serve as a foundation for future work.

The research in this paper is organized in such a way that each section builds upon the last to develop a decision-based movement model. In Section 2 the development of the energy-based value function is discussed. Section 3 describes the use of probability theory to mathematically define and manipulate the beliefs of individuals during a movement decision. Section 4 combines the value function and beliefs to enable the use of the utility theory concept of expected utility in movement decision modeling. Section 5 describes the relationship between the different individuals in the environment. Section 6 discusses the use of the decision theory, employing the knowledge of the previous sections, to determine velocity vectors with non-interacting opponents. Section 7 explores how to model interacting opponents through the use of game theory. Section 8 discusses case studies where the above described approach has been used. Section 9 concludes the paper and discusses future work.

2. Velocity alternative energy consumption

In this section, a meaningful cost is created for each velocity in the velocity space (the two-dimensional space in which the agent is choosing a vector) in order to use the VO method in a real world model of velocity vector decision making. Methods in the VO literature (Chakravarthy and Ghose, 1998; Fiorini and Shiller, 1993; Kluge and Prassler, 2006; Large et al., 2002; Shiller et al., 2001) have rudimentary cost functions consisting of simple constraints on velocities and an objective of getting to the goal as quickly as possible. This formulation typically results in a velocity magnitude at or near the constraint imposed on the magnitude. A more realistic cost would not require constraints on the velocities. Furthermore, some of the past research incorporates a “risk” parameter (a weighting variable on a portion of the function) in order to induce human behavior. This paper takes a more structured examination of risk through the use of utility theory. Each velocity is represented by an energy cost in order to create a useful function. The measurement of energy is used in this paper to represent a more holistic measure of human behavior than that of time, capturing many different physical actions and behaviors. Energy offers a novel approach to capturing movement velocity through a biological-based measurement.

The energy cost of each velocity is multifaceted, consisting of the cost of making a step using the velocity, the cost of reaching the goal after the initial step, the cost of a collision with an obstacle due to the velocity, and the cost of colliding with a wall. The cost of the initial steps and the cost of reaching the goal after the step are referred to as the non-collision cost. In this paper, the initial steps are a single velocity vector that the individual commits to for a certain number of steps. These steps take a certain amount of time to complete, which enables the calculation of distance travelled. Future work could include multiple initial steps, each with its own velocity vector. While enabling multiple initial steps to have individual velocity vectors would provide greater simulation accuracy by enabling more complex routes to take in a crowd, it would also increase the computation time greatly.

The environment in which the velocity vector decisions are made is assumed to be a building or an open environment, possibly with stationary obstacles. Regardless if the evacuation is in a building or not, individuals are basing their decisions off of similar concerns such as heading towards a goal and avoiding obstacles. While this paper explores primarily restricted space environments in the examples, the principle decision making processes are present in any environment type and the value function can be adjusted appropriately for other scenarios. Each individual in the environment has a primary objective of passing through a goal (typically a doorway). If a velocity vector forces the person to pass through another person or obstacle then the individual is stopped at first contact with the object. If a collision occurs, then the individual must make a new velocity decision immediately (instead of after a predetermined number of steps). The length of a step is defined by a specific length of time required to make the step. In the next sections, the costs associated with steps and collisions are developed.

2.1. Cost of the initial steps

The cost of the initial steps is the amount of energy consumed from moving in a certain velocity vector in a particular environment for a certain amount of time. This certain velocity vector (referred to as the initial steps velocity) is the velocity that an individual sustains for a certain number of pre-determined steps starting at his current location. The energy consumed in the movement is a combination of the movement energy and the hazard energy.

The energy consumed due to movement is determined using metabolic rate equations (Van der Walt and Wyndham, 1973) that are functions of a person's weight and velocity. It is assumed in this paper that the evacuee will run only when the metabolic equations deem running as less energy consuming than walking for a given velocity (factoring in other energy costs which will be discussed later in the paper). The most efficient velocity that results from the metabolic rate equations is 1.124 m/s. This is the velocity that consumes the least amount of energy and that individuals inherently adopt if their metabolic rate equation matches that derived in (Van der Walt and Wyndham, 1973). Since humans have different most efficient velocities, diversity in the simulation is created by randomly choosing a most efficient velocity for each individual based on their age and sex using statistical data (Knoblauch et al., 1996). Equation (1) is used as the normalized input velocity to the metabolic rate equations to force the rate equations to produce a most efficient velocity equivalent to the randomly chosen most efficient velocity.

$$v_n = v * 1.124 / v_{eff} \quad (1)$$

where, v is the individual's velocity, subscript n is the normalized input, and subscript eff is the most efficient. This modification allows for individuals to have unique metabolic rate equations dependent on their physical traits. An individual's shoulder width, height, and weight are also determined from statistical data (McDowell et al., 2008; Ogden et al., 2004; Stoudt, 1970).

A constraint is created for a person's maximum velocity due to limitations of the human body. Every individual has a different maximum velocity that is assumed to be related to their most efficient velocity. The top 1% of the fastest most efficient velocities of the population is used to develop an equation to calculate the maximum velocity for an individual in the population. The assumed maximum velocity for this top 1% is 10 m/s, which is used with a triangular distribution approximation of a normal distribution (Scherer et al., 2003) to create a maximum velocity equation dependent on the individual's most efficient velocity, seen in Eq. (2),

$$v_{max} = v_{eff} * 10 / 2.16917 = v_{eff} * 4.61 \quad (2)$$

where, subscript max is the maximum. It is seen that using this equation only those individuals with a top 1% most efficient velocity of 2.16917 m/s will be able to reach a maximum speed of 10 m/s.

There are also hazards that inflict harm on the individuals as they move in the environment. The hazard rate of the environment causes a believed loss in energy that occurs over time, due to such environment factors as fire, smoke, or toxic gas. This is the harm that the individual thinks will occur in an environment. This factor is formed to capture human decision-making concerning dangerous paths and areas. The cost of the initial steps, incorporating the movement energy and the hazard energy consumed, is calculated using Eq. (3),

$$E_{step} = \begin{cases} \text{If } |V_{step}| = 0 \text{ then} \\ \quad M_{wt}(0) * t_s * CS + H * t_s * CS \\ \text{If } |V_{step}| \neq 0 \text{ and steps through goal line then} \\ \quad M_d(|V_{step}|) * d_{Gint} + H * d_{Gint} / |V_{step}| \\ \text{If } |V_{step}| \neq 0 \text{ and does not step through goal line then} \\ \quad M_d(|V_{step}|) * |V_{step}| * t_s * CS + H * t_s * CS \end{cases} \quad (3)$$

where, E is the energy consumed, V is the velocity vector, M_{wt} is the metabolic rate for walking function, M_d is the metabolic rate for distance function, t is time, CS is the number of steps an individual commits at a velocity, H is the hazard rate, d_{Gint} is distance from initial position to where the velocity vector crosses the goal, subscript $step$ is initial steps, and subscript s is single step.

The variables of step time and the number of committed steps are functions of environment conditions and each individual's mental and physical traits. The time it takes for an individual to complete a single step (step time) is a function of their speed and such physical traits as leg length. The number of committed steps is a function of mental traits such as anxiety and anticipation, and environment conditions such as the density of others in the space. The computation of these variables in such depth is out of the scope of this paper. It is recognized that as higher fidelity models are formed for the individuals and further research on movement mechanics in different evacuation scenarios is performed these variables will be calculated. For the examples used later in this paper a value of one is assumed for the step time as well as the number of committed steps of the individuals.

2.2. Cost after the initial steps

The cost after the initial steps is the amount of energy consumed by an individual moving in a certain velocity vector from their position after the initial steps to the closest position of the goal. This energy is a combination of the movement energy, the hazard energy, and the energy spent waiting in a queue for the goal. The position of the individual after the initial steps is determined by the direction, duration, and magnitude of the initial steps velocity. An individual first moves using their chosen initial step velocity vector for a committed number of steps and then moves towards the goal, in the most direct path, using the magnitude of their chosen after initial steps velocity. Two velocity vectors are used (initial and after initial) in order to enable more complex paths for the individuals to form. More velocity vectors in the initial steps would improve model fidelity but decrease computational efficiency. For this foundational work one initial step velocity vector is used. At the time of the decision the individual determines both an initial step velocity and an after initial step velocity, in essence forming a path. After the committed number of steps the individual re-evaluates his path, deciding on a new initial step and a new after initial step velocity. A more detailed discussion on the use of the energy costs in determining velocity vectors is performed in section 6.

In the energy cost after the initial steps there are no committed number of steps. At the time of the decision it is assumed that the after initial steps velocity will be maintained from the position after the initial step until the goal is reached. The goal may be reached on the initial step or it may take many steps after to be reached. The distance, d_{as} , is the distance of an individual to a goal after their initial steps. The waiting time for an individual is determined using the arrival times of each individual choosing a specific goal and the believed flow rate of that specific goal. The arrival time for each individual is dependent on their movement velocities. The flow rate of a goal is a belief held by an individual on how many people can pass through the goal per a time increment. If the arrival times are significantly close between individuals then those individuals' wait times are averaged. Further discussion concerning wait time determination and flow rate beliefs is provided in (Mesmer and Bloebaum, 2014). The waiting time is not present in the initial step energy cost because it is captured in the after initial step energy cost. If an individual reaches the goal on their initial committed steps then no after initial

steps are needed, however there may be a wait at the goal and that wait time and the associated energy cost is captured in the after initial steps energy cost. The cost after the initial steps to the goal is calculated using Eq. (4),

$$E_{as} = \begin{cases} \text{If initial steps passed through goal then} \\ H * t_w + M_{wt}(0) * t_w \\ \text{If initial steps did not pass through goal then} \\ M_d(|V_{as}|) * d_{as} + H * (t_w + d_{as} / |V_{as}|) + M_{wt}(0) * t_w \end{cases} \quad (4)$$

where, d is distance, subscript as is after initial steps, subscript w is waiting, V_{as} is the after initial steps velocity and E_{as} is the after initial steps energy cost.

2.3. Collision cost

Collisions must be quantified in order to incorporate their effects in velocity vector decision making. To relate collisions to the energy cost method, the collision measurements must be in terms of energy. The value of the measurement is the energy lost in colliding. There is a lack of research in calculating the energy cost of collisions between humans. The assumed cost of a collision is a combination of three acts: the damage done by falling or hitting an individual, the energy spent in stabilization, and the energy spent in standing up if knocked down. This energy varies by person and depends on weight, age, and fitness. The energy also depends on the characteristics of the object that is being collided with. For this paper, an exact equation calculating the cost of collisions is left for future work and an equation based on potential energy with a modification multiplier is created. The collision cost is calculated from Eq. (5),

$$E_{Gencol} = m * g * h * 0.00239 * 3 \quad (5)$$

where, m is mass, g is acceleration of gravity, h is height, the subscript $Gencol$ is general collision, and the input variables are in SI units

Using Eq. (5), all collisions will cost the same for a specific individual. In a realistic scenario, a person is less worried about a collision that will occur 10 minutes from a certain time than a collision that will occur 10 seconds from the same time. This is due to the human nature of time preferences. With the current collision cost method, the individual will view these different timed collisions as having the same value. In this paper, a time adjustment method is used in order to account for the time preferences of individuals. This method is most commonly used in economics to determine the value of money in the future (Hazelrigg, 2012) (through the use of a discount equation) and has not previously been used for collision measurements. In the economics formulation, a future dollar's value is converted into a present dollar's value for comparisons of money from various time frames. For example, a dollar two years from now is not worth as much to an individual as a dollar they have today. The discount rate (a parameter needed in the calculation) is person specific. In this paper, to represent the diversity in a population, the discount rate is randomly chosen for each individual using Eq. (6),

$$R = .1 + .9 * p \quad (6)$$

$$p \in [0,1]$$

where, R is the discount rate and p is a random number. The cost of a collision to the individual, adjusted to his present time, is represented in Eq. (7),

$$E_{\text{coll}} = E_{\text{Gencol}} * (1 + R)^{-t_{\text{coll}}} \quad (7)$$

where, subscript coll is collision with individuals.

2.4. Collision with wall

An energy cost is associated with movements through walls to keep such impossible movements from occurring. This energy cost is related to the initial steps velocity, as it is assumed that walls are not present directly between the individual and the goal after the initial steps. The cost of colliding with a wall is high in order to make such a movement relatively impossible. In this work, the energy cost of colliding with a wall is zero nutritional calories if there is no collision and 100 nutritional calories if the initial steps velocity passes through a wall.

2.5. Velocity Vector Alternative Energy Cost

The velocity vector alternative energy cost is then determined by combining the energy consumed from taking the initial steps, making it to the goal line after the initial steps, colliding with walls, and colliding with other individuals. These four costs are a function of the individual's traits, beliefs, and velocity. The velocity vector alternative energy cost equation is shown in Eq. (8),

$$E_V = E_{\text{step}} + E_{\text{as}} + E_{\text{coll}} + E_{\text{wall}} \quad (8)$$

where, the subscript wall is collision with walls.

3. Beliefs

The individual's beliefs on obstacles in the environment are important variables in the calculation of the energy cost for a velocity vector of an individual. Beliefs on obstacles (including other individuals) can range from beliefs on their mood, to beliefs on the traction of their footwear, to beliefs on their degree of hunger. In this paper it is assumed that individuals have limited belief capacity and, for movement in crowds, the most important belief to have is how the objects will physically move. Hence, the individual's beliefs are limited to beliefs on the velocity vectors of the obstacles in the environment. For this paper, three types of obstacles are examined: random moving obstacles; obstacles moving without interactions; and obstacles moving with interactions. It is recognized that more obstacle states can exist. These three states are chosen for examination as they relate to three distinct game forms, as well as three distinct evacuee types, that can be simulated using the decision-based approach to evacuation simulation presented in this paper. When discussing "the agent" in this paper it is in reference to the current individual decision maker. Hence, many agents exist in the environment, each with separate beliefs. The next sections describe the different beliefs and updating that have been developed for this work.

3.1. Randomly moving obstacle beliefs

In this paper, a randomly moving obstacle type is used to portray a panicked person. Panicked people move about the environment without a specific goal. In this situation, the

individual making a velocity vector decision (the agent) has beliefs about the velocity vector that the panicked person (the obstacle) will next perform. This velocity is the same for the opponent's initial steps velocity and after steps velocity. While previous research incorporated beliefs into the velocity obstacle method (Kluge and Prassler, 2004), they were for the general velocity space of the obstacle and were assumed to be constant. The beliefs discussed in this paper are in the form of two discretized probability distributions – one concerning the magnitude of the obstacle's velocity vector and another concerning the direction of an obstacle's velocity vector. The probability of a specific vector occurring is the product of the vector's magnitude probability and angular probability.

3.2. Obstacles moving without interacting beliefs

Obstacles that move without interacting are viewed as distracted individuals. While these individuals move toward a goal, they do not actively avoid others in the environment. Their distraction may originate from the use of a personal communication device, from speaking with their head turned to another individual, or from any other task that keeps them from maneuvering around others. Each individual has a singular belief concerning the distracted opponent's velocity and goal. The velocity belief is the same for the opponent's initial steps velocity and the after steps velocity. The velocity and goal beliefs are formed from either a previous goal decision making process or from observations of the opponent's past velocities.

3.3. Beliefs about interacting, moving obstacles

Individuals who interact with evacuees while moving are viewed as being alert. These individuals determine their movements partly based on panicked and distracted individuals, as well as on other alert individuals (with whom they are playing a game). They know that the velocity they choose will affect the other alert individuals' decisions. Each individual has a belief on what goal the alert opponent prefers, based on the results of a previous goal decision making process or from the opponent's past movement. The velocity magnitude that the decision making individual (the agent) believes the opponent will take after the initial steps is found in the same manner as the preferred goal. The velocity magnitude belief held by the agent is used to determine the velocity vector of the opponent to the closest point on the opponent's preferred goal line, after the opponent's initial step. The agent's determination of the initial steps velocities for these opponents is found during the game interaction process that will be discussed later in the paper. As mentioned previously, this paper focuses on an initial movement followed by a movement towards a goal (for agents and opponents) examined at each movement decision point (typically after each step). The agent assumes the opponent's most efficient velocity percentile is the same as his own. The agent makes appropriate adjustments on the believed opponent's most efficient velocity due to the opponent's age and sex.

3.4. Belief initialization

The belief the agent has about an opponent is initialized when the agent first makes visual contact with the opponent. If the agent loses visual contact with the opponent and then regains contact later, then the agent's beliefs are reinitialized. For panicked opponents, the agent initially believes that all directions and magnitude are equally possible (up to a believed maximum velocity for the opponent). Hence, a uniform distribution is generated for all of the directions and possible magnitudes for the panicked opponents. Distracted or alert opponents are assumed to be stationary obstacles until the agent's singular beliefs are updated using either a

previous goal decision game or the agent has witnessed the opponent move for a certain period of time.

3.5. New information and updating

New information that the agent obtains concerning alert and distracted opponents are singular beliefs that are assumed to be completely accurate. Therefore, for these opponents the new information overwrites the previous singular beliefs. For example, the agent may have previously believed that the opponent had a velocity magnitude of 2.1 m/s. On the next observation, if the agent observes the opponent traveling at 1.8 m/s then his belief is now 1.8 m/s. In higher fidelity decision-based models the beliefs on alert and distracted opponents may be modelled as probability distributions.

For panicked opponents, as the agent witnesses the opponents move about the environment information is gathered on what movement the opponents have taken. This data can be obtained at any point and at any interval during a simulation. Probability density functions (pdfs) are created to describe this new information. The new information pdfs are defined as the probability density functions representing the likely velocity with which the obstacle actually moved. If the agent knew the obstacle's movement information exactly, then each pdf would be 0 everywhere except at the true velocity, which would have a probability of 1. In reality, this information is commonly not known with exact precision. Uncertainty arises due to the agent's inability to measure the opponent's speed and direction accurately using only his visual perception. Measurement inaccuracy of obstacle velocities must be taken into account when creating an information pdf.

The form that the new information pdfs take is a combination of a triangular and uniform distribution. The triangular distribution represents the believed measurement with the associated measurement inaccuracy. The uniform distribution represents the general uncertainty. These distributions are combined and normalized to form a single distribution. The agent's beliefs of the obstacles' movements are updated using the new information through Bayes theorem (Bayes and Price, 1763). For example, the agent's beliefs on a panicked opponent's velocity is initialized as a uniform distribution. The new information, represented as a hybrid uniform and triangular distribution are used to update the initialized beliefs, using Bayes Theorem, to form a posterior belief. On the next observation this posterior belief becomes the prior belief and updated using the newly observed information. In this manner the probabilistic beliefs of the agent are updated in a meaningful, rigorous mathematical manner. The agent's beliefs capture some of the uncertainty that impacts the decision-making process. An approach to decision-making that reflects the agent's preferences concerning uncertainty will be needed to perform a thorough analysis.

3.6. Belief Assumptions and Future Work

The beliefs that are examined in detailed and updated in this paper focus on the movement of opponents. It is recognized that the agent can have many beliefs on the opponent that will impact the movement decisions being made. The opponent's age, weight, height, anxiety level, relationship to other opponents, and risk preferences are some examples of the opponent traits and characteristics that the agent can have beliefs on. In this paper, as the focus is on the development and use of a biological-based method for movement decisions incorporating decision analysis methods, limitations are placed on the complexity of the agent's beliefs. For example, an agent will not know for certain a stranger's age. In order to properly

capture this belief a probability distribution using an appropriate range and discretization is necessary. In this paper the agent's belief on the opponent's age is singular and correct. Singular beliefs are also held by the agent on the type that an opponent is (panicked, distracted, or alert). It is assumed in this paper that the agent can correctly identify opponent types, and in future work probabilistic uncertainty on opponent types will be explored. Singular beliefs on such opponent traits as risk preference and follow rate are also assumed in this work. Environmental beliefs concerning such variables as flow rate and door width are discussed in (Mesmer and Bloebaum, 2014). Probabilistic beliefs on all of these opponent traits, types, and characteristics can be formed and incorporated directly in the movement decision methodology formed in this paper.

4. Utility Functions and Risk

Utility theory (Bernoulli, 1954; Von Neumann and Morgenstern, 1944) is used in order to incorporate the agent's risk preference into the decision of the initial steps velocity vector. Utility theory is a mathematical method for defining a person's preferences numerically. By determining the expected utility of each velocity vector, the decision that the agent ought to make concerning the movement vectors can be ascertained. For a risk neutral individual, the utility function provides a linear relationship between utility and measurement (energy consumed). A hyperbolic absolute risk aversion (HARA) utility function (Alpanda and Woglom, 2007) is used for risk averse or risk proverse individuals in this paper. This paper does not focus on forming the perfect utility function to capture human risk preferences, but instead uses the HARA utility function to demonstrate how decisions under uncertainty are possible to simulate in movement models using utility theory. The evacuees' coefficient of absolute risk aversion in the HARA function is randomly selected from a general population risk distribution. The individuals' measurement preference is to prefer consuming less energy over consuming more energy. The following sections present the utility functions developed for certain and uncertain obstacles.

4.1. Utility with Certain Obstacles

The agent is playing in a certain environment when the agent is certain of the velocities that the obstacles, or opponents, will make in the next time step, or when there are no obstacles or opponents present. The utility for each of the agent's velocities in a certain environment is then determined using Eq. (9),

$$u_{v,i,i} = u_i(-E_{v,i,i}) \quad (9)$$

where, u is the utility function and subscript i denotes an individual in a population.

4.2. Utility with Uncertain Obstacles

The agent is playing in an uncertain environment when the agent is uncertain of the velocities that the obstacles (opponents) will make in the next time step. This can occur if the agent believes an opponent is panicked. The movement of an agent in an environment involving an opponent in which the agent has a non-singular belief about the opponent's velocities has been studied previously (Fulgenzi et al., 2007; Kluge and Prassler, 2006). The utility functions used in past research were basic functions involving the probability of collision or the avoidance of any probability of collision. The research discussed in this paper expands on the notion of a

probability belief space by incorporating a higher fidelity expected utility calculation of the agent's initial steps velocity vector. The expected utility takes into account risk preference and a biologically-inspired energy cost function.

The set of all possible "environments" (in this case the set of all possible opponent velocities) must be determined in order to calculate an expected utility. Each member of the set is a different combination of believed panicked opponents' velocities (the uncertain variables). Each member has an associated probability of occurrence that is simply the product of the probability of each of the opponents' velocity vectors. The expected utility of each of the agent's velocity vectors is calculated using Eq. (10),

$$eu_{v,i,i} = \sum \text{prob}(\Omega_{\omega}) * u_i(-Ev_{i,i,\omega}) \quad (10)$$

where, Ω is the set of believed combinations of opponents' velocities and ω is a member of the set. This equation explores each possible set of panicked opponents' velocity vector combinations. The expected utilities are calculated for each of the agent's velocity vectors, and the vector with the highest expected utility is the best choice for the agent.

5. Opponent Determination

There will likely be many people in an agent's vicinity. For multiple opponent situations, it is assumed that the closest individual to the agent is the agent's primary focus. Singular velocity vector beliefs are assumed for the other (non-primary focus) individuals. For the singular velocity vector opponents, the believed velocities are determined from a previous goal decision making process, the opponent's last movement vector, or from the most likely values (magnitude and direction) of the agent's velocity beliefs about the opponent. The assumed velocity is the same for the opponent's initial steps and the after steps. In a more general definition, the singular velocity vector opponents (opponents who are not the closest to the agent) can be viewed temporarily as distracted individuals. The non-primary opponents still impact the decision making process of the agent by acting as singular velocity vector obstacles.

While this paper uses examples concerning a single primary focus opponent, multiple primary focus opponents can be examined. The larger the number of opponents, the greater the computational power required. In this paper, the use of a single primary focus opponent does not cause the other opponents to be unrepresented in the agent's decision making process, as they are simply assumed to be moving with a specific velocity magnitude. Collisions with these non-primary focus opponents, as well as other energy impacts such as waiting time influences, are still represented in the energy cost model of the agent. Hence, the methods discussed in this paper for a single primary focus opponent are scalable to multiple primary focus opponents. A single primary focus opponent is used in this paper to focus on representing the fundamental theories and formulations underlying a decision-focused model of movements.

6. Decision Theory Methods

This paper highlights four situations that an individual may encounter when determining a velocity vector. Three of these situations can be solved using decision theory, while the remaining situation is solved using game theory. The three decision theory situations occur when no other individuals are visible to the agent, when the primary focus individual (the closest opponent) is moving towards their goal but not interacting with the agent (distracted opponent), and when the primary focus individual is randomly moving (panicked opponent). The

methodology of determining the velocities of the agent in a decision theory scenario is first discussed.

6.1. After Initial Steps Velocity Determination

In order to calculate the energy consumed for a specific after steps velocity magnitude, an initial steps velocity must be defined. The energy that a specific after steps velocity consumes is found using Eq. (4) and a defined initial steps velocity. The after steps velocity is independent of uncertain obstacles, as the equation does not incorporate collisions and the wait times are only a function of distracted and alert opponents. The agent wants to maximize his overall expected utility related to the preference of minimizing the energy consumed. Since the utility function of the agent is monotonic, a lower after initial steps energy consumption is always preferred to a higher energy consumption, regardless of the values of the energies in the rest of the utility function. The notion of risk is not a factor since the after initial steps velocity measurement is independent of probabilities. This justifies minimization of the after initial steps energy when given a specific initial steps velocity.

The minimization of the energy consumed after the initial steps (given the initial steps velocity) is determined using the particle swarm optimization (PSO) method (Kennedy and Eberhart, 1995). In the PSO method, the particle's positions, velocities, and values must be defined, where the positions of the particles are the after initial steps velocity magnitudes, the velocities of the particles are the change in after initial steps velocity magnitudes, and the value of the particle is the negative of the after initial steps energy consumed, which is the negative of Eq. (4). The maximization of this problem formulation results in the minimum energy consumed after a specific initial steps velocity.

6.2. Initial Steps Velocity Determination

A maximization of the agent's expected utility must be performed in order to determine the best initial steps velocity for an agent. The utility is a function of the negative of the energy consumed using a specific initial steps velocity vector. With the initial steps velocity vector specified, the minimum after steps energy consumption is determined and used in the total energy consumed formulation. The PSO method is used, where the particles' positions are a velocity vector (i.e. initial steps velocity vector), the velocities of the particles are the changes in the velocity vector, and the values of the particles are the expected utilities, found using Eq. (10). The optimal value of the initial steps velocity PSO routine results in the best expected utility value with the corresponding best initial steps velocity for the agent.

6.3. No Opponents in Vicinity

The first decision theory situation examined is when there are no individuals in the agent's vicinity and therefore no possibility of collision. Hence, collisions are not considered in the energy consumption calculation. Additionally, there is also no waiting time at the goal line. The energy consumption of each of the agent's initial steps velocities is then a function of three sub-functions: energy consumed during initial steps, energy consumed after initial steps, and energy due to colliding with a wall. The agent will simply choose the initial steps velocity vector that has the greatest utility (as the environment is certain).

The scenario seen in Fig. 2 is a visualization of the decision space of the agent. In the figure, the width of the room is 10 feet, the door (represented by the dotted blue line) has a width of 4 feet, and the distance from the door to the agent is 8 feet. The individual in this example

Figure 2: No opponent scenario

(represented by the X) is a 40 year old male weighing 190 lbs with a most efficient velocity of 5.12 ft/s and a belief that the hazard rate of the section is zero nutritional calories per second. The individual is risk neutral with a step time of 1 second and will commit to 1 initial step. The energy consumed during the initial steps can be calculated given these parameters and the initial steps velocity.

The after initial steps energy is calculated using Eq. (4). The closer the initial steps bring the agent to the goal, the less energy is expended after the initial steps to get to the goal. There is no energy consumed after the initial steps to get to the goal if the initial steps take the agent through the goal. The wall collision measurement is 100 nutritional calories if the initial steps

Figure 3: No opponent utility space

velocity vector passes through a wall and 0 nutritional calories if it does not. The utility space can be calculated using Eq. (9) with these energy measurements. The utility space is the design space in which the agent will choose the maximum value, associated with an initial steps velocity vector. The utility space for the scenario is seen in Fig. 3 and Fig. 4, with dark red representing a high utility value. A positive V_y value indicates that the individual moves upwards in Fig. 2 and a positive V_x value indicates that the individual moves to the right. The resulting optimum utility is -0.134 utils, which correlates to an initial steps velocity vector of $[0, 5.16]$ ($[V_x, V_y]$). This velocity magnitude is approximately equivalent to the agent's most efficient velocity, which is expected in a non-hazardous environment without competition. The velocity direction minimizes distance to travel to the door (all movement along the shortest distance vector) which is also expected.

Figure 4: No opponent utility space side view

6.4. Primary Focus Opponent is Distracted

The second decision theory situation that is examined is when the agent's primary focus opponent is distracted. In this situation, there is a possibility of collision associated with the agent's initial steps velocity, which must be accounted for in the utility calculation. The energy consumption of each of the agent's velocities is then a function of four sub-functions: energy consumed during initial steps, energy consumed after initial steps, energy due to colliding with walls, and the energy due to colliding with the distracted opponent. The agent simply chooses the velocity vector that has the greatest utility (since the beliefs concerning a distracted opponent are singular).

The scenario seen in Fig. 5 was created to visualize the decision space of the agent. The width of the room is 10 feet, the doors (labeled door 0 and door 1) are both 2 feet wide, the distance from each of the individuals to the nearest door is 3 feet, and the distance between the

Figure 5 Distracted opponent scenario

individuals is 9 feet. The individuals in this example have the same traits as defined for the individual in the no opponent scenario. The individuals have a collision discount rate of 1, with a collision time in seconds. Individual X (the agent) has a goal of door 0, while individual A has a goal of door 1. These preferred goals are known by the other individual. In the simulation, the goal beliefs are determined by the agent's observing the opponents' behaviors and/or through the goal decision-making process. The goal beliefs are pre-determined and common knowledge for the purpose of this example. These goal preferences cause the individuals to maneuver around each other to reach their goals. Furthermore, the agent believes that individual A is distracted and will travel at a velocity vector of $[0, -5.11]$ (ft/s) to reach door 1.

The topic of interest in this scenario is the probability of collision and the associated collision energy for specific initial steps velocities for the agent. The probability that an agent's initial steps velocity vector collides with the opponent is either 0 or 1 since the opponent's velocity vector is known. The non-zero probability region is triangular, with an apex where the agent's velocity vector relative to the opponent's is zero. A collision cone is formed with a 100% probability of a collision occurring when the initial steps velocity vector is within the cone. This collision cone is expected from the VO method for a constant velocity obstacle.

The energy consumed colliding with the opponent (as a function of the agent's initial steps velocity vector) is non-zero where the probability of collision is non-zero. Due to the effects of the time discount on collisions, the value of each collision measurement is not identical. Collisions that happen in a relatively long time (such as the initial steps velocity $[0, -5]$ which is close to the zero relative velocity) result in lower collision costs. Collisions that happen in a relatively short time (such as the initial steps velocity $[0, 10]$) result in higher collision costs.

The utility space of the example is shown in Fig. 6, calculated using Eq. (9). As seen in the figure, the agent wishes to avoid a guaranteed collision with the opponent. The optimum utility of the utility space is -0.205 utils. This optimal utility is found at the initial steps velocity vectors of $[1.33, 4.829]$ and $[-1.33, 4.829]$. The agent chooses a maneuver that moves him to the side of the opponent, so when they pass each other they just miss making contact. This causes the least amount of energy to be expended after the initial steps, as the agent is maintaining his

movement in the most direct direction to his goal without colliding. The velocity vector chosen in the simulation is determined randomly between the two optima.

Figure 6: Distracted opponent utility space

6.5. Primary Focus Opponent is Panicked

The third decision theory situation examined in this paper is when the agent's primary focus opponent is panicked, leading to uncertainty concerning the opponent's velocity. The agent holds beliefs about the opponent's velocity, which are used to determine the probability of a collision occurring given a combination of the agent's initial step velocity and the opponent's velocity. The collision probabilities and the collision measurement for the specific velocity combinations are used in Eq. (10) to determine the expected utility space of the agent. The energy consumption of each of the agent's initial steps velocities is then a function of four sub-functions: energy consumed during initial steps, energy consumed after initial steps, energy due to colliding with walls, and the energy due to colliding with the panicked opponent. The agent chooses the initial steps velocity vector that has the greatest expected utility.

The scenario seen in Fig. 5 is used to visualize the panicked opponent example. The individuals have the same traits as defined in the distracted opponent scenario. The belief that the agent has on individual A's velocity magnitude is a triangular distribution with a most likely value of 5 ft/s and an error of ± 2.5 ft/s. The belief of the agent on individual A's velocity direction is a triangular distribution with a most likely value of -1.57 rads with an error of ± 0.785 rads (where zero rads indicates an upward direction in Fig. 5). A probability of collision space is computed using these beliefs and the individuals' locations. The probability space is represented in Fig. 7. The probability that an initial steps velocity results in a collision is the summation of the probabilities of the relative velocity vectors that would cause a collision. The collision measurement space is represented in Fig. 8. The collision measurement space is

expected to be very similar to the probability space, given the close proximity of the individuals and the fact that the agent's general collision cost is 1 nutritional calorie.

Figure 7: Panicked opponent probability space

Figure 8: Panicked opponent collision space

The expected utility space (calculated using Eq. (10)) for a risk neutral agent is shown in Fig 9. The agent prefers to avoid a collision with the opponent, preferring paths that maneuver

to the right or left of center. The expected utility space with a non-singular belief on the opponent is much smoother than that for an agent with a singular belief on the opponent due to the range of probabilities of collision (as compared to a space with 0% or 100% probability of collision). The optimum of the expected utility space is -0.29 utils with corresponding initial steps velocity vectors of [3.66, 1.63] and [-3.66, 1.63]. The risk neutral agent is willing to take a small amount of risk in this decision making scenario. The probability of collision with the opponent at the optimum utility value is approximately 5%. The agent weighs this probability of collision with the benefit of getting closer to the goal, deeming the risk acceptable.

Figure 9: Panicked opponent utility space

The expected utility space for a risk loving (proverse) agent, with a HARA risk coefficient of 4, is seen in Fig. 10. The optimum expected utility for this risk preference of the agent is 0.091 utils, with associated initial steps velocity vectors of [2.76, 3.6] and [-2.76, 3.6]. The probability of collision of these velocity vectors is 12%. The risk loving player feels that the value of reaching the goal faster is worth the higher risk of colliding with the opponent. While it makes no sense to compare optimal util values, given that the two utility functions are different, a comparison of initial steps velocity vectors resulting from the different risk preferences can be made. It is seen that the greater risk preference results in a first steps velocity vector that aims more directly at the goal. These scenarios have shown that by using the mathematical method of decision theory it is possible to capture the risk preferences of individuals in a simulation. The decision process becomes further complicated when the opponents are perceived as alert.

7. Game Theory Method

The situation when the agent and the primary focus opponent (an alert individual) interact is solved using game theory (Von Neumann and Morgenstern, 1944). Game theory is a method to mathematically represent interactions between game players. The agent and the alert opponent make decisions that may affect the decision of the other. In this case, the expected utility of an individual's initial steps velocity vector may be affected by the decision of the other individual. These interactions are examined using game theory, with the players, players' actions, and players' preferences defined in the next section.

Figure 10: Panicked opponent risk loving utility space

7.1. Players, Strategies, and Preferences

The players in an initial steps velocity vector game are the agent and any alert individuals that the agent deems to be primary foci. For this paper, a limit has been set to two players - the agent and a single primary focus opponent – as this study is intended to demonstrate the feasibility of using game theory to model decisions associated with interactions. Game Theory itself is not limited to games of two players and, with proper formulation, can be expanded to games of many players. For these higher accuracy games involving many players a trade-off occurs with computational demands that must be considered when determining the desired fidelity of the simulation. The discussion in this paper on two player games would be similar to a discussion of a many player games, leading to a straightforward expansion of the two player concept derived here for many player decision-focused movement models. The determination of alternatives and outcomes would also be similar with the primary difference being in the methods used to solve for the game's Nash Equilibria.

In this paper, the agent has specific singular beliefs on the opponent's goal, the velocity after the initial steps, and the most efficient velocity (described in the previous section). The agent believes that each opponent holds the same beliefs as the agent holds about any other opponents. The agent also believes that the opponents know the agent's goal and after initial steps velocity, both of which are determined separately in a goal decision process, discussed in (Mesmer and Bloebaum, 2014). In this game theory-based approach, the agent's after initial steps velocity is equivalent to the singular velocity determined in the goal decision process. The agent also believes that the opponents know the agent's most efficient velocity, which could be obtained by close observation. In the game, the players prefer to maximize their expected utility received.

Table 1: Example master and individual sets

Example Master Set		Example Individual Set	
Angle (radian) Positive is clockwise	Magnitude (ft/s)	Direction [x,y]	Magnitude (ft/s)
0	1	[0.707,0.707]	5
0	0.5	[0.707,0.707]	2.5
0	1.5	[0.707,0.707]	7.5
0.785	1	[1,0]	5
0.785	0.5	[1,0]	2.5
0.785	1.5	[1,0]	7.5
-0.785	1	[0,1]	5
-0.785	0.5	[0,1]	2.5
-0.785	1.5	[0,1]	7.5
0	0	[0.707, 0.707]	0

The strategies of the players are the possible initial steps velocity vectors that they may choose. The strategy sets for the players are derived from a master strategy set that is known by all players. This common knowledge of the master strategy set (populated with combinations of angles and magnitudes) results in each player knowing every other player's strategy set. The angles found in the master set relate to an angular addition to the individual's nominal direction. The individual's nominal direction is the direction of the vector pointing to the closest point on the individual's goal line (from their initial position). The magnitude in the master set is a multiplier of the magnitude of the individual's nominal movement magnitude (equivalent to the magnitude of their after initial steps velocity). For example, say an individual wishes to travel to a goal that is at a direction of 45 degrees, relative to the x axis, with an after initial steps velocity magnitude of 5 ft/s. This information combined with a master set is used to form an individual's set (such as seen in Table 1). Combining the direction and magnitude pair in an individual's set gives a specific initial steps velocity vector, such as [3.535, 3.535], for the first individual set member in the example of Table 1.

7.2. Context of the Game

The velocity vector game examined in this paper is from the agent's perspective, in that it is the game he believes he is playing. At the conclusion of the game, a pure or mixed strategy Nash equilibrium (Nash, 1950) is determined. The strategies produced using the equilibrium are the strategies that the agent believes will be made – not necessarily the strategies that the opponents will make. In the event of multiple equilibriums, an equilibrium is chosen from the set at random.

In this game, the agent assumes he knows for certain the opponent's goals, most efficient velocity, type, and preferred velocity magnitude. Furthermore, it is assumed that the opponent's know the agent's traits and beliefs. A game form using Bayesian game theory (Harsanyi, 1967) is needed if the individuals hold non-singular beliefs about these opponent characteristics. In future work, Bayesian game theory will be explored as a way to improve model fidelity by allowing incomplete information on the opponents. Bayesian games have previously been shown to be useful in goal decision games (Mesmer and Bloebaum, 2014). Normative game

theory is used in this paper as a first step in exploring the usability of game theory in decision-focused movement models. The game examined in this paper assumes that the agent believes that the opponents' risk preferences and follow rates are neutral. The opponents are also believed to have the same time step and step commitments as the agent. All of these beliefs, together with the agent's traits, are known by all players.

The games explored in this methodology are from the perspective of the agents. In this manner multiple games, with different outcomes and strategies, may occur simultaneously between a set of alert individuals. Each game is valid from the viewpoint of the game's agent, and each game may lead to a different Nash equilibrium because of the different beliefs and preferences of the agents. In a single simulation time step each individual is examined, sequentially, to determine if a decision is to be made, however, the outcomes of any decisions are not seen until the next simulation time step. Because of this, while the decisions are currently examined sequentially they can also be parallelized, improving computational efficiency. The assumption that certain information is common knowledge is made to improve computational efficiency. This paper looks to demonstrate the usefulness of the mathematically rigorous techniques of decision analysis in evacuation simulations, demonstrated in this section using the normal form of game theory. In future work these beliefs can be pulled out of the common knowledge, driving an emphasis on incomplete information games that use Bayesian Game Theory to solve.

7.3. Example

A simple example is created using the scenario seen in Fig. 5 to visualize the game theory-based approach. In this example, both of the individuals are 40 year old males, weighing 190 lbs, with most efficient velocities of 5.12 ft/s, and believe that the hazard rate of the section is zero. The individuals are risk neutral with step times of 1 second and commitments of 1 initial step. Individual X wishes to pass through door 0 and individual A wishes to pass through door 1. These traits and beliefs are known by both individuals. It is assumed that the preferred velocity magnitudes (determined in the exit decision) of the individuals are 5.12 ft/s (that happens to coincide with the most efficient velocity for the individuals in this example). For this simplified example, the player strategy sets are defined in Table 2. The master strategy set has been simplified in order to display all of the strategy combinations in the paper. In a typical application, the master strategy set would be much larger, including angles that cause the individual to go backwards from their intended direction as well as velocities that are faster than their preferred speed.

Table 2: Player strategy sets

Strategy Number	Individual X (V_x, V_y) (ft/s, ft/s)	Individual A (V_x, V_y) (ft/s, ft/s)
1	(0, 5.12)	(0, -5.12)
2	(0, 2.56)	(0, -2.56)
3	(-1.32, 4.94)	(1.32, -4.94)
4	(-0.66, 2.47)	(0.66, -2.47)
5	(1.32, 4.94)	(-1.32, -4.94)
6	(0.66, 2.47)	(-0.66, -2.47)
7	(0, 0)	(0, 0)

Table 3: Game example

Ind X, Ind A	S1	S2	S3	S4	S5	S6	S7
S1	-1.1177, -1.1177	-1.1139, -1.1247	-1.1175, -1.1221	-1.1138, -1.1262	-1.1175, -1.1221	-1.1138, -1.1262	-1.1065, -1.1495
S2	-1.1247, -1.1139	-1.1173, -1.1173	-0.2119, -0.2058	-1.1169, -1.1186	-0.2119, -0.2058	-1.1169, -1.1186	-1.0954, -1.1276
S3	-1.1218, -1.1175	-0.2055, -0.2119	-0.2055, -0.2058	-0.2055, -0.2136	-1.1216, -1.1219	-1.1178, -1.1259	-0.2055, -0.2441
S4	-1.1261, -1.1138	-1.1185, -1.1169	-0.2135, -0.2058	-0.2135, -0.2136	-1.1258, -1.1180	-1.1181, -1.1182	-0.2135, -0.2441
S5	-1.1218, -1.1175	-0.2055, -0.2119	-1.1216, -1.1219	-1.1178, -1.1259	-0.2055, -0.2058	-0.2055, -0.2136	-0.2055, -0.2441
S6	-1.1261, -1.1138	-1.1185, -1.1169	-1.1258, -1.1180	-1.1181, -1.1182	-0.2135, -0.2058	-0.2135, -0.2136	-0.2135, -0.2441
S7	-1.1495, -1.1065	-1.1276, -1.0954	-0.2441, -0.2058	-0.2441, -0.2136	-0.2441, -0.2058	-0.2441, -0.2136	-0.2441, -0.2441

The resulting normal form of the game played in this scenario is represented in Table 3, where the strategies for player X are in the first column and the strategies for player A are in the first row. The outcome cells, where the individual player strategies intersect, show the expected utilities for the players given that strategy combination (with player X's expected utilities followed by player A's). The green cells represent player X's best response to each of player A's strategies. The blue cells represent player A's best response to each of player X's strategies. The red cells represent best responses for both players. Hence, the red cells are pure strategy Nash equilibria (some of the possible solutions for the game). According to these pure strategy Nash equilibria, if one player goes straight, then the other player should also go straight. If one player veers to a side, then the other player should veer to the opposite side. These are solutions that are to be expected, given the limited player strategy sets. Mixed strategy Nash equilibria are also determined and represent alternative solutions for the game.

Further examples of the initial steps velocity vector decision method were performed on larger environments, with multiple individuals having randomly assigned traits (e.g. height, age, sex, and risk preference). These simulations were run in order to assure that randomized traits and psychological states would not result in non-realistic behaviors. The simulations showed realistic evacuee movements and confirmed proper results of the method's usage. The master strategy sets included a wide variety of angles spanning the full directional spectrum, and magnitudes that were both below and above the nominal. The individuals showed avoidance of collisions, risky decisions, and competition to reach goals first in order to reduce energy spent waiting. These are all traits that would be typically seen in an evacuation.

8. Case Studies

The examples described previously demonstrate the use of the developed velocity vector decision method to model evacuee movements. This section describes multiple case studies explored using the Vacate-GT emergency egress simulator. This simulator is formed by combining the exit decision model from previous research (Mesmer and Bloebaum, 2014) with

the movement decision model described in this paper. In Vacate-GT there are two distinct portions of the simulation loop, the decision portion and the movement portion. The decision portion is when a specific individual is updating his beliefs and making decisions. The decision flow is depicted in Fig. 11. The individual first examines his environment and determines what people are around him. He also updates his beliefs on the environment with information that he can observe, such as distances between egress alternatives, the flow rate through an alternative and so on. The egress alternative decision making process is then performed, resulting in an egress goal for the individual, a preferred velocity and the individual's believed goals and velocities for the other individuals (Mesmer and Bloebaum, 2014). The individual then updates his beliefs on the other individuals with information he has inferred from the egress alternative decision process. The velocity vector decision making process is then performed resulting in the individual's movement velocity vector. Finally, the time of the next decision is determined as the duration of a single step multiplied by the number of steps the individual is committed to.

Figure 11: Agent Decision Making Flow Chart

The other distinct portion of the simulation loop is the movement flow. The movement flow is how the position of a specific individual changes and the determination of whether new decisions must be made. The movement flow is depicted in Fig. 12. In the flow the agent first increments his position. This position change is performed using the movement velocity vector determined in the velocity vector decision process and the simulation's time interval since the last position increment. If a collision has occurred with an opponent or obstacle the positional change is reverted back. The decision making process is conducted on the next simulation interval if a collision occurs, an egress goal is crossed or the agent has completed his committed number of steps.

A case study that primarily uses movement decisions is explained in the next subsection. The case studies are not used to validate the simulator, as more data and case studies would be necessary, but instead are used to demonstrate the feasibility of decision-focused models in simulators. The simulator is also not compared to other simulations. Vacate-GT takes a fundamentally different approach to modeling evacuations. The decision making processes are

Figure 12: Agent Movement Flow Chart

modeled rather than the behaviors directly. Such an approach allows the designer to investigate why evacuees are behaving in certain ways and investigate such design variables as information (Mesmer and Bloebaum, 2015), value, and risk preferences that will impact the decision making processes of the individuals. The case studies explored in this section are to demonstrate that with the basic foundation laid out in this paper meaningful simulation results can be obtained for a variety of real-world experiments.

8.1. Stapelfeldt Trials

The Stapelfeldt Trials (Stapelfeldt, 1986) is a common case study used to validate the overall evacuation process of multiple evacuation simulators (Gwynne et al., 2005; Xue, 2009b). The trials placed 100 cadets (assumed to be males in their 20s) in a gymnasium with a single exit. Multiple trials were conducted with a variation on the exit door width. Each of the trials was performed once. The results of two of the Stapelfeldt Trials are shown in Table 4.

8.2. Simulation Setup

Table 4: Stapelfeldt Trials results

Exit Width (ft)	Experiment Exit Time (sec)	Average (3 runs) Simulation Exit Time (sec)	Standard Deviation of Simulation Exit Times (sec)
2.62	50	51.3	3.78
4.92	30	34.9	0.85

Vacate-GT is used to examine this case study by simulating two of the evacuation trials performed. The trials simulated are the 2.62 feet and 4.92 feet exit width trials. The room dimensions for the trials are 10 feet in width and 25 feet in length (Gwynne et al., 2005; Xue, 2009a). The exit door is at the center of the shortest wall. The simulation is performed three times for each of the trial situations. Before each of the simulation runs the evacuee traits are randomized, producing a different population for each run. The individuals' traits are restricted to males with ages between 20 and 30 years.

The evacuation begins immediately at the start of the simulation. The master strategy set is derived from all of the possible combinations of the angles and magnitudes seen in Table 5. The environment and initial positions of the evacuees are seen in Fig. 13 (that has a door width of 2.62 feet). The additional room after the inhabited room is required to represent the effects of individuals just past the exit impacting the movement decisions on the individuals about to exit. Since there is no known urgency given to the evacuees in the experimental trials, the individuals' hazard rates in the simulated trials are randomly chosen amongst relatively low values that do not greatly impact the evacuees' decisions.

Figure 13: Stapelfeldt Trials environment

Table 5: Stapelfeldt Trials master velocity set

Angles (degrees off of velocity vector determined in exit decision making)	Magnitude (multiple of velocity vector determined in exit decision making)
15	0
45	0.5
90	1
0	1.5
	0.216

8.3. Stapelfeldt Trials Results

The results of the Stapelfeldt simulation are shown in Table 4. The average evacuation time of the 2.62 feet exit simulations is 51.3 seconds, which is 2.6% difference from the experimental time. The average evacuation time of the 4.92 feet exit simulation is 34.9 seconds, which is 16.3% difference from the experimental time. The difference is due to many variables, such as population characteristics and traits, a low number of experimental data points, the amount of urgency addressed to the participants, and assumptions in the methodology. This simulation shows that an egress simulator that uses evacuee decisions related to energy consumption to determine behavior is a feasible approach. Furthermore, since the trials incorporate multiple evacuees in a room, the game theory-based approach is seen to be feasible for more than one opponent.

8.4. Further Case Studies

The Stapelfeldt Trials offer a case study that focuses heavily on the movement decision process of the evacuees as only one exit is available for the evacuees to choose. Vacate-GT is also examined using two other case studies, a corridor evacuation with multiple exits (Heliövaara et al., 2012) (focusing on exit decisions) and a Boeing 737-200 evacuation (Muir and Cobbett, 1996). While these case studies incorporate movement decisions (as individuals are traversing an environment), they also involve exit decisions which are not the focus of this paper. The corridor evacuation in particular is gone into further detail in (Mesmer and Bloebaum, 2014). Nonetheless, the additional case studies have been used to study Vacate-GT (which includes the movement decision process examined in this paper) as a whole and the results of the experiments and simulations are seen in Table 6. The results show that Vacate-GT can accurately capture different evacuation events through the use of decision analysis.

Table 6: Further case studies

Study	Experimental Evacuation Time (sec)	Average (3 runs) Simulated Evacuation Time (sec)	Standard Deviation of Simulation Evacuation Times (sec)
Corridor (Evacuate normally)	33.5	30.36	2.42
Corridor (Minimize individual time)	19	17.66	1.33
B737-200	30.5	29.4	1.10

9. Summary

Previous methods have been developed to model human movement in environments that contain other individuals. Many of these methods do not incorporate decision making interactions between the individuals. The movement decisions of individuals moving about an environment's population inherently involve interactions between those individuals. The decisions of one individual may, and often do, impact the decisions of other individuals. This paper has uniquely incorporated decision and game theory into the velocity obstacle method to capture these interactions. In this research, a novel utility function was introduced that captures the biological-based preference of energy minimization into evacuation movements through a value function. Risk preferences were incorporated into the utility function through proven mathematical methods (rather than objective function weights) to integrate further human characteristics. A novel modification of collision measurements was created to capture the inherent time preferences of humans. This modification used a time adjustment method to differentiate between urgent and distant collisions. The beliefs held by individuals on the environment and opponents were discussed. Probability distributions representing these beliefs allows for Bayesian updating when new information is discovered. In this paper, different types of opponents representing three human characteristics in evacuations (panicked, distracted and alert) were explored and shown to interact with decision makers in different forms. These methods were brought together to form games to determine the individual's best movement decisions (according to their individual beliefs and preferences). Multiple examples and case studies showed how a decision-based mathematical method can properly simulate the human behaviors seen in evacuations and in crowd movements in general.

Future work has been identified throughout the paper that would demonstrate the foundation and flexibility of using decision analysis and biological-based value functions in movement modelling. Future work would also examine behaviors such as trampling, pushing, blocking, aiding, and the impact of time pressures (Ben Zur and Breznitz, 1981; Busemeyer, 1985; Busemeyer and Diederich, 2002; Dror et al., 1999; Wright, 1974). These behaviors represent potential modifications to the movement decision method to improve model fidelity. This future work will not alter the basis of the method but only modify value function parameters and individual beliefs. The foundation of decision analysis and a meaningful biological based value function remain intact.

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