Optimal production for manufacturers considering consumer environmental awareness and green subsidies

Yugang Yu^a, Xiaoya Han^b, and Guiping Hu^{c,*}

^{a,b}*School of Management, University of Science and Technology of China, Hefei 230026, P. R. China*

^c*3014 Black Engineering, Iowa State University, Ames, IA 50011*

Abstract: Manufacturers provide products that have distinct green levels (i.e. higher degree of environmental friendliness) to satisfy consumer demands with different green preferences. A product with a higher green level generate fewer emissions but have higher costs. To encourage those manufacturers to produce environmentally friendly products, a government can implement subsidy policies. This paper focuses on the decision-making problem faced by manufacturers to determine which levels of green products to produce and production quantities at each green level. We develop an optimization model under oligopolistic competition considering green preferences and subsidies, with the objective of profit maximization for the manufacturers. We prove the existence and uniqueness of equilibrium and propose a converged algorithm with theory of finite dimensional variational inequality. Numerical results show that an increase of consumer environmental awareness will incentivize manufacturers to produce more green products with higher green levels, but this does not necessarily lead to higher profits for the manufacturers. Moreover, a well-designed subsidy policy can not only generate more profits for manufacturers, but also save subsidy investment for the government. In addition, with the changes of consumer environmental awareness or/and subsidy policy, manufacturers may obtain more profits even if the competition is more fierce.

Keywords: Optimal production; Green levels; Consumer environmental awareness; Green subsidy; Variational inequality

* Corresponding author: Guiping Hu, Office: (+1)515-294-8638. Fax: (+1)515-294-3524. Address: 3014 Black Engineering, Iowa State University, Ames, IA 50011, USA

E-mail addresses: ygyu@ustc.edu.cn (Yugang Yu), xyhan@mail.ustc.edu.cn (Xiaoya Han), gphu@iastate.edu (Guiping Hu).

1. Introduction

The world's environmental problems have become increasingly severe. Large amounts of emissions are generated from products such as automobiles and home appliances during the consumption process. A government can play an important role in reducing greenhouse gas emissions (Dröge and Schröder, 2005). As an environmental incentive mechanism, a government can provide manufacturers subsidies to encourage the production of green products that emit fewer greenhouse gas emissions compared with conventional counterparts (Sheu and Chen, 2012; Xu and Zhou, 2013). At the same time, more and more customers have become environmentally conscious (Hartman Group, 2007). They are willing to pay a premium for green products that are better for the environment. According to a survey from BBMG Conscious Consumer Report, 67% of Americans think that it is very important to purchase products that benefit the environment, and 51% of them are willing to pay higher prices for environmentally-friendly products (Bemporad and Baranowski, 2007). Seventy-five percent of Europeans bought green products with higher prices than their conventional alternative products in 2008, up from 31% in 2005 (Flash Eurobarometer, 2009).

Green products will become a trend with the increase of consumer awareness and government regulations. For a manufacturer which aims for sustainable development and higher long-term profits, green production can be an important part of business strategy. Ninety-three percent of CEOs (based on an interview survey of 750 CEOs around the world) feel that the future success of business depends on the development of new green products (UN Global Compact, 2010). To satisfy consumers' varying green needs and achieve higher profits, manufacturers will produce products with a variety of green levels. Currently, products of higher green levels generate fewer emissions but typically at higher production costs. Because of this, a government typically implements subsidies to support and promote green products. Products with different green levels are common in practice. At the 2008 Paris Motor Show, cars were classified into seven green levels based on carbon dioxide emissions. The green level of cars is "A" if carbon dioxide emissions are below 100g/km, and B-level is achieved when emissions are between 100g/km and 120g/km. By parity of reasoning, the level is "F" if emissions are above 250g/km. For example, Toyota simultaneously sells various green automobiles, such as conventional ones, hybrid automobiles

“PRIUS” and hydrogen fuel cell cars “MIRAI”. In addition, recent literature also begin to investigate products with different green levels. Borin et al. (2013) considered four levels of green products to study the impact of green strategies on consumers’ purchase intentions.

Determining which levels of green products and their corresponding production quantities to produce presents a realistic, timely, and challenging problem for manufacturers. Different manufacturers have different cost structures at each green level. As a result, manufacturers have their own responses to consumer environmental awareness and governmental subsidy policies. As more and more consumers are willing to pay price premiums for green products, and as a government provides more green subsidies, some manufacturers who have favorable cost structures may be willing to increase the production quantities of higher green levels. Even so, other manufacturers with high costs for green products may still produce only conventional products, despite of the increase of consumer environmental awareness and higher green subsidies. This is due to the significant manufacturing costs needed to develop lines of products with higher green levels for which price premiums and subsidies cannot yet offset the increase in production costs. Therefore, it is essential for manufacturers to be able to make optimal production decisions considering all of the relevant factors.

This paper aims to develop an optimization model which explicitly considers manufacturers’ production decisions under different levels of consumer environmental awareness and government subsidy policy. Due to the complexity of the optimization problem, we formulate the model as a variational inequality problem, and prove the existence and uniqueness of the optimal solution. We design an algorithm to obtain the supply chain equilibrium. The maximum profit of each manufacturer is achieved by optimally determining the green levels of products and the corresponding production quantities. To analyze and illustrate the influences of consumer environmental awareness and green subsidies on manufacturers’ production decisions, a case study with numerical results has been conducted. Managerial insights have also been derived from the analysis.

The remainder of the paper is organized as follows. We review the existing literature related to green production, the influence of consumers’ environmental awareness on production quantity, and government subsidy policies for green production in Section 2. In Section 3, we introduce the optimization model for

production decision making under a variety of green levels. In Section 4, we formulate the optimization model as a variational inequality problem. In Section 5, a converged algorithm is presented, which we then exemplify in Section 6 with an automobile market case study. We discuss our model by considering lower costs for greener products in Section 7. This paper concludes with a summary and conclusions in Section 8.

2. Literature review

There has been an increasing body of literature on green production, the influence of consumer environmental awareness on production quantity, and government subsidy policies on green production. The literature on green products focuses mainly on the production technologies and the green level categorization of products. Amacher et al. (2004) discussed how investments in green technologies impact the production of green products. In their study, manufacturers can generate competitive advantage by investing in the right green technologies with different cost structures. There are many studies on green technologies in the existing body of literature (Vachon and Klassen, 2007; Su et al., 2012; Krass et al., 2013). Nourira et al. (2014) developed two optimization models: one model where a firm offers a single product and a second model where the same firm offers both the original product and a variation of the product. In both of the models, the demand for and the price of the product depend on the greenness of the output products. They concluded that the firm's profit can increase significantly by offering a variation of the product compared to the situation where only one product is offered. Liu et al. (2012) studied a pricing decision problem of two manufacturers and discussed the positive influence of greenness of products on market demand. They also determined the optimal green level that manufacturers should choose as well as the relationships between greenness and green technical costs in competitive supply chains. Unlike Liu et al. (2012), we investigated a production decision problem of many manufacturers considering various green levels. In our study, the competition appears not only between different manufacturers but also in the same manufacturer. Moreover, we considered the effects of consumer environmental awareness and green subsidy policy on production decisions, which is not addressed in Liu et al. (2012).

Consumer environmental awareness has also been addressed in the literature due to

increasing industry and government attention. However, manufacturers' production decisions for products with different green levels are typically not considered together with the different consumer environmental awareness levels. Conrad (2005) studied market implications when customers have environmental awareness under product differentiation. The study investigated how consumer environmental awareness affects market share, product attributes, and market prices. Only one green level was considered in the firms' decision making. Yakita (2009) showed that if consumers' environmental awareness is high enough, a legally stipulated selection of green technologies can improve a firm's profit even when other firms do not select such green technologies. However, the influence of consumer environmental awareness on market structure was not analyzed in their study. Yalabik and Fairchild (2011) discussed how manufacturers incorporate environmentally friendly production techniques to appeal to environmentally conscious consumers in the market. They also studied the competition between two manufacturers, but did not examine manufacturers' optimal green production decisions with different environmental awareness levels. Borin et al. (2013) investigated the impact of green strategies (green, recycled products, green company processes and non-green products/processes) on consumers' purchase intentions. They concluded that consumers' willingness-to-purchase for green products is significantly higher than that for non-green products. Consumers are willing to pay higher prices for green products with the motivation to reduce environmental consequences. However, Borin et al. studied factors that influence greenness of products with an empirical study, which is different from that in our paper where we discussed the changes of green levels with the increase of consumer environmental awareness.

A government plays a very important role in manufacturers' green production with the implementation of environmental policies (Ai et al., 2011). Many studies have analyzed how government subsidy/tax policies affect manufacturers' green production. Dröge and Schröder (2005) discussed how to make a manufacturing industry green. They claimed that subsidy/tax policies can help to obtain a high ratio of green to non-green production in an industry. In their study, the authors did not differentiate green levels within products. Zhang et al. (2012) studied the relationship between environmental policies and the development of green products. They found that to incentivize manufacturers to deploy green production strategy, a government should enact effective subsidy/tax policies. With a well-designed subsidy/tax policy,

manufacturers change their conventional production strategies and produce both conventional and green products. Manufacturers' profits and green product quality can both increase (Sheu and Chen, 2012). However, Sheu and Chen (2012) did not study the risks for firms to suffer losses despite of production decisions that provide more benefits for the environment. Green supply chain design and management have been studied widely (Sarkis et al., 2011; Govindan et al., 2014; Mirhedayatian et al., 2014). Xu and Zhou (2013) indicated that a government can change firms' attitudes and improve their enthusiasm to participate in green supply chain management by implementing subsidy policies. However, Xu and Zhou's study did not consider optimal green production decisions under different subsidy policies.

To summarize, the existing literature has been separately focused on the green levels of products, the effects of consumers' environmental awareness, and government subsidies for manufacturers' green production, but there is limited literature on the impact of consumers' environmental awareness levels and government subsidy policy on the production decisions of manufacturers for products with different green levels, especially when both are present and interacting in the system simultaneously. This is the major motivation for this study. Moreover, we analyze in detail the manufacturers' optimal production decisions for products with different green levels. Green levels and the corresponding production quantities that manufacturers decide for their green products are sensitive to both consumers' environmental awareness and a government's green subsidies.

The main knowledge gaps addressed in this paper are summarized as follows. First, we study a wider range of green levels of products as often occurs in reality, whereas in previous literature, researchers have only addressed cases where there was at most one level of green product as an alternative to a conventional product or discussed different greenness with an empirical study. Second, so far there have been no publications that study how consumers' environmental awareness and/or a government's subsidy policy affects the production decisions of manufacturers at a variety of green levels. Third, we propose a converged algorithm based on variational inequality theory to solve the problem, and optimal strategies can be obtained very efficiently.

3. Model formulation

In this study, manufacturers who have different cost structures for products with different green levels are trying to determine which levels of green products to produce and the corresponding production quantities at each green level. The production decisions of manufacturers for products with different green levels will be affected by consumers' environmental awareness levels and the government's green subsidies.

In our proposed model, there are a finite number of I manufacturers, and i denotes the i th manufacturer. The manufacturers compete non-cooperatively in an oligopolistic manner. Moreover, the competition not only happens between different manufacturers but also may occur in the same manufacturer when producing products with various green levels. We allow for product differentiation by consumers, due to product greenness and brand concerns that may be related with a particular manufacturer. In other words, the products with the same green level from different manufacturers are not necessarily homogeneous. Figure 1 illustrates supply chain schematics from all the manufacturers to the market. Each manufacturer is willing to produce products with green levels $0, 1, \dots, z$ and distribute all of its products to the demand market. For example, in the automobile market, there exist conventional cars, hybrid automobiles (e.g., PRIUS, INSIGHT) and hydrogen fuel cell cars (e.g., MIRAI), which are different in greenness. In addition, each manufacturer produces green products based on the principle of first green level after production quantity priorities under the same profits. In other words, if a manufacturer gains the same profits when it chooses different levels of green products to produce, the manufacturer will select the case where green level is higher and then production quantities of green products are higher.

Figure 1. Supply chain schematics for products with different green levels

Let t ; $t=0,1,\dots,z$ denote the green level, which is exogenous and is a non-negative integer including 0. We can interpret $t=0$ as the green level for conventional products, and z represents the highest green level, which is finite and is not large due to the limitations of green technology and production cost. Larger numbers represent higher green levels. It is assumed that if manufacturer i can produce at green level t , the manufacturer can also produce at any green level lower than t .

x_i^t is a binary variable, and

$$x_i^t = \begin{cases} 1, & \text{if manufacturer } i \text{ produces products with green level } t, \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

x_i^t ; $t=0,1,\dots,z$; $i=1,\dots,I$ are grouped into the $(z+1) \times I$ -dimensional vector x , which is $(x_1^0, \dots, x_1^z, \dots, x_i^0, \dots, x_i^z, \dots, x_I^0, \dots, x_I^z)^T$. Each manufacturer can decide to produce with a variety of green levels considering all relevant factors.

d_i^t denotes the production quantity of products with green level t ; $t=0,1,\dots,z$, for manufacturer i ; $i=1,\dots,I$. The production quantities d_i^t ; $t=0,1,\dots,z$; $i=1,\dots,I$ are grouped into the $(z+1) \times I$ -dimensional vector d , and $d = (d_1^0, \dots, d_1^z, \dots, d_i^0, \dots, d_i^z, \dots, d_I^0, \dots, d_I^z)^T$, where $d \in R_+^N$, and N denotes the number of flows from all manufacturers for products with all green levels to the demand market. In addition, the production quantity of green product with level t cannot be negative:

$$d_i^t \geq 0, \quad t = 0, 1, \dots, z; \quad i = 1, \dots, I. \quad (2)$$

Let ρ_i^t represent the price of products with green level t of manufacturer i . The price function is as follows:

$$\rho_i^t = \rho_i^t(d, x, t, \lambda), \quad t = 0, 1, \dots, z; \quad i = 1, \dots, I. \quad (3)$$

According to Equation (3), the price for manufacturer i 's product with green level t depends on the production quantities of products with the same green level as well as production quantities of other green levels which can be substitutable. We consider ρ_i^t to be monotone decreasing, continuous, and continuously differentiable with respect to $d_i^t; t = 0, 1, \dots, z; i = 1, \dots, I$ (Nagurney and Yu, 2012; Masoumi et al., 2012; Yu and Nagurney, 2013), if the binary variables $x_i^t = 1; t = 0, 1, \dots, z; i = 1, \dots, I$. The price ρ_i^t also depends on the decisions of all the other manufacturers at each green level and the green level t that is only a parameter. Consumers are willing to pay a premium for eco-friendly products (Bemporad and Baranowski, 2007). Moreover, additional costs are generated by producing greener products (Conrad, 2005). Therefore, the price is positively correlated to the green level. In addition, the price is related to the consumer environmental awareness level λ which is a parameter in Equation (3). λ is a random factor with an expected value $E(\lambda) = \sigma$. In fact, σ denotes the rate of price increase per eco-friendly improvement, because consumers are willing to pay more for green products that are better for the environment. The lower bound of σ is 0, and the upper one is unique to each product (see, for example, Liu et al., 2012).

In order to address the competition among different manufacturers for the consumption of some homogeneous raw materials and resources used in the production of products, we assume that the total costs (including the environmental cost) of product with green level t produced by manufacturer i depend upon the production quantities of all the manufacturers for products with all the green levels and the decisions of all the other manufacturers at each green level. The cost function is

$$c_i^t = c_i^t(d, x, t), \quad t = 0, 1, \dots, z; \quad i = 1, \dots, I. \quad (4)$$

We assume that the cost is continuously differentiable, and convex with respect to $d_i^t; t = 0, 1, \dots, z; i = 1, \dots, I$ (Nagurney et al., 2002; Liu and Cruz, 2012; Yu and

Nagurney, 2013; Yu et al., 2015), if the binary variables $x_i^t=1; t=0,1,\dots,z; i=1,\dots,I$. In addition, the cost is positively correlated to the green level t . This is because the production of per unit of greener product may require more expensive energy sources, materials, and production and logistic operations (Conrad, 2005; Liu et al., 2012; Zhang et al., 2015; Zhang et al., 2016).

The government will provide subsidies to those manufacturers that produce green products. We define s^t denotes the governmental unit subsidy of the products with green level t , and assume the subsidy s^t depends on the green level t .

The profit of a manufacturer is the difference between the manufacturer's total revenues and total costs. The total revenues consist of the product revenue as well as the subsidies that the government provides. Manufacturers aim to maximize their profits. The profit function for the manufacturer is denoted by U_i .

The optimization model for manufacturer i can be represented with Equations (5) and (6).

Maximize

$$U_i = E[\sum_{t=0}^z [\rho_i^t(d, x, t, \lambda) d_i^t + s^t d_i^t - x_i^t c_i^t(d, x, t)]], \quad (5)$$

subject to

$$d_i^t \leq x_i^t M, \quad t = 0, 1, \dots, z, \quad (6)$$

where x_i^t is a binary variable, d_i^t is the production quantity for products with green level t for manufacturer i , which is the decision variable, and $d_i^t \in R_+$, for all i and t , and M is a sufficiently large constant. The first term of objective function (5) illustrates the product expected revenue and the second term represents the subsidies. The third term denotes the total operational costs including the environmental cost. Constraint (6) expresses that $d_i^t = 0$ if manufacturer i does not produce product with green level t (i.e. $x_i^t = 0$), and $d_i^t > 0$, otherwise (i.e. $x_i^t = 1$).

4. Solution approach

The solution approach is adapted from the variational inequality theory due to the computational complexity of the problem. The optimization model includes $2 \times (z+1) \times I$ decision variables denoted with the vector

$Y = (d_i^t, x_i^t), t = 0, 1, \dots, z; i = 1, \dots, I$. It should be noted that solving the objective function with constraints (1), (2) and (6) is challenging by simply analyzing the concavity or convexity of functions through computing the first-order and second-order derivatives of the objective function. In addition, the objective function and constraints will change when x_i^t takes 0 or 1, which means the specific objective function and constraints depend upon the values of x_i^t . All the above features lead to the optimization model solving complex and difficult. This motivates us to first adopt variational inequality theory to formulate our optimization model as a variational inequality problem at any selected green levels, through which the equilibrium production quantities can be obtained. Then, we can obtain the equilibrium green levels by using exhaustive method due to a small number of manufacturers and green levels.

According to Nagurney (2002), variational inequality theory provides a tool for formulating all kinds of equilibrium problems, and analyzing qualitatively the problems in the light of the existence and uniqueness of solutions obtained. In addition, it also provides us an approach for developing algorithms with analysis in convergence. On this basis, variational inequality theory has been extensively used in obtaining equilibrium solutions in a large number of literature (Zhang, 2006; Liu and Cruz, 2012; Nagurney and Yu, 2012; Masoumi et al., 2012; Yu et al., 2015). Nagurney et al. (2002) developed a supply chain network model and established a finite dimensional variational inequality formulation, which was then extended. Recently, Yu et al., (2013) used the theory to consider a competitive food supply chain network. Hamdouch (2011) used it to discuss the capacitated supply chain network equilibrium problem over a multi-period planning horizon.

Here, we discuss the property of our model, then apply variational inequality theory to the optimization problem. First, we define:

$$U = U(D), \quad (7)$$

where D denotes the vector of production quantities associated with all the manufacturers and U is the I -dimensional vector of all the manufacturers' profits.

To derive an equilibrium among multiple manufacturers by using variational inequality theory, we first need to formulate the optimization model of each manufacturer as a variational inequality problem. For the optimization model (5) with constraint (6), it is noted that, for each manufacturer $i; i = 1, \dots, I$, the profit function

U_i is continuously differentiable, and is a concave function with respect to the variables in d_i when green levels x_i are determined, where $d_i = (d_i^0, d_i^1, \dots, d_i^z)^T$, according to the assumptions of the price function (3) and cost function (4). In addition, we define the equilibrium/optimal production quantities at any selected green levels of each manufacturer $d_i^* \in H_{i1}$. Due to the finite market demand, $H_{i1} \equiv \{d_i | 0 \leq d_i \leq m\}$ is closed and convex. All the above properties lead to the following theorem.

Theorem 1: For each manufacturer, $d_i^* \in H_{i1}$ is the global optimal solution of optimization model (5) with constraint (6) at any selected green levels if and only if it meets the following variational inequality:

$$\sum_{t=0}^z \left[\frac{\partial c_i^t(d^*, x, t)}{\partial d_i^t} x_i^t - \rho_i^t(d^*, x, t, \sigma) - \frac{\partial \rho_i^t(d^*, x, t, \sigma)}{\partial d_i^t} d_i^{t*} - s^t \right] \times [d_i^t - d_i^{t*}] \geq 0 \quad (8)$$

$$\forall d_i \in H_{i1},$$

where $H_{i1} \equiv \{d_i | d_i \in \mathbb{R}_+^{N_i}\}$.

Theorem 1 can be proved according to Propositions 2 and 3 of the book by Nagurney (2002) and Han et al., (2006). In this paper, we assume that I manufacturers compete in a non-cooperative manner. Theorem 1 has guaranteed the global optimality of each manufacturer. The governing equilibrium/optimization concept underlying non-cooperative/competitive behavior is the same as in Nash (1951) and Cournot (1838). Each manufacturer determines its optimal production quantities at any green levels, given the optimal ones of those competitors. Therefore, the optimality/equilibrium conditions for all manufacturers can be simultaneously expressed as a variational inequality (see Gabay and Moulin, 1980; Bazaraa et al., 1993; Dafermos and Nagurney, 1987). Thus, we can derive the equilibrium among multiple manufacturers by solving a comprehensive variational inequality considering production decisions of all the manufacturers.

We consider a market mechanism in which the I manufacturers determine production quantities of green products in a non-cooperative manner, and each manufacturer tries to maximize its own profit. We aim to determine $D^* \in H = \prod_{i=1}^I H_i$ for which the I manufacturers will be in a state of equilibrium, according to the following definition.

Definition 1 (Supply chain equilibrium for products with different green levels): The equilibrium state of the supply chain under any selected green levels is one in which each manufacturer satisfies

$$U_i(D_i^*, \hat{D}_i^*) \geq U_i(D_i, \hat{D}_i^*), \quad \forall D_i \in H_i, \quad (9)$$

where D_i denotes the vector of production quantities associated with manufacturer i ; $i=1, K, I$, $\hat{D}_i^* \equiv (D_1^*, \dots, D_{i-1}^*, D_{i+1}^*, \dots, D_I^*)$ and $H_i \equiv \{D_i \mid D_i \in R_+^{N_i}\}$.

The variational inequality formulations of the above equilibrium for green products' supply chain satisfying Definition 1 can be obtained by referencing the existing literature (Zhang, 2006; Yu et al., 2013). Now we give the following theorem.

Theorem 2: For each manufacturer i ; $i=1, \dots, I$, the profit function $U_i(D)$ is continuously differentiable, and is concave with respect to D_i . Then $D^* \in H$ is a supply chain equilibrium based on Definition 1 if and only if it meets the following variational inequality:

$$-\sum_{i=1}^I \langle \nabla_{D_i} U_i(D^*)^T, D_i - D_i^* \rangle \geq 0, \quad \forall D \in H, \quad (10)$$

where $\nabla_{D_i} U_i(D)$ indicates the gradient of $U_i(D)$ with respect to D_i and $\langle \cdot, \cdot \rangle$ represents the inner product in N_i -dimensional Euclidean space. We turn optimization model (5) into variational inequality (11) by applying Equation (10): determine the vector $d^* \in H_1$ satisfying

$$\sum_{i=1}^I \sum_{t=0}^z \left[\frac{\partial c_i^t(d^*, x, t)}{\partial d_i^t} x_i^t - \rho_i^t(d^*, x, t, \sigma) - \frac{\partial \rho_i^t(d^*, x, t, \sigma)}{\partial d_i^t} d_i^{t*} - s^t \right] \times [d_i^t - d_i^{t*}] \geq 0, \quad (11)$$

$$\forall d \in H_1,$$

where $H_1 \equiv \{d \mid d \in R_+^N\}$.

Proof of Theorem 2 can be found in the Appendix.

For ease of expression in the following sections, we give the standard form of variational inequalities (10) and (11): determine the vector $D^* \in H^0$ satisfying

$$\langle G(D^*)^T, D - D^* \rangle \geq 0, \quad \forall D \in H^0. \quad (12)$$

The expression $\langle \cdot, \cdot \rangle$ represents the inner product in N -dimensional Euclidean space.

We let $D \equiv d$ and

$$G(D) \equiv \left[\frac{\partial c_i^t(d, x, t)}{\partial d_i^t} x_i^t - \rho_i^t(d, x, t, \sigma) - \frac{\partial \rho_i^t(d, x, t, \sigma)}{\partial d_i^t} d_i^t - s^t; \right. \\ \left. i = 1, \dots, I; \quad t = 0, \dots, z \right] \quad (13)$$

and $H^0 \equiv H_1$. Now, we give the Theorem about the existence of a Nash Equilibrium solution.

Theorem 3 (Existence): At least one Nash Equilibrium solution to variational inequality (11) exists, since there must be a $f > 0$, such that variational inequality (12) admits one Nash Equilibrium solution in H_f^o with

$$d^f \leq f. \quad (14)$$

Proof of Theorem 3 can be found in the Appendix. Next, we provide the Theorem about the uniqueness of the Nash Equilibrium solution.

Theorem 4 (Uniqueness): The formula $G(D)$ of variational inequality (11) is strictly monotone on $H^0 \equiv H_1$ based on our model, so the Nash Equilibrium solution to variational inequality (11) must be unique.

Proof of Theorem 4 can be found in the Appendix. Thus, the equilibrium production quantities under any selected green levels are unique. To derive the equilibrium green levels of each manufacturer, we need to compare all cases that are classified based on any different selections of green levels by using exhaustive method due to the finite and a small number of competitive manufacturers and green levels in practice. In each case, there exist a unique equilibrium solution for each manufacturer. Through comparisons among different cases, an equilibrium is established if no manufacturer can unilaterally improve its profit by changing its green levels, given the green level decisions of the other manufacturers. Moreover, the equilibrium can be unique based on the principle of first green level after production quantity priorities under the same profits. Therefore, following this approach, we can obtain the unique equilibrium levels of green products and the corresponding production quantities.

5. Algorithm

In this section, we develop an algorithm called Euler method to solve the variational inequality problem (11). The algorithm is guaranteed to converge to the optimal solution of problem (11) if a solution exists (Theorem 3), and it is unique (Theorem 4).

The Euler method was first introduced by Dupuis and Nagurney (1993), and has been widely used in the research community (Yu et al., 2013; Masoumi et al., 2012).

Specifically, we first provide the computational framework by recalling the Euler method at an iteration τ , which is shown in Dupuis and Nagurney (1993),

$$D^{\tau+1} = P_{H^0}(D^\tau - a_\tau G(D^\tau)), \quad (15)$$

where G is the function which enters variational inequality problem (12) and P_{H^0} is the projection on the feasible set H^0 . To guarantee the convergence of Equation (14), we must make the sequence $\{a_\tau\}$ satisfy: $\sum_{\tau=0}^{\infty} a_\tau = \infty, a_\tau > 0, a_\tau \rightarrow 0$, as $\tau \rightarrow \infty$ (Masoumi et al., 2012). We can get the optimal solution if the spatial distance of the difference between adjacent flows is not more than a given convergence tolerance.

Next, we apply the Euler method to the supply chain which includes green products with different levels of variational inequality (11). The explicit formula is as follows:

$$(d_i^t)^{\tau+1} = \max\{0, (d_i^t)^\tau + a_\tau \left(-\frac{\partial c_i^t(d, x, t)}{\partial d_i^t} x_i^t + \rho_i^t(d, x, t, \sigma) + \frac{\partial \rho_i^t(d, x, t, \sigma)}{\partial d_i^t} d_i^t + s^t \right) \}, \quad (16)$$

$\forall i; \forall t; i = 1, \dots, I; t = 0, \dots, z.$

The iterative Equation (16) is obtained based on fixed point theory. We consider that Equation (16) is converged when the spatial distance of the difference between successive production quantities of green products differs by no more than a given convergence tolerance.

In Section 6, we will illustrate an application of the algorithm through a case study. We then obtain the optimal solution with Matlab.

6. Case study

In this section, we present a case study for an automobile market to illustrate and validate the optimization model. In the case study, we consider two automobile manufacturers who may produce cars with three green levels. Parameters are mainly acquired from literature and website. The case study is motivated based on the real market scenario.

As is known to us, Toyota is on the first position in technical innovation towards greenness and sustainability. In fact, Toyota has been the best global green brand as it produces hybrid and hydrogen fuel cell cars and also implement various green

strategies. Honda is also trying to create a green image and running its vehicles towards the same direction in the eco-friendly way. Consumer environmental awareness levels are also rising, which motivates the automobile manufacturers to act in a sustainable manner. They are contributing to offering a protection to the environment and complying with regulations. Working with the Japanese Government, as early as 1997, Toyota first introduced PRIUS, which is the world's first hybrid automobiles. Now the selling price of 2012 Toyota PRIUS is from CNY 230,000 to CNY 270,000. Up to July 2015, the global total sales of PRIUS have been more than 3.5 million vehicles. At the end of 2014, Toyota sold hydrogen fuel cell car "MIRAI" towards consumers for the first time in the world, and the price is about CNY 380,000. In addition, Honda has also heavily invested in hybrid technology. In February 2009, Honda launched the hybrid car "INSIGHT" and the price is about CNY 210,000. In Japan, 47% of Honda auto are hybrid cars. In the following cases, we consider that conventional cars as green level 0, hybrid automobiles (such as PRIUS, INSIGHT) as green level 1, and hydrogen fuel cell cars (such as MIRAI) as green level 2.

The production of automobiles with different green levels (such as conventional cars, hybrid automobiles, hydrogen fuel cell cars) depends on consumer environmental awareness levels and government green subsidy policies. Three cases have been included to analyze optimal production of the manufacturers with different green production levels. In Case 1, we consider a base situation where an average consumer environmental awareness level exists, and the government provides a green subsidy policy. With the increasing importance of environmental protection, consumers are willing to spend more on greener cars, which is shown in Case 2. In Case 3, the government enacts a new subsidy policy in order to improve the social welfare. For all three cases, we examine the optimal situation at equilibrium, in which both manufacturers maximize their profits by deciding the production of green cars of various levels. Moreover, the total governmental subsidies and the market shares of green cars are evaluated. This allows for fair comparison for the three cases. To illustrate the significance of manufacturers' changing production decisions according to market parameters, we provide suboptimal situations for comparison. Differences in profits have been evaluated. Additionally, since we analyze results of the three cases and obtain conclusions based on relative numbers and trend, we do not show the specific units.

We provide price and cost functions for the cases. The price function is as follows:

$$\rho_i^t = a_i^t + t\sigma - \sum_{k=1}^2 \sum_{t=0}^2 x_k^t b_k^t d_k^t, \quad t = 0, 1, 2; i = 1, 2, \quad (17)$$

where the parameters $a_i^t > 0$ and $b_k^t > 0$ for all t and i or k . The product price of one manufacturer for products with green level t depends negatively on the production quantities of both manufacturers for products at each green level (Masoumi et al., 2012; Yu and Nagurney, 2013). The competition occurs not only between the two manufacturers but also in each manufacturer. For example, if the two manufacturers both produce conventional and level 1 green products, then the competition exists among the four types of products. Moreover, the price is positively correlated to the green level and consumer environmental awareness level (Conrad, 2005; Bemporad and Baranowski, 2007).

The operational cost function is

$$c_i^t = \alpha_i^t (d_i^t)^2 + \beta_i^t t^2 d_i^t, \quad t = 0, 1, 2 \quad i = 1, 2, \quad (18)$$

where the parameters $\alpha_i^t > 0$ and $\beta_i^t > 0$ for all t and i . In all the numerical cases, we assume the cost of manufacturer i for products with green level t depends only on the green level t and the production quantities of manufacturer i for products with green level t , which is a quadratic function of d_i^t and t . It is well-known that the product's environmental improvement generates an increasing marginal cost. In other words, each additional increment of environmental improvement is more difficult, hence costlier to achieve. Therefore, the quadratic cost function is commonly used (Deutch and Lester, 2004; Moraga-Gonzalez and Padron-Fumero, 2002; Harnisch et al., 1998).

6.1. Case 1

To explain the accuracy of the algorithm and how to get the optimal solution, we first consider a base situation. In this situation, we will get the optimal production quantities, prices, and profits of the two manufacturers by setting a consumers' average environmental awareness level and a government subsidy policy.

As stated in Section 5, we use the Euler algorithm to obtain the optimal solution. In the cases, we set the convergence tolerance equal to 10^{-6} , and the sequence

$a_\tau = 0.1(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \dots)$. In addition, we initialized the algorithm by setting the car

quantity for products with each green level equal to 2.5 (Yu and Nagurney, 2013).

In this case, we first set the consumers' average environmental awareness level σ to 2 (Liu et al., 2012). The subsidy is $s^0 = 0$, $s^1 = 2$, $s^2 = 5$, etc. Given consumers' environmental awareness and the government's green subsidy, Manufacturer 1 has realized the importance of producing green cars. Thus, Manufacturer 1 had its label of greenness in order to be different from conventional cars, with additional expenditures associated with the green production and processing activities. The price functions of the optimal situation are as follows:

$$\rho_1^0 = 25 - d_1^0 - 0.4d_1^1 - 0.5d_2^0;$$

$$\rho_1^1 = 28 + 1 \times 2 - d_1^1 - 0.4d_1^0 - 0.3d_2^0;$$

$$\rho_2^0 = 23 - d_2^0 - 0.5d_1^0 - 0.3d_1^1.$$

The total operational cost functions of the optimal situation are

$$c_1^0 = (d_1^0)^2;$$

$$c_1^1 = (d_1^1)^2 + 12 \times 1^2 d_1^1;$$

$$c_2^0 = (d_2^0)^2.$$

Based on these settings, the optimal results that we obtain are shown in Table 1.

Table 1

Results of optimal situation for Case 1

	Manufacturer 1		Manufacturer 2
Green level	0	1	0
Production quantity	5.2400	4.1171	4.7862
Price	15.7201	22.3511	14.3587
Profit	88.8155		45.8159
Total subsidy = 8.2341			
Proportion of green cars (%) = 29.11%			

Next, we consider the conventional situation. The price functions of the conventional situation are

$$\rho_1^0 = 25 - d_1^0 - 0.5d_2^0;$$

$$\rho_2^0 = 23 - d_2^0 - 0.5d_1^0.$$

The total operational cost functions of the conventional situation are

$$c_1^0 = (d_1^0)^2;$$

$$c_2^0 = (d_2^0)^2.$$

The results of the conventional situation are presented in Table 2.

Table 2

Results for Case 1 when Manufacturer 1 makes no cars with green level 1 (suboptimal for Manufacturer 1).

	Manufacturer 1	Manufacturer 2
Green level	0	0
Production quantity	5.6190	5.0476
Price	16.8572	15.1429
Profit	63.1474	50.9571

The production of green products can generate competitive advantage and more profits for manufacturers. It is very important that manufacturers produce green products of high level with low cost, which is also a necessary condition for the development of green products.

For example, we can see in Table 1 that Manufacturer 1 produces cars with green levels 0 and 1, and Manufacturer 2 only produces conventional (green level 0) cars. The competition occurs among the three types of cars. Manufacturer 1 earns much more profit with 88.8155 than Manufacturer 2 with 45.8159. From Table 2, if Manufacturer 1 does not produce cars with green level 1, Manufacturer 1 get the profit of 63.1474, which means it has not reached the full potential of profit maximization. Comparing the results of Table 1 and Table 2, we see that by producing green cars, both manufacturers suffer a loss of sales of conventional cars so that reducing the profits from selling conventional ones. However, because of the competitive advantage from the production of green cars, Manufacturer 1 still has a higher total profit, with an increase of 40.65%, while Manufacturer 2's profit decreases by 10.09%. According to the cost functions, Manufacturer 1 has better cost structure in green production than Manufacturer 2. Due to the high consumer average environmental awareness level and the government's green subsidy policy, Manufacturer 1 must produce green cars to obtain more profit.

6.2. Case 2

With the increase of consumer environmental awareness (Bemporad and Baranowski,

2007; Hartman Group, 2007; Flash Eurobarometer, 2009), Manufacturer 2 may also begin to produce green cars. To analyze the impact of changing consumer environmental awareness, we consider what happens when the consumer average environmental awareness level increases from $\sigma=2$ to $\sigma=5$, while the subsidy policy continues as in the Case 1 base situation. The price functions of the optimal situation are

$$\begin{aligned}\rho_1^0 &= 25 - d_1^0 - 0.4d_1^1 - 0.5d_2^0 - 0.3d_2^1; \\ \rho_1^1 &= 28 + 1 \times 5 - d_1^1 - 0.4d_1^0 - 0.3d_2^0 - 0.5d_2^1; \\ \rho_2^0 &= 23 - d_2^0 - 0.5d_1^0 - 0.3d_1^1 - 0.4d_2^1; \\ \rho_2^1 &= 26 + 1 \times 5 - d_2^1 - 0.3d_1^0 - 0.5d_1^1 - 0.4d_2^0.\end{aligned}$$

The total operational cost functions of the optimal situation are

$$\begin{aligned}c_1^0 &= (d_1^0)^2; \\ c_1^1 &= (d_1^1)^2 + 12 \times 1^2 d_1^1; \\ c_2^0 &= (d_2^0)^2; \\ c_2^1 &= (d_2^1)^2 + 24 \times 1^2 d_2^1.\end{aligned}$$

The optimal results obtained for the two manufacturers are shown in Table 3.

Table 3

Results of optimal situation for Case 2

	Manufacturer 1		Manufacturer 2	
Green level	0	1	0	1
Production quantity	5.1275	4.7869	4.6700	0.8001
Price	15.3826	24.3610	14.0101	24.4001
Profit	98.4138		44.8985	
Total subsidy = 11.1741				
Proportion of green cars (%) = 36.32%				

Now, we consider the suboptimal situation. The price functions of the suboptimal situation are

$$\begin{aligned}\rho_1^0 &= 25 - d_1^0 - 0.4d_1^1 - 0.5d_2^0; \\ \rho_1^1 &= 28 + 1 \times 5 - d_1^1 - 0.4d_1^0 - 0.3d_2^0; \\ \rho_2^0 &= 23 - d_2^0 - 0.5d_1^0 - 0.3d_1^1.\end{aligned}$$

The total operational cost functions of the suboptimal situation are the same as those in the optimal situation in Case 1. The suboptimal results are shown in Table 4.

Table 4

Results for Case 2 when Manufacturer 2 has no cars with green level 1 (suboptimal for Manufacturer 2).

	Manufacturer 1		Manufacturer 2
Green level	0	1	0
Production quantity	5.1700	4.8776	4.7379
Price	15.5100	24.6330	14.2138
Profit	101.0408		44.8960

The increase of consumer environmental awareness can force manufacturers to produce products with higher green levels, but that does not necessarily mean they can make more profit, which depends upon the market structure for different green levels. Manufacturers who do not offer more green levels of their products may see a growth in profits, even if the competition is more fierce. However, due to the increase of consumer environmental awareness, those manufacturers who begin to produce green products have a reduction in profits.

For example, Table 3 shows optimal decisions for the two manufacturers when both produce green cars of levels 0 and 1. Compared with optimal results for Case 1 (Table 1), the quantity of level 1 green cars for Manufacturer 1 increases by 16.27% and its profit increases by 10.81%, whereas Manufacturer 2, who turns to a green production mode, has a profit decrease of 2.00%. In Table 4, if the two manufacturers do not change their production decisions despite the increase of consumer environmental awareness, we find that Manufacturer 1's profit goes from 88.8155 to 101.0408, but Manufacturer 2's profit decreases slightly from 45.8159 to 44.8960.

In addition, comparing the results of Table 3 and Table 4, it is important for Manufacturer 2 to change its production decisions so that its profit will increase to 44.8985. This is because the increase of the consumers' environmental awareness, the conventional car market suffers great shrinkage while the green car market continuously expands. Both manufacturers suffer a loss of sales of conventional cars. However, since the competition mechanism has been changed and happens among four types of cars (conventional and level 1 green cars of the two manufacturers), the production of Manufacturer 2's green cars generates a competitive advantage compared with conventional ones, and decreases sales volume of Manufacturer 1's green cars. Therefore, Manufacturer 1 sees a reduction in profits, and Manufacturer 2 obtains a higher profit by beginning to produce green cars, which can remedy its loss

of conventional cars and the relatively high cost of retooling for green car production. Figure 2 shows the effects of consumer environmental awareness σ on the two manufacturers' profits. Figure 3 presents the relationship between σ and the proportion of green cars.

Figure 2. Effect of σ on manufacturers' profits.

From Figure 2, there exists a critical point $\sigma = 4.9935$. When $2 \leq \sigma < 4.9935$, Manufacturer 2 will not produce green cars. The competition occurs among three types of cars (conventional and level 1 green cars of Manufacturer 1 and conventional cars of Manufacturer 2). On this basis, with the increase of σ , Manufacturer 2's profit decreases and Manufacturer 1's profit increases. When $4.9935 \leq \sigma \leq 7$, Manufacturer 2 will choose to produce green cars. The competition mechanism is changed by adding level 1 green cars of Manufacturer 2. Finally, the two manufacturers' profits both increase with the increased σ .

Figure 3. Effect of σ on the proportion of green cars.

Figure 3 shows that the proportion of level 1 green cars in market will continually increases with the increase of σ .

6.3. Case 3

On October 28, 2011, China commodity net has reported that Japanese Government will provide about CNY 60,000 per unit for those manufacturers who produce electric automobiles. To analyze the effects of change in the government's green subsidy policy on manufacturer production decisions, we consider what happens if the government changes its green subsidy policy from $s^0 = 0$, $s^1 = 2$, $s^2 = 5$, etc. to $s^0 = s^1 = 0$, $s^2 = 10$, etc. We keep the same consumer average environmental awareness level $\sigma = 5$ in Case 2. The price functions of the optimal situation are

$$\rho_1^0 = 25 - d_1^0 - 0.4d_1^1 - 0.3d_1^2 - 0.5d_2^0;$$

$$\rho_1^1 = 28 + 1 \times 5 - d_1^1 - 0.4d_1^0 - 0.4d_1^2 - 0.3d_2^0;$$

$$\rho_1^2 = 32 + 2 \times 5 - d_1^2 - 0.3d_1^0 - 0.4d_1^1 - 0.2d_2^0;$$

$$\rho_2^0 = 23 - d_2^0 - 0.5d_1^0 - 0.3d_1^1 - 0.2d_1^2.$$

The total cost functions in the optimal situation are

$$\begin{aligned}
c_1^0 &= (d_1^0)^2; \\
c_1^1 &= (d_1^1)^2 + 12 \times 1^2 d_1^1; \\
c_1^2 &= (d_1^2)^2 + 10 \times 2^2 d_1^2; \\
c_2^0 &= (d_2^0)^2.
\end{aligned}$$

The optimal results obtained for the two manufacturers are shown in Table 5.

Table 5

Results of optimal situation for Case 3

	Manufacturer 1			Manufacturer 2
Green level	0	1	2	0
Production quantity	5.1010	5.3322	0.3885	4.6930
Price	15.3030	24.0641	37.0097	14.0791
Profit	103.4925			44.0491
Total subsidy = 3.8845				
Proportion of green cars (%) = 36.87%				

Next, we consider the suboptimal situation. The price functions and the total operational cost functions of the suboptimal situation are the same as those in the optimal situation in Case 2. The suboptimal results obtained for the two manufacturers are shown in Table 6.

Table 6

Results for Case 3 when two manufacturers maintain Case 2 production decisions despite new subsidy policy.

	Manufacturer 1		Manufacturer 2	
Green level	0	1	0	1
Production quantity	5.1984	4.3315	4.7409	0.3447
Price	15.5952	24.9946	14.2226	25.0337
Profit	91.5707		45.1893	

A well-designed subsidy policy can bring more profit for manufacturers and increase both the quantity of green products in the market and their green levels, while also allowing the government to offer low subsidies. At the same time, due to the change of subsidy policy, the competition mechanism may be changed, which may lead to absolute competitive advantage for the manufacturers by producing greener products. Moreover, those manufacturers who need to invest a large amount to produce green products may only produce conventional products and therefore will see their profits dwindle.

For example, in Table 5 we can see optimal decisions for the two manufacturers when Manufacturer 1 produces cars with green levels 0, 1, and 2, and Manufacturer 2 only produces conventional cars. Manufacturer 1 earns more profit, an increase of 5.16%, and Manufacturer 2's profit decreases by 1.89%, compared with optimal results for Case 2 (Table 3). In addition, by comparing Table 3 and Table 5, we can also find that more green levels bring more profits for Manufacturer 1, but less total subsidies for the government, which means that different varieties of greenness are beneficial to both Manufacturer 1 and government.

The production decisions of manufacturers can be affected greatly by the green subsidy policy. From Table 6, we see that if Manufacturer 1 do not change its production decision, then it suffer big losses from 98.4138 to 91.5707, compared with the optimal results of Case 2 (Table 3). By comparing results from Tables 5 and 6, we see that changing production (Manufacturer 1 increases the green levels of cars, and Manufacturer 2 gives up production of green cars) means Manufacturer 1's profit increases to 103.4925 and Manufacturer 2's profit decreases to 44.0491. Since no subsidy is given to cars of green level 1, and a higher unit subsidy is given to cars with green levels higher than 1, Manufacturer 1 has much more competitive advantage due to its better cost structure compared to Manufacturer 2. Moreover, there are still some consumers willing to pay higher prices to purchase cars with green level 1, and only Manufacturer 1 produces green cars with this level. To achieve more profit, Manufacturer 1 must produce cars of higher green levels, and Manufacturer 2 gives up producing green cars to reduce its loss. Therefore, with the change of green subsidy policy, the competition mechanism will be changed and it happens among conventional, level 1 and level 2 green cars of Manufacturer 1 and conventional cars of Manufacturer 2. Figure 4, 5 and 6 show the effects of unit subsidy for cars with green level 2 on manufacturers' profits, total subsidies and the proportion of green cars in the market.

Figure 4. Effect of s^2 on manufacturers' profits.

From Figure 4, there exists a critical point $s^2 = 8.6271$. When $5 \leq s^2 < 8.6271$, Manufacturer 1 will not produce cars with green level 2 and Manufacturer 2 will only produce conventional cars. Hence, the competition mechanism is the same with that of optimal situation in Case 1. The profits of the two manufacturers are both unchanged with the increased s^2 . When $8.6271 \leq s^2 \leq 14$, Manufacturer 1 chooses to produce cars with green level 2. The competition will be changed by adding level 2 green cars. On this basis, with the increase of s^2 , Manufacturer 1's profit decreases and then increases, while Manufacturer 2's profit continuously decreases.

Figure 5. Effect of s^2 on total subsidies.

Figure 5 shows that total subsidies of the government will increase with the increased s^2 . Moreover, if the unit subsidy s^2 ranges from 8.6271 to 11.9358, Manufacturer 1 is willing to produce cars with higher green levels to earn more profit, and the market share of green cars (i.e. those of green level greater than 0) will be higher, yet the total subsidies given to manufacturers by the government will be lower compared with the optimal situation in Case 2 (Table 3).

Figure 6. Effect of s^2 on the proportion of green cars.

From Figure 6, when $5 \leq s^2 < 8.6271$, the proportion of green cars does not change. When $8.6271 \leq s^2 \leq 14$, the proportion of green cars increases with the increased s^2 .

7. Discussion

In the previous sections, assumption has been made that greener products need higher costs. However, in practice, the relationship between the cost and green level may be different depending on the types of products. For example, Lippiatt and Boyles (2001) concluded that environmentally preferable building products don't necessarily cost more. Dwivedi et al. (2016) investigated how small sized budget hotels can become greener as well as more cost effective. Ren et al. (2016) indicated that the marginal cost of production of green products is lower than that of normal ones with the development of green products.

In this section, we analyze the scenario where the production cost decreases with the increasing green level. We consider that the specific cost function is

$$c_i^t = \alpha_i^t (d_i^t)^2 + \beta_i^t (\gamma_i^t - \phi_i^t t^2) d_i^t, \quad t = 0, 1, 2; i = 1, 2, \quad (19)$$

where the parameters $\gamma_i^t > 0$ and $\phi_i^t > 0$. The cost of manufacturer i for products with green level t is a decreasing quadratic function of t ; $t = 0, 1, 2$. We consider that parameter values are the same with those in Case 1. The price functions of the optimal situation are as follows:

$$\rho_1^0 = 25 - d_1^0 - 0.4d_1^1 - 0.3d_1^2 - 0.5d_2^0 - 0.3d_2^1;$$

$$\rho_1^1 = 28 + 1 \times 2 - d_1^1 - 0.4d_1^0 - 0.4d_1^2 - 0.3d_2^0 - 0.5d_2^1;$$

$$\rho_1^2 = 32 + 2 \times 2 - d_1^2 - 0.3d_1^0 - 0.4d_1^1 - 0.2d_2^0 - 0.3d_2^1;$$

$$\rho_2^0 = 23 - d_2^0 - 0.5d_1^0 - 0.3d_1^1 - 0.2d_1^2 - 0.4d_2^1;$$

$$\rho_2^1 = 26 + 1 \times 2 - d_2^1 - 0.3d_1^0 - 0.5d_1^1 - 0.3d_1^2 - 0.4d_2^0.$$

The total operational cost functions of the optimal situation are

$$c_1^0 = (d_1^0)^2;$$

$$c_1^1 = (d_1^1)^2 + 12 \times (3 - 2 \times 1^2) d_1^1;$$

$$c_1^2 = (d_1^2)^2 + 10 \times (10 - 2 \times 2^2) d_1^2;$$

$$c_2^0 = (d_2^0)^2;$$

$$c_2^1 = (d_2^1)^2 + 24 \times (1 - 0.375 \times 1^2) d_2^1.$$

Based on these settings, the results that we obtain are shown in Table 7.

Table 7

Results of optimal situation when the cost decreases with the increased green level.

	Manufacturer 1			Manufacturer 2	
Green level	0	1	2	0	1
Production quantity	4.8688	3.4903	4.1487	4.4530	2.1922
Price	14.6064	20.4708	27.4463	13.3590	19.5763
Profit	106.1983			49.2688	
Total subsidy = 32.1085					
Proportion of green cars (%) = 51.33%					

Table 7 shows that optimal decisions for the two manufacturers when Manufacturer 1 produces cars with green levels 0, 1, and 2, and Manufacturer 2 produces cars with green levels 0 and 1. The two manufacturers both earn more profits, an increase of 19.57% for Manufacturer 1, and 7.54% for Manufacturer 2, compared to optimal results for Case 1 (Table 1). In addition, it is observed that the two manufacturers are

willing to produce greener cars because the total cost decreases with the increase of the green level. Therefore, the relationship between the cost and green level plays an important role in determining the levels of green cars and production quantities at the green levels for manufacturers.

8. Conclusions

In this paper, a mathematical model has been formulated to analyze a market where each manufacturer provides a variety of product types distinguished by green levels to satisfy consumer demands for various green preferences. Products with higher green levels generate fewer emissions, but usually cost more to produce. To encourage manufacturers to produce products with high green levels, the government promotes subsidy policies in which products with higher green levels receive higher unit subsidy. Another consideration for manufacturers is that environmentally conscious consumers are willing to pay higher prices for more eco-friendly products. This paper focuses on the decision problem of a manufacturer: how to determine which green levels of products to produce and production quantities for each green level. We build an optimization model considering consumer environmental awareness and green subsidy policies, with an objective of profit maximization for the manufacturers. Due to the complexity of the problem, we formulate the model as a variational inequality problem that can be solved by an Euler algorithm. The optimal strategies can be obtained very efficiently.

Managerial insights can be derived from the case study. First, the increase of consumers' environmental awareness can force manufacturers to produce more products with high green levels, but this does not necessarily mean manufacturers can obtain more profit, which depends on market structure. In a society where environmental awareness continues to grow, manufacturers must change their conventional production modes by considering green levels, in order to follow sustainable development paths and obtain more long-term profits. Second, a well-designed subsidy policy can generate more profit for manufacturers who have good cost structure in green production, improve quantity and green level of products in the market, and also make government subsidies low. Therefore, manufacturers should adjust their green production decisions according to changes in green subsidy policy, but increase green levels of products by improving their own situations (cost

structure) rather than depending heavily on external factors (subsidies). Third, with the changes of consumer environmental awareness or/and subsidy policy, manufacturers may obtain more profits even if the competition is more fierce. Moreover, the model can assist manufacturers to determine when they should produce or give up greener products.

This study is subject to various limitations which present future research directions. First, the subsidy for green manufacturers is exogenous in this analysis. More realistic analysis could be conducted by considering the government as a separate decision maker from the manufacturers, similar to a Stackelberg game. This will allow us to not only analyze the influence between different manufacturers, but also to study the interactions between the government and the manufacturers. Second, this study has not explicitly considered coordination between supply chains, and it may be important to consider not only competition between manufacturers but also possible cooperation between the manufacturers. Third, we only consider the scenario where subsidies are provided to the manufacturers. However, subsidies directly provided to the consumers can increase their willingness to pay for green products, including investigating different levels of consumer willingness-to-buy affected by subsidies.

Acknowledgements

This research is supported by grants from the Thousand Young Scholar Program, the National Science Foundation of China (NSFC) for Distinguished Youth Scholars (No. 71225002) and the National Natural Science Foundation of China (No. 71401067). The authors thank the Editor and two anonymous reviewers for their valuable comments and suggestions for improving the quality of the paper.

Appendix A. Proof

Proof of Theorem 2: Variational inequality (10) follows directly from Gabay and Moulin (1980). Note the following equation

$$\nabla_{D_i} U_i(D) = \left[\frac{\partial U_i}{\partial d_i^t}; t = 0, 1, K; i = 1, K, I \right]. \quad (\text{A.1})$$

Since λ is a random factor and $E(\lambda) = \sigma$, according to Equation (5), we also have the following equation:

$$U_i = \sum_{t=0}^z [\rho_i^t(d, x, t, \sigma) d_i^t + s^t d_i^t - x_i^t c_i^t(d, x, t)], \quad (\text{A.2})$$

For each $t; t=0,1,K,z$ and $i; i=1,K,I$, we obtain

$$\begin{aligned} \frac{\partial U_i}{\partial d_i^t} &= \frac{\partial \left[\sum_{t=0}^z [\rho_i^t(d, x, t, \sigma) d_i^t + s^t d_i^t - x_i^t c_i^t(d, x, t)] \right]}{\partial d_i^t} \\ &= \sum_{t=0}^z \left[\rho_i^t(d, x, t, \sigma) + \frac{\partial \rho_i^t(d, x, t, \sigma)}{\partial d_i^t} d_i^t + s^t - \frac{\partial c_i^t(d, x, t)}{\partial d_i^t} x_i^t \right]. \end{aligned} \quad (\text{A.3})$$

Multiplying the expression in Equation (A.3) by a minus sign and by the term $(d_i^t - d_i^{t*})$, for each manufacturer, we obtain variational inequality (8). Moreover, there are I competitive manufacturers in our study, and the objective function of each manufacturer can be formulated as variational inequality (8). Thus, we have I number of variational inequalities. To obtain the equilibrium/optimal solution for each manufacturer, the I number of variational inequalities need to be satisfied simultaneously. According to the properties of variational inequality that is used to derive an equilibrium among multiple manufacturers, we only need to solve variational inequality (11) by summing over all i and using the definition of the feasible set H_1 .

We can find that I number of variational inequality (8) and the summing variational inequality (11) are equivalent. It is obvious that variational inequality (11) can be obtained according to I number of variational inequality (8). On the contrary, if variational inequality (11) is satisfied, we can derive variational inequality (8) for each manufacturer. The reason is that for $\forall i$, we can set $\hat{d}_i = \hat{d}_i^*$, where $\hat{d}_i \equiv (d_1, \dots, d_{i-1}, d_{i+1}, \dots, d_I)$. The proof is complete.

Proof of Theorem 3: Since the population is finite, the market demand is finite. This requires that production quantity d_i^t of manufacturer i of green product with level t ($i=1, \dots, I; t=0, 1, \dots, z$) must be bounded. Therefore, we get that

$$H_f^0 \equiv \{d \mid 0 \leq d \leq f\}, \quad (\text{A.4})$$

in which $f > 0$ and $d \leq f$ means $d_i^t \leq f$ for all $i=1, \dots, I; t=0, 1, \dots, z$. Then we can get that H_f^0 , which is a subset of H becomes bounded, closed, and convex.

Thus, the variational inequality

$$\left\langle G(D^f)^T, D - D^f \right\rangle \geq 0, \quad \forall D \in H_f^0, \quad (\text{A.5})$$

admits at least one Nash Equilibrium solution $d^f \in H_f^0$, since G is continuous and H_f^0 is compact.

Proof of Theorem 4: We consider the price ρ_i^t to be monotone decreasing, continuous, and continuously differentiable with respect to the production quantities d_i^t ; $i = 1, \dots, I$; $t = 0, 1, \dots, z$, and the cost c_i^t is continuously differentiable, and convex with respect to the production quantity d_i^t (Yu and Nagurney 2013). Therefore, $G(D)$ of variational inequality (11) is strictly monotone on $H^0 \equiv H_1$, that is,

$$\left\langle (G(D^1) - G(D^2))^T, D^1 - D^2 \right\rangle > 0, \quad \forall D^1, D^2 \in H^0, D^1 \neq D^2, \quad (\text{A.6})$$

which means the Nash Equilibrium solution to variational inequality (11) must be unique.

References

- Xu, A., Hu, X., Gao, S., 2011. A three-player game model for the green supply chain in the home appliance industry. In Proceedings of the 8th International Conference on Innovation & Management (pp. 578-583): Wuhan University of Technology Press.
- Amacher, G. S., Koskela, E., Ollikainen, M., 2004. Environmental quality competition and eco-labeling. *Journal of Environmental Economics and Management* 47, 284-306.
- Bazaraa, M.S., Sherali, H.D., Shetty, C.M., 1993. *Nonlinear Programming: Theory and Algorithms*. Wiley, New York.
- Bemporad, R., Baranowski, M., 2007. Conscious consumers are changing the rules of marketing. Are you ready? Highlights from the BBMG conscious consumer report. Available at <<http://www.bbmng.com>>.
- Borin, N., Lindsey-Mullikin, J., Krishnan, R., 2013. An analysis of consumer reactions to green strategies. *Journal of Product & Brand Management* 22, 118-128.
- Cournot, A.A., 1838. *Researches into the Mathematical Principles of the Theory of Wealth*. MacMillan, UK (English translation).
- Conrad, K., 2005. Price competition and product differentiation when consumers care for the

- environment. *Environmental and Resource Economics* 31, 1-19.
- Dafermos, S., Nagurney, A., 1987. Oligopolistic and competitive behavior of spatially separated markets. *Regional Science and Urban Economics* 17, 245–254.
- Deutch, J.M., Lester, P.K., 2004. *Making Technology Work: Applications in Energy and the Environment*. Cambridge University Press, Cambridge, UK.
- Dröge, S., Schröder, P. J., 2005. How to turn an industry green: taxes versus subsidies. *Journal of Regulatory Economics* 27, 177-202.
- Dupuis, P., Nagurney, A., 1993. Dynamical systems and variational inequalities. *Annals of Operations Research* 44, 7-42.
- Dwivedi, R.K., Pandey, M., Vashisht, A., 2016. Simplifying Cost Effective Green Practices For Indian Budget Hotels. *Global Journal For Research Analysis* 5(4).
- Eurobarometer, F., 2009. Europeans' attitudes towards the issue of sustainable consumption and production. Flash Eurobarometer 256. Available at http://www.ec.europa.eu/public_opinion/flash/fl_256_en.pdf.
- Gabay, D., Moulin, H., 1980. On the uniqueness and stability of Nash equilibria in noncooperative games. In: *Applied Stochastic Control of Econometrics and Management Science*, Bensoussan, A., Kleindorfer, P., Tapiero, C. S. (Eds), North-Holland, Amsterdam, The Netherlands, pp. 271-294.
- Govindan, K., Sarkis, J., Jabbour, C. J. C., Zhu, Q. H., Geng, Y., 2014. Eco-efficiency based green supply chain management: Current status and opportunities. *European Journal of Operational Research* 233(2), 293-298.
- Hamdouch, Y., 2011. Multi-period supply chain network equilibrium with capacity constraints and purchasing strategies. *Transportation Research Part C: Emerging Technologies* 19(5): 803-820.
- Han, J.Y., Xiu, N. H., Qi, H.D., 2006. *Nonlinear complementarity theory and algorithm*. Shanghai Science and Technology Press, Shanghai (in Chinese).
- Harnisch, J., I.S. Wing, H.D. Jacoby, R.G. Prinn., 1998. Primary aluminum production: Climate policy, emissions and costs. MIT Joint Program on the Science and Policy of Global Change, Report 44. Available at http://www.globalchange.mit.edu/files/document/MITJPSPGC_Rpt44.pdf.
- Hartman, G., 2007. *Hartman Report on Sustainability: Understanding the Consumer Perspective*, Hartman Group, Bellevue, Washington. Available at <http://www.sustainableinnovations.com/PDF/hartman>.
- Moraga-Gonzalez, J.L., Padron-Fumero, N., 2002. Environmental policy in a green market. *Environmental and Resource Economics* 22, 419–447.
- Krass, D., Nedorezov, T., Ovchinnikov, A., 2013. Environmental taxes and the choice of green technology. *Production and Operations Management* 22(5), 1035-1055.
- Lippiatt, B.C., Boyles, A.S., 2001. Using BEES to select cost-effective green products. *The*

- International Journal of Life Cycle Assessment 6(2), 76-80.
- Liu, Z., Cruz, J.M., 2012. Supply chain networks with corporate financial risks and trade credits under economic uncertainty. *International Journal of Production Economics* 137(1), 55-67.
- Liu, Z. L., Anderson, T. D., Cruz, J. M., 2012. Consumer environmental awareness and competition in two-stage supply chains. *European Journal of Operational Research* 218, 602-613.
- Masoumi, A. H., Yu, M., Nagurney, A., 2012. A supply chain generalized network oligopoly model for pharmaceuticals under brand differentiation and perishability. *Transportation Research Part E: Logistics and Transportation Review* 48, 762-780.
- Mirhedayatian, S.M., Azadi, M., Farzipoor Saen, R., 2014. A novel network data envelopment analysis model for evaluating green supply chain management. *International Journal of Production Economics* 147, 544-554.
- Nagurney, A., 2002. *Variational Inequality*, Isenberg School of Management University of Massachusetts.
- Nagurney, A., Dong, J., Zhang, D., 2002. A Supply Chain Network Equilibrium Model. *Transportation Research Part E: Logistics and Transportation Review* 38 (5): 281-303.
- Nagurney, A., Yu, M., 2012. Sustainable fashion supply chain management under oligopolistic competition and brand differentiation. *International Journal of Production Economics* 135, 532-540.
- Nash, J.F., 1951. Noncooperative games. *Annals of Mathematics* 54, 286-298.
- Nouira, I., Frein, Y., Hadj-Alouane, A. B., 2014. Optimization of manufacturing systems under environmental considerations for a greenness-dependent demand. *International Journal of Production Economics* 150, 188-198.
- Ren, X., Liu, Q., Zhang, Y., 2016. The Optimal Strategy in Remanufacturing Green Supply Chains. Available at <http://onlinepresent.org/proceedings/vol121_2016/29.pdf>.
- Sarkis, J., Zhu, Q., Lai, K.-h., 2011. An organizational theoretic review of green supply chain management literature. *International Journal of Production Economics* 130, 1-15.
- Sheu, J.-B., Chen, Y. J., 2012. Impact of government financial intervention on competition among green supply chains. *International Journal of Production Economics* 138, 201-213.
- Su, J. C., Wang, L., Ho, J. C., 2012. The impacts of technology evolution on market structure for green products. *Mathematical and Computer Modelling* 55, 1381-1400.
- UN Global Compact, 2010. Accenture release findings of largest CEO research study on corporate sustainability. Available at <www.unglobalcompact.org/news/42-06-22-2010> (accessed 25 May 2011).
- Vachon, S., Klassen, R. D., 2007. Supply chain management and environmental technologies: the role of integration. *International Journal of Production Research* 45, 401-423.
- Xu, A., Zhou, Z., 2013. A Game Model for the Pricing of the Government's Subsidy in the Green

-
- Supply Chain for Home Appliances Industry in China. WHICEB 2013 Proceedings. Paper 10. Available at <<http://aisel.aisnet.org/whiceb2013/10>>.
- Yakita, A., 2009. Technology choice and environmental awareness in a trade and environment context. *Australian Economic Papers* 48, 270-279.
- Yalabik, B., Fairchild, R. J., 2011. Customer, regulatory, and competitive pressure as drivers of environmental innovation. *International Journal of Production Economics* 131, 519-527.
- Yu, M., Nagurney, A., 2013. Competitive food supply chain networks with application to fresh produce. *European Journal of Operational Research* 224, 273-282.
- Yu, Y., Han, X., Liu, J., et al., 2015. Supply chain equilibrium among companies with offline and online selling channels. *International Journal of Production Research* 53(22), 6672-6688.
- Zhang, D., 2006. A network economic model for supply chain versus supply chain competition. *Omega* 34, 283-295.
- Zhang, L., Wang J., You J., 2015. Consumer environmental awareness and channel coordination with two substitutable products. *European Journal of Operational Research* 241(1), 63-73.
- Zhang, Q., Tang W., Zhang J., 2016. Green supply chain performance with cost learning and operational inefficiency effects. *Journal of Cleaner Production* 112, 3267-3284.
- Zhang, X., Xu, X., He, P., 2012. New product design strategies with subsidy policies. *Journal of Systems Science and Systems Engineering* 21, 356-371.