Multi-task Ranking SVM for Image Cosegmentation

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Abstract

Cosegmentation is the task of simultaneously segmenting multiple images that contain common or similar foreground objects. The assumption that common objects appear in multiple images provides a weak form of supervised prior information, thus cosegmentation usually performs better than unsupervised segmentation methods. Recently, a multi-task learning based cosegmentation method was proposed, and it can simultaneously segment more than two images and easily add different types of prior. However, it has the shortcoming of information loss in the initialization of the multi-task classification model. To ameliorate this problem, in this paper, we propose a novel multi-task ranking SVM model which incorporates multi-task learning and learning to rank into a unified framework. The proposed model is trained using the relative order information between the cosaliency score of pixel pairs. In addition, an optimization algorithm is proposed to optimize the multi-task ranking SVM model based on the alternative direction method of multipliers (ADMM), which ensures that the proposed method is faster than most of state-of-the-art cosegmentation approaches. Finally, the proposed method is evaluated on two widely used benchmark datasets, i.e. CMU iCoseg and MSRC. The experiment results show that the proposed approach is effective and performs better than most of the state-of-the-art works.

Keywords: Image cosegmentation, Multi-task learning, Ranking SVM, Visual saliency, ADMM

1. Introduction

Image cosegmentation is the task of simultaneously segmenting multiple related images into different parts. Different with unsupervised single image segmentation approaches, image cosegmentation makes the assumption that these related images contain common or similar objects, which provides a form of weakly supervised prior information. Therefore, image cosegmentation can usually achieve better performance than those unsupervised single image segmentation approaches.

The idea of cosegmentation was first introduced in [1], in which a MRF based generative model was proposed to simultaneously segment two images. The work [2] later proposed a discriminative cosegmentation model which can simultaneously segment more than two images and deal with more greater variation foreground appearance. The work [3] tried to deal with a more challenge cosegmentation task, multiple objects cosegmentation. The works [4, 5] used the supervised image annotation to do cosegmentation. The above methods can be considered as low level feature based methods[6, 7]. Recently, some object-based cosegmentation methods are proposed, and the details of these works can be found in [8-11].

There are several limits in the above works. The work [1] can segment only two images. Although [2] can segment more than two images, its computational complexity is too high. The methods [4, 5] need fully supervised human annotation. In order to solve these limitations, the work [12] proposed a multi-task learning

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based image cosegmentation method. In this framework, the segmentation of each image is treated as a single task with independent classifier. The prior information shared among these images is modeled by the multi-task regularization term. Specifically, features are firstly extracted for each image, and then these features are used to compute the cosaliency score of these images, in which the computed cosaliency score is in the interval $[0, 1]$. Third, the cosaliency score is binarized by a threshold, and then these binarized score is used to initialize the multi-task classification model. Finally, the output of multi-task classification model is used to initialize the GrabCut algorithm which outputs the final segmentation results.

The above multi-task learning based cosegmentation model has some merits. First, it can simultaneously segment more than two images. Second, its computational complexity is lower than previous methods when segmenting more than two images. Third, it's easy and flexible to model the shared prior information among these images by the multi-task regularization term. However, there is still weak points in [12]. In particular, after the cosaliency score of those images is computed, it is binarized by a fixed threshold and used to initialize the multi-task classifier model. However, it is difficult to choose the fixed threshold. Inappropriate threshold may lead to information loss. Thus how to combine the cosaliency score with the framework of multi-task learning remains a problem.

In this paper, we focus on the framework of multi-task learning based cosegmentation methods and propose a novel multi-task Ranking SVM (MTR-SVM) model to solve the above problems. The proposed MTR-SVM is based on the frameworks of both multi-task learning [13-15] and learning to rank [16-18], and is directly trained using the relative order information between the cosaliency score of pixel pairs instead of the binarized form of cosaliency score. MTR-SVM can flexibly model the relationships or shared features among these tasks by the regularization term. In addition, we propose an ADMM based algorithm to optimize the MTR-SVM model and it ensures that MTR-SVM is faster than those state-of-the-art cosegmentation approaches. Finally, we test the performance of the proposed approach on two widely used benchmark datasets, the CMU iCoseg dataset and the MSRC dataset. The experiment results show that the proposed approach improves the segmentation accuracy significantly and costs less computational time, as compared with recent state-of-the-art cosegmentation techniques.

The rest of this paper is organized as follows. Section 2 introduces the related works and the MTR-SVM model is proposed on Section 3. Section 4 designs a novel ADMM based learning algorithm for MTR-SVM model. The experiment evaluation is given in Section 5. The conclusion and further works are discussed in Section 6.

2. Related works

2.1 Learning to rank

Learning to rank [17, 18] is a popular method in Information Retrieval and it aims to learn a ranking function from the training data. The existing learning to rank algorithms can be roughly divided into three categories: the point-wise, pair-wise and list-wise approaches. The work [19] proposed a regression based point-wise ranking model which learns a regression function between the ranking score and the features. A SVM based pair-wise ranking model was introduced in [16], where the ranking problem is transformed into a two class classification problem. In [20], the list of objects is treated as an instance, and a list-wise loss function on the basis of probability model is formulated. Besides, there are other works about ranking models, and these can be found in [21, 22].

2.2 Multi-task learning

Multi-task learning [14, 15] is the task of simultaneously learning multiple related tasks by exploiting task

relationships or shared features among these tasks. Multi-task learning usually can achieve better performance than single-task learning methods, especially when the number of training data of each task is limited. The current multi-task learning methods can be divided into two categories: methods are based on sharing task relationships [23] and that are based on sharing features [13]. Most of multi-task learning approaches share the common information by the multi-task regularization term. The work [14] proposed a low rank multi-task learning method which assumes that the parameter vectors of those tasks are located in a low rank subspace. In [13], a $\ell_{2,1}$ regularization term is used to constrain the parameter matrix, so that the parameter vectors of each task share common features. The work [23] proposed a kernel function based multi-task learning model which can automatically learn the task relationships.

3. Multi-Task ranking SVM for image cosegmentation

In this part, we introduce a novel multi-task Ranking SVM model for image cosegmentation. The general framework of the proposed method is illustrated in figure 1. In order to improve the computational speed, SLIC [24] is used to over-segment each image into 200 superpixels.

Fig. 1 The overview of the proposed algorithm. The up row shows the segmentation flow and the down row shows the specific structure of the proposed MTR-SVM model.

3.1 Feature extraction and visual cosaliency

In this part, color features, texture features, geometry measurements and edges features are firstly extracted for each superpixel [25]. And we use the symbol X_i^j to represent the extracted features of the j -th superpixel in i -th image. Then the similarities between superpixels within each image and across different images are computed, and it is useful to the computation of the cosaliency. We use the symbol $A(i, j; p, q)$ to represent the similarity between the j -th superpixel in i -th image and q -th superpixel in p -th image.

Visual saliency is the task of finding the visual attention parts of the whole visual field and has attracted a lot of researches [26-29]. Visual saliency is usually used to initialize the foreground parts in image segmentation task.

For multiple images, visual cosaliency aims to find the saliency part that is not only salient in its own image but also similar to salient part in other images. In this paper, the cosaliency for each image is computed by the robust visual cosaliency method proposed in [12]. More specifically, symbol s_i^j is used to represent the cosaliency score of superpixel X_i^j and is computed by an iteration method,

$$s_i^j(t+1) = (1-\alpha)\hat{s}_i^j + \alpha \sum_{p,q:s_p^q(t) \geq \tau} s_p^q(t) \times A(i,j;p,q) \quad (1)$$

where $s_i^j(t+1)$ is the cosaliency score of X_i^j in the $t+1$ iteration. $\hat{s}_i^j$ is the saliency score of X_i^j and is computed by a PCA based saliency computation method [26]. τ is a threshold. α is a weight parameter and is in interval $[0,1]$.

3.2 MTR-SVM model

When the cosaliency score of each image has been computed, we choose a pair-wise learning to rank model (ranking SVM) to directly fit these cosaliency score, instead of binarizing the cosaliency score to initialize the multi-task classification model in [12]. And the learning to rank based method can avoid the difficult choice of the threshold and reduce information loss.

Specifically, for a single image i , assume that there are n_i superpixels. Then a linear ranking function $f_i(x) = \langle W_i, x \rangle$ will be learned to fit these features and cosaliency score of superpixels in image i , in which W_i is the parameter vector holding the same dimension with feature vector x . In addition, function f_i should keep the ranking order of these superpixels, which can be formulated as the following constraints:

$$s_i^j > s_i^k \Leftrightarrow f_i(X_i^j) > f_i(X_i^k) \Leftrightarrow \langle W_i, X_i^j - X_i^k \rangle > 0 \quad (2)$$

The constraints in Eq. (2) are equivalent to a two-class classification problem with the following training instances:

$$\begin{aligned} & \{(X_i^j - X_i^k, Y_i^{jk}), j, k \in \{1, 2, \dots, n_i\}\} \\ & \text{where } Y_i^{jk} = \begin{cases} +1, & \text{if } s_i^j > s_i^k \\ -1, & \text{otherwise} \end{cases} \end{aligned} \quad (3)$$

Next, we utilize the SVM model to fit these classification data. And the objective function of the classification problem can be written as:

$$\begin{aligned} & \min_{w_i} \frac{1}{2} \|W_i\|^2 + C \sum_{j,k=1}^{n_i} \xi_{j,k} \\ & \text{s.t. } \xi_{j,k} \geq 0 \\ & Y_i^{j,k} \langle W_i, X_i^j - X_i^k \rangle \geq 1 - \xi_{j,k}, j, k \in \{1, \dots, n_i\} \end{aligned} \quad (4)$$

Note that Eq. (4) is equivalent to Eq. (5):

$$\min_{w_i} \sum_{j,k=1}^{n_i} \left[1 - Y_i^{j,k} \langle W_i, X_i^j - X_i^k \rangle \right]_+ + \lambda \|W_i\|^2 \quad (5)$$

where $\lambda = \frac{1}{2C}$, subscript “+” means the positive part.

After Eq. (5) is optimized, we use the symbol W_i^* to represent the optimized parameter vector. Then the ranking score of superpixel X_i^j can be reevaluated by $f_i(X_i^j) = \langle W_i^*, X_i^j \rangle$.

The segmentation of each image can be considered as a single task, and the segmentation of multiple images can be treated as a multi-task segmentation problem. The prior information that these images share same or similar foreground objects can be modeled by the regularization term of the multi-task learning models. Assuming there are T similar images needed to be segmented, and for each image t , there are $n_t^2 - n_t$ different pairwise instances $\{(X_t^j - X_t^k, Y_t^{jk}), j, k \in \{1, 2, \dots, n_t\}\}$. Then the general formula of regularization multi-task learning is:

$$\min_W \sum_{t=1}^T \sum_{j,k=1}^{n_t} L(Y_t^{jk}, f_t(X_t^j - X_t^k)) + \lambda_1 \|W\|_2^2 + \lambda_2 R(W) \quad (6)$$

where W_i is the parameter vector of i -th task and it has the same dimension with feature vector X_i^j .

$W = [W_1, W_2, \dots, W_T]$ is the parameter matrix. $L(\cdot, \cdot)$ is the loss function. Here we use hinge loss function. $R(\cdot)$ is the multi-task regularization term. In this paper, three types of regularization terms are used, **Mean graph**

regularization, $R(W) = \sum_{t=1}^T \left\| W_t - \frac{1}{T} \sum_{s=1}^T W_s \right\|_2^2$, which assumes that the task parameter vectors are drawn from the

same distribution and all of them are close to their mean; **Low rank regularization**, $R(W) = \|W\|_*$, which assumes that the task parameter vectors share a common low rank subspace, and each parameter vector can be linearly represented by others; $\ell_{2,1}$ **regularization**, $R(W) = \|W\|_{2,1}$, which assumes that the task parameter vectors share common features.

Finally, combining Eq. (5) with Eq. (6), the object function of Multi-task Ranking SVM model is formulated as follows:

$$\min_W \sum_{t=1}^T \sum_{j,k=1}^{n_t} \left[1 - Y_t^{jk} \langle W_t, X_t^j - X_t^k \rangle \right]_+ + \lambda_1 \sum_{t=1}^T \|W_t\|_2^2 + \lambda_2 R(W) \quad (7)$$

For the t -th image, the pairwise training dataset $\{(X_t^j - X_t^k, Y_t^{jk}), j, k \in \{1, 2, \dots, n_t\}\}$ totally contains $n_t^2 - n_t$ different training instances. And in the following, for the convenience of deduction, we use $\{(x_t^i, y_t^i), i \in \{1, 2, \dots, N_t\}\}$ to represent the training dataset $\{(X_t^j - X_t^k, Y_t^{jk}), j, k \in \{1, 2, \dots, n_t\}\}$, where

$N_t = n_t^2 - n_t$. Thus, Eq. (7) can be rewritten as,

$$\min_W \sum_{t=1}^T \sum_{i=1}^{N_t} \left[1 - y_t^i \langle W_t, x_t^i \rangle \right]_+ + \lambda_1 \sum_{t=1}^T \|W_t\|_2^2 + \lambda_2 R(W) \quad (8)$$

In the following section, we propose an ADMM based optimization method to optimize the above object function. After the MTR-SVM model has been optimized, the ranking function of each image can be achieved. Then the saliency score of superpixels in all images can be reevaluated by applying the ranking function to these

superpixels, $f_t(X_t^i) = \langle W_t, X_t^i \rangle$. Lastly, the GrabCut algorithm is used to achieve the final segmentation results

[30], and it can be initialized by the following method: for each superpixel X_t^i , if its score is smaller than 80% of the mean scores of all the superpixels in all images, it will be labeled as background.

4. The alternative direction method of multipliers for MTR-SVM model

In this section, the alternative direction method of multipliers (ADMM) [31-35] is used to optimize the problem in Eq. (8). Although the alternative direction method has been developed for decades of years, how to use it to solve a specific problem is not mathematically trivial. In the following, we derive the solution of the problem in Eq. (8).

4.1 ADMM

Consider the following optimization problem:

$$\begin{aligned} \min_{x,y} & f_1(x) + f_2(y) \\ \text{s.t.} & Ax + By = c \end{aligned} \quad (9)$$

where $x \in R^{n_1}$ and $y \in R^{n_2}$ are optimization variables, $A \in R^{m \times n_1}$ and $B \in R^{m \times n_2}$ are linear operators, $c \in R^m$ is a vector of data. ADMM is an iterative method to solve problem in Eq. (9) and it minimizes the following augmented Lagrangian:

$$\begin{aligned} L(x, y, \lambda) = & f_1(x) + f_2(y) - \langle \lambda, Ax + By - c \rangle \\ & + \frac{\tau}{2} \|c - Ax - By\|^2 \end{aligned} \quad (10)$$

where τ is a penalty parameter and λ is the Lagrange multiplier. Then Eq. (10) is equivalent to the following two sub-problems:

$$\begin{aligned} \min_x & f_1(x) - \langle \lambda, Ax + By - c \rangle + \frac{\tau}{2} \|c - Ax - By\|^2 \\ \min_y & f_2(y) - \langle \lambda, Ax + By - c \rangle + \frac{\tau}{2} \|c - Ax - By\|^2 \end{aligned} \quad (11)$$

The updating rule of the Lagrange multiplier is: $\lambda_{new} = \lambda_{old} - \tau(Ax + By - c)$. The complete steps of ADMM are described in algorithm 1.

Algorithm 1. ADMM

Initialize: $y_0 \in R^{n_2}, \lambda_0 \in R^m, \tau > 0$

1: for $k = 0, 1, 2, \dots$ do

2: $x_{k+1} = \arg \min_x L(x, y_k, \lambda_k)$

3: $y_{k+1} = \arg \min_y L(x_{k+1}, y, \lambda_k)$

4: $\lambda_{k+1} = \lambda_k - \tau(Ax_{k+1} + By_{k+1} - c)$

5: end for

4.2 ADMM to solve problem in Eq. (8) with mean graph regularization

In this section, we derive the optimization problem in Eq. (8) with the mean graph multi-task regularization:

$R(W) = \sum_{t=1}^T \left\| W_t - \frac{1}{T} \sum_{s=1}^T W_s \right\|_2^2$. First, we use an auxiliary variable $e_t^j = y_t^j - \langle W_t, x_t^j \rangle, t=1, \dots, T; j=1, \dots, N_t$ to

transform Eq. (8) into the following form:

$$\begin{aligned} \min_W \sum_{t=1}^T \sum_{j=1}^{N_t} [y_t^j e_t^j]_+ + \lambda_1 \sum_{t=1}^T \|W_t\|^2 + \lambda_2 \sum_{t=1}^T \left\| W_t - \frac{1}{T} \sum_{s=1}^T W_s \right\|_2^2 \\ \text{s.t. } e_t^j = y_t^j - \langle W_t, x_t^j \rangle, t=1, \dots, T; j=1, \dots, N_t. \end{aligned} \quad (12)$$

Then the augmented Lagrangian function in Eq. (12) is defined as:

$$\begin{aligned} \min_{W, e, \alpha, \tau} L(e, W, \alpha, \tau) = \sum_{t=1}^T \sum_{j=1}^{N_t} [y_t^j e_t^j]_+ + \lambda_1 \sum_{t=1}^T \|W_t\|^2 \\ + \frac{\tau}{2} \sum_{t=1}^T \sum_{j=1}^{N_t} \left(\langle W_t, x_t^j \rangle + e_t^j - y_t^j + \frac{\alpha_t^j}{\tau} \right)^2 \\ + \lambda_2 \sum_{t=1}^T \left\| W_t - \frac{1}{T} \sum_{s=1}^T W_s \right\|_2^2 \end{aligned} \quad (13)$$

where α is the Lagrange multiplier.

At the k -th iteration, when parameter matrix W is fixed, Eq. (13) is reduced to the following problem:

$$\min_e \sum_{t=1}^T \sum_{j=1}^{N_t} [y_t^j e_t^j]_+ + \frac{\tau}{2} \sum_{t=1}^T \sum_{j=1}^{N_t} \left(\langle W_t, x_t^j \rangle + e_t^j - y_t^j + \frac{\alpha_t^j}{\tau} \right)^2 \quad (14)$$

The optimization of Eq. (14) can be decomposed into $\sum_{t=1}^T N_t$ independent single variable minimization problem,

and each is a single variable 2-piece piecewise function [33]. After the deduction, we get the explicit solution of variable e :

$$e_t^j = \begin{cases} t_t^j, & \text{if } y_t^j t_t^j < 0 \\ 0, & \text{if } 0 \leq y_t^j t_t^j \leq \frac{1}{\tau} \\ t_t^j - \frac{1}{\tau} y_t^j, & \text{if } y_t^j t_t^j \geq \frac{1}{\tau} \end{cases} \quad (15)$$

where $t_t^j = y_t^j - \langle X_t^j, W_t \rangle_t - \frac{\alpha_t^j}{\tau}$, $t=1, \dots, T; j=1, \dots, N_t$.

When the auxiliary variable e is fixed, Eq. (13) is reduced to the following problem:

$$\begin{aligned} \min_W \lambda_1 \sum_{t=1}^T \|W_t\|^2 + \lambda_2 \sum_{t=1}^T \left\| W_t - \frac{1}{T} \sum_{s=1}^T W_s \right\|_2^2 \\ + \frac{\tau}{2} \sum_{t=1}^T \sum_{j=1}^{N_t} \left(\langle W_t, x_t^j \rangle + e_t^j - y_t^j + \frac{\alpha_t^j}{\tau} \right)^2 \end{aligned} \quad (16)$$

This sub-problem is convex and has closed form solution. The closed form solution of parameter W can be derived by letting the gradient of Eq. (16) equal to zero.

$$W_t = - \left(2\lambda_1 + 2\lambda_2 - \frac{2\lambda_2}{T} \right)^{-1} \left[\tau x_t \left(e_t - y_t + \frac{\alpha_t}{\tau} \right) - \frac{2\lambda_2}{T} \sum_{i=1, i \neq t}^T W_i \right], \quad t = 1, \dots, T \quad (17)$$

where $x_t = [x_t^1, \dots, x_t^{N_t}]$ is the feature matrix of t -th image. $e_t = [e_t^1, \dots, e_t^{N_t}]^T$ is the vector form of auxiliary variable in t -th image. Besides, $y_t = [y_t^1, \dots, y_t^{N_t}]^T$ is the label vector and $\alpha_t = [\alpha_t^1, \dots, \alpha_t^{N_t}]^T$ is the vector form of Lagrange multiplier.

The update of Lagrangian multiplier α is:

$$\alpha_t^{(k)} = \alpha_t^{(k-1)} + \tau^{(k-1)} (\langle x_t, W_t \rangle + e_t - y_t), t = 1, \dots, T \quad (18)$$

The augmented penalty parameter τ is monotonically non-decreasing over the iterative process and has an upper bound. It can be updated by following formula:

$$\tau^{(k)} = \rho \tau^{(k-1)} \quad (19)$$

where $\rho \geq 1$ is a hyperparameter.

The overview of the algorithm is illustrated in Algorithm 2.

Algorithm 2. ADMM for MTR-SVM with mean graph regularization

Initialize: $W_0, \alpha_0, \tau > 0, \rho \geq 1$
1: for $k = 0, 1, 2, \dots$ do
2: Update e by equation (15);
3: Update W by equation (17);
4: Update α by equation (18);
5: Update τ by $\tau = \rho \tau$;
6: end for

4.3 ADMM to solve problem in Eq. (8) with low rank regularization

In this part, we derive the optimization problem in Eq. (8) with the low rank regularization: $R(W) = \|W\|_*$.

First, we use the auxiliary variable $e_t^j = y_t^j - \langle W_t, x_t^j \rangle, t = 1, \dots, T; j = 1, \dots, N_t$ and $U = W$ to transform Eq. (8) into following form:

$$\begin{aligned} \min_W \quad & \sum_{t=1}^T \sum_{j=1}^{N_t} [y_t^j e_t^j]_+ + \lambda_1 \sum_{t=1}^T \|W_t\|^2 + \lambda_2 \|U\|_* \\ \text{s.t.} \quad & U = W \\ & e_t^j = y_t^j - \langle W_t, x_t^j \rangle, t = 1, \dots, T; j = 1, \dots, N_t. \end{aligned} \quad (20)$$

Then, the augmented Lagrangian function of Eq. (20) is defined as:

$$\begin{aligned}
\min_{W, e, \alpha, \tau} L(e, W, U, \alpha, \tau) &= \sum_{t=1}^T \sum_{j=1}^{N_t} [y_t^j e_t^j]_+ + \lambda_1 \sum_{t=1}^T \|W_t\|^2 \\
&+ \frac{\tau}{2} \sum_{t=1}^T \sum_{j=1}^{N_t} \left(\langle W_t, x_t^j \rangle + e_t^j - y_t^j + \frac{\alpha_t^j}{\tau} \right)^2 + \lambda_2 \|U\|_* \\
&+ \frac{\tau}{2} \left\| U - W + \frac{\beta}{\tau} \right\|_F^2
\end{aligned} \tag{21}$$

At the k -th iteration, when parameter matrix W is fixed, Eq. (21) is reduced to the following two sub-problems:

$$\min_e \sum_{t=1}^T \sum_{j=1}^{N_t} [y_t^j e_t^j]_+ + \frac{\tau}{2} \sum_{t=1}^T \sum_{j=1}^{N_t} \left(\langle W_t, x_t^j \rangle + e_t^j - y_t^j + \frac{\alpha_t^j}{\tau} \right)^2 \tag{22}$$

$$\min_U \frac{\tau}{2} \left\| U - W + \frac{\beta}{\tau} \right\|_F^2 + \lambda_2 \|U\|_* \tag{23}$$

The sub-problem in Eq. (22) is equal to Eq. (14), and it has the same solution with Eq. (14). For the sub-problem in Eq. (23), according to [36], its globally optimal solution is given by the singular value thresholding

$$U = Z \Sigma_{\frac{\lambda_2}{\tau}} V \tag{24}$$

where $W - \frac{\beta}{\tau} = Z \Sigma V$ is the SVD of $W - \frac{\beta}{\tau}$, and $\Sigma_{\frac{\lambda_2}{\tau}} = \text{Diag} \left\{ \left(\Sigma_{ii} - \frac{\lambda_2}{\tau} \right)_+ \right\}$.

When auxiliary e and parameter matrix U are fixed, Eq. (21) is reduced to the following sub-problem:

$$\begin{aligned}
\min_W &\lambda_1 \sum_{t=1}^T \|W_t\|^2 + \frac{\tau}{2} \sum_{t=1}^T \sum_{j=1}^{N_t} \left(\langle W_t, x_t^j \rangle + e_t^j - y_t^j + \frac{\alpha_t^j}{\tau} \right)^2 \\
&+ \frac{\tau}{2} \left\| U - W + \frac{\beta}{\tau} \right\|_F^2
\end{aligned} \tag{25}$$

The sub-problem in Eq. (25) is convex and has closed form solution. The solution of parameter matrix W can be derived by letting the gradient of Eq. (25) equal to zero,

$$\begin{aligned}
W_t &= \left(2\lambda_1 + \tau x_t x_t^T + \tau \right)^{-1} \left[\tau \left(U_t + \frac{\beta_t}{\tau} \right) \right. \\
&\quad \left. - \tau x_t \left(e_t - y_t + \frac{\alpha_t}{\tau} \right) \right], t = 1, \dots, T
\end{aligned} \tag{26}$$

where $x_t = [x_t^1, \dots, x_t^{N_t}]$ is the feature matrix of t -th image. $e_t = [e_t^1, \dots, e_t^{N_t}]^T$ is the vector form of auxiliary variable in t -th image. Besides, $y_t = [y_t^1, \dots, y_t^{N_t}]^T$ is the label vector, $\alpha_t = [\alpha_t^1, \dots, \alpha_t^{N_t}]^T$ is the vector form of Lagrange multiplier and β_t is the t -th column of β .

The update rule of Lagrangian multiplier is:

$$\alpha_t^{(k)} = \alpha_t^{(k-1)} + \tau^{(k-1)} (\langle x_t, W_t \rangle + e_t - y_t), t = 1, \dots, T \quad (27)$$

$$\beta^{(k)} = \beta^{(k-1)} + \tau^{(k-1)} (U^{(k-1)} - W^{(k-1)}) \quad (28)$$

The augmented penalty parameter τ is monotonically non-decreasing over the iterative process and has an upper bound. It can be updated by following formula:

$$\tau^{(k)} = \rho \tau^{(k-1)} \quad (29)$$

where $\rho \geq 1$ is a hyperparameter.

The overview of the algorithm is illustrated in Algorithm 3.

Algorithm 3. ADMM for MTR-SVM with Low Rank Regularization

Initialize: $W_0, \alpha_0, \tau > 0, \rho \geq 1$

- 1: for $k = 0, 1, 2, \dots$ do
 - 2: Update e by equation (15);
 - 3: Update U by $U = Z \sum_{\frac{\lambda_2}{\tau}} V$;
 - 4: Update W by equation (26);
 - 5: Update α by equation (27);
 - 6: Update β by equation (28);
 - 7: Update τ by $\tau = \rho \tau$;
 - 8: end for
-

4.4 ADMM to solve problem in Eq. (8) with $\ell_{2,1}$ regularization

In this part, we derive the optimization problem in Eq. (8) with the $\ell_{2,1}$ regularization: $R(W) = \|W\|_{2,1}$. First, we use the auxiliary variable $e_t^j = y_t^j - \langle W_t, x_t^j \rangle, t = 1, \dots, T; j = 1, \dots, N_t$ and $U = W$ to transform Eq. (8) into the following form:

$$\begin{aligned} \min_W \quad & \sum_{t=1}^T \sum_{j=1}^{N_t} [y_t^j e_t^j]_+ + \lambda_1 \sum_{t=1}^T \|W_t\|^2 + \lambda_2 \|U\|_{2,1} \\ \text{s.t.} \quad & e_t^j = y_t^j - \langle W_t, x_t^j \rangle, t = 1, \dots, T; j = 1, \dots, N_t. \\ & U = W. \end{aligned} \quad (30)$$

Then, the augmented Lagrangian function of Eq. (30) is defined as:

$$\begin{aligned} \min_{W, e, \alpha, \tau} \quad & L(e, W, \alpha, \tau) = \sum_{t=1}^T \sum_{j=1}^{N_t} [y_t^j e_t^j]_+ + \lambda_1 \sum_{t=1}^T \|W_t\|^2 \\ & + \frac{\tau}{2} \sum_{t=1}^T \sum_{j=1}^{N_t} \left(\langle W_t, x_t^j \rangle + e_t^j - y_t^j + \frac{\alpha_t^j}{\tau} \right)^2 + \lambda_2 \|U\|_{2,1} \\ & + \frac{\tau}{2} \left\| U - W + \frac{\beta}{\tau} \right\|_F^2 \end{aligned} \quad (31)$$

At the k -th iteration, when parameter matrix W is fixed, Eq. (31) is reduced to the following two sub-problems:

$$\min_e \sum_{t=1}^T \sum_{j=1}^{N_t} [y_t^j e_t^j]_+ + \frac{\tau}{2} \sum_{t=1}^T \sum_{j=1}^{N_t} \left(\langle W_t, x_t^j \rangle + e_t^j - y_t^j + \frac{\alpha_t^j}{\tau} \right)^2 \quad (32)$$

$$\min_U \frac{\tau}{2} \left\| U - W + \frac{\beta}{\tau} \right\|_F^2 + \lambda_2 \|U\|_{2,1} \quad (33)$$

The sub-problem in Eq. (32) is equal to Eq. (14) and its solution is the same with Eq. (15). For the sub-problem in Eq. (33), according to [37], its global optimal solution is given by,

$$U_g = \left(1 - \frac{\lambda_2}{\tau \|V_g\|} \right)_+ V_g \quad (34)$$

where, $V = W - \frac{\beta}{\tau}$, U_g means the g -th row of matrix U and V_g means the g -th row of matrix V .

When the auxiliary variable e and U is fixed, Eq. (31) is reduced to the following sub-problem:

$$\begin{aligned} \min_W \frac{\tau}{2} \sum_{t=1}^T \sum_{j=1}^{N_t} \left(\langle W_t, x_t^j \rangle + e_t^j - y_t^j + \frac{\alpha_t^j}{\tau} \right)^2 + \lambda_1 \sum_{t=1}^T \|W_t\|^2 \\ + \frac{\tau}{2} \left\| U - W + \frac{\beta}{\tau} \right\|_F^2 \end{aligned} \quad (35)$$

The sub-problem in Eq. (35) is equal to Eq. (25) and its solution is the same with Eq. (26). Finally, the updating rule of Lagrangian multiplier and augmented penalty parameter are separately:

$$\alpha_t^{(k)} = \alpha_t^{(k-1)} + \tau^{(k-1)} (\langle x_t, W_t \rangle + e_t - y_t), t = 1, \dots, T \quad (36)$$

$$\beta^{(k)} = \beta^{(k-1)} + \tau^{(k-1)} (U^{(k-1)} - W^{(k-1)}) \quad (37)$$

$$\tau^{(k)} = \rho \tau^{(k-1)} \quad (38)$$

where $\rho \geq 1$ is a hyperparameter.

The overview of the algorithm is illustrated in Algorithm 4.

Algorithm 4: ADMM for MTR-SVM with

$\ell_{2,1}$ regularization

Initialize: $W_0, \alpha_0, \tau > 0, \rho \geq 1$

1. for $k = 0, 1, 2, \dots$ do
 2. Update e by equation (15);
 3. Update U by equation (34);
 4. Update W by equation (26);
 5. Update α by equation (36);
 6. Update β by equation (37);
 7. Update τ by $\tau = \rho \tau$;
 8. end for
-

5. Experiments

In this section, we test the performance of the proposed MTR-SVM algorithm on two publicly available datasets, CMU iCoseg dataset and MSRC dataset. These two datasets are both widely used by previous cosegmentation approaches to evaluate segmentation performance. The proposed method aims at improving the accuracy of cosegmentation and reducing the computation time. Thus we separately evaluate these two points on the above two datasets. The segmentation accuracy is used to quantitatively analyze the segmentation results, and it is defined as the percentage of the well classified pixels (foreground and background) over manually labeled ground truth ($p = \frac{GT \cap R}{GT \cup R}$). In the following, three symbols, ‘Mean MTL’, ‘Low rank MTL’ and ‘L2,1 MTL’ are used to represent the three models used in [12].

Fig. 2 The original images of Christ are presented in the first row; in the second row the saliency map are displayed, the output map of MTL are presented in the third row; in the last row the output map of the proposed approach are displayed.

Fig. 3 The original images of Statue are presented in the first row; in the second row the saliency map are displayed, the output map of MTL are presented in the third row; in the last row the output map of the proposed approach are displayed.

5.1 Experiments on CMU iCoseg dataset

In this part, we evaluate the segmentation accuracy and computation time of the proposed method on CMU iCoseg dataset, which is composed of 38 object classes and contains 4 to 41 images in each image class. We fixed the parameter λ_1 to 0.01 for the proposed MTR-SVM with three different multi-task regularization terms. The qualitative result on iCoseg dataset is displayed in Table 1. For the comparison experiments, we rerun the codes provided by the authors on a PC with 3.6 GHz CPU and 12 GB RAM and record the running time of these compared approaches.

Table 1 Cosegmentation performance on iCoseg dataset

	Class	# of images (used/total)	[2]	[38]	[39]	MTL			MTR-SVM		
						Mean	LR	L2,1	Mean	LR	L2,1
1	Alaskan bear	19/19	74.83	70.95	70.76	83.79	84.41	83.56	82.43	83.57	83.93
2	Red sox players	25/25	59.55	66.19	72.57	97.20	97.22	97.00	93.95	94.22	94.29
3	Stonehenge	5/5	55.66	70.82	72.19	78.62	79.01	78.23	89.52	87.48	89.79
4	Stonehenge2	18/18	60.10	89.05	66.30	60.56	61.26	61.65	73.99	72.25	74.12
5	Liverpool	33/33	66.96	82.97	86.92	90.43	90.22	89.98	90.04	89.02	89.70
6	Ferrari	11/11	78.58	88.87	63.52	92.57	90.73	89.70	87.92	88.50	88.19
7	Taj Mahal	5/5	61.75	77.81	67.23	92.78	85.13	84.78	91.71	84.92	84.73
8	Taj Mahal2	5/5	81.20	72.57	64.52	78.29	69.35	72.87	77.69	75.77	79.62
9	Pyramids	10/10	61.96	77.97	74.64	75.59	79.68	75.91	80.08	77.48	81.14
10	Elephants	15/15	54.64	62.27	74.17	80.17	81.89	79.86	84.37	86.24	86.36
11	Goose	31/31	97.28	83.96	69.71	77.96	78.76	79.92	89.73	78.86	80.24
12	Pandas	25/25	93.35	86.58	69.60	65.41	65.67	66.42	71.39	68.70	69.46
13	Airshows1	12/12	69.20	92.87	87.56	98.38	98.27	95.94	95.03	94.96	95.19
14	Airshows2	39/39	95.04	93.27	93.57	97.61	97.58	97.68	96.79	96.71	96.76
15	Airshows3	22/22	56.67	85.27	80.81	99.60	99.57	99.51	99.13	99.09	99.08
16	Cheetah	33/33	58.92	90.21	86.00	73.99	73.33	72.20	76.20	76.24	76.41
17	Pandas2	21/21	54.89	79.16	69.76	55.64	55.42	56.40	61.09	63.43	63.55
18	Kite1	18/18	98.14	70.77	86.60	95.87	95.08	95.28	96.44	95.96	96.03
19	Kite2	10/10	59.84	84.49	68.17	77.90	77.01	77.12	82.72	83.16	83.20
20	Kite3	7/7	61.74	81.86	69.10	73.24	71.08	73.00	79.09	78.36	78.72
21	Kite4	11/11	93.92	77.37	70.67	97.73	97.76	97.89	97.21	97.15	97.24
22	Gymnastics1	6/6	85.58	98.21	87.24	94.38	94.31	94.53	96.41	96.25	96.56
23	Gymnastics2	4/4	71.73	92.00	81.59	95.38	95.69	94.93	95.73	96.20	96.18
24	Gymnastics3	6/6	76.36	96.91	81.82	92.54	92.58	92.96	95.21	95.44	96.29
25	Skating1	11/11	66.39	80.47	72.45	81.19	79.06	78.39	82.45	83.78	83.54
26	Skating2	12/12	94.72	56.86	88.60	97.67	97.50	96.83	96.32	96.30	96.91
27	Woman players1	36/36	95.91	91.46	81.28	90.70	90.10	90.50	90.58	90.13	90.37
28	Woman players2	16/16	97.42	82.03	72.20	82.79	82.74	82.70	86.65	84.95	84.89
29	Monks	17/17	57.89	86.03	62.15	86.82	84.48	85.81	88.88	82.51	83.44
30	Hot balloons	24/24	72.26	84.29	79.76	92.14	90.78	89.69	90.26	92.05	93.55
31	Statue	41/41	72.32	89.17	86.60	78.53	80.54	78.04	87.90	86.40	88.55
32	Christ	13/13	68.20	86.91	73.60	70.20	70.49	70.16	86.13	85.76	85.66
33	Speedskating	13/13	94.77	59.08	78.15	89.82	91.85	89.60	93.28	93.22	93.24
34	Track field	5/5	55.22	67.57	66.87	81.72	81.76	81.14	82.67	83.99	83.47
35	Windmill	18/18	67.61	60.67	82.19	92.81	92.82	92.52	92.44	92.21	92.15
36	Kendo1	30/30	72.31	67.16	68.81	93.76	92.86	95.31	95.15	90.30	93.63
37	Kendo2	11/11	75.31	73.24	70.76	93.83	92.54	96.23	95.91	91.22	94.64
38	Brown bear	5/5	66.70	87.76	84.15	77.71	78.09	79.22	88.37	83.02	85.16
	Average		73.34	80.13	75.86	85.14	84.67	84.57	87.79	86.73	87.53

For each image class, all of the images are input into the cosegmentation algorithm. The segmentation accuracy of each image class is averaged over the selected images. It can be seen that the proposed approach achieves the best performance on most image classes. For each multi-task regularization term, the mean segmentation accuracy of the proposed MTR-SVM algorithm is higher than that of [12], which shows that the proposed method can reasonably utilize the saliency information of each image to achieve better cosegmentation results. In Figure 2 and figure 3, we show the saliency map, the output map of MTL in [12] and the output map of the proposed MTR-SVM on Christ image class and Statue image class respectively. And we can see that the proposed approach

Fig. 4 The comparison of segmentation results on Christ and Stonehenge image classes.

can achieve more bright and complete objects than the original saliency map and the output map of MTL. Thus, MTR-SVM can easily distinct the foreground objects from the background and achieve better cosegmentation results.

Figure 4 shows some segmentation examples that demonstrate the good performance of the proposed method. Figure 5 demonstrates that the proposed MTR-SVM yields fine results, even though individual images are challenging.

In the following, we evaluate the computation time of the proposed method and those compared methods. The time cost of the proposed method and those compared methods is listed in Table 2. It can be seen that [38] cost the longest time, because it used the deformable shape templates shared by input images to align common objects between these images and the computation of these shape templates is very time consuming. In [2], multiple images are modeled into one graph and a Laplace matrix must be calculated in the process of optimization. Thus, when the number of images is increasing, the time spent on the calculating of the Laplace matrix is increasing. [39] proposed a distributed cosegmentation approach for highly variable large-scale image collection, and it cost relatively little time than the above two methods. Both [12] and the proposed method are based on the framework of multi-task learning in which each image is treated as a single segmentation task. And the cosegmentation can be treated as simultaneously optimizing multiple little scale problems. Thus, compared with others methods, the multi-task learning based methods cost less time. In addition, in the proposed approach, an ADMM based optimization algorithm is proposed to optimize the MTR-SVM model and each sub-problem of ADMM has closed form solution. Thus the proposed MTR-SVM cost far less time than [12].

Table 2 Time cost on iCoseg dataset

Method		Time (seconds)
[2]		15300
[39]		2123
[38]		87837
MTL	Mean	666
	Low Rank	1750
	L2,1	153
MTR-SVM	Mean	60
	Low Rank	65
	L2,1	67

Table 4 Time cost on MSRC dataset

Method		Time (seconds)
[2]		25087
[3]		17752
[39]		1465
[40]		93903
MTL	Mean	3029
	Low Rank	4249
	L2,1	40
MTR-SVM	Mean	1403
	Low Rank	876
	L2,1	17

5.2 Experiments on MSRC dataset

In this part, we evaluate the segmentation accuracy and computation time of the proposed method on MSRC dataset. MSRC is composed of 14 object classes and contains about 30 images in each image class. For the proposed MTR-SVM with three different multi-task regularization terms, we fixed the parameter λ_1 to 0.01. The quantitative results on MSRC dataset are displayed in Table 3. For the comparison experiments, we rerun the code provided by the authors on a PC with 3.6 GHz CPU and 12 GB RAM and record the running time of those compared methods.

Table 3 compares the accuracy of the proposed MTR-SVM with those conventional algorithms. For each image class, all of the images are input into the cosegmentation algorithm. The segmentation accuracy of each image class is computed by averaging the segmentation accuracy of each single image. It can be seen that the mean segmentation accuracy of [40] is the highest. Although [40] can achieve relatively high accuracy, it incorporates

both image saliency information and the edge map information into cosegmentation. And the computation of edge map for each image cost quit a long time which is not practical in reality. For each multi-task regularization term, the mean segmentation accuracy of the proposed MTR-SVM algorithm is higher than that of [12]. In Figure 6, we show some segmentation examples on MSRC, and these examples demonstrate that the proposed MTR-SVM yields fine results, even though individual images are challenging.

Table 3 Cosegmentation performance on MSRC dataset

	Class	# of images (used/total)	[2]	[3]	[39]	[40]	MTL			MTR-SVM		
							Mean	LR	L2,1	Mean	LR	L2,1
1	Bike	30/30	58.63	67.17	60.82	63.96	64.05	64.98	64.57	64.50	67.89	64.91
2	Bird	32/32	60.73	68.21	84.68	90.77	87.19	88.31	89.02	86.16	87.18	87.18
3	Car	30/30	75.82	76.59	65.72	71.50	68.62	67.79	68.80	73.13	73.28	73.79
4	Cat	24/24	59.29	66.15	75.64	82.87	72.89	73.88	73.91	78.49	76.75	73.56
5	Chair	30/30	76.57	72.54	70.74	80.96	72.78	74.89	75.98	73.61	73.99	73.39
6	Cow	30/30	79.03	83.75	74.74	91.80	87.89	88.53	89.76	87.93	90.22	88.33
7	Dog	30/30	76.55	78.20	70.25	85.58	79.30	79.96	81.16	79.50	76.69	79.29
8	Face	30/30	63.69	60.47	66.25	74.22	76.04	76.82	77.16	76.50	78.22	78.94
9	Flower	32/32	70.13	69.61	72.02	84.21	75.85	75.39	76.06	79.87	78.68	78.07
10	House	30/30	68.15	68.45	68.45	80.46	72.85	73.15	73.83	72.88	67.90	75.60
11	Plane	30/30	55.57	55.27	73.47	80.66	72.72	73.45	70.87	74.51	76.04	74.57
12	Sheep	30/30	80.24	89.24	74.45	91.81	87.89	87.64	88.54	89.87	89.54	89.86
13	Sign	30/30	77.83	76.79	74.22	84.36	86.76	85.85	85.82	83.06	84.39	84.10
14	Tree	30/30	84.52	83.98	73.34	84.78	72.66	72.49	73.03	76.94	77.28	77.48
	Average		70.48	72.60	71.77	82.00	76.96	77.36	77.75	78.35	78.43	78.51

The time cost of the proposed method and those compared methods is listed in Table 4, which demonstrates the efficiency of the proposed MTR-SVM approach.

6. Conclusions and future works

In this paper, we propose a multi-task ranking SVM model to simultaneously segment multiple images. MTR-SVM combines the framework of multi-task learning with learning to rank. It can simultaneously deal with more than two images and add different types of cosegmentation prior by the multi-task regularization term. In addition, we propose an efficient ADMM based algorithm to optimize the proposed model. Experiment results demonstrate that the proposed approach outperforms the state-of-the-art techniques significantly both on the iCoseg dataset and MSRC dataset. In future works, we look forward to exploring more multi-task regularization terms, such as nonconvex regularization and other low rank regularization [41, 42]. Furthermore, we will investigate an automatic mechanism to construct the multi-task regularization term.

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Fig. 5 Images and segmentation results on iCoseg dataset. There are 4 groups of images. In each group, the top row represents the original images, and the bottom row represents the segmentation result.

Fig. 6 Images and segmentation results on MSRC dataset. There are 4 groups of images. In each group, the top row represents the original images, and the bottom row represents the segmentation result.

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