Non-fragile H_∞ State Estimation for Nonlinear Networked System with Probabilistic Diverging Disturbance and Multiple Missing Measurements *

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Abstract: This paper is concerned with the non-fragile H_∞ state estimation problem for a class of discrete-time networked system with probabilistic diverging disturbance and multiple missing measurements. The measurement missing phenomenon is assumed to occur randomly and the missing probability for each sensor is governed by an individual random variable satisfying a certain probabilistic distribution over the interval $[0, 1]$. The aim of this paper is to estimate the networked system by designing a non-fragile H_∞ estimator such that the augmented estimation error system is asymptotically mean square stable with a prescribed H_∞ disturbance attention level γ . By using the Lyapunov method and stochastic analysis, we derive a sufficient condition for the existence of the desired estimator. By solving the linear matrix inequalities (LMIs), the estimator gain matrix is given. Two numerical examples are employed to demonstrate the effectiveness and applicability of the proposed design technique.

Keywords: H_∞ state estimator, probabilistic diverging disturbance, multiple missing measurements, non-fragile filter.

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1 Introduction

In recent decades, nonlinear networked system has attracted many researchers' attention[1]-[9]. In order to understand the nonlinear networked system well, it is essential to estimate the system states based on available measurements. The state estimation problem is very important because the system states are not completely available in the system outputs in real applications.

In the actual condition, the system outputs often partially lost information due to various reasons, such as a failure in the measurement, intermittent sensor failures, accidental loss of some collected data, and so on. On the one hand, the measurement output of the nonlinear networked system may be transmitted through non-ideal channels with communication constrains. The measurement information possibly misses in extreme cases. In most of the existing works, the missing measurements probability of sensors was assumed as the uniform distribution by default, which was described by the Bernoulli distribution over the interval $[0, 1]$ [16, 17, 26]. In [27], Wei studied missing measurements with randomly occurring nonlinearities (RONs). In [13], Zhang considered missing measurements with randomly occurring interval delays (RODs). Actually, the missing measurements of each sensor behaves in a different probabilistic characteristic derived from the statistical strategy and their occurrence probability of the missing measurements is different from each other. Thus, the missing probability of each sensor is described by different probability distribution, which can be any discrete form over the interval $[0, 1]$. And based on this, we study the missing measurements for neural networks with random time delay (See RODs in [10], [13]-[15]) which, to the best of our knowledge, has not been fully investigated and still remains challenging.

On the other hand, the disturbances are randomly divided into the original system and state estimator due to the environmental disturbances in practice. The designed estimator may be sensitive to perturbations. So, it is important to design an estimator which is to be insensitive against numerical round off errors, limited word length of the computer, imprecision in analogue-digital conversion, i.e. the designed estimator is expected to be insensitive or non-fragile. In [10, 19], a non-fragile H_∞ filter for nonlinear system with Channel Fadings has been studied, devising an non-fragile H_∞ filter via using a combination of the stochastic analysis and Lyapunov function approach. Therefore, designing a non-fragile H_∞ estimator has drawn a great deal of attentions. However, the non-fragile H_∞ state estimation problem for a class of discrete-time nonlinear networked system with RODs, probabilistic diverging disturbance, multiple missing measurements, and RONs

has not been investigated yet.

Two main contributions of this paper are summarized as follows. First, for multiple missing measurements, the missing probability for each sensor is governed by an individual random variable satisfying a certain probabilistic distribution over the interval $[0, 1]$. Then a probabilistic distribution could be any commonly used discrete distribution over the interval $[0, 1]$, and the measurement output includes exogenous input disturbance. Second for a such non-fragile H_∞ estimator, the newly constructed error dynamics of the filtering process is asymptotically mean square stable.

This paper is organized as follows: In **Section 2**, we show a discrete-time nonlinear networked system with RONS, RODs, probabilistic diverging disturbance and multiple missing measurements. The sufficient conditions for the filtering process asymptotically mean square stable and the H_∞ performance index of the filtering error system are derived in **Section 3**. **Section 4** provides two examples to verify the effectiveness of the proposed criteria. **Section 5** concludes the paper.

Notations: The superscript T stands for matrix transposition. $l_2[0, \infty]$ is the space of square integrable vectors. R^n denotes n dimensional Euclidean space, and $R^{n \times m}$ is the set of all $n \times m$ real matrices. $I, 0$ denotes identity matrix and zero matrices with proper dimension respectively. $*$ denotes the symmetric parts. $diag\{\dots\}$ stands for a block-diagonal matrix. The notations $X > Y$ and $X \geq Y$, where X and Y are matrices of the same dimension, mean that the matrix $X - Y$ is positive definite and positive semi-definite, respectively. If $X \in R^p$ and $Y \in R^q$, $C(X; Y)$ denotes the space of all continuous functions mapping $R^p \rightarrow R^q$. $E\{x\}$ stands for the expectation of x . $Pro(\cdot)$ means the occurrence probability of the event. N denotes the set of all nonnegative integers.

2 Preliminaries and Problem Formulation

Consider the following discrete-time networked system with probabilistic diverging disturbance:

$$\begin{cases} x(k+1) = Ax(k) + A_dg(x(k)) + Bx(k - \tau(k)) + (\rho(k) + (1 - \rho(k))\frac{\varrho(k)}{u})D\omega(k) \\ y(k) = C_yx(k) + D_yv(k) \\ z(k) = C_zx(k) \end{cases} \quad (1)$$

with the initial state $x_0 = x(s), s \in [-\tau_M, 0]$, where $x(k) \in R^n$ is the state vector, $\tau(k) \in [\tau_m, \tau_M]$ is the time-varying delay, $\omega(k) \in R^m$ is the disturbance input function, $\omega(k) \in l_2[0, +\infty)$. $y(k) \in R^l$ is the measurement

output, $v(k) \in R^s$ is exogenous input disturbance, $v(k) \in l_2[0, +\infty)$. $z(k) \in R^r$, which needs to be estimated, is a linear combination of the states. A, B, D, C_y, D_y , and C_z are certain constant matrices with appropriate dimension.

To characterize the phenomenon of probabilistic diverging disturbance, $\rho(k)$ is a probabilistic process representing the disturbances process defined by

$$\rho(k) = \begin{cases} 1 & \text{if the disturbances are not diverged} \\ 0 & \text{if the disturbances are diverged by } 100 \frac{\varrho(k)}{u} \% \text{ and } 100(1 - \frac{\varrho(k)}{u}) \% \end{cases}$$

$\rho(k) \in R$ satisfies $Pro\{\rho(k) = 1\} = \bar{\rho}$ and $Pro\{\rho(k) = 0\} = 1 - \bar{\rho}$.

$\varrho(k) \in R$ is a binomial distributed sequence satisfying

$$E\{(\varrho(k))^2\} = u\bar{\varrho} - u\bar{\varrho}^2 + (u\bar{\varrho})^2, \quad E\{\varrho(k)\} = u\bar{\varrho}, \quad \text{where } u \text{ is the number of changes.}$$

The measurement with sensor data missing and time delay is described by

$$\begin{aligned} y(k) &= \Theta C_y x(k) + \sum_{j=1}^l \beta_j B_y x(k-j) + D_y v(k) \\ &= \sum_{j=1}^l \alpha_j C_{yj} x(k) + \sum_{j=1}^l \beta_j B_y x(k-j) + D_y v(k) \end{aligned} \quad (2)$$

where $\Theta = diag\{\alpha_1, \alpha_2, \dots, \alpha_l\}$, $C_{yj} = diag\{\underbrace{0, \dots, 0}_{j-1}, 1, \underbrace{0, \dots, 0}_{l-j}\} \times C_y (j = 1, \dots, l)$ are known constant matrices. It is assumed that α_j and β_j have the probabilistic density function $p_j(s), \delta_j(t)$ on the interval $[0, 1]$ with mathematical expectation $\bar{\alpha}_j, \bar{\beta}_j$ and variance $\sigma_j, \bar{\sigma}_j$ respectively.

Remark 1. The measurement output data could miss at a fraction rather than 0 or 1 [10]. The random time delays also describe the partial time delay information of the measurement output.

In the networked world nowadays, the nonlinear disturbance and the communication delays may be subject to random change in environmental disturbances, for instance, network-induced random failures and changing subsystem interconnections. Then RONS and RODs should be taken into the networked system.

The nonlinear networked system to be investigated in this paper is as follows:

$$\begin{cases} x(k+1) = Ax(k) + a(k)A_d g(x(k)) + \sum_{i=1}^q b_i(k)Bx(k - \tau_i(k)) + (\rho(k) + (1 - \rho(k))\frac{\varrho(k)}{u})D\omega(k) \\ y(k) = \sum_{j=1}^l \alpha_j C_{yj} x(k) + \sum_{j=1}^l \beta_j B_y x(k-j) + D_y v(k) \\ z(k) = C_z x(k) \end{cases} \quad (3)$$

$a(k)g(x(k))$ means RONS, where $g(x(\cdot)) = [g_1(x_1(\cdot)), g_2(x_2(\cdot)), \dots, g_2(x_2(\cdot))]$ is a nonlinear function, the $a(k) \in R$ is a stochastic variable satisfying

$$Pro(a(k) = 1) = \bar{a}, Pro(a(k) = 0) = 1 - \bar{a}$$

The variables $\tau_i(k) (i = 1, 2, \dots, q)$ denote the time-varying delays satisfying $\tau_i(k) \in [\underline{\tau}_i, \bar{\tau}_i]$, where $\underline{\tau}_i$ and $\bar{\tau}_i$ ($\tau_m = \underline{\tau}_1, \bar{\tau}_i = \underline{\tau}_{i+1}, \bar{\tau}_q = \tau_M$) are constant positive integers representing the lower and upper bounds on the communication delay, respectively. The stochastic variables $b_i(k) \in R (i = 1, \dots, q)$ are Bernoulli distributed white sequences satisfying the following probability distribution:

$$Pro(b_i(k) = 1) = \bar{b}_i, Pro(b_i(k) = 0) = (1 - \bar{b}_i) (i = 1, \dots, q)$$

Remark 2. In [29], Zhang considered the nonlinear networked system with RODs and RONS, in [4], Park took probabilistic diverging disturbance into neural network. The disturbances from estimator can not be ignored in neural network [4]. Probabilistic diverging disturbance has not been used in the nonlinear networked system.

Section 4 shows that the disturbances from estimator can not be ignored in the nonlinear networked system.

We construct the following state estimator as follows:

$$\left\{ \begin{array}{l} \hat{x}(k+1) = A\hat{x} + a(k)A_dg(\hat{x}(k)) + \sum_{i=1}^q b_i(k)B\hat{x}(k - \tau_i(k)) + K(k)[y(k) - UC_y\hat{x}(k) - \sum_{j=1}^l \bar{\beta}_j B_y\hat{x}(k-j)] \\ \quad + (\rho_k + (1 - \rho_k)(1 - \frac{\rho_k}{v}))D\omega(k) \\ \hat{z}(k) = C_z\hat{x}(k) \end{array} \right. \quad (4)$$

where $K(k) = K + \Delta K(k)$, $K \in R^{n \times l}$ is the estimator gain matrix, $\Delta K(k)$ represents the norm-bounded parameter uncertainty. $\Delta K(k) = MF(k)H, F^T(k)F(k) \leq I$, M, H are certain matrices with suitable dimension.

$U = diag\{\bar{\alpha}_1, \bar{\alpha}_2, \dots, \bar{\alpha}_l\}$. $\hat{x}(k)$ denotes the estimate of $x(k)$.

Using (3), (4) and the error vector $\bar{x}(k) = x(k) - \hat{x}(k)$, we obtain the error dynamical equations as follow:

$$\left\{ \begin{array}{l} \bar{x}(k+1) = \tilde{A}(k)\bar{x}(k) - K(k)(\Theta - U)C_yx(k) + (a(k) - \bar{a})A_d f(\bar{x}) + \bar{a}A_d f(\bar{x}) + \sum_{i=1}^q (b_i(k) - \bar{b}_i)B\bar{x}(k - \tau_i(k)) \\ \quad + \sum_{i=1}^q \bar{b}_i B\bar{x}(k - \tau_i(k)) - \sum_{j=1}^l \bar{\beta}_j K(k)B_y\bar{x}(k-j) - \sum_{j=1}^l (\beta_j - \bar{\beta}_j)K(k)B_yx(k-j) \\ \quad - (1 - \rho(k))(1 - 2\frac{\rho(k)}{u})D\omega(k) - K(k)D_yv(k) \\ \bar{z}(k) = C_z\bar{x}(k) \end{array} \right. \quad (5)$$

where $\tilde{A}(k) = A - K(k)UC_y$, $f(\bar{x}) = g(x(k)) - g(\hat{x}(k))$, $\bar{z}(k) = z(k) - \hat{z}(k)$.

Taking the augmented vectors as $e(\cdot) = [x^T(\cdot), \bar{x}^T(\cdot)]^T$, $\tilde{z}(k) = [z^T(k), \bar{z}^T(k)]^T$ according to (3), (5), we can obtain

$$\left\{ \begin{array}{l} e(k+1) = \bar{A}(k)e(k) + N(k)e(k) + (a(k) - \bar{a})\bar{A}_d\phi(e(k)) + \bar{a}\phi(e(k)) + \sum_{i=1}^q (b_i(k) - \bar{b}_i)\bar{B}e(k - \tau_i(k)) \\ \quad + \sum_{i=1}^q \bar{b}_i\bar{B}e(k - \tau_i(k)) + \sum_{j=1}^l \bar{\beta}_j B_{y1}(k)e(k - j) + \sum_{j=1}^l (\beta_j(k) - \bar{\beta}_j)B_{y2}(k)e(k - j) \\ \quad + \bar{D}\omega(k) + \bar{D}_y(k)v(k) \\ \tilde{z}(k) = \tilde{C}_z e(k) \end{array} \right. \quad (6)$$

where $\phi(e(k)) = [g^T(x(k)), f^T(\bar{x}(k))]^T$, $\bar{A}(k) = \text{diag}\{A(k), \tilde{A}(k)\}$, $\bar{B} = \text{diag}\{B, B\}$, $B_{y1}(k) = \text{diag}\{0, -K(k)B_y\}$,

$\bar{D} = [(\rho(k) + (1 - \rho(k))(\frac{\varrho(k)}{u}))D^T, -(1 - \rho(k))(1 - 2\frac{\varrho(k)}{u})D^T]^T$, $\bar{D}_y(k) = [0^T, -(K(k)D_y)^T]^T$,

$$N(k) = \begin{bmatrix} 0 & 0 \\ -K(k)(\Theta - U)C_y & 0 \end{bmatrix}, \quad B_{y2}(k) = \begin{bmatrix} 0 & 0 \\ -K(k)B_y & 0 \end{bmatrix}, \quad \tilde{C}_z = \text{diag}\{C_z, C_z\}.$$

Before obtaining the main results of the paper, we state the following preliminaries.

Definition 1. For all non-zero $\omega(k) \in l_2[0, \infty)$, $v(k) \in l_2[0, \infty)$, the augmented system (6) is said to be asymptotically mean square stable with its estimation error satisfying a prescribed H_∞ disturbance attention level γ , if the following conditions are satisfied:

- 1) with zero disturbance, the system (6) is asymptotically mean square stable;
- 2) There exists a positive scalar γ that satisfies the following inequality:

$$J = E\left\{\sum_{k=0}^{\infty} \{\bar{z}^T(k)\bar{z}(k) - \gamma^2\omega^T(k)\omega(k) - \gamma^2v^T(k)v(k)\}\right\} < 0 \quad (7)$$

under the zero-initial condition for all nonzero $\omega(k) \in l_2[0, \infty)$, $v(k) \in l_2[0, \infty)$.

Lemma 1.[10] Let $N = N^T$, H and E be real matrices with appropriate dimension, and $F^T(k)F(k) \leq I$.

Then the inequality $N + HF(k)E + (HF(k)E)^T < 0$, if only if there exists a positive scalar ϵ such that

$N + \epsilon HH^T + \epsilon^{-1}E^TE < 0$ or

$$\begin{bmatrix} N & \epsilon H & E^T \\ \epsilon H^T & -\epsilon I & 0 \\ E & 0 & -\epsilon I \end{bmatrix} < 0 \quad (8)$$

Assumption 1. The stochastic variables $a(k), b_i(k), \rho(k), \varrho(k)$ and $\alpha_j, \beta_j, (i = 1, \dots, q, j = 1, \dots, l)$ are mutually independent.

Assumption 2. The nonlinear functions in (3) is globally Lipschitz, i.e. the following inequality holds:

$$h_i^- \leq \frac{g_i(\alpha) - g_i(\beta)}{\alpha - \beta} \leq h_i^+ \quad (9)$$

where $0 < h_i^- < h_i^+, \forall \alpha, \beta \in R, \alpha \neq \beta, i = 1, 2, \dots, n$.

Remark 3. In **Assumption 2**, if $0 \leq h_i^- < h_i^+$ holds. According to (9), we have

$$\begin{aligned} g_i^2(x_i) - h_i g_i(x_i) x_i &\leq 0 \\ f_i^2(\bar{x}_i) - h_i f_i(\bar{x}_i) \bar{x}_i &\leq 0 \end{aligned} \quad (10)$$

where $h_i = h_i^- + h_i^+$

3 Main results

In this section, we firstly consider the augmented system (6) without uncertain($\Delta K(k) = 0$) is asymptotically mean square stable. A sufficient condition is given based on the estimation error satisfying a prescribed H_∞ disturbance attenuation level γ .

Theorem 1. For given scalars $\tau_M > \tau_m \geq 0, \bar{a} > 0, \bar{b}_i > 0, \bar{\alpha}_j > 0, \bar{\beta}_j > 0, \bar{\rho} > 0, \bar{\varrho} > 0$, positive integers q, l , the system (6) without uncertain($\Delta K(k) = 0$) is asymptotically mean square stable with the estimation error satisfying a prescribed H_∞ disturbance attenuation level γ , if there exist symmetric positive definite matrices $P, S_1, S_2, G_i, T_j, (i = 1, \dots, q, j = 1, \dots, l)$, diagonal positive semi-definite matrices F and Σ such that

$$Z = \begin{bmatrix} G & Z_1 \\ * & Z_2 \end{bmatrix} < 0 \quad (11)$$

where

$$G = \begin{bmatrix} G_{11} & 0 & 0 & G_{14} & G_{15} & G_{16} \\ * & G_{22} & 0 & 0 & 0 & 0 \\ * & * & G_{33} & 0 & 0 & 0 \\ * & * & * & G_{44} & G_{45} & G_{46} \\ * & * & * & * & G_{55} & G_{56} \\ * & * & * & * & * & G_{66} \end{bmatrix} \quad (12)$$

$$G_{11} = \bar{A}^T(k) P \bar{A}(k) + \sum_{j=1}^l (\sigma_j^2 N_j^T(k) P N_j(k) + T_j) - P + S_1 + \sum_{i=1}^q (\Delta \tau_i + 1) G_i + \bar{C}_z^T \bar{C}_z,$$

$$G_{14} = [\bar{b}_1, \dots, \bar{b}_q] \bar{A}^T(k) P \bar{B}, G_{15} = \bar{a} \bar{A}^T P \bar{A}_d + F \Sigma, G_{16} = [\bar{\beta}_1, \dots, \bar{\beta}_l] \bar{A}^T P \bar{B}_{y1}, G_{22} = S_2 - S_1, G_{33} = -S_2,$$

$$\begin{aligned}
G_{44} &= \begin{bmatrix} (\bar{b}_1^2 + \bar{b}_1)\bar{B}^T P \bar{B} - G_1 & \bar{b}_1 \bar{b}_2 \bar{B}^T P \bar{B} & \dots & \bar{b}_1 \bar{b}_q \bar{B}^T P \bar{B} \\ \bar{b}_2 \bar{b}_1 \bar{B}^T P \bar{B} & (\bar{b}_2^2 + \bar{b}_2)\bar{B}^T P \bar{B} - G_2 & \dots & \bar{b}_2 \bar{b}_q \bar{B}^T P \bar{B} \\ \vdots & \vdots & \dots & \vdots \\ \bar{b}_q \bar{b}_1 \bar{B}^T P \bar{B} & \bar{b}_q \bar{b}_2 \bar{B}^T P \bar{B} & \dots & (\bar{b}_q^2 + \bar{b}_q)\bar{B}^T P \bar{B} - G_q \end{bmatrix}, \\
G_{45} &= [\bar{a}\bar{b}_1, \dots, \bar{a}\bar{b}_q]^T \bar{B}^T P \bar{A}_d, \quad G_{46} = \begin{bmatrix} \bar{b}_1 \bar{\beta}_1 \bar{B}^T P B_{y1} & \bar{b}_2 \bar{\beta}_1 \bar{B}^T P B_{y1} & \dots & \bar{b}_q \bar{\beta}_1 \bar{B}^T P B_{y1} \\ \bar{b}_1 \bar{\beta}_2 \bar{B}^T P B_{y1} & \bar{b}_2 \bar{\beta}_2 \bar{B}^T P B_{y1} & \dots & \bar{b}_q \bar{\beta}_2 \bar{B}^T P B_{y1} \\ \vdots & \vdots & \dots & \vdots \\ \bar{b}_1 \bar{\beta}_l \bar{B}^T P B_{y1} & \bar{b}_2 \bar{\beta}_l \bar{B}^T P B_{y1} & \dots & \bar{b}_q \bar{\beta}_l \bar{B}^T P B_{y1} \end{bmatrix}, \\
G_{55} &= (\bar{a} + \bar{a}^2)\bar{A}_d^T P \bar{A}_d - 2\Sigma, \quad G_{56} = [\bar{a}\bar{\beta}_1, \dots, \bar{a}\bar{\beta}_l]\bar{A}_d^T P B_{y1},
\end{aligned}$$

$$\begin{aligned}
G_{66} &= \begin{bmatrix} \bar{\sigma}_1^2 B_{y2}^T P B_{y2} + \bar{\beta}_1^2 B_{y1}^T P B_{y1} - T_1 & \bar{\beta}_1 \bar{\beta}_2 B_{y1}^T P B_{y1} \\ \bar{\beta}_2 \bar{\beta}_1 B_{y1}^T P B_{y1} & \bar{\sigma}_2^2 B_{y2}^T P B_{y2} + \bar{\beta}_2^2 B_{y1}^T P B_{y1} - T_2 \\ \vdots & \vdots \\ \bar{\beta}_l \bar{\beta}_1 B_{y1}^T P B_{y1} & \bar{\beta}_l \bar{\beta}_2 B_{y1}^T P B_{y1} \\ \dots & \bar{\beta}_1 \bar{\beta}_l B_{y1}^T P B_{y1} \\ \dots & \bar{\beta}_2 \bar{\beta}_l B_{y1}^T P B_{y1} \\ \dots & \vdots \\ \dots & \bar{\sigma}_l^2 B_{y2}^T P B_{y2} + \bar{\beta}_l^2 B_{y1}^T P B_{y1} - T_l \end{bmatrix},
\end{aligned}$$

$$\begin{aligned}
Z_1 &= \begin{bmatrix} \tilde{D}^T P \bar{A}(k) & 0 & 0 & [\bar{b}_1, \dots, \bar{b}_q] \tilde{D}^T P \bar{B} & \bar{a} \tilde{D}^T P A_d & [\bar{\beta}_1, \dots, \bar{\beta}_l] \bar{D}^T P B_{y1} \\ D_y^T P \bar{A}(k) & 0 & 0 & [\bar{b}_1, \dots, \bar{b}_q] D_y^T P \bar{B} & D_y^T P \bar{\pi} & [\bar{\beta}_1, \dots, \bar{\beta}_l] D_y^T P B_{y1} \end{bmatrix}^T, \\
Z_2 &= \begin{bmatrix} \tilde{D}^T P \tilde{D} + \check{D}^T P \check{D} & \tilde{D}^T P D_y \\ * & D_y^T P D_y \end{bmatrix}, \\
N_j &= \begin{bmatrix} 0 & 0 \\ -K(k)C_{yj} & 0 \end{bmatrix}, \quad \bar{C}_z = \begin{bmatrix} 0 & 0 \\ 0 & C_z^T C_z \end{bmatrix}, \quad \tilde{D} = [(\bar{\rho} + (1 - \bar{\rho})\bar{\varrho})\bar{\varrho} D^T, -(1 - \bar{\rho})(1 - 2\bar{\varrho})D^T]^T, \\
\check{D} &= [\sqrt{(\bar{\rho} + (1 - \bar{\rho})\frac{(1 - \bar{\varrho})\bar{\varrho} + u\bar{\varrho}^2}{u})} D^T, -\sqrt{(1 - \bar{\rho})(1 - 4\bar{\varrho} + 4\frac{(1 - \bar{\varrho})\bar{\varrho} + u\bar{\varrho}^2}{u})} D^T]^T, \quad \Lambda_1 = \{\lambda_{11}, \lambda_{12}, \dots, \lambda_{1n}\}, \quad \Lambda_2 = \{\lambda_{21}, \lambda_{22}, \dots, \lambda_{2n}\}, \quad \Sigma = \text{diag}\{\Lambda_1, \Lambda_2\},
\end{aligned}$$

$$F = \text{diag}\{\underbrace{h_1, \dots, h_n}_{F_1}, \underbrace{h_1, \dots, h_n}_{F_1}\}, \quad \Delta\tau_i = \bar{\tau}_i - \underline{\tau}_i.$$

Proof. We consider (6) with $\omega(k) = 0$ and $v(k) = 0$

$$\begin{aligned} e(k+1) = & \bar{A}(k)e(k) + N(k)e(k) + (a(k) - \bar{a})\bar{A}_d\phi(e(k)) + \bar{a}\bar{A}_d\phi(e(k)) + \sum_{i=1}^q (b_i(k) - \bar{b}_i)\bar{B}e(k - \tau_i(k)) \\ & + \sum_{i=1}^q \bar{b}_i\bar{B}e(k - \tau_i(k)) + \sum_{j=1}^l \bar{\beta}_j B_{y1}(k)e(k-j) + \sum_{j=1}^l (\beta_j(k) - \bar{\beta}_j)B_{y2}(k)e(k-j) \end{aligned} \quad (13)$$

The Lyapunov function is constructed as follows:

$$V(k) = V_1(k) + V_2(k) + V_3(k) + V_4(k) \quad (14)$$

where

$$\begin{aligned} V_1(k) &= e^T(k)Pe(k) \\ V_2(k) &= \sum_{i=k-\tau_m}^{k-1} e^T(i)S_1e(i) + \sum_{i=k-\tau_M}^{k-1-\tau_m} e^T(i)S_2e(i) \\ V_3(k) &= \sum_{j=1}^l \sum_{i=k-j}^{k-1} e^T(i)T_je(i) \\ V_4(k) &= \sum_{i=1}^q \sum_{j=k-\tau_i(k)}^{k-1} e^T(j)G_ie(j) + \sum_{i=1}^q \sum_{\rho=-\bar{\tau}_i}^{-\tau_i} \sum_{j=k+\rho}^{k-1} e^T(j)G_ie(j) \end{aligned}$$

The difference of $V_1(k)$ along the solution of (13) is as follows:

$$\begin{aligned} E\{\Delta V_1(k)\} &= E\{V_1(k+1) - V_1(k)\} \\ &= E\{(\bar{A}(k)e(k) + N(k)e(k) + (a(k) - \bar{a})\bar{A}_d\phi(e(k)) + \bar{a}\bar{A}_d\phi(e(k)) + \sum_{i=1}^q (b_i(k) - \bar{b}_i)\bar{B}e(k - \tau_i(k)) \\ &\quad + \sum_{i=1}^q \bar{b}_i\bar{B}e(k - \tau_i(k)) + \sum_{j=1}^l \bar{\beta}_j B_{y1}(k)e(k-j) + \sum_{j=1}^l (\beta_j(k) - \bar{\beta}_j)B_{y2}(k)e(k-j))^T P \\ &\quad (\bar{A}(k)e(k) + N(k)e(k) + (a(k) - \bar{a})\bar{A}_d\phi(e(k)) + \bar{a}\bar{A}_d\phi(e(k)) + \sum_{i=1}^q (b_i(k) - \bar{b}_i)\bar{B}e(k - \tau_i(k)) \\ &\quad + \sum_{i=1}^q \bar{b}_i\bar{B}e(k - \tau_i(k)) + \sum_{j=1}^l \bar{\beta}_j B_{y1}(k)e(k-j) + \sum_{j=1}^l (\beta_j(k) - \bar{\beta}_j)B_{y2}(k)e(k-j)) - e^T(k)Pe(k)\} \\ &= e^T(k)\bar{A}(k)^T P \bar{A}(k)e(k) + 2e^T(k)\bar{A}^T(k)P\bar{a}\bar{A}_d\phi(e(k)) + 2e^T(k)\sum_{i=1}^q \bar{b}_i\bar{A}^T(k)P\bar{B}e(k - \tau_i(k)) \\ &\quad + 2e^T(k)\sum_{j=1}^l \bar{\beta}_j\bar{A}^T(k)PB_{y1}(k)e(k-j) + e^T(k)\sum_{j=1}^l \sigma_j^2 \bar{N}_j^T(k)P\bar{N}_j(k)e(k) + \bar{a}(1 - \bar{a})\phi^T(e(k))\bar{A}_d^T P \bar{A}_d\phi(e(k)) \\ &\quad + \bar{a}^2\phi^T(e(k))\bar{A}_d^T P \bar{A}_d\phi(e(k)) + 2\sum_{i=1}^q \bar{a}\bar{b}_i\phi^T(e(k))\bar{A}_d^T P \bar{B}e(k - \tau_i(k)) + 2\sum_{j=1}^l \bar{a}\bar{b}_j\phi^T(e(k))\bar{A}_d^T P B_{y1}(k)e(k-j) \\ &\quad + \sum_{i=1}^q \bar{b}_i(1 - \bar{b}_i)e^T(k - \tau_i(k))\bar{B}^T P \bar{B}e(k - \tau_i(k)) + \sum_{i=1}^q \sum_{j=1}^q \bar{b}_i\bar{b}_je^T(k - \tau_i(k))\bar{B}^T P \bar{B}e(k - \tau_j(k)) \end{aligned}$$

$$\begin{aligned}
& + 2 \sum_{i=1}^q \sum_{j=1}^l \bar{b} \bar{\beta}_j e^T(k - \tau_j(k)) \bar{B}^T P \bar{B}_{y1}(k) e(k - j) - e^T(k) P e(k) \\
& + \sum_{j=1}^l \sum_{i=1}^l \bar{\beta}_j \bar{\beta}_i e^T(k - j) B_{y1}^T(k) P B_{y1}(k) e(k - i) + \sum_{i=1}^l \bar{\sigma}_i^2 e^T(k - j) B_{y2}^T(k) P B_{y2}(k) e(k - j)
\end{aligned} \tag{15}$$

$$E\{\Delta V_2(k)\} = E\{e^T(k) S_1 e(k) - e^T(k - \tau_m)(S_1 - S_2) e(k - \tau_m) - e^T(k - \tau_M) S_2 e(k - \tau_M)\} \tag{16}$$

$$E\{\Delta V_3(k)\} = E\left\{\sum_{j=1}^l (e^T(k) T_j e(k) - e^T(k - j) T_j e(k - j))\right\} \tag{17}$$

$$\begin{aligned}
E\{\Delta V_4(k)\} &= E\{V_4(k+1) - V_4(k)\} \\
&= E\left\{\sum_{i=1}^q \sum_{j=k+1-\tau_i(k+1)}^k e^T(j) G_i e(j) - \sum_{i=1}^q \sum_{j=k-\tau_i(k)}^{k-1} e^T(j) G_i e(j) \right. \\
&\quad \left. + \sum_{i=1}^q \sum_{\rho=-\bar{\tau}_i}^{-\underline{\tau}_i} \sum_{j=k+1+\rho}^k e^T(j) G_i e(j) - \sum_{i=1}^q \sum_{\rho=-\bar{\tau}_i}^{-\underline{\tau}_i} \sum_{j=k+\rho}^{k-1} e^T(j) G_i e(j)\right\} \\
&= E\left\{\sum_{i=1}^q (e^T(k) G_i e(k) - e^T(k - \tau_i(k)) G_i e(k - \tau_i(k))) + \sum_{i=1}^q \sum_{j=k+1-\tau_i(k)}^{k-\underline{\tau}_i} e^T(j) G_i e(j) \right. \\
&\quad \left. + \sum_{i=1}^q \sum_{j=k-\underline{\tau}_i+1}^{k-1} e^T(j) G_i e(j) - \sum_{i=1}^q \sum_{j=k-\tau_i(k)+1}^{k-1} e^T(j) G_i e(j) + \sum_{i=1}^q \sum_{\rho=-\bar{\tau}_i}^{-\underline{\tau}_i} \sum_{j=k+1+\rho}^k e^T(j) G_i e(j) \right. \\
&\quad \left. - \sum_{i=1}^q \sum_{\rho=-\bar{\tau}_i}^{-\underline{\tau}_i} \sum_{j=k+\rho}^{k-1} e^T(j) G_i e(j)\right\} \\
&\leq E\left\{\sum_{i=1}^q (e^T(k) G_i e(k) - e^T(k - \tau_i(k)) G_i e(k - \tau_i(k))) + \sum_{i=1}^q \sum_{j=k-\bar{\tau}_i}^{k-\underline{\tau}_i} e^T(j) G_i e(j) \right. \\
&\quad \left. + \sum_{i=1}^q \sum_{j=k-\underline{\tau}_i+1}^{k-1} e^T(j) G_i e(j) \right. \\
&\quad \left. - \sum_{i=1}^q \sum_{j=k-\underline{\tau}_i+1}^{k-1} e^T(j) G_i e(j) + \sum_{i=1}^q (\bar{\tau}_i - \underline{\tau}_i) e^T(k) G_i e(k) - \sum_{i=1}^q \sum_{j=k-\bar{\tau}_i}^{k-\underline{\tau}_i} e^T(j) G_i e(j)\right\} \\
&= E\left\{\sum_{i=1}^q ((\Delta \tau_i + 1) e^T(k) G_i e(k) - e^T(k - \tau_i(k)) G_i e(k - \tau_i(k)))\right\}
\end{aligned} \tag{18}$$

According to **Remark 3**, the following inequalities hold for any positive diagonal matrices

$$\Lambda_1 = \text{diag}\{\lambda_{11}, \lambda_{12}, \dots, \lambda_{1n}\}, \Lambda_2 = \text{diag}\{\lambda_{21}, \lambda_{22}, \dots, \lambda_{2n}\}$$

$$\begin{aligned}
0 &\leq - \sum_{j=1}^n 2(\lambda_{1j} g_j^2(x_j(k)) - h_j \lambda_{1j} g_j(x_j(k)) x_j(k)) \\
0 &\leq - \sum_{j=1}^n 2(\lambda_{2j} f_j^2(\bar{x}_j(k)) - h_j \lambda_{2j} f_j(\bar{x}_j(k)) \bar{x}_j(k))
\end{aligned} \tag{19}$$

Using (15), (16), (17), (18), (19) , we get

$$E\{\Delta V(k)\} \leq \Psi^T G \Psi \quad (20)$$

where $\Psi(k) = [e^T(k), \quad e^T(k - \tau_m), \quad e^T(k - \tau_M), \quad e^T(k - \tau_1(k)), \dots, \quad e^T(k - \tau_q(k)), \quad \phi^T(k),$
 $e^T(k - 1), \dots, \quad e^T(k - l)]^T$

$G < 0$ since $Z < 0$ and G is the principal minor of the matrix Z , which means (13) is asymptotically mean square stable without disturbances.

Next, we prove that the estimation error satisfies a prescribed H_∞ disturbance attenuation level γ . We choose the same Lyapunov function, the following inequality can be obtained

$$E\{V(k+1)\} - E\{V(k)\} + E\{\bar{z}^T(k)\bar{z}(k) - \gamma^2\omega^T(k)\omega(k) - \gamma^2v^T(k)v(k)\} = \Upsilon(k)^T Z \Upsilon(k) < 0 \quad (21)$$

where $\Upsilon(k) = [e^T(k), \quad e^T(k - \tau_m), \quad e^T(k - \tau_M), \quad e^T(k - \tau_1(k)), \dots, \quad e^T(k - \tau_q(k)), \quad \phi^T(k),$
 $e^T(k - 1), \quad \dots, \quad e^T(k - l), \quad \omega^T(k), \quad v^T(k)]^T$

Summing from 0 to ∞ on both sides of (21), we get

$$\sum_{k=0}^{\infty} E\{\bar{z}^T(k)\bar{z}(k) - \gamma^2\omega^T(k)\omega(k) - \gamma^2v^T(k)v(k)\} + E\{V(\infty)\} - E\{V(0)\} < 0$$

Considering the zero initial condition $\Psi(0) = 0$, i.e $V(0) = 0$, we get that J in (7) is equivalent to

$$\begin{aligned} J &= \sum_{k=0}^{\infty} E\{\bar{z}^T(k)\bar{z}(k) - \gamma^2\omega^T(k)\omega(k) - \gamma^2v^T(k)v(k)\} \\ &= \sum_{k=0}^{\infty} E\{\bar{z}^T(k)\bar{z}(k) - \gamma^2\omega^T(k)\omega(k) - \gamma^2v^T(k)v(k) + \Delta(V(k))\} - E\{V(\infty)\} + E\{V(0)\} \\ &\leq \sum_{k=0}^{\infty} E\{\bar{z}^T(k)\bar{z}(k) - \gamma^2\omega^T(k)\omega(k) - \gamma^2v^T(k)v(k) + \Delta(V(k))\} \\ &= \sum_{k=0}^{\infty} E\{e^T(k)\bar{C}_z^T \bar{C}_z e(k) - \gamma^2\omega^T(k)\omega(k) - \gamma^2v^T(k)v(k) + \Delta(V(k))\} < 0 \end{aligned} \quad (22)$$

Since $Z < 0$, $J < 0$ holds for any $\Upsilon \neq 0, v(k), \omega(k) \in l_2[0, \infty)$. According to **Definition 1**, the system (6) is asymptotically mean square stable with the estimation error satisfying a prescribed H_∞ disturbance attenuation level γ . The proof is completed.

Based on **Theorem 1**, we can give the following theorem to obtain an non-fragile H_∞ estimator for the nonlinear networked system (3).

Theorem 2. For given scalars $\tau_M > \tau_m \geq 0$, $\bar{a} > 0, \bar{b}_i > 0, \bar{\alpha}_j > 0, \bar{\beta}_j > 0, \bar{\rho} > 0, \bar{\varrho} > 0$, positive integers q, l , the system (6) is asymptotically mean square stable with the estimation error satisfying a prescribed H_∞

disturbance attenuation level γ , if there exist positive matrices $P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$, $S_1 = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{13} \end{bmatrix}$, $S_2 = \begin{bmatrix} S_{21} & S_{22} \\ * & S_{23} \end{bmatrix}$, $G_i = \begin{bmatrix} G_{i1} & G_{i2} \\ * & G_{i3} \end{bmatrix}$, $T_j = \begin{bmatrix} T_{j1} & T_{j2} \\ * & T_{j3} \end{bmatrix}$ ($i = 1, 2, \dots, q, j = 1, 2, \dots, l$), diagonal positive semi-definite matrices $\tilde{\Sigma}$ and F and matrix S such that the following inequalities hold

$$\Gamma = \begin{bmatrix} \tilde{Z} & \epsilon_1 \Sigma_2^T + \epsilon_2 \Sigma_4^T + \epsilon_3 \Sigma_5^T & \Sigma_1^T + \Sigma_3^T + \Sigma_6^T \\ * & -(\epsilon_1 + \epsilon_2 + \epsilon_3)I & 0 \\ * & * & -(\epsilon_1 + \epsilon_2 + \epsilon_3)I \end{bmatrix} < 0 \quad (23)$$

$$\tilde{Z} = \begin{bmatrix} \tilde{Z}_{11} & 0 & 0 & 0 & 0 & \tilde{Z}_{16} & \tilde{Z}_{17} & \tilde{Z}_{18} & \tilde{Z}_{19} & 0 & 0 \\ * & \tilde{Z}_{22} & \tilde{Z}_{23} & 0 & 0 & \tilde{Z}_{26} & \tilde{Z}_{27} & \tilde{Z}_{28} & \tilde{Z}_{29} & \tilde{Z}_{210} & 0 \\ * & * & \tilde{Z}_{33} & 0 & 0 & 0 & \tilde{Z}_{37} & 0 & 0 & 0 & \tilde{Z}_{311} \\ * & * & * & \tilde{Z}_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \tilde{Z}_{55} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \tilde{Z}_{66} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \tilde{Z}_{77} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \tilde{Z}_{88} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \tilde{Z}_{99} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \tilde{Z}_{1010} & 0 \\ * & * & * & * & * & * & * & * & * & * & \tilde{Z}_{1111} \end{bmatrix} \quad (24)$$

where $\tilde{Z}_{11} = \tilde{Z}_{22} = \begin{bmatrix} -P_1 & 0 \\ 0 & -P_2 \end{bmatrix}$, $\tilde{Z}_{16} = \left[\begin{bmatrix} \sqrt{\bar{b}_1} P_1 B & 0 \\ 0 & \sqrt{\bar{b}_1} P_2 B \end{bmatrix}, \dots, \begin{bmatrix} \sqrt{\bar{b}_q} P_1 B & 0 \\ 0 & \sqrt{\bar{b}_q} P_2 B \end{bmatrix} \right]$,

$\tilde{Z}_{17} = \begin{bmatrix} \sqrt{\bar{a}} P_1 A_d & 0 \\ 0 & \sqrt{\bar{a}} P_2 A_d \end{bmatrix}$, $\tilde{Z}_{18} = \left[\begin{bmatrix} 0 & 0 \\ -\bar{\sigma}_1 S B_y & 0 \end{bmatrix}, \dots, \begin{bmatrix} 0 & 0 \\ -\bar{\sigma}_l S B_y & 0 \end{bmatrix} \right]$,

$\tilde{Z}_{19} = \begin{bmatrix} \sqrt{(\bar{\rho} + (1 - \bar{\rho}) \frac{(1 - \bar{\varrho}) \bar{\varrho} + u \bar{\varrho}^2}{u})} P_1 D & 0 \\ 0 & -\sqrt{(1 - \bar{\rho})(1 - 4\varrho + 4 \frac{(1 - \bar{\varrho}) \bar{\varrho} + u \bar{\varrho}^2}{u})} P_2 D \end{bmatrix}$,

$\tilde{Z}_{23} = \begin{bmatrix} P_1 A & 0 \\ 0 & P_2 A - SUC_y \end{bmatrix}$, $\tilde{Z}_{27} = \begin{bmatrix} \bar{a} A_d P_1 & 0 \\ 0 & \bar{a} A_d P_2 \end{bmatrix}$,

$$\begin{aligned}
\tilde{Z}_{26} &= \left[\begin{bmatrix} \bar{b}_1 P_1 B_2 & 0 \\ 0 & \bar{b}_1 P_2 B_2 \end{bmatrix}, \dots, \begin{bmatrix} \bar{b}_q P_1 B_2 & 0 \\ 0 & \bar{b}_q P_2 B_2 \end{bmatrix} \right], \\
\tilde{Z}_{28} &= \left[\begin{bmatrix} 0 & 0 \\ 0 & -\bar{\beta}_1 S B_y \end{bmatrix}, \dots, \begin{bmatrix} 0 & 0 \\ 0 & -\bar{\beta}_l S B_y \end{bmatrix} \right], \\
\tilde{Z}_{29} &= \begin{bmatrix} \bar{\rho} + (1 - \bar{\rho})\bar{\varrho} P_1 D & 0 \\ 0 & -(1 - \bar{\rho})(1 - 2\bar{\varrho}) P_2 D \end{bmatrix}, \quad \tilde{Z}_{210} = \begin{bmatrix} 0 \\ -S D_y \end{bmatrix}, \\
\tilde{Z}_{33} &= \begin{bmatrix} -P_1 + S_{11} + \sum_{i=1}^q (\Delta\tau_i + 1)G_{i1} + \sum_{j=1}^l T_{j1} & S_{12} + \sum_{i=1}^q (\Delta\tau_i + 1)G_{i2} + \sum_{j=1}^l T_{j2} \\ * & P_2 + S_{13} + \sum_{i=1}^q (\Delta\tau_i + 1)G_{i3} + \sum_{j=1}^l T_{j3} \end{bmatrix}, \\
\tilde{Z}_{37} &= \begin{bmatrix} \Lambda_1 F 1 & 0 \\ 0 & \Lambda_2 F 1 \end{bmatrix}, \quad \tilde{Z}_{311} = \left[\begin{bmatrix} -\sigma_1 C_{y1}^T S^T & 0 \\ * & 0 \end{bmatrix}, \dots, \begin{bmatrix} -\sigma_l C_{yl}^T(k) S^T(k) & 0 \\ * & 0 \end{bmatrix} \right], \\
\tilde{Z}_{44} &= \begin{bmatrix} S_{21} - S_{11} & S_{22} - S_{12} \\ * & S_{23} - S_{13} \end{bmatrix}, \\
\tilde{Z}_{55} &= \begin{bmatrix} -S_{21} & -S_{22} \\ * & -S_{23} \end{bmatrix}, \quad \tilde{Z}_{66} = \text{diag} \left\{ \begin{bmatrix} -G_{11} & -G_{12} \\ * & -G_{13} \end{bmatrix}, \dots, \begin{bmatrix} -G_{q1} & -G_{q2} \\ * & -G_{q3} \end{bmatrix} \right\}, \\
\tilde{Z}_{77} &= \begin{bmatrix} -2\Sigma \end{bmatrix}, \quad \tilde{Z}_{88} = \text{diag} \left\{ \begin{bmatrix} -T_{11} & -T_{12} \\ * & -T_{13} \end{bmatrix}, \dots, \begin{bmatrix} -T_{l1} & -T_{l2} \\ * & -T_{l3} \end{bmatrix} \right\}, \\
\tilde{Z}_{99} &= -\gamma^2 I, \quad \tilde{Z}_{1010} = -\gamma^2 I, \\
\tilde{Z}_{1111} &= \text{diag} \left\{ \underbrace{\begin{bmatrix} -P_1 & 0 \\ * & -P_2 \end{bmatrix}, \dots, \begin{bmatrix} -P_1 & 0 \\ * & -P_2 \end{bmatrix}}_l \right\},
\end{aligned}$$

$$\Sigma_1 = [0, -(P_2 M)^T, \underbrace{0, \dots, 0}_{12+2q+4l}],$$

$$\Sigma_2 = [\underbrace{0, \dots, 0}_{12+2q}, \underbrace{\bar{\sigma}_1 H B_y, 0, \dots, \bar{\sigma}_l H B_y}_{2l}, \underbrace{0, 0, \dots, 0}_{2+2l}],$$

$$\Sigma_3 = [0, 0, 0, -(P_2 M)^T, \underbrace{0, \dots, 0}_{10+2q+4l}],$$

$$\Sigma_4 = [0, 0, 0, 0, 0, H U C_y, \underbrace{0, \dots, 0}_{4+2q}, 0, 0, 0, \underbrace{0, \bar{\beta}_1 H B_y, \dots, 0, \bar{\beta}_l H B_y}_{2l}, 0, H D_y, \underbrace{0, \dots, 0}_{2l}],$$

$$\Sigma_5 = [0, 0, 0, 0, -H C_y, \underbrace{0, \dots, 0}_{9+2q+4l}],$$

$$\Sigma_6 = [\underbrace{0, \dots, 0}_{14+2q+2l}, \underbrace{\sigma_1(P_2 M)^T, 0, \dots, \sigma_q(P_2 M)^T}_{2l}, 0].$$

then the H_∞ estimator gain matrix is given by

$$K = P_2^{-1} S \quad (25)$$

Proof. We substitute the following matrices:

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}, S_1 = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{13} \end{bmatrix},$$

$$S_2 = \begin{bmatrix} S_{21} & S_{22} \\ * & S_{23} \end{bmatrix}, G_i = \begin{bmatrix} G_{i1} & G_{i2} \\ * & G_{i3} \end{bmatrix}, T_i = \begin{bmatrix} T_{i1} & T_{i2} \\ * & T_{i3} \end{bmatrix}.$$

into Z , and define

$$P_2 K = S \quad (26)$$

the LMIs (11) are equivalent to the following inequality by virtue of Schur complement in [25].

$$\tilde{Z} + \tilde{\Sigma}_1^T \Delta K \tilde{\Sigma}_2 + \tilde{\Sigma}_2^T \Delta K^T \tilde{\Sigma}_1 + \tilde{\Sigma}_3^T \Delta K \tilde{\Sigma}_4 + \tilde{\Sigma}_4^T \Delta K^T \tilde{\Sigma}_3 + \tilde{\Sigma}_5^T \Delta K^T \tilde{\Sigma}_6 + \tilde{\Sigma}_6^T \Delta K \tilde{\Sigma}_5 \leq 0$$

where

$$\tilde{\Sigma}_1 = [0, -P_2^T, \underbrace{0, \dots, 0}_{12+4q+2l}],$$

$$\tilde{\Sigma}_2 = [\underbrace{0, \dots, 0}_{12+2q}, \underbrace{\bar{\sigma}_1 B_y, 0, \dots, \bar{\sigma}_l B_y}_{2l}, \underbrace{0, 0, \dots, 0}_{2+2q}],$$

$$\tilde{\Sigma}_3 = [0, 0, 0, -P_2^T, \underbrace{0, \dots, 0}_{10+4q+2l}],$$

$$\tilde{\Sigma}_4 = [0, 0, 0, 0, 0, UC_y, \underbrace{0, \dots, 0}_{4+2q}, 0, 0, 0, \underbrace{0, \bar{\beta}_1 B_y, \dots, 0, \bar{\beta}_l B_y}_{2l}, 0, D_y, \underbrace{0, \dots, 0}_{2q}],$$

$$\tilde{\Sigma}_5 = [0, 0, 0, 0, -C_y, \underbrace{0, \dots, 0}_{9+4q+2l}],$$

$$\tilde{\Sigma}_6 = [\underbrace{0, \dots, 0}_{14+2q+2l}, \underbrace{\sigma_1 P_2, 0, \dots, \sigma_q P_2}_{2q}, 0].$$

Using **Lemma 1**, we get the (23). Thus, the proof is completed.

Remark 4. If the H_∞ disturbance attenuation level γ is given, the LMIs (23) is feasible, than the following optimization problem can be solved and a minimal disturbance attenuation level γ can be obtained:

min γ

s.t $\Gamma < 0$.

4 Numerical examples

In this section, two numerical examples are provided to illustrate the effectiveness of the proposed criteria.

Example 1. Consider the following nonlinear networked system (1) with

$$A = \begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 0.3 & 0 \\ 0 & 0 & 0.6 \end{bmatrix} \quad A_d = \begin{bmatrix} -0.2 & 0.2 & 0.1 \\ 0.1 & -0.3 & 0.2 \\ 0.4 & 0.1 & -0.2 \end{bmatrix}, \quad B = \begin{bmatrix} -0.2 & 0.2 & 0.1 \\ 0.1 & -0.3 & 0.2 \\ 0.4 & 0.1 & -0.2 \end{bmatrix},$$

$$B_y = \begin{bmatrix} -0.2 & 0.1 & 0.15 \\ 0.2 & -0.3 & 0.1 \\ 0.1 & 0.2 & -0.3 \end{bmatrix}, \quad D = \begin{bmatrix} -1 & 0.5 & 2 \\ 0.7 & 1 & -0.5 \\ 0.5 & 0.5 & 0.7 \end{bmatrix}$$

$$M = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & -0.3 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} \quad H = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & -0.3 \end{bmatrix}$$

$$F(k) = \text{diag}\{-\sin(k), \cos(k), \sin(k)\}$$

$$x(k) = [x_1(k), x_2(k), x_3(k)]^T$$

$g(x) = \tanh(0.5x)$, $u = 10$, also $\text{Pro}(a(k) = 1) = 0.5$ is assumed, and the disturbance $\omega(k)$ are defined as follows : $\omega(k) = [\omega_1(k), \omega_2(k), \omega_3(k)]^T$, where

$$\omega_1(k) = \begin{cases} 0.5\sin(k) & 10 \leq k \leq 50 \\ 0 & \text{otherwise} \end{cases}$$

$$\omega_2(k) = \begin{cases} 0.5\sin(k) & 70 \leq k \leq 120 \\ 0 & \text{otherwise} \end{cases}$$

$$\omega_3(k) = \begin{cases} 0.5\sin(k) & 150 \leq k \leq 200 \\ 0 & \text{otherwise} \end{cases}$$

We take $\tau(k) = \sin(0.5\pi k) + 3$ with the upper bound $\tau_M = 4$ and the lower bound $\tau_m = 2$. The initial conditions are chosen as $x_0(k) = [-1, 0, 1]^T$, for $-4 \leq k \leq 0$.

We suppose $\Theta = \text{diag}\{\alpha_1, \alpha_2, \alpha_3\}$, $\alpha_i (i = 1, 2, 3)$ obey the Bernoulli distribution $p_i(s) (i = 1, 2, 3)$ as follow:

$$p_1(s) = \begin{cases} 0.2 & s = 0 \\ 0.8 & s = 1 \end{cases}, \quad p_2(s) = \begin{cases} 0.3 & s = 0 \\ 0.7 & s = 1 \end{cases}, \quad p_3(s) = \begin{cases} 0.4 & s = 0 \\ 0.6 & s = 1 \end{cases}$$

It easy to obtain that

$$\begin{aligned} \bar{\alpha}_1 &= 0.8 & \bar{\alpha}_2 &= 0.7 & \bar{\alpha}_3 &= 0.6 \\ \sigma_1^2 &= 0.16 & \sigma_2^2 &= 0.21 & \sigma_3^2 &= 0.24 \end{aligned} \tag{27}$$

therefore (28)

$$U = \text{diag}\{0.8, 0.7, 0.6\}$$

$$\sigma^2 = \text{diag}\{0.16, 0.21, 0.24\}$$

$\beta_i (i = 1, 2, 3)$ obey $\delta_i(t) (i = 1, 2, 3)$ as follows:

$$\delta_1(s) = \begin{cases} 0.2 & t = 0 \\ 0.8 & t = 1 \end{cases}, \quad \delta_2(s) = \begin{cases} 0.3 & t = 0 \\ 0.4 & t = 0.5 \\ 0.3 & t = 1 \end{cases}, \quad \delta_3(s) = \begin{cases} 0.2 & t = 0 \\ 0.5 & t = 0.7 \\ 0.3 & t = 1 \end{cases}$$

It easy to obtain that

$$\begin{aligned} \bar{\beta}_1 &= 0.52 & \bar{\beta}_2 &= 0.5 & \bar{\beta}_3 &= 0.87 \\ \bar{\sigma}_1^2 &= 0.1656 & \bar{\sigma}_2^2 &= 0.15 & \bar{\sigma}_3^2 &= 0.1225 \end{aligned}$$

We use matlab of LMI Control Toolbox to obtain following K , see in table.

$$D_y = \begin{bmatrix} 2 & 0 & 1.2 \\ 1.5 & 0.25 & 0.2 \\ -0.2 & -0.8 & 1.6 \end{bmatrix}, \quad C_z = [0, 1, 0], \quad \omega(k) \neq v(k) = [ke^{-k}, ke^{-k}, ke^{-k}]^T, \quad \text{we obtain following}$$

Figure 1.

Figure 1: state trajectories of original system and estimator

Table 1: minimal γ with $\tau_1 \in [2, 3]$, $\tau_2 \in [3, 4]$ and $u = 10$ for various $\bar{\rho}$ and $\bar{\varrho}$

cases	ρ_0	ϱ_0	K	γ_{min}
1	0.9	0.25	$\begin{bmatrix} 0.4143 & -0.2269 & -0.0024 \\ -0.6752 & 0.7832 & 0.1834 \\ 0.6734 & -0.2671 & -0.1696 \end{bmatrix}$	0.2862
2	0.7	0.3	$\begin{bmatrix} -0.8506 & 0.9826 & -0.4039 \\ 0.6319 & -0.7430 & -0.8100 \\ -0.2154 & -1.1564 & 2.7654 \end{bmatrix}$	0.2145
3	0.3	0.7	$\begin{bmatrix} 0.4254 & -0.2364 & -0.0046 \\ -0.6623 & 0.7718 & 0.1812 \\ 0.6710 & -0.2662 & -0.1653 \end{bmatrix}$	2.0375

Figure 2: error trajectories

Figure 3: disturbances in original system(left) and estimator(right)

The estimation performance for $x_2(k)$ are shown in **Figure 1**, and **Figure 1** shows the **case 1** in **Table 1**. **Figure 2** shows the estimation performance for $x_1(k), x_2(k), x_3(k)$, which shows that estimation errors asymptotically converge to zeros. In the **Table 1**, according to the H_∞ disturbance attenuation level γ are the larger the more down cases, we can conclude that the disturbance in estimator plays a leading role. We can not ignore the disturbances of estimator in nonlinear networked system. The disturbances are divided into more original systems and estimators, the higher the H_∞ disturbance attenuation level γ . Now, we only consider the networked system with time-varying delays.

Example 2. Consider the nonlinear networked system in [30]

We employ the Lyapunov functional (14) in **Theorem 1** to analyze stable condition of the nonlinear networked system in [30]. For $q = 2$, the optimum of parameter γ is 0.9709.

Table 2: minimal γ with $\tau_m = 2$ and $\tau_M = 3$

Method	γ_{min}
Theorem 3 [30] as $q = 2$	5.2072
our method as $q = 2$	0.9709

Comparing the results in [30], it is clear that our results are less conservative from **Table2**.

5 Conclusion

In this paper, we have considered the problem of non-fragile H_∞ state estimation for a class of discrete-time neural networks with RODs, probabilistic diverging disturbance, multiple missing measurements, as well

as RONS. The sufficient conditions which guarantee that error system is asymptotically mean square stable have been given via Lyapunov method and stochastic analysis. Then we have obtained the parameters of the non-fragile filter by solving a set of LMIs. Finally, a numerical simulation example has been provided to verify the effectiveness of the proposed estimator design approach. It has been concluded that the disturbances in estimator in state estimation for nonlinear networked system can not be ignored.

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