Robust adaptive neural control for a class of non-affine nonlinear systems

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Abstract

This paper addresses the adaptive neural tracking control problem for a class of uncertain non-affine nonlinear system with non-affine function being semi-bounded and possibly non-differentiable. Compared with traditional control schemes, the proposed scheme can be applied to a more general class of non-affine nonlinear system, and relaxes constraint conditions as follows: firstly, the assumption that non-affine function must be differentiable is canceled, and only a continuous condition for non-affine function is required to guarantee the controllability of the considered system, secondly, the assumption that non-affine function is completely bounded is relaxed, and the non-affine function is constrained by a semi-bounded condition with the bounds being unknown functions. Then, an adaptive neural tracking controller is designed based on an invariant set. In the control design process, minimal learning parameter (MLP) technique is used to reduce the number of adaptive parameters, and a smooth robust compensator is employed to circumvent the influences of approximation error and external disturbance. Furthermore, it is proven that all the closed-loop signals are semi-globally uniformly ultimately bounded. Finally, simulation examples are provided to demonstrate the effectiveness of the designed method.

Keywords: Adaptive control, Neural network, Non-affine nonlinear systems, Invariant set

1. Introduction

In recent years, a lot of efforts have been made in the field of approximation-based adaptive control schemes for uncertain nonlinear systems [1-11]. Due to the universal approximation, learning and adaptation abilities of neural network (NN), the adaptive controllers involving NN approximation have achieved fairly good stability and accurate trajectory tracking for unknown nonlinear dynamics. However, most of them are applicable to the relatively simple nonlinear systems in affine form. In practice, there exist many systems with non-affine structure, such as chemical reaction [12], dynamic model in pendulum control [13], etc. Owing to few choices of mathematical tools in handling non-affine appearance of the control input, the adaptive control problem for non-affine nonlinear systems still remains challenging.

For non-affine nonlinear system, the main difficulty of controller design lies in that the control input does not appear in an affine form, and therefore, the control methods developed for affine nonlinear system cannot be directly employed to design controller for non-affine

nonlinear system in general. To overcome the difficulty, some representative adaptive control schemes have been presented for non-affine nonlinear systems. In [14-16], by using a Taylor series expansion in the neighborhood of an unknown control input trajectory, the non-affine nonlinear system is transformed into an affine form, and then, adaptive fuzzy controllers are designed. However, the Taylor series expansion is only valid locally around a specified point. The mean value theorem is therefore employed to convert the non-affine function into a model of affine form since it is valid globally [17-19]. Recently, the implicit function theorem is used to demonstrate the existence of an ideal controller that can achieve control objective, and then a NN or fuzzy system is used to construct this unknown ideal implicit controller for non-affine nonlinear systems [20-23]. In [24-29], by combining the implicit function theorem with mean value theorem, adaptive NN controllers are proposed for non-affine nonlinear systems. However, it is worth noting that the common assumption of the aforementioned approaches is that non-affine function must be differentiable with respect to the control input. And to assure the controllability of the system, the partial

derivative of non-affine function with respect to the control input is always assumed to be strictly positive or negative. Moreover, it is noticeable that for most of the existing approaches, the partial derivative of non-affine function with respect to the control input is constrained by both upper and lower bounds. Unfortunately, a priori knowledge of the plant dynamics is always difficult to be acquired for the bounds of the partial derivative of non-affine function, and it is well known that dead-zone and hysteresis exist widely in practical systems, which results in non-differentiable [7, 19, 30-34].

On the other hand, an approximation error will inevitably occur if a NN is used to approximate unknown nonlinear function of a given system [35-37], and external disturbances exist everywhere in actual systems, which may degrade the performance of the closed-loop control system, or even worse, deteriorate the stability. Recently, in [38], the tracking control problem for non-affine nonlinear systems is solved based on a self-organizing approximation approach. In [25], based on dynamic learning scheme, adaptive NN control is presented for a class of non-affine nonlinear systems. However, only the effect of NN approximation error is compensated in these methods. In [24], direct adaptive neural control is developed for a class of uncertain non-affine nonlinear systems based on disturbance observer. A direct self-constructing neural controller is designed for a class of uncertain non-affine nonlinear systems [21]. An adaptive control scheme is proposed for uncertain non-affine nonlinear systems with input saturation using neural networks [7]. Although NN approximation error and external disturbance are considered in [7, 21, 24], there are a large number of neural weights to be tuned online in these approaches, which increases the computational burden. Therefore, MLP technique is always employed to deal with the problem in some references on affine systems. In [39], combining DSC with MLP technique, a robust adaptive NN tracking control scheme has been proposed for a class of affine nonlinear systems.

Motivated by the above discussion, a novel adaptive neural tracking control scheme is proposed for a class of non-affine nonlinear system with non-affine function being semi-bounded and possibly non-differentiable. The main contributions of this paper are summarized as follows:

1) For uncertain non-affine function, the restrictive differential condition with respect to the control input is removed, and only a continuous condition is presented to guarantee the controllability of uncertain non-affine nonlinear system.

2) Different from most of the existing researches, the

non-affine function is constrained by a semi-bounded condition with the bounds being unknown functions, which makes the controller design much more difficult. A compact set is then constructively applied in the controller design for non-affine nonlinear systems to deal with the difficulty.

3) By introducing a robust term, the effects of NN approximation error and external disturbance are compensated such that the proposed approach is able to handle a certain degree of uncertainties. Furthermore, MLP technique is employed to reduce the number of adaptive parameters.

This paper is organized as follows. In Section 2, the problem formulation and preliminaries are provided. The adaptive neural tracking control design approach and stability analysis of the closed-loop system are presented in Section 3. Two simulation examples are given to demonstrate the effectiveness of the proposed method in Section 4. We conclude the work in Section 5.

2. Problem formulation and preliminaries

2.1. Problem formulation

Consider a class of uncertain SISO non-affine nonlinear systems as follows

$$\begin{cases} \dot{x}_i = x_{i+1}, & i = 1, 2, \dots, n-1 \\ \dot{x}_n = f(x, u) + d(t) \\ y = x_1 \end{cases} \quad (1)$$

where $x = [x_1, x_2, \dots, x_n]^T \in R^n$, $u \in R$ and $y \in R$ are the system state, control input and output, respectively; $f(x, u)$ is an unknown non-affine continuous function, and $d(t)$ is an unknown external disturbance.

The control objective of this paper is to design an adaptive neural tracking controller for system (1) such that the output y follows the desired trajectory y_d and all the signals in the closed-loop system remain semi-globally uniformly ultimately bounded in the presence of external disturbance.

Throughout this paper, we make the following assumptions and lemmas.

Assumption 1: Define a function $L(x, u)$ as follows

$$L(x, u) = f(x, u) - f(x, 0) \quad (2)$$

And for all x and u , there exist unknown constants b_1 and b_2 such that

$$\begin{cases} L(x, u) \geq H_1(x)u + b_1, & u \geq 0 \\ L(x, u) \leq H_2(x)u + b_2, & u < 0 \end{cases} \quad (3)$$

where $H_1(x)$ and $H_2(x)$ are unknown continuous functions satisfying $H_1(x) > 0, H_2(x) > 0$.

Remark 1: It is noticeable that $f(x, u)$ is commonly assumed to satisfy $0 < g_1 \leq \partial f(x, u)/\partial u \leq g_2$, ($g_1, g_2 \in R$) such as in [6], [21], [22], which can be seen as the controllability condition of their systems. However, in this paper, Assumption 1 is used to guarantee the controllability of system (1), while the assumption that non-affine function must be differentiable is canceled. Furthermore, by the partial integration for $\partial f(x, u)/\partial u$ with respect to u in [6], [21], [22], we can obtain

$$\begin{cases} g_1 u \leq f(x, u) - f(x, 0) \leq g_2 u, & u \geq 0 \\ g_2 u \leq f(x, u) - f(x, 0) \leq g_1 u, & u < 0 \end{cases} \quad (4)$$

Then, compared with (4), Assumption 1 relaxes the constraint condition as continuous and semi-bounded condition, and is a reasonable relaxation of (4). Therefore, Assumption 1 is valid in this paper. Moreover, due to the presence of unknown continuous function $H_1(x)$ and $H_2(x)$, the bounds of semi-bounded condition are also unknown continuous functions, which makes the design of system controller more difficult. Thus, a compact set is introduced later to overcome this difficulty.

Assumption 2 : For $\forall t > 0$, there exists unknown positive constant d^* such that $|d(t)| \leq d^*$.

Assumption 3 : The desired trajectory y_d is bounded and sufficiently smooth, namely, there exist $\dot{y}_d, \ddot{y}_d, \dots, y_d^{(n)}$ included in the compact set $\Omega_0 := \{(y_d, \dot{y}_d, \dots, y_d^{(n)}) : \sum_{i=0}^n (y_d^{(i)})^2 \leq A_0\}$, where $y_d^{(0)}$ denotes y_d ; A_0 is an unknown positive constant. For simplicity, we define $x_d = [y_d, \dot{y}_d, \dots, y_d^{(n-1)}]^T$ in this paper.

Lemma 1 [40] : Consider the dynamic system in the form of

$$\dot{\chi}(t) = -\varpi \chi(t) + \vartheta v(t) \quad (5)$$

where ϖ and ϑ are positive constants and $v(t)$ is a positive function. Given any bounded initial condition $\chi(0) \geq 0$, then $\chi(t) \geq 0$ holds for all $\forall t \geq 0$.

Lemma 2 [41] : The hyperbolic tangent function $\tanh(\cdot)$ is continuous and differentiable, and it fulfils that for $\forall q \in R$ and $\forall \varsigma > 0$

$$0 \leq |q| - q \tanh\left(\frac{q}{\varsigma}\right) \leq 0.2785\varsigma \quad (6)$$

Lemma 3 (Young's inequality [42]) : For $\forall (x, y) \in R^2$, the following inequality holds

$$xy \leq \frac{\varepsilon^p}{p} |x|^p + \frac{1}{q\varepsilon^q} |y|^q \quad (7)$$

where $\varepsilon > 0$, $p > 1$, $q > 1$ and $(p-1)(q-1) = 1$. We choose $p = q = 2$ in this paper.

2.2. RBF neural network approximation

In this paper, the radial basis function neural network (RBFNN) will be applied to approximate the continuous function $g(Z): R^n \rightarrow R$

$$g_{nn}(Z) = W^T \phi(Z) \quad (8)$$

where $Z = [Z_1, Z_2, \dots, Z_n]^T \in \Omega_Z \subset R^n$ is the input vector; $W = [W_1, W_2, \dots, W_l]^T \in R^l$ is the weight vector; l denotes the NN node number; and $\phi(Z) = [\phi_1(Z), \phi_2(Z), \dots, \phi_l(Z)]^T \in R^l$, with $\phi_i(Z)$ being chosen as the commonly used Gaussian functions as follows

$$\phi_i(Z) = \exp\left[\frac{-(Z - \mu_i)^T (Z - \mu_i)}{\sigma_i^2}\right], \quad i = 1, 2, \dots, l \quad (9)$$

where $\mu_i = [\mu_{i1}, \mu_{i2}, \dots, \mu_{in}]^T$ is the center of the receptive field and σ_i is the width of the Gaussian function.

As proven in [43], by choosing enough neural nodes, the neural network (8) can approximate any continuous function over a compact set $\Omega_Z \subset R^n$ with arbitrary accuracy.

$$g(Z) = W^{*T} \phi(Z) + \varepsilon(Z), \forall Z \in \Omega_Z \quad (10)$$

where W^* is the ideal constant weight vector, and $\varepsilon(Z)$ is the approximation error which satisfies $|\varepsilon(Z)| \leq \varepsilon^*$ with $\varepsilon^* > 0$ being an unknown constant.

For simplicity, in this paper, ε denotes $\varepsilon(Z)$, $\|x\|$ denotes the 2-norm of vector x , and define $\tilde{\delta} = \delta - \hat{\delta}$ with $\hat{\delta}$ being the estimate of unknown constant δ and $\tilde{\theta} = \theta - \hat{\theta}$ with $\hat{\theta}$ being the estimate of unknown constant θ .

3. Controller methodology

3.1. Error dynamics and system transformation

Define the system state tracking error as

$$e = [e_1, e_2, \dots, e_n]^T, \quad e_i = x_i - y_d^{(i-1)}, e \in R^n \quad (11)$$

From (11), we have

$$x = e + [y_d, \dot{y}_d, \dots, y_d^{(n-1)}]^T \quad (12)$$

Define the filtering error as

$$s = \left(\frac{d}{dt} + q\right)^{n-1} e_1 = [\lambda_1, \lambda_2, \dots, \lambda_{n-1}, 1]e, s \in R \quad (13)$$

where $\lambda_i = C_{n-1}^{i-1} q^{n-i}$, ($i = 1, \dots, n-1$), C_{n-1}^{i-1} denotes the sign of permutations and combinations defined as $C_{n-1}^{i-1} = \frac{(n-1)!}{(i-1)!(n-i)!}$, and q is a positive design

parameter. For the definition of (13), the following Lemma 4 is introduced for the control design.

Lemma 4 [44]: The definition (13) for s have the following properties:

i) If $s = 0$, then $[\lambda_1, \lambda_2, \dots, \lambda_{n-1}, 1]e = 0$, which is a hyperplane in R^n . And the tracking error e_1 converges to zero asymptotically on this hyperplane.

ii) If $|s(t)| \leq B$ for $\forall t \geq 0$ with B being a positive constant, then $e(t)$ will converge to the following compact set

$$\Omega_e = \{e \mid |e_i| \leq 2^{i-1} q^{i-n} B, i = 1, 2, \dots, n\}, \forall t \geq T_0 \quad (14)$$

where $T_0 \geq 0$ is a computable constant.

According to Lemma 4, in order to confine the tracking error e , we need to guarantee the convergence of the filtering error s . By recalling (1), (11) and (13), the dynamics of filtering error s can be written as

$$\begin{aligned} \dot{s} &= [0, \lambda_1, \lambda_2, \dots, \lambda_{n-1}]e + \dot{e}_n \\ &= f(x, u) + d(t) - y_d^{(n)} + \sum_{i=1}^{n-1} \lambda_i e_{i+1} \\ &= f(x, u) + d(t) + \Phi \end{aligned} \quad (15)$$

where $\Phi = -y_d^{(n)} + \sum_{i=1}^{n-1} \lambda_i e_{i+1}$.

Consider the following Lyapunov function candidate

$$V_s = \frac{1}{2} s^2 \quad (16)$$

Define a compact set $\Omega_s = \{s \mid V_s \leq p\}$, where p is an arbitrary positive constant. By noting (13), we can know that Ω_s is equivalent to

$$\Omega_s = \{(e_1, e_2, \dots, e_n) \mid (\sum_{i=1}^{n-1} \lambda_i e_i + e_n)^2 \leq 2p\} \quad (17)$$

From (12), it is easily seen that x can be viewed as a continuous function with respect to $e_1, e_2, \dots, e_n, y_d, \dot{y}_d, \dots, y_d^{(n-1)}$. Therefore, $L(x, 0)$, $H_1(x)$ and $H_2(x)$ are the continuous functions of $e_1, e_2, \dots, e_n, y_d, \dot{y}_d, \dots, y_d^{(n-1)}$. Then, they can be expressed as follows

$$\begin{cases} L(x, 0) = \kappa_{L_0}(e_1, e_2, \dots, e_n, y_d, \dot{y}_d, \dots, y_d^{(n-1)}) \\ H_1(x) = \kappa_{H_1}(e_1, e_2, \dots, e_n, y_d, \dot{y}_d, \dots, y_d^{(n-1)}) \\ H_2(x) = \kappa_{H_2}(e_1, e_2, \dots, e_n, y_d, \dot{y}_d, \dots, y_d^{(n-1)}) \end{cases} \quad (18)$$

where $\kappa_{L_0}(\cdot)$, $\kappa_{H_1}(\cdot)$ and $\kappa_{H_2}(\cdot)$ are unknown continuous functions.

Since Ω_s and Ω_0 are compact sets, $\Omega_s \times \Omega_0$ is also a compact set. It is shown from (18) that all the variables of $\kappa_{L_0}(\cdot)$, $\kappa_{H_1}(\cdot)$ and $\kappa_{H_2}(\cdot)$ are included in the compact

set $\Omega_s \times \Omega_0$. Therefore, $\kappa_{L_0}(\cdot)$, $\kappa_{H_1}(\cdot)$ and $\kappa_{H_2}(\cdot)$ have maximums and minimums on $\Omega_s \times \Omega_0$. Namely, based on $\Omega_s \times \Omega_0$, we have

$$\begin{cases} |L(x, 0)| = \kappa_{L_0}(e_1, e_2, \dots, e_n, y_d, \dot{y}_d, \dots, y_d^{(n-1)}) \leq N \\ H_1(x) = \kappa_{H_1}(e_1, e_2, \dots, e_n, y_d, \dot{y}_d, \dots, y_d^{(n-1)}) \geq H_{1m} > 0 \\ H_2(x) = \kappa_{H_2}(e_1, e_2, \dots, e_n, y_d, \dot{y}_d, \dots, y_d^{(n-1)}) \geq H_{2m} > 0 \end{cases} \quad (19)$$

where N is the maximum for $\kappa_{L_0}(\cdot)$ on $\Omega_s \times \Omega_0$; H_{1m} is the minimum for $\kappa_{H_1}(\cdot)$ on $\Omega_s \times \Omega_0$; H_{2m} is the minimum for $\kappa_{H_2}(\cdot)$ on $\Omega_s \times \Omega_0$.

Remark 2: In this paper, we will design the adaptive controller u to be the continuous function of $\hat{\theta}, \hat{\delta}, e_1, e_2, \dots, e_n, y_d, \dot{y}_d, \dots, y_d^{(n)}$, and it can be expressed as follows

$$u = u(\hat{\theta}, \hat{\delta}, e_1, e_2, \dots, e_n, y_d, \dot{y}_d, \dots, y_d^{(n)}) \quad (20)$$

Furthermore, by noting (12), it can be seen that x is a continuous function with respect to $e_1, e_2, \dots, e_n, y_d, \dot{y}_d, \dots, y_d^{(n-1)}$. Then, we can know that $L(x, u)$ is a continuous function with respect to u and x , and it can be expressed as follows

$$L(x, u) = \kappa_L(\hat{\theta}, \hat{\delta}, e_1, e_2, \dots, e_n, y_d, \dot{y}_d, \dots, y_d^{(n)}) \quad (21)$$

where $\kappa_L(\cdot)$ is an unknown continuous function.

Considering $f(x, 0)$ is an unknown function, we use a RBFNN to approximate it over a compact set Ω_x as follows

$$f(x, 0) = W^{*T} \phi(x) + \varepsilon \quad (22)$$

where W^* is the unknown ideal weight vector, and ε is the approximation error satisfying $|\varepsilon| \leq \varepsilon^*$ with $\varepsilon^* > 0$. To reduce the number of adaptive parameters, based on MLP technique, we define the constant

$$\theta = H_m^{-1} \|W^*\|^2 \quad (23)$$

where $H_m = \min\{H_{1m}, H_{2m}\}$ is an unknown positive constant.

Consider the following Lyapunov function

$$V = V_s + \frac{H_m}{2\gamma} \tilde{\delta}^2 + \frac{H_m}{2\beta} \tilde{\theta}^2 \quad (24)$$

where $\gamma > 0$ and $\beta > 0$ are design parameters, and $\delta = H_m^{-1}(\Delta^* + \varepsilon^* + d^*)$, $\Delta^* = \max\{N, |b_1|, |b_2|\}$ are unknown positive constants.

Define a compact set $\Omega_1 := \{(\hat{\theta}, \hat{\delta}, s) \mid V \leq p\}$. Noting (13) and (24), Ω_1 can be rewritten as

$$\Omega_1 = \left\{ (\hat{\theta}, \hat{\delta}, e_1, e_2, \dots, e_n) \mid \left(\sum_{i=1}^{n-1} \lambda_i e_i + e_n \right)^2 + \frac{H_m}{\gamma} \tilde{\delta}^2 + \frac{H_m}{\beta} \tilde{\theta}^2 \leq 2p \right\} \quad (25)$$

Since the sets $\Omega_0 \in R^{n+1}$ and $\Omega_1 \in R^{n+2}$ are compact, $\Omega_2 = \Omega_1 \times \Omega_0 \in R^{2n+3}$ is also a compact set. From (21), we can see that Ω_2 includes all the variables of continuous function $\kappa_L(\cdot)$. Therefore, $\kappa_L(\cdot)$ has a maximum M on Ω_2 , namely, on the compact set Ω_2 , we have

$$|L(x, u)| \leq M \quad (26)$$

From (26), we can obtain

$$-M \leq L(x, u) \leq M \quad (27)$$

Remark 3: It should be pointed out that (26) is satisfied on the set Ω_2 and (19) is also satisfied on the set Ω_2 because $\Omega_2 \subset \Omega_s \times \Omega_0 \times R^2$. Since that compact set Ω_1 can be written as $\Omega_1 = \{(\hat{\theta}, \hat{\delta}, e_1, e_2, \dots, e_n) | V \leq p\}$ by noting (24), we know that the initial position of $(\hat{\theta}, \hat{\delta}, e_1, e_2, \dots, e_n, y_d, \dot{y}_d, \dots, y_d^{(n)})$ is in Ω_2 according to the initial condition $V(0) \leq p$. Therefore, similar as [45], if $\dot{V} \leq 0$ for $V = p$ based on the inequalities (19) and (26), then we can conclude that $V \leq p$ for $\forall t > 0$, namely, Ω_2 is an invariant set, and hence the stability of the whole system is achieved. Thus, in the following part, the stability of system (1) will be proven on the basis of (19) and (26). It also should be mentioned that (19) and (26) are satisfied based on Ω_2 , though $L(x, u)$ is completely bounded in these inequalities, which does not contradict with Assumption 1.

It follows from (3) and (27) that

$$\begin{cases} H_1(x)u + b_1 \leq L(x, u) \leq M, & u \geq 0 \\ -M \leq L(x, u) \leq H_2(x)u + b_2, & u < 0 \end{cases} \quad (28)$$

which can be rewritten as

$$\begin{cases} H_1(x)u + b_1 \leq L(x, u) \leq \frac{M - b_1}{u}u + b_1, & u > 0 \\ \frac{-M - b_2}{u}u + b_2 \leq L(x, u) \leq H_2(x)u + b_2, & u < 0 \end{cases} \quad (29)$$

Using (29), one has

$$\begin{cases} H_1(x)u + b_1 \leq \frac{M - b_1}{u}u + b_1, & u > 0 \\ \frac{-M - b_2}{u}u + b_2 \leq H_2(x)u + b_2, & u < 0 \end{cases} \quad (30)$$

Define $h_1(u) = (M - b_1)/u$ and $h_2(u) = (-M - b_2)/u$. From (30), we can know

$$\begin{cases} h_1(u) \geq H_1(x) \geq H_{1m}, & \text{for } \forall u > 0 \\ h_2(u) \geq H_2(x) \geq H_{2m}, & \text{for } \forall u < 0 \end{cases} \quad (31)$$

Noting (29) and the definition of $h_1(u)$ and $h_2(u)$, we have

$$\begin{cases} H_1(x)u + b_1 \leq L(x, u) \leq h_1(u)u + b_1, & u > 0 \\ h_2(u)u + b_2 \leq L(x, u) \leq H_2(x)u + b_2, & u < 0 \end{cases} \quad (32)$$

Then, it can be learned from (32) that there exist functions $\alpha_1(t)$ and $\alpha_2(t)$ taking values in the closed interval $[0, 1]$ and satisfying

$$\begin{cases} L(x, u) = \alpha_1(t)(H_1(x)u + b_1) \\ \quad + (1 - \alpha_1(t))(h_1(u)u + b_1), & u > 0 \\ L(x, u) = \alpha_2(t)(h_2(u)u + b_2) \\ \quad + (1 - \alpha_2(t))(H_2(x)u + b_2), & u < 0 \end{cases} \quad (33)$$

To facilitate the controller design, define the piecewise functions $h(x, u, t)$ and $\Delta(t)$ as follows

$$\begin{aligned} h(x, u, t) &= \begin{cases} \alpha_1(t)H_1(x) + (1 - \alpha_1(t))h_1(u), & u > 0 \\ H_1(x), & u = 0 \\ \alpha_2(t)h_2(u) + (1 - \alpha_2(t))H_2(x), & u < 0 \end{cases} \\ \Delta(t) &= \begin{cases} b_1, & u > 0 \\ L(x, 0), & u = 0 \\ b_2, & u < 0 \end{cases} \end{aligned} \quad (34)$$

Utilizing (31) and (34), we have

$$h(x, u, t) \geq H_m > 0 \quad (35)$$

According to (19) and (34), we can get

$$\Delta^* \geq |\Delta(t)| > 0 \quad (36)$$

From (33) and (34), $L(x, u)$ can be rewritten as

$$L(x, u) = h(x, u, t)u + \Delta(t) \quad (37)$$

Then, with the help of (37) and (2), system (1) can be rewritten in the form of

$$\begin{cases} \dot{x}_i = x_{i+1}, & i = 1, 2, \dots, n-1 \\ \dot{x}_n = h(x, u, t)u + \Delta(t) + f(x, 0) + d(t) \\ y = x_1 \end{cases} \quad (38)$$

Remark 4: It can be seen from (28)-(38) that the control gain function $h(x, u, t)$ is a positive function, namely, the control direction is positive, which guarantees the controllability of system (1) without the condition $\partial f(x, u)/\partial u$ existing. Furthermore, from the above-mentioned derivation process, it can be observed that (38) hold based on Assumption 1. Therefore, Assumption 1 is the controllability condition of system (1).

3.2. Controller design

In this subsection, an adaptive neural tracking controller is proposed for system (1). To facilitate readers' comprehension, the overall block diagram of the proposed control scheme is presented in Fig.1.

Figure 1: Block diagram of the adaptive neural control system

To begin with this design, the time derivative of V_s along (15) is

$$\dot{V}_s = sf(x, u) + sd(t) + s\Phi \quad (39)$$

According to Assumption 2, we obtain

$$\dot{V}_s \leq sf(x, u) + |s|d^* + s\Phi \quad (40)$$

Noting (2), we can rewrite (40) as

$$\dot{V}_s \leq s[L(x, u) + f(x, 0)] + |s|d^* + s\Phi \quad (41)$$

Substituting (22) into (41) yields

$$\begin{aligned} \dot{V}_s &\leq sL(x, u) + s(W^{*T}\phi(x) + \varepsilon) + |s|d^* + s\Phi \\ &\leq sL(x, u) + sW^{*T}\phi(x) + |s|\varepsilon^* + |s|d^* + s\Phi \end{aligned} \quad (42)$$

Applying Lemma 3, we get

$$sW^{*T}\phi(x) \leq \frac{s^2 \|W^*\|^2}{2a^2} \phi^T(x)\phi(x) + \frac{a^2}{2} \quad (43)$$

Substituting (43) into (42), one has

$$\begin{aligned} \dot{V}_s &\leq sL(x, u) + \frac{s^2 \|W^*\|^2}{2a^2} \phi^T(x)\phi(x) + \frac{a^2}{2} \\ &\quad + s\Phi + |s|(\varepsilon^* + d^*) \end{aligned} \quad (44)$$

It follows from (24) and (44) that the time derivative of V is

$$\begin{aligned} \dot{V} &\leq sL(x, u) + \frac{s^2 \|W^*\|^2}{2a^2} \phi^T(x)\phi(x) + \frac{a^2}{2} + s\Phi \\ &\quad + |s|(\varepsilon^* + d^*) + \frac{H_m}{\gamma} \tilde{\delta} \dot{\delta} + \frac{H_m}{\beta} \tilde{\theta} \dot{\theta} \end{aligned} \quad (45)$$

Substituting (37) into (45), we have

$$\begin{aligned} \dot{V} &\leq sh(x, u, t)u + s\Delta(t) + \frac{s^2 \|W^*\|^2}{2a^2} \phi^T(x)\phi(x) + \frac{a^2}{2} \\ &\quad + s\Phi + |s|(\varepsilon^* + d^*) + \frac{H_m}{\gamma} \tilde{\delta} \dot{\delta} + \frac{H_m}{\beta} \tilde{\theta} \dot{\theta} \end{aligned} \quad (46)$$

Noting $\delta = H_m^{-1}(\Delta^* + \varepsilon^* + d^*)$ and using (36), we can further have

$$\begin{aligned} \dot{V} &\leq sh(x, u, t)u + \frac{s^2 \|W^*\|^2}{2a^2} \phi^T(x)\phi(x) + \frac{a^2}{2} + s\Phi \\ &\quad + |s|(\Delta^* + \varepsilon^* + d^*) + \frac{H_m}{\gamma} \tilde{\delta} \dot{\delta} + \frac{H_m}{\beta} \tilde{\theta} \dot{\theta} \\ &\leq sh(x, u, t)u + \frac{s^2 \|W^*\|^2}{2a^2} \phi^T(x)\phi(x) + \frac{a^2}{2} + s\Phi \\ &\quad + |s|H_m\delta + \frac{H_m}{\gamma} \tilde{\delta} \dot{\delta} + \frac{H_m}{\beta} \tilde{\theta} \dot{\theta} \end{aligned} \quad (47)$$

Using $\dot{\delta} = -\hat{\delta}$, $\dot{\theta} = -\hat{\theta}$ and $\theta = H_m^{-1} \|W^*\|^2$, we have

$$\begin{aligned} \dot{V} &\leq sh(x, u, t)u + \frac{s^2 H_m \theta}{2a^2} \phi^T(x)\phi(x) + \frac{a^2}{2} \\ &\quad + s\Phi + |s|H_m\delta - \frac{H_m}{\gamma} \tilde{\delta} \dot{\delta} - \frac{H_m}{\beta} \tilde{\theta} \dot{\theta} \end{aligned} \quad (48)$$

Choose the adaptive laws as follows

$$\dot{\delta} = \gamma s \tanh\left(\frac{s}{\varsigma}\right) - v_1 \gamma \hat{\delta} \quad (49)$$

$$\dot{\theta} = \frac{\beta s^2}{2a^2} \phi^T(x)\phi(x) - v_2 \beta \hat{\theta} \quad (50)$$

where $\varsigma > 0$, $v_1 > 0$, $v_2 > 0$ and $a > 0$ are design parameters.

Remark 5: Note that (49) and (50) satisfy the conditions of Lemma 1, therefore we have $\hat{\theta}(t) \geq 0$, $\hat{\delta}(t) \geq 0$ for $\forall t > 0$ by choosing $\hat{\theta}(0) \geq 0$, $\hat{\delta}(0) \geq 0$, respectively. This property is very important and useful in our design. Furthermore, to make use of the property of Lemma 2 later, we introduce the hyperbolic tangent function $\tanh(\cdot)$ in (49).

Substituting (49) and (50) into (48) yields

$$\begin{aligned} \dot{V} &\leq sh(x, u, t)u + \frac{s^2 H_m \theta}{2a^2} \phi^T(x)\phi(x) + \frac{a^2}{2} \\ &\quad + |s|H_m\delta - \frac{H_m}{\gamma} \tilde{\delta} \left[\gamma s \tanh\left(\frac{s}{\varsigma}\right) - v_1 \gamma \hat{\delta} \right] \\ &\quad + s\Phi - \frac{H_m}{\beta} \tilde{\theta} \left[\frac{\beta s^2}{2a^2} \phi^T(x)\phi(x) - v_2 \beta \hat{\theta} \right] \end{aligned} \quad (51)$$

Using $\tilde{\delta} = \delta - \hat{\delta}$ and $\tilde{\theta} = \theta - \hat{\theta}$, we can further obtain

$$\begin{aligned} \dot{V} \leq & sh(x, u, t)u + \frac{s^2 H_m \hat{\theta}}{2a^2} \phi^T(x) \phi(x) + \frac{a^2}{2} \\ & + H_m \delta \left[|s| - s \tanh\left(\frac{s}{\varsigma}\right) \right] + s H_m \hat{\delta} \tanh\left(\frac{s}{\varsigma}\right) \\ & + \left[|s\Phi| - s\Phi \tanh\left(\frac{s\Phi}{\varsigma}\right) \right] + s\Phi \tanh\left(\frac{s\Phi}{\varsigma}\right) \\ & + v_1 H_m \tilde{\delta} \hat{\delta} + v_2 H_m \tilde{\theta} \hat{\theta} \end{aligned} \quad (52)$$

From Lemma 2, we can further have

$$\begin{aligned} \dot{V} \leq & sh(x, u, t)u + \frac{s^2 H_m \hat{\theta}}{2a^2} \phi^T(x) \phi(x) + s H_m \hat{\delta} \tanh\left(\frac{s}{\varsigma}\right) \\ & + s\Phi \tanh\left(\frac{s\Phi}{\varsigma}\right) + 0.2785\varsigma(H_m \delta + 1) \\ & + v_1 H_m \tilde{\delta} \hat{\delta} + v_2 H_m \tilde{\theta} \hat{\theta} + \frac{a^2}{2} \end{aligned} \quad (53)$$

Using $\hat{\delta} = \delta - \tilde{\delta}$ and $\hat{\theta} = \theta - \tilde{\theta}$, we can rewrite (53) as

$$\begin{aligned} \dot{V} \leq & sh(x, u, t)u + \frac{s^2 H_m \hat{\theta}}{2a^2} \phi^T(x) \phi(x) + s H_m \hat{\delta} \tanh\left(\frac{s}{\varsigma}\right) \\ & + s\Phi \tanh\left(\frac{s\Phi}{\varsigma}\right) + 0.2785\varsigma(H_m \delta + 1) \\ & + v_1 H_m (\tilde{\delta} \delta - \tilde{\delta}^2) + v_2 H_m (\tilde{\theta} \theta - \tilde{\theta}^2) \end{aligned} \quad (54)$$

Applying Lemma 3 to $\tilde{\delta} \delta$ and $\tilde{\theta} \theta$, we can obtain

$$\begin{cases} \tilde{\delta} \delta \leq \frac{\delta^2}{2} + \frac{\tilde{\delta}^2}{2} \\ \tilde{\theta} \theta \leq \frac{\theta^2}{2} + \frac{\tilde{\theta}^2}{2} \end{cases} \quad (55)$$

Substituting (55) into (54), we get

$$\begin{aligned} \dot{V} \leq & sh(x, u, t)u + s H_m \left[\frac{s \hat{\theta}}{2a^2} \phi^T(x) \phi(x) + \hat{\delta} \tanh\left(\frac{s}{\varsigma}\right) \right. \\ & \left. + H_m^{-1} \Phi \tanh\left(\frac{s\Phi}{\varsigma}\right) \right] + 0.2785\varsigma(H_m \delta + 1) - \frac{v_1 H_m}{2} \tilde{\delta}^2 \\ & - \frac{v_2 H_m}{2} \tilde{\theta}^2 + H_m \left(v_1 \frac{\delta^2}{2} + v_2 \frac{\theta^2}{2} \right) + \frac{a^2}{2} \end{aligned} \quad (56)$$

Based on (56), the adaptive neural control law is designed as

$$\begin{aligned} u = & -ks - \frac{\hat{\theta}s}{2a^2} \phi^T(x) \phi(x) - \hat{\delta} \tanh\left(\frac{s}{\varsigma}\right) \\ & - \xi \Phi \tanh\left(\frac{s\Phi}{\varsigma}\right) \end{aligned} \quad (57)$$

where $k > 0$ and $\xi \geq H_m^{-1}$ are design parameters.

Remark 6: In reality, the term $-\frac{\hat{\theta}s}{2a^2} \phi^T(x) \phi(x)$ of (57) is designed to eliminate the effect of unknown function $f(x, 0)$; the term $-\hat{\delta} \tanh\left(\frac{s}{\varsigma}\right)$ of (57) is designed to eliminate the effects of NN approximation error and external disturbance, which will enhance the robustness of the closed-loop control system; the term $-\xi \Phi \tanh\left(\frac{s\Phi}{\varsigma}\right)$ of (57) is designed to eliminate the effect of function Φ generated in the process of controller design. Therefore, the effects of the second term in the right side of (56) will be offset by the latter three terms of control law (57), and then the linear feedback term $-ks$ will be used to guarantee the fast convergence of tracking error.

By recalling Remark 5, we have $\hat{\theta}(t) \geq 0$, $\hat{\delta}(t) \geq 0$ for $\forall t > 0$ by choosing $\hat{\theta}(0) \geq 0$, $\hat{\delta}(0) \geq 0$, respectively. Moreover, it can be easily found that

$$\begin{cases} \frac{s^2}{2a^2} \phi^T(x) \phi(x) \geq 0 \\ s \tanh\left(\frac{s}{\varsigma}\right) \geq 0 \\ s\Phi \tanh\left(\frac{s\Phi}{\varsigma}\right) \geq 0 \end{cases} \quad (58)$$

Therefore, by substituting (57) into (56) and noting $h(x, u, t) \geq H_m > 0$ and $\xi \geq H_m^{-1}$, we can rewrite (56) as

$$\begin{aligned} \dot{V} \leq & -k H_m s^2 + 0.2785\varsigma(H_m \delta + 1) - \frac{v_1 H_m}{2} \tilde{\delta}^2 \\ & - \frac{v_2 H_m}{2} \tilde{\theta}^2 + H_m \left(v_1 \frac{\delta^2}{2} + v_2 \frac{\theta^2}{2} \right) + \frac{a^2}{2} \end{aligned} \quad (59)$$

$\dot{V}$ will become negative by appropriately adjusting design parameters.

3.3. Stability analysis

In this subsection, the main results of this paper is stated in the following theorem, and the stability analysis of the closed-loop system is given.

Theorem 1: Consider the uncertain non-affine non-linear system (1) under Assumption 1-3, the control law (57), and the adaptation laws (49), (50). Then, Given any $p > 0$, for bounded initial conditions satisfying $\hat{\theta}(0) \geq 0$, $\hat{\delta}(0) \geq 0$ and $V(0) \leq p$, there exist $k, a, \varsigma, v_1, v_2, \gamma, \beta$ and ξ such that

1) $V(t) \leq p$ for $\forall t > 0$, and all the signals of closed-loop system are semi-globally uniformly ultimately bounded;

2) the state vector x always remains in compact set Ω_x defined as

$$\Omega_x = \{x(t) \mid \|x - x_d\| \leq \eta, [x_d, y_d^{(n)}]^T \in \Omega_0\}, \forall t \geq 0 \quad (60)$$

where η is a positive constant depending on initial conditions and design parameters;

3) the tracking error e will converge to the compact set Ω_e , which is defined as

$$\Omega_e = \{e \mid |e_i| \leq 2^{i-1} q^{i-n} B, \quad i = 1, 2, \dots, n\} \quad (61)$$

where B is a positive constant depending on design parameters, which can be made arbitrarily small by appropriately adjusting design parameters. Therefore, the compact set Ω_e can be made arbitrarily small.

Proof: In this paper, we choose the Lyapunov function as (24), and therefore (59) can be rewritten as

$$\dot{V} \leq -\mu V + \rho \quad (62)$$

where

$$\begin{cases} \mu = \min\{2kH_m, v_1\gamma, v_2\beta\} \\ \rho = 0.2785\varsigma(H_m\delta + 1) + H_m\left(v_1\frac{\delta^2}{2} + v_2\frac{\theta^2}{2}\right) + \frac{a^2}{2} \end{cases} \quad (63)$$

It can be seen from (63) that ρ/μ can be made arbitrarily small by reducing ς , v_1 , v_2 , a , and meanwhile, increasing k , β , γ . By choosing the appropriate design parameters, we can have $\rho/\mu \leq p$. Then, according to (62), we have that $\dot{V} \leq 0$ holds for $V = p$. Therefore, by the analysis of Remark 3, we can know that Ω_2 is an invariant set and the closed-loop system is stable. Thus, the property 1) of Theorem 1 is proven.

Multiplying (62) by $e^{\mu t}$, it becomes

$$(\dot{V}(t) + \mu V(t))e^{\mu t} \leq \rho e^{\mu t} \quad (64)$$

Further, one has

$$\frac{d}{dt}(V(t)e^{\mu t}) \leq \rho e^{\mu t} \quad (65)$$

Integrating (65) over $[0, t]$, we get

$$V(t) \leq (V(0) - \Gamma)e^{-\mu t} + \Gamma \quad (66)$$

where $\Gamma = \rho/\mu$.

According to (66), we can know that $V(t)$ is bounded and therefore, $s, \tilde{\delta}, \tilde{\theta}$ is semi-globally uniformly ultimately bounded. From (24) and (66), we have

$$\frac{1}{2}s^2 \leq V(t) \leq V(0) + \Gamma \quad (67)$$

Further, we have

$$|s| \leq \sqrt{2(V(0) + \Gamma)} \quad (68)$$

Define $\omega = [e_1, e_2, \dots, e_{n-1}]^T$ and $\Lambda = [\lambda_1, \lambda_2, \dots, \lambda_{n-1}]^T$, we can have

$$\begin{cases} s = \Lambda^T \omega + e_n \\ \dot{\omega} = A_s \omega + b_s s \end{cases} \quad (69)$$

$$\text{where } A_s = \begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \\ -\lambda_1 & -\lambda_2 & \dots & -\lambda_{n-1} \end{bmatrix}, b_s = [0, 0, \dots, 1]^T.$$

Noting that A_s is a Hurwitz matrix, hence, there exists a positive constant c_1 satisfying $\|e^{A_s t}\| \leq c_1 e^{-\lambda_1 t}$. From (69), we can get

$$\omega(t) = e^{A_s t} \omega(0) + \int_0^t e^{A_s(t-\tau)} b_s s(\tau) d\tau \quad (70)$$

Using (68), we have

$$\begin{aligned} \|\omega(t)\| &\leq c_1 \|\omega(0)\| e^{-\lambda_1 t} + c_1 \int_0^t e^{-\lambda_1(t-\tau)} |s(\tau)| d\tau \\ &\leq c_1 \|\omega(0)\| + \frac{c_1 \sqrt{2(V(0) + \Gamma)}}{\lambda_1} \end{aligned} \quad (71)$$

Noting $s = \Lambda^T \omega + e_n$ and $e = [\omega^T, e_n]^T$, we have

$$\|e\| \leq \|\omega\| + |e_n| \leq (1 + \|\Lambda\|) \|\omega\| + |s| \quad (72)$$

Substituting (68) and (71) into (72), we get

$$\|e\| \leq \eta \quad (73)$$

where

$$\begin{aligned} \eta &= c_1 (1 + \|\Lambda\|) \|\omega(0)\| \\ &\quad + \left[1 + \frac{(1 + \|\Lambda\|)c_1}{\lambda_1} \right] \sqrt{2(V(0) + \Gamma)} \end{aligned} \quad (74)$$

and further we know that η depends on the initial conditions and design parameters. Therefore, the property 2) of Theorem 1 is proven.

Furthermore, from (16), (24) and (66), we have

$$\begin{aligned} \lim_{t \rightarrow +\infty} |s| &\leq \lim_{t \rightarrow +\infty} \sqrt{2V(t)} \\ &\leq \lim_{t \rightarrow +\infty} \sqrt{2[(V(0) - \Gamma)e^{-\mu t} + \Gamma]} \end{aligned} \quad (75)$$

Noting $\lim_{t \rightarrow +\infty} (V(0) - \Gamma)e^{-\mu t} = 0$, therefore we have

$$\lim_{t \rightarrow +\infty} |s| \leq B \quad (76)$$

where $B = \sqrt{2\Gamma}$. According to the property ii) of Lemma 4, it is easy to get that the property 3) of Theorem 1 holds. In conclusion, Γ can be made arbitrarily small by appropriately choosing design parameters.

Thus, the compact set Ω_e can be made arbitrarily small. This completes the proof.

Remark 7: There are different design parameters that can have various effects on the performance of the proposed scheme. Note that adaptation gains γ and β in adaptation laws (49) and (50) that are used to tune the convergence rate of the adaptation process, and higher adaptation gains can make the convergence rate faster. Moreover, the design parameters ς and a can also determine the convergence rate of the adaptation process and smaller ς and a can lead to faster convergence. The small positive constants ν_1 and ν_2 are e-modification factors which can improve the stability of (49) and (50). The parameter k is set to guarantee that tracking error e converges to compact set Ω_e asymptotically in finite time and higher k can lead to faster convergence. For design parameter $\xi \geq H_m^{-1}$, it does not influence the size of tracking error, and we can get its value by the method of trial and error in the simulation because of unknown positive constant H_m . It can be seen from the property 3) of Theorem 1 that the size of Ω_e is determined by $B = \sqrt{2\Gamma}$, which can be decreased by reducing ς , ν_1 , ν_2 , a , and meanwhile, increasing k , β , γ .

4. Simulation

In this section, two examples are given to illustrate the effectiveness of the proposed scheme. It is necessary to emphasize that external disturbance and parameter uncertainty are considered in two simulation examples.

Example 1: Consider the following third-order van-der-Pol oscillator described by[25] :

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = f(x, u) + d(t) \\ y = x_1 \end{cases} \quad (77)$$

where $d(t) = \sin(2\pi t)$ is an external disturbance, and $f(x, u)$ is described as follows

$$\begin{aligned} f(x, u) = & -x_1 - 0.5x_2 + 0.7(1 - (x_1 + 0.5x_2)^2)x_3 \\ & + (r_0(x) + \Delta r(x))(\psi(u) + \frac{1}{3}\psi(u)^3 + \sin(\psi(u))) \end{aligned} \quad (78)$$

where $r_0(x)$ and $\Delta r(x)$ are the nominal part and the parameter variation part of uncertain parameter, respectively. To facilitate the simulation, $r_0(x) = 2$ and $\Delta r(x) = \sin(x_1 x_2 x_3)$ are arranged.

It is worth noting that, for comparison purpose later, we choose $\psi(u) = u$ without dead-zone or backlash

nonlinearities that will make $f(x, u)$ non-differentiable with respect to u . However, system (77) still satisfies Assumption 1. Furthermore, the desired trajectory $y_d = \sin(0.5t) + \cos(t)$.

Based on Theorem 1, the adaptive neural tracking controller is designed as (57) and the adaptation laws are given as (49), (50). In this simulation, the design parameters are taken as $k = 2$, $\varsigma = 0.1$, $\nu_1 = 0.1$, $\nu_2 = 0.2$, $\lambda_1 = \lambda_2 = 4$, $\gamma = 5$, $\beta = 4$, $\xi = 1$ and $a = 0.25$. The Gaussian RBF neural network $\hat{W}^T \phi(x)$ is constructed by using 27 nodes, with the centers μ_i evenly spaced in interval $[-4, 4] \times [-4, 4] \times [-4, 4]$ and with width $\sigma_i = 2.5$. The initial conditions are chosen as $[x_1(0), x_2(0), x_3(0)]^T = [0.5, 0, 0]^T$, $\hat{\delta}(0) = 0$ and $\hat{\theta}(0) = 0$. In the simulation, $d(t)$ is added in the closed-loop system from $t = 25s$ to observe the effect of external disturbance. The simulation results are shown as Figs. 2-5.

Figure 2: Output y and desired trajectory y_d for Example 1

Figure 3: States x_2 , x_3 and reference trajectories $\dot{y}_d$, y_d for Example 1

Figure 4: Control input u for Example 1

Figure 5: Adaptive parameters $\hat{\delta}$ and $\hat{\theta}$ for Example 1

It can be seen from Figs. 2-3 that fairly good tracking performance is obtained. The boundedness of control input u is presented in Fig. 4. The response curves of adaptive parameters $\hat{\delta}$, $\hat{\theta}$ are depicted in Fig. 5. The simulation results demonstrate the excellent tracking capability of the proposed controller and its effectiveness for tracking control of non-affine nonlinear system (77).

Note that $f(x, u)$ is a smooth non-affine function in Example 1, and therefore, we can compare the response of the proposed approach with the direct adaptive neural control (DANC) approach in [24] to further testify the effectiveness of the proposed approach. The output responses under two methods are shown in Figs. 6 and 7. Moreover, the tracking performances of the two schemes are summarized in Table 1, where the max error is chosen as the max $|e_1|$, the average error is defined as $Me_1 = \sum_{t=0}^{50} |e_1|/50$ and the standard deviation is defined as $\sum_{t=0}^{50} (y - Me_1)^2/50$. The results show that the proposed controller can track the desired trajectory as good as the DANC. However, only 2 adaptive param-

eters are required by using MLP technique in the proposed method, and the DANC method needs 7 adaptive parameters at least, namely, the number of adaptive parameters used in the proposed method is much less than in the DANC method.

Figure 6: Output y of two control schemes and desired trajectory y_d with $(x(0) = [1, 0, 0]^T)$

Figure 7: Tracking errors of two control schemes with $(x(0) = [1, 0, 0]^T)$

Table 1: Performance Comparison of Two Method

Methods	Max	Average	Standard deviation
DANC[24]	0.0897	0.0074	0.0136
Proposed method	0.0922	0.0073	0.0137

Remark 8: When dead-zone or backlash nonlinearities are considered in the control input, non-affine function $f(x, u)$ will be non-differentiable with respect to u . In this case, the DANC scheme is no longer applicable, but the proposed approach is still applicable. Therefore, the proposed approach can be applied to the non-affine nonlinear systems with non-affine function being possi-

bly non-differentiable, which make the controller design more difficult. To further illustrate the applicability of the proposed scheme, the backlash nonlinearity is considered in Example 2.

Example 2: Consider the following continuous-time non-affine nonlinear system [26] :

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x, u) + d(t) \\ y = x_1 \end{cases} \quad (79)$$

where $d(t) = \sin(2\pi t)$ is an external disturbance, and $f(x, u)$ is described as follows

$$f(x, u) = x_1 x_2^2 + x_2 e^{-1-x_1^2} - x_1 x_2 + \cos(0.1\psi(u)) + (r_0(x) + \Delta r(x))\psi(u) \quad (80)$$

where $r_0(x)$ and $\Delta r(x)$ are defined similar as in Example 1. To facilitate the simulation, $r_0(x) = 2$ and $\Delta r(x) = 0.3 \sin x_2^2$ are arranged. Furthermore, $\psi(u)$ is given as follows

$$\psi(u) = \begin{cases} u - 0.5 & \dot{u} > 0 \text{ and } \psi(u) = u - 0.5 \\ u + 0.5 & \dot{u} < 0 \text{ and } \psi(u) = u + 0.5 \\ \psi(t_-) & \text{otherwise} \end{cases} \quad (81)$$

where u is the actual control input. It can be easily seen that the continuous non-affine function $f(x, u)$ is non-differentiable with respect to u because of the presence of backlash nonlinearity. However, system (79) satisfies Assumption 1. The desired trajectory $y_d = \sin(0.5t) + \cos(t)$.

Based on Theorem 1, the adaptive neural tracking control law is designed as (57) and the adaptation laws are given as (49), (50). In this simulation, the design parameters are taken as $k = 2.5$, $\varsigma = 0.25$, $v_1 = 0.15$, $v_2 = 0.4$, $\lambda_1 = 2$, $\gamma = 5$, $\beta = 3$, $\xi = 1$ and $a = 0.25$. The Gaussian RBF neural network $\hat{W}^T \phi(x)$ is constructed by choosing 9 nodes, with the centers μ_i evenly spaced in interval $[-4, 4] \times [-4, 4]$ and with width equal to 2. The initial conditions are given as $[x_1(0), x_2(0)]^T = [0.5, 0]^T$, $\hat{\delta}(0) = 0$ and $\hat{\theta}(0) = 0$. In the simulation, $d(t)$ is added in the closed-loop system from $t = 25s$ for the same aim as Example 1. The simulation results are shown in Figs. 8-11.

From Figs. 8-9, it is observed clearly that output y and state x_2 converge rapidly to the reference signals y_d and $\dot{y}_d$, respectively. As seen from Figs. 10-11, control input u and adaptive parameters $\hat{\delta}$, $\hat{\theta}$ are bounded. The simulation results indicate that the proposed controller achieves good tracking performance even though there exist backlash nonlinearity and external disturbance in the non-affine nonlinear system (79).

Figure 8: Output y and desired trajectory y_d for Example 2

Figure 9: State x_2 and reference trajectory $\dot{y}_d$ for Example 2

Figure 10: Control input u for Example 2

To investigate the effect of different design parameters on the dynamic response of the system, three sets of design parameters are considered:

Figure 11: Adaptive parameters $\hat{\delta}$ and $\hat{\theta}$ for Example 2

Figure 12: Output y under 3 parameter sets and desired trajectory y_d with $(x(0) = [1, 0]^T)$

$$\begin{aligned}
 I: & \begin{cases} \varsigma = 0.35, v_1 = 0.25, v_2 = 0.5, a = 0.35 \\ k = 2, \beta = 2.5, \gamma = 4.5 \end{cases} \\
 II: & \begin{cases} \varsigma = 0.25, v_1 = 0.15, v_2 = 0.4, a = 0.25 \\ k = 2.5, \beta = 3, \gamma = 5 \end{cases} \\
 III: & \begin{cases} \varsigma = 0.15, v_1 = 0.05, v_2 = 0.3, a = 0.15 \\ k = 3, \beta = 3.5, \gamma = 5.5 \end{cases}
 \end{aligned}$$

where the second set is from Example 2.

Based on the closed-loop control system of Example 2, we conduct the simulation by using the three sets of design parameters. And the other parameters are the same as Example 2. By the last simulation, the output responses under the three sets are shown in Figs. 12 and 13. Moreover, the tracking performances are summarized in Table 2, in which the definition of max error, average error and standard deviation of output tracking error e_1 is the same as the ones of Table 1. Finally, the simulation results show that the compact set Ω_e can be made small by reducing ς , v_1 , v_2 , a , and meanwhile, increasing k , β , γ . Therefore, the adjusting method of design parameters is validity.

Table 2: Output Tracking Error under Three Sets

Parameter sets	Max	Average	Standard deviation
<i>I</i>	0.2025	0.0715	0.0591
<i>II</i>	0.1306	0.0496	0.0418
<i>III</i>	0.0671	0.0271	0.0232

5. Conclusion

By modelling the non-affine function appropriately, a novel adaptive neural tracking control scheme has been

Figure 13: Tracking errors under 3 parameter sets with $(x(0) = [1, 0]^T)$

presented for a more general class of uncertain non-affine nonlinear system. The assumption on the non-affine function is more relaxed as only a semi-bounded and continuous condition is required. The MLP technique has also been used to reduce the number of adaptive parameters, and the adaptive neural tracking controller has been designed based on the compact set. Finally, it is proven that this compact set is an invariant set for appropriate design parameters, which results in the stability of the closed-loop system. Therefore, the proposed controller can guarantee that all the signals in the closed-loop system are semi-global uniformly ultimately boundedness and the tracking error converges to a small residual set asymptotically. Finally, simulation results have demonstrated the effectiveness of the proposed approach.

Acknowledgment

This work was supported by National Science Foundation of China (Grant no. 61304120 and 61473307) and Aeronautical Science Foundation of China (Grant no. 20155896026). The authors are grateful to the editor and the anonymous reviewers for constructive comments that helped to improve the quality and presentation of this paper.

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