Image Segmentation and Bias Correction Using Local Inhomogeneous iNtensity Clustering (LINC): A Region-based Level Set Method [☆]

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Abstract

Image segmentation is still an open problem due to the existing of intensity inhomogeneity and noise. To accurately segment images with these biases, a local inhomogeneous intensity clustering (LINC) model is proposed. In LINC, a linear combination of a given set of smooth orthogonal basis functions is used to estimate the bias field. A local clustering criterion function is first defined to cluster the nearly homogeneous intensities in a relatively small neighborhood of each pixel. An energy functional is then defined by integrating the function with respect to the neighborhood center. This energy together with a regularization term and an arc length term are incorporated into a variational level set formulation in which de-noising is implicitly included due to the implied convolution. Image segmentation and bias correction can be simultaneously achieved by updating variables of the final energy functional iteratively till it is stable or a predetermined iteration number is reached. The proposed model LINC has been extensively tested on both synthetic and real images. Experimental results and comparison with state-of-the-art methods demonstrate the advantages of the proposed model in terms of segmentation accuracy, bias field correction, dealing with noise, and robustness to initialization.

Keywords: level set, image segmentation, bias correction, orthogonal basis function, energy minimization

1. Introduction

Image segmentation plays an important and fundamental role in the field of computer vision, pattern recognition, and neural computing because of its usefulness for object detection and recognition [1]. The objective of image segmentation is to separate an image into several meaningful parts where inner pixels are

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considered as the representation of an object with respect to a certain characteristic such as shape, structure, or texture [2]. In the field of neural computing, the sub-parts are may further used to interpret relationship of the desired objects in the image or tissues for medical images [3, 4]. Image segmentation has been studied for decades and many efforts have been devoted in the literature to proposing effective segmentation methods [5, 6]. In particular, many methods have been proposed in the field of neural computing in recent years [7, 8]. The existing methods can be generally classified into model based methods [9, 10], clustering methods [11, 12], machine learning methods [13, 13, 14], graph-cut based methods [15], and active contour and level set methods [16, 17, 18, 19, 20]. However, it is still challenging to accurately extract objects from a complex image especially if the image is ambiguous due to the existing of noise and intensity inhomogeneity (also known as bias field) [20, 21, 22].

Due to intensity overlaps of different interested objects caused by the bias field, those widely used intensity based segmentation methods may not be applicable to accurately segment images with intensity inhomogeneity [23]. However, as one of the most popular methods, the level set method has been increasingly applied to image segmentation since it was first introduced by Osher and Sethian in [24] because of the following desirable advantages [25]: 1) level set methods can provide smooth and closed contours as object boundaries in sub-pixel accuracy [26]; 2) they are easily formulated on a fixed Cartesian grid and are easy to be extended to a higher dimension under a principled energy minimization framework [14]; and 3) they allow to incorporate various prior knowledge, such as shape, into their energy formulation for a particular segmentation target[27]. In level set methods, image segmentation is transformed to an energy optimization problem where the energy functional defined on the so called level set function is minimized[28]. The zero level set, which is used to represent the desired object boundaries in one dimension lower, implicitly moves toward the boundaries till the optimal level set function which minimizes the energy functional is achieved [29].

Existing level set methods are generally classified into edge-based and region-based methods in the literature [30, 31, 32]. Edge-based level set methods usually utilize an edge indicator to attract the zero level set toward desired object boundaries [1]. As edge indicators are mostly based on image gradient, edge-based level set methods are particularly efficient to segment images with sharp gradient. However, they may seriously suffer from boundary leakage problem in the vicinity of weak boundaries. The drawback is overcome in region-based level set methods where region descriptors are usually used to guide the motion of the zero level set [33]. As one of the most popular region-based level set methods, the Chan-Vese (CV) model has been successfully applied in image segmentation [34]. In the CV model, the image domain is assumed to be composed of two intensity homogeneous parts. Therefore, the CV model is incapable to deal with difficulties caused by intensity inhomogeneity. To improve the capability of region-based level set methods in dealing with intensity inhomogeneity, local region information has been incorporated in the literature. We call the improved methods locally region-based level set methods. The methods generally rely on the

assumption that intensity-inhomogeneous images are homogeneous in local regions of each pixel. Two of the most representative ones are Region-Scalable Fitting (RSF) model and Local Intensity Clustering (LIC) model proposed by Li *et al.* in [35] and [36], respectively. In the RSF model, two fitting functions are used to locally approximate image intensities on both sides of the zero level set contour. A data term energy is defined based on the functions, which is further incorporated into a variational level set formulation to segment images with intensity inhomogeneity. However, the RSF model does not have the capability to estimate the bias field and remove it from the image to be segmented. Based on the well known image model where an observed image is viewed as the product of the true image and the bias field, Li *et al.* proposed a local clustering criterion function derived from the clustering property of image intensities in the neighborhood of each pixel in [36]. The local clustering criterion function is then integrated with respect to the neighborhood center to give a global criterion of image segmentation. The LIC model has shown its powerful capability for image segmentation and bias correction. But no constraints on the bias field are introduced to ensure it is slowly and smoothly varying. Hence, Li *et al.* proposed a Multiplicative Intrinsic Component Optimization (MICO) model in [37] for image segmentation and bias correction where the slowly and the smoothly varying property of the bias field is ensured by a linear combination of a given set of smooth basis functions. But the MICO model is not a level set method and is sensitive to noise.

In this paper, we propose a local inhomogeneous intensity clustering (LINC) model for image segmentation and bias correction. In the proposed model, the slowly and smoothly varying property of the bias field is ensured by a linear combination of a given set of smooth basis functions as proposed in [37]. Values of the bias field in a relatively small neighborhood of each pixel are regarded as being equal to the bias value at that pixel. A local clustering criterion function is defined for intensities in the neighborhood based on the clustering property of local inhomogeneous intensities pointed by Li *et al.* in [36]. The clustering criterion function is then used to define an energy functional by integrating itself with respect to the neighborhood center. This energy is finally incorporated into a variational level set formulation together with a level set function regularization term and an arc length term of the zero level set contour. Image segmentation and bias correction can be simultaneously achieved by updating variables of the final energy functional iteratively till it is stable or a predetermined iteration number is reached.

The rest of this paper is organized as follows. In section 2, we briefly review four well-known models for image segmentation. The details of the proposed model LINC are presented in Section 3. Experimental results of LINC on several synthetic and real images and comparison of LINC with state-of-the-art methods are given in Section 4. We discuss the robustness of LINC to initialization, the relationship of LINC with the CV model, and its extension to multiple phase level set formulation in Section 5. This paper is finally summarized in Section 6.

2. Related Work

Given an Image I defined on a continuous domain $\Omega \subset R^2$, its intensities can be viewed as a function $I : \Omega \rightarrow R$. Let $\phi : \Omega \rightarrow R$ be a *level set function* defined on Ω . We denote the 0-level set contour of the level set function ϕ by C , i.e. $C \triangleq \{\mathbf{x} : \phi(\mathbf{x}) = 0\}$. This contour separates the image domain Ω into two regions: Ω_1 and Ω_2 , which are regarded as *inside* and *outside* of the 0-level set contour C , respectively. We let the level set function ϕ take negative and positive values inside and outside the 0-level set contour C in this paper, i.e. $\Omega_1 \triangleq \{\mathbf{x} : \phi(\mathbf{x}) < 0\}$ and $\Omega_2 \triangleq \{\mathbf{x} : \phi(\mathbf{x}) > 0\}$.

2.1. Chan-Vese Model

In the well known Chan-Vese (CV) model, average intensities of the given image I inside and outside C are denoted by two constants c_1 and c_2 , respectively [34]. A data term F , which we call *global intensity fitting* energy, was defined to attract the 0-level set contour C toward the object boundaries in the following form:

$$F(c_1, c_2, \phi) = \lambda_1 \int |I(\mathbf{x}) - c_1|^2 (1 - H(\phi(\mathbf{x}))) d\mathbf{x} + \lambda_2 \int |I(\mathbf{x}) - c_2|^2 H(\phi(\mathbf{x})) d\mathbf{x} \quad (1)$$

where λ_1 and λ_2 are positive weighting coefficients and H is the Heaviside function. As well known, an area term $\mathcal{A}$ and an arc length term $\mathcal{L}$ are also included in the final energy functional E of the CV model to control the smoothness of C . These two terms can be written as

$$\mathcal{A}(\phi) = \int (1 - H(\phi(\mathbf{x}))) d\mathbf{x} \quad (2)$$

and

$$\mathcal{L}(\phi) = \int |\nabla H(\phi(\mathbf{x}))| d\mathbf{x} = \int \delta(\phi(\mathbf{x})) |\nabla \phi(\mathbf{x})| d\mathbf{x} \quad (3)$$

where δ is the derivative of Heaviside function H .

For fixed c_1 and c_2 , we minimize the energy functional $E = F + \alpha\mathcal{A} + \nu\mathcal{L}$ with respect to ϕ using the standard gradient descent method and obtain the following evolution equation of the level set function:

$$\frac{\partial \phi}{\partial t} = \delta(\phi) \left(\lambda_1 (I - c_1)^2 - \lambda_2 (I - c_2)^2 + \alpha + \nu \operatorname{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right) \right) \quad (4)$$

where α and ν are two positive weighting coefficients.

For fixed ϕ , we take partial derivative of energy functional E with respect to c_1 and c_2 , respectively. By setting the results with zeros and resolving the equations, we obtain

$$c_1 = \frac{\int I(\mathbf{x})(1 - H(\phi(\mathbf{x}))) d\mathbf{x}}{\int (1 - H(\phi(\mathbf{x}))) d\mathbf{x}} \quad (5)$$

and

$$c_2 = \frac{\int I(\mathbf{x})H(\phi(\mathbf{x}))d\mathbf{x}}{\int H(\phi(\mathbf{x}))d\mathbf{x}}. \quad (6)$$

The constants c_1 and c_2 are updated in the evolution of the level set function ϕ until the optimal ones (which minimize the energy functional E) are obtained or the predetermined maximum iteration number reached.

It is obvious that the constants c_1 and c_2 are intensity averages of pixels inside and outside the zero level set contour C . Due to the global averages of image intensity as shown in Eq. (5) and Eq. (6), the CV model is robust to some extent with respect to noise [34, 38] and is also robust to the initialization as shown in Fig. 1. But it obviously lacks the capability to deal with intensity inhomogeneity as reported by Li *et al.* in [35, 36, 39], which can also be seen from Fig. 1.

2.2. Region-Scalable Fitting Model

To overcome difficulties caused by intensity inhomogeneity on image segmentation, a region-based level set model, named Region-Scalable Fitting (RSF), was proposed by Li *et al.* in [35, 39]. Let $M_1(\phi(\mathbf{x})) = 1 - H(\phi(\mathbf{x}))$ and $M_2(\phi(\mathbf{x})) = H(\phi(\mathbf{x}))$ be the membership functions of Ω_1 and Ω_2 and K be a normalized even function with the property $K(\mathbf{u}) \geq K(\mathbf{v})$, if $|\mathbf{u}| < |\mathbf{v}|$, and $\lim_{|\mathbf{u}| \rightarrow \infty} K(\mathbf{u}) = 0$. For a given point $\mathbf{y} \in \Omega$, the following *local intensity fitting* energy is defined in the RSF model:

$$\mathcal{E}_{\mathbf{y}}(\phi, f_1(\mathbf{y}), f_2(\mathbf{y})) = \sum_{i=1}^2 \lambda_i \int K(\mathbf{x} - \mathbf{y}) |I(\mathbf{x}) - f_i(\mathbf{y})|^2 M_i(\phi(\mathbf{x})) d\mathbf{x} \quad (7)$$

where λ_1 and λ_2 are positive weighting coefficients, $f_1(\mathbf{y})$ and $f_2(\mathbf{y})$ are two values that approximate image intensities in Ω_1 and Ω_2 , respectively. The fitting values f_1 and f_2 will optimally approximate local intensities on the two sides of C when the contour C is exactly on the desired object boundaries, i.e. the energy $\mathcal{E}_{\mathbf{y}}$ is minimized for all $\mathbf{y}$ in the image domain Ω . This can be achieved by minimizing the integral of $\mathcal{E}_{\mathbf{y}}$ over all the center points $\mathbf{y}$ in the image domain Ω , i.e., minimizing

$$\mathcal{E} = \int \mathcal{E}_{\mathbf{y}} d\mathbf{y} = \int \left(\sum_{i=1}^2 \lambda_i \int K(\mathbf{x} - \mathbf{y}) |I(\mathbf{x}) - f_i(\mathbf{y})|^2 M_i(\phi(\mathbf{x})) d\mathbf{x} \right) d\mathbf{y}. \quad (8)$$

The following regularization term $\mathcal{P}$ is used in RSF to preserve the stability of the level set function ϕ :

$$\mathcal{P}(\phi) = \int \frac{1}{2} (|\nabla \phi(\mathbf{x})| - 1)^2 d\mathbf{x}. \quad (9)$$

Note that this term was first introduced by Li *et al.* in [40] to intrinsically maintain the regularity of the level set function during its evolution.

The final energy functional of RSF is defined by $\mathcal{F} = \mathcal{E} + \mu\mathcal{P} + \nu\mathcal{L}$, where μ and ν are positive weighting coefficients. For fixed f_1 and f_2 , we minimize this energy functional with respect to ϕ using the standard gradient descent method and obtain

$$\frac{\partial \phi}{\partial t} = \delta(\phi)(\lambda_1 e_1 - \lambda_2 e_2) + \mu \left(\nabla^2 \phi - \operatorname{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right) \right) + \nu \delta(\phi) \operatorname{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right) \quad (10)$$

120 where

$$e_i(\mathbf{x}) = \int K(\mathbf{x} - \mathbf{y}) |I(\mathbf{x}) - f_i(\mathbf{y})|^2 d\mathbf{y}, i = 1, 2. \quad (11)$$

121 For fixed ϕ , the energy functional $\mathcal{F}$ can be minimized by

$$f_i = \frac{K * (IM_i(\phi))}{K * M_i(\phi)}, i = 1, 2. \quad (12)$$

122 The RSF model has shown powerful capability for segmenting images with intensity inhomogeneity [35].
 123 However, it is sensitive to the size of local scalable-region and the location of initial contour as pointed by
 124 Wang *et. al* in [2], which can be seen from Fig. 2. In fact, if the size of the local scalable-region is not large
 125 enough to ensure pixels inside are belongs to two interested objects or the zero level set contour is initialized
 126 far from the boundaries, the image will be miss-segmented or over-segmented by RSF as shown in Fig. 2. In
 127 addition, the RSF model has no capability to remove bias fields from images with intensity inhomogeneity.

128 2.3. Local Intensity Clustering Model

129 To simultaneously segment images with intensity inhomogeneity and estimate the bias field, a region-
 130 based Local Intensity Clustering (LIC) model was proposed by Li *et al.* in [36]. The LIC model is based
 131 on the assumptions that the bias field b is slowly and smoothly varying and the true image approximately
 132 takes distinct constant values c_1 and c_2 in disjoint regions Ω_1 and Ω_2 . The image intensities in a circular
 133 neighborhood at each point $\mathbf{y} \in \Omega$ are used to define the following *local intensity clustering* criterion function

$$\mathcal{E}_{\mathbf{y}}(\phi, b, \mathbf{c}) = \sum_{i=1}^2 \int K(\mathbf{x} - \mathbf{y}) |I(\mathbf{x}) - b(\mathbf{y})c_i|^2 M_i(\phi(\mathbf{x})) d\mathbf{x} \quad (13)$$

134 where $\mathbf{c}$ is a vector defined by $\mathbf{c} = (c_1, c_2)$.

135 The local clustering criterion function $\mathcal{E}_{\mathbf{y}}$ evaluates the classification of intensities in the neighborhood
 136 at point $\mathbf{y} \in \Omega$. The smaller the energy $\mathcal{E}_{\mathbf{y}}$, the better the classification. To obtain the best segmentation,
 137 we minimize $\mathcal{E}_{\mathbf{y}}$ for all $\mathbf{y} \in \Omega$, i.e., minimizing

$$\mathcal{E} = \int \mathcal{E}_{\mathbf{y}} d\mathbf{y} = \int \left(\sum_{i=1}^2 \int K(\mathbf{x} - \mathbf{y}) |I(\mathbf{x}) - b(\mathbf{y})c_i|^2 M_i(\phi(\mathbf{x})) d\mathbf{x} \right) d\mathbf{y}. \quad (14)$$

138 The above-defined energy $\mathcal{E}$ is used as the data term in the final energy functional $\mathcal{F}$. In LIC, the final
 139 energy functional is defined by $\mathcal{F} = \mathcal{E} + \mu\mathcal{P} + \nu\mathcal{L}$ where μ and ν are positive weighting coefficients and $\mathcal{P}$
 140 and $\mathcal{L}$ are the regularization term and the arc length term, respectively.

141 For fixed b and $\mathbf{c}$, we minimize the final energy functional $\mathcal{F}$ using the standard gradient descent method
 142 and obtain

$$\frac{\partial \phi}{\partial t} = \delta(\phi)(e_1 - e_2) + \mu \left(\nabla^2 \phi - \operatorname{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right) \right) + \nu \delta(\phi) \operatorname{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right) \quad (15)$$

143 where

$$e_i(\mathbf{x}) = \int K(\mathbf{x} - \mathbf{y}) |I(\mathbf{x}) - b(\mathbf{y})c_i|^2 d\mathbf{y}, \quad i = 1, 2. \quad (16)$$

144 For fixed ϕ and b , the optimal $\mathbf{c}$ that minimizes the final energy functional $\mathcal{F}$ is given by

$$c_i = \frac{\int I(\mathbf{x})M_i(\phi(\mathbf{x}))(b * K)(\mathbf{x})d\mathbf{x}}{\int M_i(\phi(\mathbf{x}))(b^2 * K)(\mathbf{x})d\mathbf{x}}, \quad i = 1, 2. \quad (17)$$

145 For fixed ϕ and $\mathbf{c}$, the optimal b that minimizes the final energy functional $\mathcal{F}$ is given by

$$b = \frac{\left(I \sum_{i=1}^2 c_i M_i(\phi)\right) * K}{\sum_{i=1}^2 c_i^2 M_i(\phi) * K}. \quad (18)$$

146 The LIC model has shown its powerful capability for simultaneously segmenting the image and estimating
 147 the bias field [36]. However, as pointed by Wang *et al.* in [2], drawbacks associated with the RSF model
 148 in sensitivities to the size of local scalable-region and the location of initial contour still exist in the model
 149 as shown in Fig. 3. In particular, there is no specific constraint on the bias field to ensure its slowly and
 150 smoothly varying property. Therefore, as shown in Fig. 3, the vessel can be obviously seen in the estimated
 151 fields (not smooth enough) of LIC for the vessel image with different initializations.

152 2.4. Multiplicative Intrinsic Component Optimization Model

153 To ensure the slowly and smoothly varying property of the bias field b , Li *et al.* represented b as a
 154 linear combination of a given set of smooth basis functions $g_1, g_2, \dots$, and g_M with weighting coefficients $w_1,$
 155 $w_2, \dots, w_M$ in [37], i.e.,

$$b(\mathbf{x}) = \mathbf{w}^T G(\mathbf{x}) \quad (19)$$

156 where $(\cdot)^T$ is the transpose operator, $G(\mathbf{x})$ and $\mathbf{w}$ are column vectors defined by $G(\mathbf{x}) = (g_1(\mathbf{x}), g_2(\mathbf{x}), \dots,$
 157 $g_M(\mathbf{x}))^T$ and $\mathbf{w} = (w_1, w_2, \dots, w_M)^T$, respectively.

158 Assume that the true image (image without intensity inhomogeneity) is piecewise constant and there are
 159 N different types of objects in the image domain Ω . Let c_i be the constant intensity of the i -th object and u_i
 160 be the membership function with $\sum_{i=1}^N u_i(\mathbf{x}) = 1$. Li *et al.* proposed an Multiplicative Intrinsic Component
 161 Optimization (MICO) model and tried to minimize the following energy function to simultaneously segment
 162 a image with intensity inhomogeneity and estimate the bias field in [37]:

$$F(\mathbf{u}, \mathbf{c}, \mathbf{w}) = \int \sum_{i=1}^N \lambda_i |I(\mathbf{x}) - \mathbf{w}^T G(\mathbf{x})c_i|^2 u_i^q(\mathbf{x})d\mathbf{x} \quad (20)$$

163 where $q \geq 1$, λ_i is a positive weighting coefficient for clustering the i -th object, and $\mathbf{c}$ and $\mathbf{u}$ are column
 164 vectors defined by $\mathbf{c} = (c_1, c_2, \dots, c_N)^T$ and $\mathbf{u} = (u_1, u_2, \dots, u_N)^T$, respectively.

165 For fixed $\mathbf{w}$ and $\mathbf{u}$, the energy $F(\mathbf{u}, \mathbf{c}, \mathbf{w})$ can be minimized with respect to $\mathbf{c}$ by resolving $\frac{\partial F}{\partial c_i} = 0, i =$
 166 $1, 2, \dots, N$, respectively. Thus, the energy function $F(\mathbf{u}, \mathbf{c}, \mathbf{w})$ is minimized at $\mathbf{c}$ with

$$c_i = \frac{\int I(\mathbf{x})b(\mathbf{x})u_i^q(\mathbf{x})d\mathbf{x}}{\int b^2(\mathbf{x})u_i^q(\mathbf{x})d\mathbf{x}}, \quad i = 1, 2, \dots, N. \quad (21)$$

For fixed $\mathbf{c}$ and $\mathbf{u}$, the energy $F(\mathbf{u}, \mathbf{c}, \mathbf{w})$ can be minimized with respect to $\mathbf{w}$ by resolving $\frac{\partial F}{\partial \mathbf{w}} = \mathbf{0}$ where $\mathbf{0}$ is a constant function with value 0. Thus, the optimal $\mathbf{w}$ that minimizes $F(\mathbf{u}, \mathbf{c}, \mathbf{w})$ is given by

$$\mathbf{w} = \left(\int G(\mathbf{x}) G^T(\mathbf{x}) \left(\sum_{i=1}^N c_i^2 u_i^q(\mathbf{x}) \right) d\mathbf{x} \right)^{-1} \int G(\mathbf{x}) I(\mathbf{x}) \left(\sum_{i=1}^N c_i u_i^q(\mathbf{x}) \right) d\mathbf{x}. \quad (22)$$

For fixed $\mathbf{w}$ and $\mathbf{c}$, the energy $F(\mathbf{u}, \mathbf{c}, \mathbf{w})$ can be minimized with respect to $\mathbf{u} = (u_1, u_2, \dots, u_N)^T$ subject to the constraint that $\sum_{i=1}^N u_i(\mathbf{x}) = 1$. For the case $q > 1$, the energy functional $F(\mathbf{u}, \mathbf{c}, \mathbf{w})$ is minimized at $\mathbf{u} = (u_1, u_2, \dots, u_N)^T$, given by

$$u_i(\mathbf{x}) = \frac{(\lambda_i | I(\mathbf{x}) - \mathbf{w}^T G(\mathbf{x}) c_i |^2)^{\frac{1}{1-q}}}{\sum_{j=1}^N (\lambda_j | I(\mathbf{x}) - \mathbf{w}^T G(\mathbf{x}) c_j |^2)^{\frac{1}{1-q}}} \quad i = 1, 2, \dots, N. \quad (23)$$

For $q = 1$, it can be shown that the minimizer $\mathbf{u} = (u_1, u_2, \dots, u_N)^T$ is given by

$$u_i(\mathbf{x}) = \begin{cases} 1, & i = i_{\min}(\mathbf{x}) \\ 0, & i \neq i_{\min}(\mathbf{x}) \end{cases} \quad i = 1, 2, \dots, N \quad (24)$$

where

$$i_{\min}(\mathbf{x}) = \underset{i}{\operatorname{argmin}} (\lambda_i | I(\mathbf{x}) - \mathbf{w}^T G(\mathbf{x}) c_i |^2). \quad (25)$$

As the final energy functional F is convex in each of its variables, the MICO model is able to simultaneously segment images with intensity inhomogeneity and correct the bias field [37]. As shown in Fig. 4, the MICO model is robust to the initialization. But as MICO is pixel-based, it is sensitive to noise, which can be seen from Fig. 4.

3. Problem Formulation

As well known, the intensity of an observed image I with the intensity inhomogeneity at location $\mathbf{x}$ can be viewed as a product of the true image J and the bias field b at this location, i.e.,

$$I(\mathbf{x}) = b(\mathbf{x})J(\mathbf{x}) + n(\mathbf{x}) \quad (26)$$

where n is additive noise with zero-mean. The true image J characterizes an intrinsic physical property of objects being imaged, which ideally takes a specific intensity c_i for the i -th type of objects and is therefore assumed to be piecewise constant. That is to say, the true image J approximately takes N distinct constant values $c_1, c_2, \dots$, and c_N in disjoint regions $\Omega_1, \Omega_2, \dots$, and Ω_N , respectively. The bias field b is assumed to be slowly and smoothly varying as well known in the literature. The problem of image segmentation and bias correction is therefore considered as finding the specific intensity c_i for the i -th type of objects and estimating the bias field b .

3.1. Local Inhomogeneous Intensity Clustering

Consider a relatively small circular neighborhood with a radius ρ centered at a given point $\mathbf{y} \in \Omega$, defined by $\mathcal{O}_{\mathbf{y}} \triangleq \{\mathbf{x} : \|\mathbf{x} - \mathbf{y}\| \leq \rho\}$. The bias field in the neighborhood can be ignored due to its slowly and smoothly varying property. That is, values of $b(\mathbf{x})$ are close to $b(\mathbf{y})$ for all $\mathbf{x}$ in the small circular neighborhood $\mathcal{O}_{\mathbf{y}}$, i.e.

$$b(\mathbf{x}) \approx b(\mathbf{y}) \quad \text{for } \mathbf{x} \in \mathcal{O}_{\mathbf{y}}. \quad (27)$$

The partition $\{\Omega_i\}_{i=1}^N$ of the entire image domain Ω induces a partition of the neighborhood $\mathcal{O}_{\mathbf{y}}$, i.e. $\mathcal{O}_{\mathbf{y}} = \{\Omega_i \cap \mathcal{O}_{\mathbf{y}}\}_{i=1}^N$ forms a partition of $\mathcal{O}_{\mathbf{y}}$. Taking into account the constant intensity c_i of the true image J in Ω_i , intensities $b(\mathbf{x})J(\mathbf{x})$ in each subregion $\Omega_i \cap \mathcal{O}_{\mathbf{y}}$ are close to the constant $b(\mathbf{y})c_i$, i.e.

$$b(\mathbf{x})J(\mathbf{x}) \approx b(\mathbf{y})c_i \quad \text{for } \mathbf{x} \in \Omega_i \cap \mathcal{O}_{\mathbf{y}}. \quad (28)$$

Taking into account Eq.(19), the above equation can be rewritten as

$$b(\mathbf{y})c_i = \mathbf{w}^T G(\mathbf{y})c_i \quad \text{for } \mathbf{x} \in \Omega_i \cap \mathcal{O}_{\mathbf{y}}. \quad (29)$$

In consideration of the image model given in Eq.(26), we have

$$I(\mathbf{x}) = b(\mathbf{x})J(\mathbf{x}) + n(\mathbf{x}) \approx \mathbf{w}^T G(\mathbf{y})c_i + n(\mathbf{x}) \quad \text{for } \mathbf{x} \in \Omega_i \cap \mathcal{O}_{\mathbf{y}}. \quad (30)$$

As mentioned earlier, $n(\mathbf{x})$ is additive zero-mean noise. Therefore, intensities in the set

$$\mathbf{I}_{\mathbf{y}}^i = \{\mathbf{x} : \mathbf{x} \in \Omega_i \cap \mathcal{O}_{\mathbf{y}}\} \quad (31)$$

form a cluster with cluster center approximately being equal to $\mathbf{w}^T G(\mathbf{y})c_i$. This local intensity clustering property indicates that inhomogeneous intensities in the neighborhood $\mathcal{O}_{\mathbf{y}}$ can be classified into N clusters. This allows us to apply the standard K-means algorithm in the following continuous form to classify these local inhomogeneous intensities in the neighborhood $\mathcal{O}_{\mathbf{y}}$:

$$F_{\mathbf{y}} = \sum_{i=1}^N \int_{\mathcal{O}_{\mathbf{y}}} |I(\mathbf{x}) - \mathbf{w}^T G(\mathbf{y})c_i|^2 u_i(\mathbf{x}) d\mathbf{x} \quad (32)$$

where u_i is the membership function of Ω_i , i.e. $u_i(\mathbf{x}) = 1$ for $\mathbf{x} \in \Omega_i$ and $u_i(\mathbf{x}) = 0$ otherwise. On account of the inherent property of the membership function u_i in representing the subregion Ω_i , $F_{\mathbf{y}}$ can be rewritten as

$$F_{\mathbf{y}} = \sum_{i=1}^N \int_{\mathcal{O}_{\mathbf{y}} \cap \Omega_i} |I(\mathbf{x}) - \mathbf{w}^T G(\mathbf{y})c_i|^2 d\mathbf{x}. \quad (33)$$

Remark 1: In the case that the neighborhood $\mathcal{O}_{\mathbf{y}}$ intersects only with parts of the subregions $\Omega_1, \Omega_2, \dots$, and Ω_N (not all of them), intersections of $\mathcal{O}_{\mathbf{y}}$ with respect to the rest subregions will be null sets. Thus, minimizing energy $F_{\mathbf{y}}$ in Eq. (33) is in fact clustering intensities in the neighborhood of point $\mathbf{y}$ with respect to the intersection subregions.

210 3.2. Energy Formulation

211 In view of the normalized even function K with the property $K(\mathbf{u}) \geq K(\mathbf{v})$, if $\|\mathbf{u}\| < \|\mathbf{v}\|$, and
 212 $\lim_{\|\mathbf{u}\| \rightarrow \infty} K(\mathbf{u}) = 0$ as used in both RSF and LIC models in [35] and [36], we define

$$\mathcal{E}_{\mathbf{y}} = \sum_{i=1}^N \lambda_i \int_{\mathcal{O}_{\mathbf{y}} \cap \Omega_i} K(\mathbf{x} - \mathbf{y}) |I(\mathbf{x}) - \mathbf{w}^T G(\mathbf{y}) c_i|^2 d\mathbf{x} \quad (34)$$

213 where λ_i is a positive weighting coefficient for clustering the i -th object. We use K as a window function
 214 such that $K(\mathbf{x} - \mathbf{y}) = 0$ for $\mathbf{x} \notin \mathcal{O}_{\mathbf{y}}$ and rewrite $\mathcal{E}_{\mathbf{y}}$ as

$$\mathcal{E}_{\mathbf{y}} = \sum_{i=1}^N \lambda_i \int_{\Omega_i} K(\mathbf{x} - \mathbf{y}) |I(\mathbf{x}) - \mathbf{w}^T G(\mathbf{y}) c_i|^2 d\mathbf{x}. \quad (35)$$

215 The local inhomogeneous intensity clustering function $\mathcal{E}_{\mathbf{y}}$ evaluates the classification of intensities in the
 216 neighborhood $\mathcal{O}_{\mathbf{y}}$ by giving a partition $\{\Omega_i \cap \mathcal{O}_{\mathbf{y}}\}_{i=1}^N$ of $\mathcal{O}_{\mathbf{y}}$. To ensure the partition $\{\Omega_i\}_{i=1}^N$ of the entire
 217 domain Ω to be the one such that $\mathcal{E}_{\mathbf{y}}$ is minimized for all $\mathbf{y}$ in Ω , we minimize the integral of $\mathcal{E}_{\mathbf{y}}$ with respect
 218 to $\mathbf{y}$ over the entire image domain Ω and define

$$\mathcal{E} = \int \mathcal{E}_{\mathbf{y}} d\mathbf{y} = \int \left(\sum_{i=1}^N \lambda_i \int_{\Omega_i} K(\mathbf{x} - \mathbf{y}) |I(\mathbf{x}) - \mathbf{w}^T G(\mathbf{y}) c_i|^2 d\mathbf{x} \right) d\mathbf{y}. \quad (36)$$

219 3.3. Two-Phase Level Set Formulation

220 In the case that the 0-level set contour of the level set function ϕ separates the image domain Ω into
 221 two regions Ω_1 and Ω_2 , membership functions of the regions can be written as $M_1(\phi(\mathbf{x})) = 1 - H(\phi(\mathbf{x}))$
 222 and $M_2(\phi(\mathbf{x})) = H(\phi(\mathbf{x}))$, the same as mentioned in section 2.2. Thus, for the case $N = 2$, we rewrite the
 223 energy $\mathcal{E}$ described in Eq.(36) into the following level set formulation:

$$\mathcal{E} = \int \left(\sum_{i=1}^2 \lambda_i \int K(\mathbf{x} - \mathbf{y}) |I(\mathbf{x}) - \mathbf{w}^T G(\mathbf{y}) c_i|^2 M_i(\phi(\mathbf{x})) d\mathbf{x} \right) d\mathbf{y}. \quad (37)$$

224 By exchanging the integral order, we rewrite the above energy as

$$\mathcal{E} = \int \left(\sum_{i=1}^2 \lambda_i \int K(\mathbf{x} - \mathbf{y}) |I(\mathbf{x}) - \mathbf{w}^T G(\mathbf{y}) c_i|^2 d\mathbf{y} \right) M_i(\phi(\mathbf{x})) d\mathbf{x}. \quad (38)$$

225 It is obvious that the energy $\mathcal{E}$ is a function of variables the level set function ϕ , the vector $\mathbf{c} = (c_1, c_2)$, and
 226 the weight coefficients of the basis functions. Therefore, we rewrite the energy $\mathcal{E}$ as

$$\mathcal{E}(\phi, \mathbf{w}, \mathbf{c}) = \int \sum_{i=1}^2 \lambda_i e_i(\mathbf{x}) M_i(\phi(\mathbf{x})) d\mathbf{x} \quad (39)$$

227 where

$$e_i(\mathbf{x}) = \int K(\mathbf{x} - \mathbf{y}) |I(\mathbf{x}) - \mathbf{w}^T G(\mathbf{y}) c_i|^2 d\mathbf{y}, \quad i = 1, 2. \quad (40)$$

Let $*$ be the convolution operator. The functions e_i can be computed using the following equivalent expression:

$$e_i(\mathbf{x}) = I^2(\mathbf{x})(\mathbf{1} * K)(\mathbf{x}) - 2c_i I(\mathbf{x})(\mathbf{w}^T G * K)(\mathbf{x}) + c_i^2 ((\mathbf{w}^T G)^2 * K)(\mathbf{x}), \quad i = 1, 2 \quad (41)$$

where $\mathbf{1}$ is the constant function with value 1.

The energy $\mathcal{E}(\phi, \mathbf{w}, \mathbf{c})$ defined in Eq. (39) is used as the data term of the final energy functional of the proposed level set formulation, defined by

$$\mathcal{F}(\phi, \mathbf{w}, \mathbf{c}) = \mathcal{E}(\phi, \mathbf{w}, \mathbf{c}) + \mu \mathcal{P}(\phi) + \nu \mathcal{L}(\phi) \quad (42)$$

where $\mathcal{P}(\phi)$ is the regularization term defined in Eq.(9) used here to maintain the regularity of the level set function, $\mathcal{L}(\phi)$ is the arc length term defined in Eq.(3) used to smooth the 0-level set contour of the level set function, and μ and ν are positive weighting coefficients of $\mathcal{P}(\phi)$ and $\mathcal{L}(\phi)$, respectively.

Remark 2: The proposed model LINC is essentially different from both LIC and MICO. The main difference of LINC with respect to LIC is that an explicit constraint on the bias field is introduced into LINC to ensure the slowly and smoothly varying property of the bias field. The essential difference of LINC with respect to MICO is that it is a region-based level set method and the introduction of window function K .

3.4. Energy Minimization

Results of image segmentation and bias correction from LINC, given by the level set function ϕ and the optimal weighting coefficients $\mathbf{w}$ of the given set of basis functions, can be obtained by minimizing the energy functional $\mathcal{F}(\phi, \mathbf{w}, \mathbf{c})$ defined in Eq.(42). The energy minimization is achieved by an iterative process where the energy functional $\mathcal{F}(\phi, \mathbf{w}, \mathbf{c})$ with respect to each of its variables ϕ , $\mathbf{w}$, and $\mathbf{c}$ is minimized in each iteration by giving the other two updated in previous iteration. The solution to minimize the energy functional $\mathcal{F}(\phi, \mathbf{w}, \mathbf{c})$ with respect to each of its variables are given as follows.

For fixed $\mathbf{w}$ and $\mathbf{c}$, we minimized the energy functional $\mathcal{F}(\phi, \mathbf{w}, \mathbf{c})$ with respect to ϕ using the standard gradient descent method and obtain

$$\frac{\partial \phi}{\partial t} = \delta(\phi)(\lambda_1 e_1 - \lambda_2 e_2) + \mu \left(\nabla^2 \phi - \text{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right) \right) + \nu \delta(\phi) \text{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right). \quad (43)$$

For fixed ϕ and $\mathbf{c}$, we minimized the energy functional $\mathcal{F}(\phi, \mathbf{w}, \mathbf{c})$ with respect to $\mathbf{w}$. By solving the equation $\frac{\partial \mathcal{F}}{\partial \mathbf{w}} = \mathbf{0}$, we obtain

$$\mathbf{w} = A^{-1} \mathbf{v} \quad (44)$$

where A is an $M \times M$ matrix given by

$$A = \int \left(K * \left(\sum_{i=1}^2 \lambda_i c_i^2 M_i(\phi) \right) \right) (\mathbf{y}) G(\mathbf{y}) G^T(\mathbf{y}) d\mathbf{y} \quad (45)$$

253 and $\mathbf{v}$ is an M -dimensional column vector given by

$$\mathbf{v} = \int \left(K * \left(I \sum_{i=1}^2 \lambda_i c_i M_i(\phi) \right) \right) (\mathbf{y}) G(\mathbf{y}) d\mathbf{y}. \quad (46)$$

254 For fixed ϕ and $\mathbf{w}$, the energy functional $\mathcal{F}(\phi, \mathbf{w}, \mathbf{c})$ can be minimized by the optimal $\mathbf{c}$ with

$$c_i = \frac{\int I(\mathbf{x}) M_i(\phi(\mathbf{x})) (K * (\mathbf{w}^T G))(\mathbf{x}) d\mathbf{x}}{\int M_i(\phi(\mathbf{x})) (K * (\mathbf{w}^T G)^2)(\mathbf{x}) d\mathbf{x}}, \quad i = 1, 2. \quad (47)$$

255 **Remark 3:** The main difference of the proposed model LINC with respect to MICO in optimization
 256 of $\mathbf{w}$ and $\mathbf{c}$ is the convolution operation in Eq. (45), Eq. (46), and Eq. (47), which indicates that LINC is
 257 non-sensitive to noise. To estimate the bias field b using weighting coefficients computed in Eq. (44) and the
 258 basis functions, the matrix A must be non-singular to ensure that its inverse matrix A^{-1} exists and Eq. (44)
 259 has a unique solution. Let $h_m(\mathbf{y}) \triangleq \sqrt{K * \left(I \sum_{i=1}^2 \lambda_i c_i M_i(\phi) \right)}(\mathbf{y}) g_m(\mathbf{y})$, then the (m, k) entry of matrix A
 260 can be expressed as inner product of h_m and h_k given by $\langle h_m, h_k \rangle = \int h_m(\mathbf{y}) h_k(\mathbf{y}) d\mathbf{y}$ where m and k are
 261 integers in $[1, M]$. It is obvious that matrix A is the Gramian matrix of $h_1, h_2, \dots$, and h_M , which indicates
 262 that matrix A is non-singular if and only if $h_1, h_2, \dots$, and h_M are linearly independent. From the definition
 263 of h_m , it is easy to see that $h_1, h_2, \dots$, and h_M are linearly independent, which implies the non-singularity
 264 of A .

265 3.5. Implementation

266 In implementation of LINC, the choice of K is important. But it is flexible as long as the chosen K is a
 267 normalized even function and satisfies the property that $K(\mathbf{u}) \geq K(\mathbf{v})$, if $|\mathbf{u}| < |\mathbf{v}|$, and $\lim_{|\mathbf{u}| \rightarrow \infty} K(\mathbf{u}) = 0$.
 268 In this paper, the following Gaussian kernel function is chosen as function K :

$$K_\sigma(\mathbf{u}) = \frac{1}{(\sqrt{2\pi}\sigma)^n} e^{-|\mathbf{u}|^2/2\sigma^2} \quad (48)$$

269 where n is the dimension number of variable $\mathbf{u}$ and σ is the standard deviation, which determines the width
 270 of the Gaussian kernel. We set $\sigma = 3$ in our implementation.

271 In our numerical implementation, the stepped Heaviside function H is approximated by a smoothed
 272 Heaviside function H_ϵ with $\epsilon = 1$, defined by

$$H_\epsilon(x) = \frac{1}{2} \left[1 + \frac{2}{\pi} \arctan \left(\frac{x}{\epsilon} \right) \right]. \quad (49)$$

273 The derivative of H_ϵ is used to approximate the Dirac delta function δ , which can be written as

$$\delta_\epsilon(x) = H'_\epsilon(x) = \frac{1}{\pi} \frac{\epsilon}{\epsilon^2 + x^2}. \quad (50)$$

274 In the implementation of the proposed model LINC, 15 orthogonal four order Legendre polynomial
 275 functions are used as the basis functions to approximately estimate the bias field, i.e., $M = 15$ in this paper.

276 In fact, for each $\mathbf{x} \in \Omega$, we can rewrite $\mathbf{x}$ as $\mathbf{x} = (x_1, x_2)$ where x_1 and x_2 are directional components of the
 277 given two-dimensional image I defined on Ω . The smooth basis functions $g_1, g_2, \dots$, and g_{15} used in this paper
 278 are defined by $g_1(\mathbf{x}) = 1$, $g_2(\mathbf{x}) = x_1$, $g_3(\mathbf{x}) = (3x_1^2 - 1)/2$, $g_4(\mathbf{x}) = (5x_1^3 - 3x_1)/2$, $g_5(\mathbf{x}) = (35x_1^4 - 30x_1^2 + 3)/8$,
 279 $g_6 = x_2$, $g_7 = x_1x_2$, $g_8(\mathbf{x}) = (3x_1^2 - 1)x_2/2$, $g_9(\mathbf{x}) = (5x_1^3 - 3x_1)x_2/2$, $g_{10}(\mathbf{x}) = (3x_2^2 - 1)/2$, $g_{11}(\mathbf{x}) =$
 280 $x_1(3x_2^2 - 1)/2$, $g_{12}(\mathbf{x}) = (3x_1^2 - 1)(3x_2^2 - 1)/4$, $g_{13}(\mathbf{x}) = (5x_2^3 - 3x_2)/2$, $g_{14}(\mathbf{x}) = x_1(5x_2^3 - 3x_2)/2$, $g_{15}(\mathbf{x}) =$
 281 $(35x_2^4 - 30x_2^2 + 3)/8$. The column vector $G(\mathbf{x})$ can be therefore written as $G(\mathbf{x}) = (g_1(\mathbf{x}), g_2(\mathbf{x}), \dots, g_{15}(\mathbf{x}))^T$.
 282 The implementation of LINC can be straightforwardly expressed as follows.

- 283 • Step 1. Initialize $\mathbf{w}$ and ϕ .
- 284 • Step 2. Update $\mathbf{c}$ in Eq.(47).
- 285 • Step 3. Update ϕ in Eq.(43).
- 286 • Step 4. Update $\mathbf{w}$ in Eq.(44).
- 287 • Step 5. Check convergence criterion and iteration number. If convergence has been reached or the
 288 iteration number exceeds a predetermined maximum number, stop the iteration, otherwise, go to Step 2.

289 The convergence criterion used in Step 5 is $\|\mathbf{c}^{(n+1)}(t) - \mathbf{c}^{(n)}(t)\| < 0.001$, where $\mathbf{c}^{(n)}$ is the vector $\mathbf{c}$
 290 updated in Step 2 at the n -th iteration.

291 4. Experimental Results

292 The proposed model LINC has been extensively tested on synthetic and real images in Matlab R2011b on
 293 a computer with Intel(R) Core(TM) i5-3230M 2.6 GHz CPU, 4 GB RAM, and Windows 7 64-bit operating
 294 system. In this section, we first demonstrated the effectiveness of LINC on four distinctive images. We then
 295 validate the robustness of LINC to noise and its capability to segment images with intensity inhomogeneity
 296 and estimate the bias field. We finally compare LINC with four state-of-the-art methods mention in Section
 297 2 on several synthetic and real images. For images used in this paper, we initialized the weighting coefficient
 298 vector $\mathbf{w}$ with a constant column vector with value 1. The level set function was initialized as a binary step
 299 function, defined by $\phi(\mathbf{x}) = -c$ for $\mathbf{x}$ inside the initial zero-level contour of ϕ and $\phi(\mathbf{x}) = c$ otherwise. Unless
 300 otherwise specified, we set $c = 2.0$, $\Delta t = 0.1$, $\lambda_1 = \lambda_2 = 1.0$, $\mu = 1.0$ in this paper.

301 4.1. Effectiveness of LINC

302 To show the effectiveness of LINC, four distinctive images and segmentation results of LINC for the
 303 images are given in Fig. 5. The first image shows a cup, which may result in errors on segmentation of
 304 the image due to the shadow of the cup. There is also a protruding decoration strip between the cover
 305 and the wall, which may lead segment methods to separate the cup into two parts. The second image is a
 306 flower with dewdrops on it. The petals and tangle-some background increase the difficulty of segmentation
 307 methods on extracting the whole flower from the image. In the third image, there are two bottles with

different intensities and illumination. The intensity differences of the two bottles and the light change may mislead segmentation methods. The fourth image is an X-ray image of a human hand with bones and joints both clearly displayed. The obviously boundaries of bones and the gaps between joints may obstruct segmentation of the whole hand from the image.

In this experiment, we initialized the level set function for each image as a binary step function with its zero-level set contour estimated by a random circle. The initial zero-level set contours are given in green circles in the first row of Fig. 5. The weighting coefficients ν of LINC for the images are set to be $0.1 \times 255 \times 255$, $0.5 \times 255 \times 255$, $0.3 \times 255 \times 255$, and $0.02 \times 255 \times 255$, respectively. The segmentation results of LINC are given in the second row in red curves. It can be seen that the cup was successfully segmented as a whole object for the first image in spite of the disturbance of the shadow of the cup and protruding decoration strip between the cover and wall. For the second image, the whole flower was accurately segmented in spite of the existing of gaps among petals and the messy grass in the background. The two bottles in the third image were also well segmented, which proves that LINC can overcome difficulties brought by intensity differences of the bottles and the light change. For the fourth X-ray image, boundaries of bones and the gaps between joints are not able to obstruct LINC from providing an exact segmentation of the whole hand. Therefore, the proposed model LINC is effective in extracting desired objects from above mentioned complex images.

4.2. Robustness of LINC To Noise

Three synthetic and one real noisy images are selected in this subsection to show the robustness of LINC to noise. The weighting coefficients ν of LINC for the images are set to be $0.3 \times 255 \times 255$, $0.3 \times 255 \times 255$, $0.1 \times 255 \times 255$, and $0.3 \times 255 \times 255$, respectively. The original images with initial contours and final segmentation results of LINC for the images are shown in the first and second rows of Fig. 6, respectively. The first image is the same with the one used by Chan and Vese to show the robustness of their C-V model to noise in [34]. While the second synthetic image contains a concave heptagon, the third one contain both concave and convex shapes as shown in Fig. 6. The fourth image used in this experiment is a real image with three leaves with irregular boundaries and slender petioles, which may be miss-segmented due to the existing of noise.

The noise in the second, third, and fourth images are additive poisson noise, speckle noise, salt and pepper noise, and gaussian noise generated by matlab *imnoise* function with default parameters. The level set function for each image was initialized as a binary step function with the zero-level set contour estimated by a random circle as shown in green curves in the first row of Fig. 6. From the final segmentation results of LINC for the images as shown in the second row of Fig. 6, it can be seen that although segmentation accuracy may effected negatively at sharp corners due to the existing of salt and pepper noise as shown in the second and third columns of Fig. 6, the proposed model LINC can provide well segmentation results

even if desired object boundaries of the images are disturbed by noise, which proves the robustness of LINC in segmenting noisy images.

4.3. Capability of LINC in Correcting Bias Field

To validate the capability of LINC to segment images with intensity inhomogeneity and estimate the bias field, we first applied it to two synthetic images and two real images in this subsection. The segmentation and bias correction of LINC for these images are given in Fig. 7. We then applied LINC to another four images with both intensity inhomogeneity and noise and give the segmentation and bias correction results of LINC for these four images in Fig. 8. We finally applied LINC to segment four medical images and show the results in Fig. 9.

In Fig. 7, we demonstrate the capability of LINC to segment two synthetic images and two real images with intensity inhomogeneity. Shapes in the first two images are the same with the second and third images used in Fig. 6. The third image is a T-shaped object with severe intensity inhomogeneity in the left side and shadow at apex of the T-shape. There are three leaves in the fourth image with cuspidal leaf apices, slender petioles, and venation. Note that intensities of these images are all biased because of the existing of bias field. In the same way, we initialized the level set function for each image as a binary step function with its zero-level set contour estimated by a random circle as shown in green curves in the first column of Fig. 7. The weighting coefficients ν of LINC for these images are all set to be $0.05 \times 255 \times 255$. The results of LINC for these images are given in four rows with original images and initial contours shown in the first column. The estimated bias fields, bias corrected images, and final segmentation results are given in the second, third, and fourth columns respectively. As shown in the second column of Fig. 7, it can be seen that the estimated bias field for each image is slowly and smoothly varying and is consistent with intensity inhomogeneity existing in the image. Thus, it can be seen that intensities in the bias corrected images become much more homogeneous as shown in the third column of Fig. 7. From segmentation results shown in the fourth column of Fig. 7, we can see that the proposed model LINC can segment each image well even though there are severely inhomogeneous intensities. In particular, LINC considers cuspidal leaf apices, slender petioles, and venation as parts of the leaves for the fourth image which agrees with the fact that they are parts of the leaves.

We demonstrate the capability of LINC to segment images with both intensity inhomogeneity and noise in Fig. 8 where results of LINC for three synthetic images and one real image are shown in four rows. The first image is generated by adding a simulated bias field on the first image used in Fig. 6. Although shapes in the second and third images are the same with shapes in the first and second images of Fig. 7, bias fields in these images are different not only in intensity but also position. The fourth image is the same with the fourth image in Fig. 7 except the poisson noise, speckle noise, salt and pepper noise, and gaussian noise added by matlab *imnoise* function with default parameters. The weighting coefficients ν of LINC for the

images are set to be $0.2 \times 255 \times 255$, $0.2 \times 255 \times 255$, $0.1 \times 255 \times 255$, and $0.3 \times 255 \times 255$, respectively. The level set function for each image was initialized as a binary step function with its zero-level set contour estimated by a random circle as shown in green curves in the first column of Fig. 8. From the estimated bias fields and the corrected images as shown in the second and third columns of Fig. 8, we can see that the estimated bias field is consist consistent with the intensity inhomogeneity for each image. It can be seen from the final segmentation results shown in the fourth column of Fig. 8 in red curves that LINC segmented these images well except at sharp corners of the five-pointed star in the third image and cuspidal leaf apices and slender petioles in the fourth image which are obscure due to the noise.

We finally applied LINC to segment four medical images and demonstrate the results in rows in Fig. 9. The first two images are X-ray images of vessels with intensity inhomogeneity and noise both in foreground and background. The third one is a microscope image of two cells with quite blurry boundaries. The fourth image is a cardiac Cine MR short axis image with obvious intensity inhomogeneity and noise. The weighting coefficients ν of LINC for the images are set to be $0.0045 \times 255 \times 255$, $0.003 \times 255 \times 255$, $0.005 \times 255 \times 255$, and $0.01 \times 255 \times 255$, respectively. The original images with initial contours, the estimated bias fields, the corrected images, and final segmentation results are shown in columns. We initialized the level set function for each image as a binary step function with its zero-level set contour estimated by a random circle shown in green curves in the first column of Fig. 9. It is obvious that the estimated bias field is consist consistent with the intensity inhomogeneity for each image as shown in the second column of Fig. 9. Therefore, we can see that intensities of the corrected images shown in the third column of Fig. 9 are much more homogeneous. From the final segmentation results shown with the original images in the fourth column of Fig. 9, it can be seen that LINC segmented the medical images well even though there are inhomogeneous intensities, noise, and quite blurry boundaries.

All above, it is obvious that the proposed model LINC is able to segment images with only intensity inhomogeneity, images with both intensity inhomogeneity and noise, and real medical images and it has the capability of correcting bias fields and dealing with noise in the images.

4.4. Qualitative Evaluation of Segmentation Accuracy

In this subsection, we first select four images from all the images mentioned above randomly and give the curve evolution process of LINC in segmenting the images in Fig. 10. The initial zero level contours are given in the first column. From the second to fifth columns, the evolving curves are zero level contours of the level set funcionts after 5, 10, 15, and 20 iterations. It is obvious that the zero level contours evolve to the desired object boundaries as the iteration time increases. We then compare the proposed model LINC with CV, RSF, LIC, and MICO on three real images and give the results in Fig. 11, Fig. 12, and Fig. 13, respectively. The first image used in this experiment is an X-ray image of a hand with severe intensity inhomogeneity, which is the same with the fourth image in Fig. 5. The second image with both intensity

inhomogeneity and simulated noise is the same one in the fourth row of Fig. 6. The third image is the fourth image shown in Fig. 6, which is an X-ray image of vessel with both intensity inhomogeneity and noise. For each image, the level set functions of CV, RSF, LIC, and LINC are all initialized as the same binary step function by estimating the zero-level set contour with the green circular curves shown in the first row of each figure. For MICO, the membership functions and intensity constants of desired segmentation objects are both initialized as random numbers. While segmentation results and energy curves of CV and RSF are given in the second row of each figure, results of LIC, MICO, and LINC are shown in the third to fifth rows with the estimated bias field, corrected image, final segmentation result, and the energy curve of each model given from left to right. We use the same smoothed Heaviside function H_ϵ with $\epsilon = 1$ and its derivative δ_ϵ in Eq. (49) and Eq. (50) for CV, RSF, LIC, and LINC in this comparison experiment. The normalized even function K used in RSF, LIC, and LINC are all approximated by the Gaussian kernel function K_σ as given in Eq. (48) with $\sigma = 3$. The time step for the segmentation model used in this comparison are all set to $\Delta t = 0.1$. The basis functions used in MICO are the same with that used in LINC, which are 15 orthogonal four order Legendre polynomial functions, i.e., $M = 15$. For images used in this subsection, we only change the parameter ν of CV by setting the others $\lambda_1 = \lambda_2 = 1.0$, $\alpha = 1.0$. For RSF, parameters λ_1 , λ_2 , and μ are fixed with $\lambda_1 = \lambda_2 = 1.0$, $\mu = 1.0$. The parameter μ of LIC is set to $\mu = 1.0$ and parameter q of MICO is set to $q = 1.0$ for the images used in this subsection.

To segment the first image, parameters ν of CV, RSF, LIC, and LINC are all set to be $0.02 \times 255 \times 255$. From results shown in Fig. 11, it can be seen that for the same initialization given in the first row, segmentation results of CV, RSF, LIC, MICO and LINC are diverse from each other. Specifically, CV and LIC extracted boundaries of bones but not the whole hand well. RSF segmented the image wrongly at wrist of the hand. MICO has successfully extracted boundaries of the whole hand, but there are errors at corners of the image. Due to the lack of a specific constraint on the bias field to ensure its slowly and smoothly varying property as mentioned in section 2, we can see the hand from the bias field estimated by LIC as shown in the first column and the third row of Fig. 11. Although the bias field estimated by MICO is much more smooth than the one estimated by LIC, the estimation is inadequate especially at corners of the image which results in errors for the segmentation result. From the energy curves of CV, RSF, LIC, MICO, and LINC, it can be seen that the energy functional of LINC converged in 15 iterations, which is close to MICO, but CV, RSF, and LIC iterated 300 times to achieve the results shown in Fig. 11.

For the second image, which includes three leaves with severe intensity inhomogeneity and noise, parameters ν of CV, LIC, and LINC are all set to be $0.3 \times 255 \times 255$. The parameter ν of RSF is set to $\nu = 0.05 \times 255 \times 255$. The comparison of LINC with CV, RSF, LIC, and MICO for this image is given in Fig. 12. It is obvious that CV, RSF, and MICO are sensitive to noise in segmenting this image. Although the final segmentation result of LIC is almost correct, it can be seen from the estimated bias field of LIC that there are three leaves in the image. This indicates that the bias field estimated by LIC for this image

is not smooth enough. From the energy curves of CV, RSF, LIC, MICO, and LINC, we can see that the energy functional of LINC converged in 20 iterations, which is much faster than CV, RSF, and LIC (all converged in more than 100 iterations). Although the energy functional of LINC converged slowly (in 20 iterations) than MICO (in 10 iterations) for this image, LINC is much more robust to noise.

For the third X-ray image of vessel, we set parameters ν of CV, RSF, and LIC with $\nu = 0.001 \times 255 \times 255$ and set the parameter ν of LINC with $\nu = 0.003 \times 255 \times 255$. From the results of CV, RSF, LIC, MICO, and LINC shown in Fig. 13, we can see that CV can not segment this image correctly. In fact, no matter what values parameters of CV are set to be, parts of background/foreground of this image is incorrectly identified as foreground/background, as shown in Fig. 13. From segmentation results of RSF, LIC, and LINC, it can be seen that RSF missed parts of the vessel at down-right corner of the image, but LIC and MICO considered parts of non-vessel tissues as the vessel. From the estimated bias fields of LIC, MICO, and LINC as shown in the first column of Fig. 13 from the third row to the fifth row, it can be obviously seen that the bias field from LINC is the most smooth. From energy curves of CV, RSF, LIC, MICO, and LINC as shown in Fig. 13, we can see that the energy functional of LINC converged in 20 iterations, which is much faster than CV, RSF, and LIC (all converged in more than 200 iterations). But it converged slowly than MICO (in 10 iterations) for this image. However, the estimation of LINC for the bias field is better than MICO especially at left side and up-right corner of the image which results in errors for the segmentation result of MICO.

The iteration number and CPU times of CV, RSF, LIC, MICO, and LINC for segmenting the images in Fig. 11, Fig. 12, and Fig. 13 are listed in Table 1. The numbers in Table 1 were recorded from our experiments with Matlab code run on a computer with Intel(R) Core(TM) i5-3230M 2.6 GHz CPU, 4 GB RAM, and Windows 7 64-bit operating system. It is obvious that for the images, the propose model LINC is slower than MICO but faster than all the others except CV for the image given in Fig. 12.

4.5. Quantitative Evaluation of Segmentation Accuracy

To evaluate segmentation accuracy of the proposed model quantitatively, we compare segmentation results of LINC with CV, RSF, LIC, and MICO in terms of false positive ratio (FPR), false negative ratio (FNR), and dice similarity coefficient (DSC) on all the images mentioned above. To produce the ground truth for each image, non-local means denoising and N3 bias correction are first used to reduce noise and correct bias field from the image [41, 42]. The image is then segmented by FCM followed by manual revision to provide a desired result as the ground truth.

For image segmentation, a false positive (FP) occurs when a segmentation result indicates that a pixel belongs to a tissue, but it does not in fact. On the contrary, a false negative (FN) occurs when the segmentation result indicates that a pixel does not belong to a tissue, but it does in fact. Let NFP and NFN

be the number of FP and FN and A be the ground truth, FPR and FNR can then be defined by

$$\text{FPR} = \frac{\text{NFP}}{|I| - |A|} \times 100\% \quad (51)$$

and

$$\text{FNR} = \frac{\text{NFN}}{|A|} \times 100\%, \quad (52)$$

respectively, where $|*|$ denotes the size of region $*$. Obviously, values of FPR and FNR both range from 0 and 1 with a smaller value indicating a better match between the segmentation result and ground truth.

To compare segmentation accuracy of the proposed framework with state-of-the-art methods more objectively and precisely, we also evaluate segmentation results quantitatively with the metric of dice similarity coefficient in this subsection. It is well known that DSC is defined as twice of the quotient between intersection size of a pairwise variables and sum of their sizes where the variables are a segmentation result and the ground truth for image segmentation. Let B be the segmentation, definition of DSC can therefore be written as

$$\text{DSC} = \frac{2|A \cap B|}{|A| + |B|} \quad (53)$$

where $\cap$ is the intersection operator. It is obvious that values of DSC are in the interval of $[0, 1]$ with a higher value indicating a better match between the segmentation result B and the ground truth A .

We calculate FPR, FNR, and DSC values for the segmentation results and show the results with boxplots in Fig. 14. Note that a better match between a segmentation result and the ground truth will result in lower values not only for FPR but also for FNR. Obviously, the boxes of FPR and FNR values shown in Fig 14 for LINC are relatively shorter and lower than the others, which exhibits desirable stability of LINC in segmenting images mentioned above. Furthermore, although there are outliers in DSC values, the boxes of LINC are relatively shorter and higher than all the others. Therefore, segmentation results of LINC matched with the ground truth better than segmentation results of CV, RSF, LIC, and MICO.

5. Discussions

5.1. Impact of Weighting Coefficients and Their Setting

In this subsection, we first discuss impact of weighting coefficients of LINC on segmentation results. From Eq. (48), we can see that $K(\mathbf{u}) > 0$ for any $\mathbf{u}$ in its definition domain. Similarly, $|I(\mathbf{x}) - \mathbf{w}^T G(\mathbf{y})c_i|^2$ in Eq. (40) is not less than 0 for any $\mathbf{x} \in \Omega$ and its neighbourhood pixels $\mathbf{y}$. Therefore, $e_i(\mathbf{x}) \geq 0$ for $\mathbf{x} \in \Omega$. In addition, it is obvious that $\delta(\phi) > 0$ as shown in Eq. (50). That is to say, $\lambda_i e_i \delta(\phi)$ is monotone increasing if $\lambda_i > 0$. Therefore, the first term $\delta(\phi)(\lambda_1 e_1 - \lambda_2 e_2)$ on the right hand of Eq. (43) is monotone increasing and decreasing for any λ_1 and λ_2 , respectively, only if they take positive values. Thus, given a positive increment on λ_1 and λ_2 , the level set function ϕ will be increased or decreased much harder in

each iteration. As mentioned in section 2, we let the level set function ϕ take negative and positive values inside and outside the 0-level set contour C in this paper. Hence, for all the others fixed, the greater the coefficient λ_1 is, the smaller the region inside the 0-level set contour is, and vice versa. Similarly, the greater the coefficient λ_2 is, the smaller the region outside the 0-level set contour is, and vice versa.

As mentioned earlier, the regularization term and arc length term are used to maintain the regularity of the level set function and smooth the 0-level set contour of the level set function. That is to say, the greater values the parameters μ and ν take, the level set function is much more close to sign distance function and the smoother the 0-level set contour is. In practice, we can find the best weighting coefficients of LINC with training images from the dataset. We think image features, such as shape, size, and texture, can be used in our future work to determine the best parameters of LINC for a given specific image.

5.2. Robustness of LINC to Initialization

To validate the robustness of the proposed model LINC to initialization in segmenting images with intensity inhomogeneity and noise, we applied it to one synthetic image and two real images in four initialization strategies and give the results in Fig. 15, Fig. 16, and Fig. 17, respectively. In each of these figures, each initialization strategy is given in a row with original images with initial contours, the estimated bias fields, the corrected images, and final segmentation results shown in columns. Images shown in these figures are the same with the first one in Fig. 8, the fourth one in Fig. 6, and the first one in Fig. 9, respectively. For each image, the level set function was initialized as a binary step function with its zero-level set contour estimated by a random circle shown in green circular curves in the first column of each figure. It can be seen that there are not obvious differences between arbitrary two initialization strategies in the estimated bias fields, the corrected images, and the final segmentation results for each image, which proves that the proposed model LINC is robust to initialization.

5.3. Relationship with Chan-Vese Model

It is worth pointing out that the proposed energy $\mathcal{E}$ in Eq. (38) reduces to the *global intensity fitting* energy in Chan-Vese model when the bias field is a constant function $\mathbf{1}$, i.e. $b(\mathbf{x}) = \mathbf{w}^T G(\mathbf{x}) = 1$. That is to say the proposed model in the two-phase level set formulation in Eq. (38) is a generalization of the well-known Chan-Vese model [34]. To show this, we need the fact that K is normalized and even, i.e. $\int K(\mathbf{x} - \mathbf{y}) d\mathbf{y} = \int K(\mathbf{y} - \mathbf{x}) d\mathbf{y} = \int K(\mathbf{x} - \mathbf{y}) d\mathbf{x} = 1$ and recall that $M_1(\phi(\mathbf{x})) = 1 - H(\phi(\mathbf{x}))$ and $M_2(\phi(\mathbf{x})) = H(\phi(\mathbf{x}))$. For the case of $b(\mathbf{y}) = \mathbf{w}^T G(\mathbf{y}) = 1$, the energy $\mathcal{E}$ in Eq. (38) can be rewritten as

$$\begin{aligned} \mathcal{E} &= \int \sum_{i=1}^2 \lambda_i \left(\int K(\mathbf{x} - \mathbf{y}) |I(\mathbf{x}) - c_i|^2 d\mathbf{y} \right) M_i(\phi(\mathbf{x})) d\mathbf{x} \\ &= \int \sum_{i=1}^2 \lambda_i \left(|I(\mathbf{x}) - c_i|^2 \int K(\mathbf{x} - \mathbf{y}) d\mathbf{y} \right) M_i(\phi(\mathbf{x})) d\mathbf{x} \end{aligned}$$

$$\begin{aligned}
&= \int \sum_{i=1}^2 \lambda_i |I(\mathbf{x}) - c_i|^2 M_i(\phi(\mathbf{x})) d\mathbf{x} \\
&= \lambda_1 \int |I(\mathbf{x}) - c_1|^2 (1 - H(\phi(\mathbf{x}))) d\mathbf{x} \\
&+ \lambda_2 \int |I(\mathbf{x}) - c_1|^2 H(\phi(\mathbf{x})) d\mathbf{x}
\end{aligned} \tag{54}$$

which is exactly the *global intensity fitting* energy in Chan-Vese model in Eq. (1). Hence, the proposed model LINC in the two-phase level set formulation in Eq. (38) is a generalization of the well-known Chan-Vese model as given in Eq. (1).

5.4. Extension to Multiphase Level Set Formulation

For the case $N \geq 3$, we define N membership functions M_i to represent the regions of Ω_i using two or more level set functions, i.e., $M_i(\phi_1(\mathbf{x}), \phi_2(\mathbf{x}), \dots, \phi_k(\mathbf{x})) = 1$ for $\mathbf{x} \in \Omega_i$ and $M_i(\phi_1(\mathbf{x}), \phi_2(\mathbf{x}), \dots, \phi_k(\mathbf{x})) = 0$ otherwise, $i = 1, 2, \dots, N$. By defining $\Phi = (\phi_1, \phi_2, \dots, \phi_k)$, the energy $\mathcal{E}$ described in Eq. (36) can be rewritten as the following multiphase level set formulation:

$$\mathcal{E}(\Phi, \mathbf{w}, \mathbf{c}) = \int \sum_{i=1}^N \lambda_i e_i(\mathbf{x}) M_i(\Phi(\mathbf{x})) d\mathbf{x}. \tag{55}$$

For the function $\Phi = (\phi_1, \phi_2, \dots, \phi_k)$, we define $\mathcal{P}(\Phi) = \sum_{j=1}^k \mathcal{P}(\phi_j)$ and $\mathcal{L}(\Phi) = \sum_{j=1}^k \mathcal{L}(\phi_j)$, where $\mathcal{P}(\phi_j)$ and $\mathcal{L}(\phi_j)$ are defined in Eq. (9) and Eq. (3) for each level set function ϕ_j , respectively. To extend the propose model LINC to multiphase level set segmentation, we rewrite the energy functional $\mathcal{F}$ in Eq. (42) into the following form :

$$\mathcal{F}(\Phi, \mathbf{w}, \mathbf{c}) = \mathcal{E}(\Phi, \mathbf{w}, \mathbf{c}) + \mu \mathcal{P}(\Phi) + \nu \mathcal{L}(\Phi). \tag{56}$$

While minimization of $\mathcal{F}(\Phi, \mathbf{w}, \mathbf{c})$ with respect to $\mathbf{w}$ and $\mathbf{c}$ can be achieved by the same procedure as in the two-phase case given in section 3.4, minimization of $\mathcal{F}(\Phi, \mathbf{w}, \mathbf{c})$ with respect to ϕ_j can be performed by solving the following gradient flow equations:

$$\frac{\partial \phi_j}{\partial t} = - \sum_{i=1}^N \frac{\partial M_i(\Phi)}{\partial \phi_j} \lambda_i e_i + \mu \left(\nabla^2 \phi_j - \operatorname{div} \left(\frac{\nabla \phi_j}{|\nabla \phi_j|} \right) \right) + \nu \delta(\phi_j) \operatorname{div} \left(\frac{\nabla \phi_j}{|\nabla \phi_j|} \right), \quad j = 1, 2, \dots, k. \tag{57}$$

To validate the correctness of multiphase level set formulation defined in Eq. (56), we applied it to extract white matter and gray matter from two brain MR images with obvious intensity inhomogeneity. We use two level set function ϕ_1 and ϕ_2 to represent regions of white matter, gray matter, and background by setting $M_1(\phi_1, \phi_2) = (1 - H(\phi_1))(1 - H(\phi_2))$, $M_2(\phi_1, \phi_2) = (1 - H(\phi_1))H(\phi_2)$, and $M_3(\phi_1, \phi_2) = H(\phi_1)$. The results are shown in two rows in Fig. 18. For the first image, parameters of LINC are set to $\lambda_1 = 1.2$, $\lambda_2 = 1.0$, $\lambda_3 = 0.8$, $\mu = 1.0$, and $\nu = 0.005 \times 255 \times 255$. For the second image, these parameters are set to $\lambda_1 = 1.2$, $\lambda_2 = 1.0$, $\lambda_3 = 0.7$, $\mu = 1.0$, and $\nu = 0.005 \times 255 \times 255$. In Fig. 18, the original images with initial contours, estimated bias fields, bias corrected images, and final segmentation results are shown in

the first, second, third, and fourth column, respectively. The level set functions ϕ_1 and ϕ_2 are initialized as binary step functions with their zero-level set contours estimated by random circles shown in the first column with red and blue curves, respectively. On one hand, from the segmentation results, we can see that the white matter and gray matter are well segmented. On the other hand, the improvement of the image quality in terms of intensity homogeneity can not only be seen from the bias corrected images but also be demonstrated by comparing the histograms of the original images and the bias corrected images as given in the fifth and sixth columns of Fig. 18. It is obvious that there are not well-separated peaks in histograms of the original images due to the mixture of intensity distribution caused by the bias, but three well-defined and well-separated peaks are in histograms of the bias corrected image, each corresponding to the background or a tissue. This demonstrates the capability of multiphase LINC in correcting bias field from images with intensity inhomogeneity.

6. Conclusion and Future Work

Image segmentation is an important but challenging task especially when there are bias field and noise in the image. A local inhomogeneous intensity clustering (LINC) model has been proposed for simultaneously segmenting the image and estimating the bias field. In the proposed model, a given set of smooth orthogonal basis functions are used to estimate the slowly and smoothly varying bias field with a linear combination. Intensities in a sufficiently small neighborhood of each pixel can be considered as nearly homogeneous because of the similarity of the bias field in the neighborhood. A local clustering criterion function is defined to cluster intensities in the neighborhood. An energy functional was then defined based on the function by integrating it with respect to the neighborhood center. This energy together with a regularization term and an arc length term are incorporated into a variational level set formulation. In this formulation, the noise in the images can be implicitly held by the implied convolution in the update of variables of LINC during the evolution of the level set function. Experiments and comparison with state-of-the-art methods have demonstrated the advantages of the proposed model in terms of segmentation accuracy, bias field correction, dealing with noise, and robustness to initialization. In fact, the two-phase level set formulation of LINC is a generalization of the well-known Chan-Vese model, which can be demonstrated by setting the linear combination of the given set of smooth orthogonal basis functions to be a constant function $\mathbf{1}$. As an extension, the proposed model LINC has been improved into multiphase and its three-phase extension has been applied to segment brain tissues from MR images and correct the bias field with promising results. In the future, we will further improve the proposed model to segment images in 3D and try to apply the multiphase formulation to segment brain tissues on public image repositories.

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691 Figure Captions

Figure 1: Segmentation error of CV for an X-ray image of vessel with $\lambda_1 = \lambda_2 = \alpha = 1.0$ and $\nu = 0.001 \times 255 \times 255$ after 300 iterations: initializations (green curves) and corresponding segmentation results (red curves) are given in the 1st and 2nd rows, respectively.

Figure 2: Segmentation error of RSF for an X-ray image of vessel with $\lambda_1 = \lambda_2 = \mu = 1.0$ and $\nu = 0.001 \times 255 \times 255$ after 300 iterations: initializations (green curves) and corresponding segmentation results (red curves) are given in the 1st and 2nd rows, respectively.

Figure 3: Segmentation error of LIC for an X-ray image of vessel with $\mu = 1.0$, $\nu = 0.001 \times 255 \times 255$ after 300 iterations: initializations (green curves) and corresponding estimated bias fields, corrected images, and segmentation results (red curves) are given in the 1st, 2nd, 3rd, and 4th rows, respectively.

Figure 4: Segmentation error of MICO for an X-ray image of vessel with $\mu = 1.0$, $\nu = 0.001 \times 255 \times 255$ after 300 iterations: initializations (green curves) and corresponding estimated bias fields, corrected images, and segmentation results (red curves) are given in the 1st, 2nd, 3rd, and 4th rows, respectively. Note that we added zero mean Gaussian noise with 0.0005 variance to original images in the last two columns.

Figure 5: Segmentation of LINC for four distinctive images with initial contours and final segmentation results given in two rows in green and red curves, respectively.

Figure 6: Segmentation of LINC for noisy images with initial contours and final segmentation results given in two rows in green and red curves, respectively.

Figure 7: Segmentation and bias correction of LINC for images with intensity inhomogeneity. From left to right: original images with initial contours in green circular curves, estimated bias fields, corrected images, and final segmentation results in red curves.

Figure 8: Segmentation and bias correction of LINC for images with both intensity inhomogeneity and noise. From left to right: original images with initial contours in green circular curves, estimated bias fields, corrected images, and final segmentation results in red curves.

Figure 9: Application of LINC to medical images. From left to right: original images with initial contours in green circular curves, estimated bias fields, corrected images, and final segmentation results in red curves.

Figure 10: Results of LINC on four distinctive images given in four rows. The curve evolution process from the initial zero level contour (shown in green circles in the 1st column) to the final zero level contour (shown in red curves in the 5th column) is shown in each row for the corresponding image. Zero level contours after 5, 10, 15, 20 iterations are shown in the 2nd, 3rd, and 4th column, respectively.

Figure 11: Comparison of LINC with CV, RSF, LIC, and MICO on an X-ray image of a hand with severe intensity inhomogeneity. The original image with initial contour is in the 1st row in green curves. The segmentation results and energy curves of CV and RSF are in the 2nd row. The estimated bias fields, the corrected images, final segmentation results in red curves, and energy curves of LIC, MICO, and LINC are in the 3rd, 4th, and 5th rows, respectively.

Figure 12: Comparison of LINC with CV, RSF, LIC, and MICO on a real image with both intensity inhomogeneity and noise. The original image with initial contour is in the 1st row in green curves. The segmentation results and energy curves of CV and RSF are in the 2nd row. The estimated bias fields, the corrected images, final segmentation results in red curves, and energy curves of LIC, MICO, and LINC are in the 3rd, 4th, and 5th rows, respectively.

Figure 13: Comparison of LINC with CV, RSF, LIC, and MICO on an X-ray image of vessel with both intensity inhomogeneity and noise. The original image with initial contour is in the 1st row in green curves. The segmentation results and energy curves of CV and RSF are in the 2nd row. The estimated bias fields, the corrected images, final segmentation results in red curves, and energy curves of LIC, MICO, and LINC are in the 3rd, 4th, and 5th rows, respectively.

Figure 14: Quantitative comparison of LINC with CV, RSF, LIC, and MICO on images used in this paper.

Figure 15: Segmentation and bias correction of LINC for a synthetic image in four initialization strategies. From left to right: original images with initial contours in green circular curves, estimated bias fields, corrected images, and final segmentation results in red curves.

Figure 16: Segmentation and bias correction of LINC for a real image in four initialization strategies. From left to right: original images with initial contours in green circular curves, estimated bias fields, corrected images, and final segmentation results in red curves.

Figure 17: Segmentation and bias correction of LINC for a medical image in four initialization strategies. From left to right: original images with initial contours in green circular curves, estimated bias fields, corrected images, and final segmentation results in red curves.

Figure 18: Segmentation and bias correction of LINC for two MR brain images from BrainWeb [43] in rows using three phase level set function formulate. From left to right: original images with initial contours of the zero-level contours of ϕ_1 in red and ϕ_2 in blue, estimated bias fields, bias corrected images, final zero-level contours of ϕ_1 in red and ϕ_2 in blue, histograms of the original images, and histograms of the bias corrected images images.

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Table 1: Iterations and CPU time (in second) of CV, RSF, LIC, MICO, and LINC for the images in Fig. 11, Fig. 12, and Fig. 13.

Figure 1

Figure 1: Segmentation error of CV for an X-ray image of vessel with $\lambda_1 = \lambda_2 = \alpha = 1.0$ and $\nu = 0.001 \times 255 \times 255$ after 300 iterations: initializations (green curves) and corresponding segmentation results (red curves) are given in the 1st and 2nd rows, respectively.

Figure 2

Figure 2: Segmentation error of RSF for an X-ray image of vessel with $\lambda_1 = \lambda_2 = \mu = 1.0$ and $\nu = 0.001 \times 255 \times 255$ after 300 iterations: initializations (green curves) and corresponding segmentation results (red curves) are given in the 1st and 2nd rows, respectively.

Figure 3

Figure 3: Segmentation error of LIC for an X-ray image of vessel with $\mu = 1.0$, $\nu = 0.001 \times 255 \times 255$ after 300 iterations: initializations (green curves) and corresponding estimated bias fields, corrected images, and segmentation results (red curves) are given in the 1st, 2nd, 3rd, and 4th rows, respectively.

Figure 4

Figure 4: Segmentation error of MICO for an X-ray image of vessel with $\mu = 1.0$, $\nu = 0.001 \times 255 \times 255$ after 300 iterations: initializations (green curves) and corresponding estimated bias fields, corrected images, and segmentation results (red curves) are given in the 1st, 2nd, 3rd, and 4th rows, respectively. Note that we added zero mean Gaussian noise with 0.0005 variance to original images in the last two columns.

Figure 5

Figure 5: Segmentation of LINC for four distinctive images with initial contours and final segmentation results given in two rows in green and red curves, respectively.

Figure 6

Figure 6: Segmentation of LINC for noisy images with initial contours and final segmentation results given in two rows in green and red curves, respectively.

Figure 7

Figure 7: Segmentation and bias correction of LINC for images with intensity inhomogeneity. From left to right: original images with initial contours in green circular curves, estimated bias fields, corrected images, and final segmentation results in red curves.

Figure 8

Figure 8: Segmentation and bias correction of LINC for images with both intensity inhomogeneity and noise. From left to right: original images with initial contours in green circular curves, estimated bias fields, corrected images, and final segmentation results in red curves.

Figure 9

Figure 9: Application of LINC to medical images. From left to right: original images with initial contours in green circular curves, estimated bias fields, corrected images, and final segmentation results in red curves.

Figure 10

Figure 10: Results of LINC on four distinctive images given in four rows. The curve evolution process from the initial zero level contour (shown in green circles in the 1st column) to the final zero level contour (shown in red curves in the 5th column) is shown in each row for the corresponding image. Zero level contours after 5, 10, 15, 20 iterations are shown in the 2nd, 3rd, and 4th column, respectively.

Figure 11

Figure 11: Comparison of LINC with CV, RSF, LIC, and MICO on an X-ray image of a hand with severe intensity inhomogeneity. The original image with initial contour is in the 1st row in green curves. The segmentation results and energy curves of CV and RSF are in the 2nd row. The estimated bias fields, the corrected images, final segmentation results in red curves, and energy curves of LIC, MICO, and LINC are in the 3rd, 4th, and 5th rows, respectively.

Figure 12

Figure 12: Comparison of LINC with CV, RSF, LIC, and MICO on a real image with both intensity inhomogeneity and noise. The original image with initial contour is in the 1st row in green curves. The segmentation results and energy curves of CV and RSF are in the 2nd row. The estimated bias fields, the corrected images, final segmentation results in red curves, and energy curves of LIC, MICO, and LINC are in the 3rd, 4th, and 5th rows, respectively.

Figure 13

Figure 13: Comparison of LINC with CV, RSF, LIC, and MICO on an X-ray image of vessel with both intensity inhomogeneity and noise. The original image with initial contour is in the 1st row in green curves. The segmentation results and energy curves of CV and RSF are in the 2nd row. The estimated bias fields, the corrected images, final segmentation results in red curves, and energy curves of LIC, MICO, and LINC are in the 3rd, 4th, and 5th rows, respectively.

Figure 14

Figure 14: Quantitative comparison of LINC with CV, RSF, LIC, and MICO on images used in this paper.

Figure 15

Figure 15: Segmentation and bias correction of LINC for a synthetic image in four initialization strategies. From left to right: original images with initial contours in green circular curves, estimated bias fields, corrected images, and final segmentation results in red curves.

Figure 16

Figure 16: Segmentation and bias correction of LINC for a real image in four initialization strategies. From left to right: original images with initial contours in green circular curves, estimated bias fields, corrected images, and final segmentation results in red curves.

Figure 17

Figure 17: Segmentation and bias correction of LINC for a medical image in four initialization strategies. From left to right: original images with initial contours in green circular curves, estimated bias fields, corrected images, and final segmentation results in red curves.

Figure 18

Figure 18: Segmentation and bias correction of LINC for two MR brain images from BrainWeb [26] using three phase level set function formulate. From left to right: original images with initial contours of the zero-level contours of ϕ_1 in red and ϕ_2 in blue, estimated bias fields, bias corrected images, final zero-level contours of ϕ_1 in red and ϕ_2 in blue, histograms of the original images, and histograms of the bias corrected images images.

Table 1

Table 1: Iterations and CPU time (in second) of CV, RSF, LIC, MICO, and LINC for the images in Fig. 11, Fig. 12, and Fig. 13.

method	The Image in Fig. 11		The Image in Fig. 12		The Image in Fig. 13	
	iterations	time	iterations	time	iterations	time
CV	300	2.49	100	0.87	200	0.61
RSF	300	6.80	100	2.25	200	1.75
LIC	300	7.80	100	2.99	200	1.95
MICO	10	0.26	10	0.26	10	0.08
LINC	15	0.91	20	1.21	20	0.39