Finite-time H_∞ synchronization for complex dynamical networks with mixed mode-dependent time delays

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Abstract

This paper investigated the finite-time H_∞ synchronization control for complex dynamical networks with mixed mode-dependent time delays. The complex dynamical networks system contains the general coupling term and the time-delay coupling term. The state feedback control is designed to ensure that the complex dynamical networks system is finite time H_∞ synchronized. The sufficient conditions are established in the form of linear matrix inequalities by constructing a suitable Lyapunov-Krasovskii functional with triple integral terms and using Kronecker product. In the last, a numerical example is given to demonstrate the effectiveness of proposed approach.

Key words: Complex dynamical networks; Finite time H_∞ synchronization; Mixed mode-dependent time delays; Linear matrix inequality.

1. Introduction

During last decades, complex dynamical networks have received increasing research interest in different fields for its large size and complex topology [1-3]. Synchronization has become one of the most interesting phenomenon in real-world systems and has attracted many researchers to investigate because of its potential applications in many fields, such as image processing [4], neural networks [5] and secure communication [6]. In this regard, many literatures [7-12] have paid more attention to the synchronization problem of complex dynamical networks.

In many practical applications, it is often required that synchronization be reached in a finite time interval [13-21]. Authors in [13], the problem of finite-time synchronization has been investigated for uncertain Markov jump neural networks with input constraints. The parameter uncertainties under consideration are assumed to belong to fixed convex poly tope. Since finite time synchronization means the optimality in convergence time and has better robustness and disturbance rejection properties. The finite-time state consensus problems in [14] were investigated for multi-agent systems and presented one framework for constructing effective distributed protocols. Authors of [15], have investigated the finite -time synchronization of Markovian jump complex networks with partially unknown transition rates, by constricting the suitable

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stochastic Lyapunov-Krasovskii functional, using finite-time stability theorem, inequality technique and the pinning control technique. Recently, by combining the advantages of both finite-time control technique and finite-time stability theorem, [20-21] have investigated the finite-time synchronization of complex dynamical networks. Recently, the analysis and design of finite-time control, finite-time H_∞ control and finite-time H_∞ filtering were also tackled for varieties of systems in [22-25]. Motivated by the above state, therefore, it becomes important to investigate the synchronization for complex dynamical networks in the finite convergence time.

On the other hand, it has been well-known that external disturbance is quite common in real-world systems, as well as complex dynamical networks. The presence of external disturbance will degrade the system performance and might even destroy synchronization among the nodes. Therefore, it is of significant important to consider the disturbance rejection problem when a given complex dynamical networks is subject to external disturbance. In [26], authors researched H_∞ synchronization for complex dynamical networks with coupling delays, which used distributed impulsive control. In the paper, two type of time-varying coupling delays are considered. H_∞ synchronization criteria are establish to guarantee the synchronization of the system with respect to disturbance. Stabilization of complex dynamical networks with noise disturbance under performance constraint was considered in [27], and single controller was constructed for achieving certain prescribed H_∞ performance. In the paper [28], the problem of complex dynamical networks concerning the disturbance has been researched. It is shown that the disturbance rejection problem of a dynamical network can be solved by analysis the H_∞ control problem of a set of independent systems whose dimensions are equal to that of a single node. Authors in [29] concerned with the robust H_∞ synchronization problem for a class of complex dynamical networks by applying the observer-based control. The proposed feedback control scheme is developed to ensure the asymptotic stability of the augmented system, to reconstruct the non-measurable state variables of each node and to improve the H_∞ performance related to the synchronization error and observation error despite the external disturbance. In [30], the robust H_∞ control problem of uncertain complex delayed dynamical networks with non-identical nodes is investigated. The dynamic output feedback controllers are designed to ensure robustly global stability and a smaller prescribed H_∞ cluster synchronization for CDNs with mixed time delays was studied in [31], and a H_∞ cluster synchronization criterion was obtained in the form of LMIs. It is noted that the study of finite-time H_∞ synchronization for complex dynamic system with mixed mode-dependent time delays and external disturbance in the literature [32-38].

In this paper, the problem of state feedback and robust finite-time H_∞ synchronization control is studied for a class of complex dynamical networks with mixed mode-dependent time delays and external disturbance. Firstly, sufficient criteria of finite-time synchronization and finite-time H_∞ synchronization are provided for a class of complex dynamical network systems by utilizing Kronecker product and LMI methods. Then, a controller is designed such that the class of complex dynamical networks are finite-time H_∞ synchronization, and have an H_∞ attention performance level. Finally, an example is also given to illustrate the effectiveness of obtained results. The contribution of this paper lies in the following two aspects: (i) A new class of delayed complex dynamical networks is introduced whose the coupling configuration matrix is considered stochastic. (ii) By applying the existing Matlab LMI toolbox and Kronecker product, finite-time H_∞ synchronization is provided for the complex dynamical network system.

Notation: The notation in this paper is standard. $\mathbb{R}^n$ denotes the n dimensional Euclidean space; $\mathbb{R}^{m \times n}$ represents the set of all $m \times n$ real matrices. For a real asymmetric matrix X and Y , the notation $X \geq Y$ (respectively, $X > Y$) means $X - Y$ is semi-positive definite (respectively, positive definite). The superscript T denotes matrix transposition. Moreover, in symmetric block matrices, $*$ is used as an ellipsis for the terms that are introduced by asymmetry and $diag\{\dots\}$ denotes a block-diagonal matrix. The notation $A \otimes B$ stands for the Kronecker product of matrices A and B , Let $\tau > 0$ and $C([- \tau, 0], \mathbb{R}^n)$ denotes the family of continuous function ϕ from $[- \tau, 0]$ to $\mathbb{R}^n$. $(\Omega, F, \{F_t\}, P)$ is a complete probability space with a filtration, $\{F_t\}_{t \geq 0}$ satisfying the usual conditions. $\mathcal{E}\{\cdot\}$ represents the mathematical expectation. $\lambda_{\min}(\cdot)$, $\lambda_{\max}(\cdot)$ mean the minimum and the maximum eigenvalue of the corresponding matrix, respectively. If not explicitly stated matrices are assumed to have compatible dimensions.

2. Description of the problem

Considering the following delayed complex dynamic networks consisting of N identical modes with diffusively couplings, in which each node is a n -dimensional dynamical system:

$$\begin{cases} \dot{x}_k(t) = -Ax_k(t) + Bf(x_k(t)) + Cf(x_k(t - \tau_1(t))) + \sigma_1 \sum_{j=1}^N v_{kj}(r(t))\Gamma_1 x_j(t) \\ \quad + \sigma_2 \sum_{j=1}^N l_{kj}(r(t))\Gamma_2 x_j(t - \tau_1(t)) + \int_{t-\tau_2(t)}^t Mg(x_i(s))ds + D\omega_k(t) + u_k(t) \\ z_k(t) = Ex_k(t) \end{cases} \quad (1)$$

where $x_k(t) = (x_{k1}(t), x_{k2}(t), \dots, x_{kn}(t))^T \in \mathbb{R}^n$, represents the state vector of the k -th nodes at time t ; A is a positive diagonal matrix; B and C are weight matrix and delayed weight matrix, respectively; the constant $\sigma_1 > 0, \sigma_2 > 0$ denotes the coupling strength; D is a known matrix; E is a known matrix representing the parameters of output equation; $z_k(t) \in \mathbb{R}^n$ is the output of the node i ; the bounded function $\tau_1(t)$ and $\tau_2(t)$ represent unknown the discrete time delay and the distributed delay of the system with $0 \leq \tau_1(t) \leq \tau_1, 0 \leq \tau_2(t) \leq \tau_2, 0 \leq \dot{\tau}_1(t) \leq d_1, 0 \leq \dot{\tau}_2(t) \leq d_2$; $\Gamma_1 > 0, \Gamma_2 > 0$ represents the inner coupling matrix of the complex networks; $\omega_k(t) \in \mathbb{R}^p$ is the system external disturbance which belongs to $\mathcal{L}_2[0, \infty)$ and satisfied $\int_0^\infty \omega^T(t)\omega(t)dt \leq \mathcal{W}$, where $\omega(t) = [\omega_1(t), \omega_2(t), \dots, \omega_N(t)]^T$; $V_i = (v_{kj}(r(t)))_{n \times n}$, $L_i = (l_{kj}(r(t)))_{n \times n}$ describe the linear coupling configuration of the networks at time t at the mode $r(t)$, and are defined as $v_{kj}(r(t)) = v_{jk}(r(t)) \geq 0, l_{kj}(r(t)) = l_{jk}(r(t)) \geq 0$ for $k \neq j, v_{kk}(r(t)) = -\sum_{k \neq j}^n v_{kj}(r(t))$,

$l_{kk}(r(t)) = -\sum_{k \neq j}^n l_{kj}(r(t)), k, j = 1, 2, \dots, n$. $f(x_k(t)), g(x_k(t))$ denote the continuous nonlinear vector functions, satisfying the following assumption.

Assumption 1 ([39]) *Nonlinear function $f(\cdot)$ and $g(\cdot)$ are bounded functions satisfying $f_i(0) = g_i(0) = 0$, and for $i \in \{1, 2, \dots, N\}$, there exist constants l_i^-, l_i^+, c_i^- and c_i^+ such that:*

$$l_i^- \leq \frac{f_i(s_1) - f_i(s_2)}{s_1 - s_2} \leq l_i^+, c_i^- \leq \frac{g_i(s_1) - g_i(s_2)}{s_1 - s_2} \leq c_i^+, (i = 1, 2, \dots, N)$$

for all $s_1, s_2 \in \mathbb{R}, s_1 \neq s_2$, where l_i^-, l_i^+, c_i^- and c_i^+ are some fixed constant.

Remark 1 In Assumption 1, the constants l_i^+, l_i^-, c_i^+ and c_i^- are allowed positive, negative or zero. Then, the proposed nonlinear function maybe non-monotonic and more general than the usual sigmoid functions and Lipschitz-type conditions. By using this type assumption, it would be possible to reduce the conservatism of the main results.

Let $\{r(t)(t \geq 0)\}$ be a right-continuous Markovian chain on the probability space $(\Omega, \mathcal{F}, \{F_t\}_{t \geq 0}, P)$ taking value in the finite space $\mathcal{S} = \{1, 2, \dots, m\}$, with generator $\Pi = \{\pi_{ij}\}_{m \times m} (i, j \in \mathcal{S})$ given as follows:

$$Pr(r_{t+\Delta t} = j | r_t = i) = \begin{cases} \pi_{ij}\Delta t + o(\Delta t), & i \neq j, \\ 1 + \pi_{ij}\Delta t + o(\Delta t), & i = j, \end{cases}$$

where $\Delta t > 0, \lim_{\Delta t \rightarrow 0} (o(\Delta t)/\Delta t) = 0$, and π_{ij} is the transition rate from mode i to mode j satisfying $\pi_{ij} \geq 0$

for $i \neq j$ with $\pi_{ij} = -\sum_{j=1, j \neq i}^m \pi_{ij} (i, j \in \mathcal{S})$.

The H_∞ control problem to be addressed here is formulated as follows, design state feedback control law

$$u_k(t) = K_k x_k(t), \quad k = 1, 2, \dots, n. \quad (2)$$

such that the system is finite-time H_∞ synchronization. Where K_k is the k th node controller to be described.

For the sake of simplicity, we denote associated with the k -th modes by $\Delta_i = \Delta(r(t) = i) (i \in \mathcal{S})$, where Δ can be any matrix.

Let

$$K = [K_1, K_2, \dots, K_N]^T, \quad x(t) = [x_1(t), x_2(t), \dots, x_N(t)]^T,$$

$$\omega(t) = [\omega_1(t), \omega_2(t), \dots, \omega_N(t)]^T, \quad z(t) = [z_1(t), z_2(t), \dots, z_N(t)]^T,$$

$$F(x(t)) = [f(x_1(t)), f(x_2(t)), \dots, f(x_N(t))]^T, \quad G(x(t)) = [g(x_1(t)), g(x_2(t)), \dots, g(x_N(t))]^T,$$

From (1) to (2) the complex dynamical networks with the matrix Kronecker product, (1) can be represent as follows:

$$\begin{cases} \dot{x}(t) = (I_N \otimes (K - A))x(t) + (I_N \otimes B)F(x(t)) + (I_N \otimes C)F(x(t - \tau_1(t))) + \sigma_1(V_i \otimes \Gamma_1)x(t) \\ \quad + \sigma_2(L_i \otimes \Gamma_2)x(t - \tau_1(t)) + (I_N \otimes M) \int_{t-\tau_2(t)}^t G(x(s))ds + (I_N \otimes D)\omega(t) \\ z(t) = (I_N \otimes E)x(t) \end{cases} \quad (3)$$

Remark 2 This article considered the complex dynamic network system with the linear coupling strength on the mode at time t . While the rest of the coefficient is constant matrix, in the process of jump. This makes the complex dynamical network system is more general.

Throughout this paper, we make the following lemmas and definitions.

Lemma 1 ([40]) For a positive matrix M , scalar $h_U > h_L > 0$, such that the following integrations are well defined, it holds that

$$\begin{aligned} (1) & -(h_U - h_L) \int_{t-h_U}^{t-h_L} x^T(s) M x(s) ds \leq -(\int_{t-h_U}^{t-h_L} x^T(s) ds) M (\int_{t-h_U}^{t-h_L} x^T(s) ds), \\ (2) & -(\frac{h_U^2 - h_L^2}{2}) \int_{t-h_U}^{t-h_L} \int_s^t x^T(u) M x(u) du ds \leq -(\int_{t-h_U}^{t-h_L} \int_s^t x^T(u) du ds) M (\int_{t-h_U}^{t-h_L} \int_s^t x(u) du ds). \end{aligned}$$

Lemma 2 ([41]) If for any constant matrix $R \in \mathbb{R}^{m \times m}$, $R = R^T > 0$, scalar $\gamma > 0$ and a vector function $\phi : [0, \gamma] \rightarrow \mathbb{R}^m$ such that the integrations concerned are well defined, the following inequality hold:

$$-\gamma \int_{t-\gamma}^t \dot{\phi}^T(s) R \dot{\phi}(s) ds \leq \begin{pmatrix} \phi(t) \\ \phi(t-\gamma) \end{pmatrix}^T \begin{pmatrix} -R & R \\ * & -R \end{pmatrix} \begin{pmatrix} \phi(t) \\ \phi(t-\gamma) \end{pmatrix}.$$

Lemma 3 ([42]) Let $\otimes$ denotes the Kronecker product. A , B , C and D are matrices with appropriate dimensions. The following properties are hold:

- (1) $(cA) \otimes B = A \otimes (cB)$, for any constant c ,
- (2) $(A + B) \otimes C = A \otimes C + B \otimes C$,
- (3) $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$,
- (4) $(A \otimes B)^T = A^T \otimes B^T$.

Lemma 4 ([43]) Let $U = (u_{ij})_{N \times N}$, $P \in \mathbb{R}^{n \times n}$ and $y = (y_1^T, y_2^T, \dots, y_N^T)^T$ with $x_k, y_k \in \mathbb{R}^n$ ($k = 1, 2, \dots, N$), if $U = U^T$ and the sum of entries in each row of U is zero, then:

$$x^T(U \otimes P)y = - \sum_{1 \leq i < j \leq N} u_{ij}(x_i - x_j)^T P(y_i - y_j).$$

Lemma 5 ([44]) For the positive definite matrix S , $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{bmatrix}$, where $S_{11} \in \mathbb{R}^{n \times r}$, then the following

three conditions are equivalent

- (1) $S < 0$,
- (2) $S_{11} < 0$, $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$,
- (3) $S_{22} < 0$, $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Definition 1 ([45]) Given the time constant $t_p > 0$, and denote $x_{kl}(t) = x_k(t) - x_l(t)$, the complex networks (3) is said to be mean-square stochastically finite-time synchronized with respect $(c_1, c_2, t_p, \mathcal{W})$, if there exist two scalar $c_1 > 0$, $c_2 > c_1$, such that the following condition holds for $i, l = 1, 2, \dots, N$ and any $t \in [0, t_p]$:

$$\sum_{1 \leq k < l \leq N} \sup_{-\tau \leq \theta \leq 0} \mathcal{E} \left\{ \|x_{kl}(\theta)\|^2 \right\} \leq c_1 \Rightarrow \sum_{1 \leq k < l \leq N} \mathcal{E} \left\{ \|x_{kl}(t)\|^2 \right\} \leq c_2$$

Definition 2 ([45]) Given two scalars $\gamma > 0$, $t_p > 0$, and denoted $z_{kl}(t) = z_k(t) - z_l(t)$, the complex networks (3) is said to be mean-square stochastically finite-time H_∞ synchronization, if (I) the complex networks (3) is mean-square stochastically finite-time synchronized in the sense of definition 1 and (I) under zero initial state, the following condition is satisfied:

$$\sum_{1 \leq k < l \leq N} \mathcal{E} \left\{ \int_0^\infty [z_{kl}^T(t) z_{kl}(t) - \gamma^2 \omega_{kl}^T(t) \omega_{kl}(t)] dt \right\} < 0.$$

3. Main results

In this section, a design problem of the feedback controller for the finite-time H_∞ synchronization of a complex dynamical networks with mixed mode-dependent time delays will be studied via a continuous Lyapunov functional approach.

Theorem 1 For given scalar $0 < c_1 < c_2$, $0 < \tilde{\gamma}$, and $0 < \rho$, the complex dynamical networks (3) is mean-square stochastically finite-time H_∞ synchronized, if there exist matrices $P_i > 0$, $Q_r > 0$, ($r = 1, 2, 3, 4$), such that the following matrix inequalities holds for $i \in \mathcal{S}$:

$$\Theta(t) < 0, \quad (4)$$

$$c_1(a + b) + \tilde{\gamma}^2 e^{\rho t_p} \mathcal{W} < \lambda_1 c_2, \quad (5)$$

where

$$\Theta(t) = \begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} & P_i B + \Phi_1 C & \Theta_{15} & 0 & 0 & P_i M + \Phi_1 M & \Theta_{19} & \Theta_{110} \\ * & \Theta_{22} & 0 & 0 & 0 & 0 & 0 & 0 & \Theta_{29} & 0 \\ * & * & \Theta_{33} & 0 & 0 & 0 & 0 & 0 & (\Phi_2 B)^T & 0 \\ * & * & * & -(1 - d_1)Q_2 & 0 & 0 & 0 & 0 & (\Phi_2 C)^T & 0 \\ * & * & * & * & \Theta_{55} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -Q_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -Q_2 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\frac{1}{\tau_1}Q_3 & (\Phi_2 M)^T & 0 \\ * & * & * & * & * & * & * & * & \tau_1^2 Q_4 - 2\Phi_2 & \Phi_2 D \\ * & * & * & * & * & * & * & * & * & -\tilde{\gamma}I \end{bmatrix},$$

$$\lambda_1 = \min(\lambda_{\min}(P_i), i \in \mathcal{S}), \lambda_2 = \max(\lambda_{\max}(P_i), i \in \mathcal{S}),$$

$$\lambda_3 = \lambda_{\max}(Q_1), \lambda_4 = \lambda_{\max}(Q_2), \lambda_5 = \lambda_{\max}(Q_3), \lambda_6 = \lambda_{\max}(Q_4), \lambda_7 = \lambda_{\max}(l^+ l^+), \lambda_8 = \lambda_{\max}(c^+ c^+),$$

$$a = \lambda_2 + (\tau_1 + \frac{\tau_1^3}{2})\lambda_3 + 4\tau_1\lambda_6, b = (\tau_1 + \frac{\tau_1^3}{2})\lambda_4\lambda_7 + (\frac{\tau_2^2}{2} + \frac{\tau_1^3}{6})\lambda_5\lambda_8,$$

$$l^+ = \text{diag}\{l_1^+, l_2^+, \dots, l_N^+\}, c^+ = \text{diag}\{c_1^+, c_2^+, \dots, c_N^+\},$$

$$\Theta_{11} = P_i(K - A) + (K - A)^T P_i + 2\sigma_1 N V_{kli} P_i \Gamma_1 + (1 + \tau_1^2)Q_1 - Q_4 + \sum_{j=1}^S \pi_{ij} P_j - \Lambda_1 S \Lambda_2$$

$$- \Lambda_1 T \Lambda_2 + E^T E - \rho P_i + 2\Phi_1(K - A) + 2\sigma_1 N V_{kli} \Phi_1 \Gamma_1,$$

$$\Theta_{12} = \sigma_1 N L_{kli} P_i \Gamma_2 + Q_4 + \sigma_2 N L_{kli} \Phi_2 \Gamma_2,$$

$$\Theta_{13} = P_i B + \Phi_1 B + \frac{1}{2}S(\Lambda_1 + \Lambda_2),$$

$$\Theta_{15} = \frac{1}{2}T(\Lambda_1 + \Lambda_2),$$

$$\Theta_{19} = -\Phi_1 + (\Phi_2(K - A))^T + (\sigma_1 N V_{kli} \Phi_2 \Gamma_2)^T,$$

$$\Theta_{110} = P_i D + \Phi_1 D,$$

$$\Theta_{22} = -(1 - d_1)Q_1 - Q_4,$$

$$\Theta_{29} = (\sigma_2 N L_{kli} \Phi_2 \Gamma_2)^T,$$

$$\Theta_{33} = (1 + \tau_1^2)Q_2 - S,$$

$$\Theta_{55} = d_2 Q_3 + \frac{\tau_2^2}{2} Q_3 - T,$$

Proof: For $i \in \mathcal{S}$, choose the following Markovian switched Lyapunov functional candidate as

$$V(x(t), i, t) = V_1(x(t), i, t) + V_2(x(t), i, t) + V_3(x(t), i, t) + V_4(x(t), i, t) + V_5(x(t), i, t), \quad (6)$$

$$\begin{aligned} V_1(x(t), i, t) &= x^T(t)(U \otimes P_i)x(t), \\ V_2(x(t), i, t) &= \int_{t-\tau_1(t)}^t x^T(s)(U \otimes Q_1)x(s)ds + \tau_1 \int_0^{\tau_1} \int_{t-\theta}^t x^T(s)(U \otimes Q_1)x(s)dsd\theta, \\ V_3(x(t), i, t) &= \int_{t-\tau_1(t)}^t F^T(x(s))(U \otimes Q_2)F(x(s))ds + \tau_1 \int_0^{\tau_1} \int_{t-\theta}^t F^T(x(s))(U \otimes Q_2)F(x(s))dsd\theta, \\ V_4(x(t), i, t) &= \int_0^{\tau_2(t)} \int_{t-\theta}^t G^T(x(s))(U \otimes Q_3)G(x(s))dsd\theta + \int_0^{\tau_2} \int_0^u \int_{t-\theta}^t G^T(x(s))(U \otimes Q_3)G(x(s))dsd\theta du, \\ V_5(x(t), i, t) &= \tau_1 \int_0^{\tau_1} \int_{t-\theta}^t \dot{x}^T(s)(U \otimes Q_4)\dot{x}(s)dsd\theta, \end{aligned}$$

$$U = \begin{bmatrix} N-1 & -1 & \cdots & -1 \\ -1 & N-1 & \cdots & -1 \\ \vdots & \vdots & \cdots & \vdots \\ -1 & -1 & \cdots & N-1 \end{bmatrix}_{N \times N}.$$

Then, as noted in [45-47], Let $\mathcal{L}$ be the weak infinitesimal generator of the random process $\{x(t), i, t \geq 0\}$, along the system (3) defined by

$$\mathcal{L}V(x(t), i, t) = \lim_{\Delta \rightarrow 0^+} \frac{1}{\Delta} [\mathcal{E}[V(x(t+\Delta), t+\Delta, r_{t+\Delta})] | x(t), r_{t=i}] - V(x(t), i, t).$$

Then, it can be obtained from (6) that

$$\mathcal{L}V(x(t), i, t) = \sum_{i=1}^5 \mathcal{L}V_i(x(t), i, t), \quad (7)$$

$$\begin{aligned} \mathcal{L}V_1(x(t), i, t) &= \dot{x}^T(t)(U \otimes P_i)x(t) + x^T(t)(U \otimes P_i)\dot{x}(t) + x^T(t) \sum_{j=1}^s \pi_{ij}(U \otimes P_j)x(t) \\ &= 2x^T(t)(U \otimes P_i)\dot{x}(t) + x^T(t) \sum_{j=1}^s \pi_{ij}(U \otimes P_j)x(t) \\ &= 2x^T(t)(U \otimes P_i) [(I_N \otimes (K - A))x(t) + (I_N \otimes B)F(x(t)) + (I_N \otimes C)F(x(t - \tau_1(t)))] \\ &\quad + \sigma_1(V_i \otimes \Gamma_1)x(t) + \sigma_2(L_i \otimes \Gamma_2)x(t - \tau_1(t)) + (I_N \otimes D)\omega(t) + x^T(t) \sum_{j=1}^s \pi_{ij}(U \otimes P_j)x(t) \\ &= 2x^T(t)(U \otimes P_i(K - A))x(t) + 2x^T(t)(U \otimes P_i B)F(x(t)) \\ &\quad + 2x^T(t)(U \otimes P_i C)F(x(t - \tau_1(t))) + 2\sigma_1 x^T(t)(U V_i \otimes P_i \Gamma_1)x(t) \\ &\quad + 2\sigma_2 x^T(t)(U L_i \otimes P_i \Gamma_2)x(t - \tau_1(t)) + 2x^T(t)(U \otimes P_i M) \int_{t-\tau_2(t)}^t G(x(s))ds \\ &\quad + 2x^T(t)(U \otimes P_i D)\omega(t) + x^T(t) \sum_{j=1}^s \pi_{ij}(U \otimes P_j)x(t), \end{aligned} \quad (8)$$

$$\begin{aligned}
\mathcal{L}V_2(x(t), i, t) &= x^T(t)(U \otimes Q_1)x(t) - (1 - \dot{\tau}(t))x^T(t - \tau_1(t))(U \otimes Q_1)x(t - \tau_1(t)) \\
&\quad + \tau_1 \int_0^{\tau_1} x^T(t)(U \otimes Q_1)x(t)d\theta - \tau_1 \int_0^{\tau_1} x^T(t - \theta)(U \otimes Q_1)x(t - \theta)d\theta \\
&\leq x^T(t)[(1 + \tau_1^2)(U \otimes Q_1)]x(t) - (1 - d_1)x^T(t - \tau_1(t))(U \otimes Q_1)x(t - \tau_1(t)) \\
&\quad - \tau_1 \int_{t-\tau_1}^t x^T(s)(U \otimes Q_1)x(s)ds,
\end{aligned} \tag{9}$$

$$\begin{aligned}
\mathcal{L}V_3(x(t), i, t) &= F^T(x(t))(U \otimes Q_2)F(x(t)) - (1 - \dot{\tau}_1(t))F^T(x(t - \tau_1(t)))(U \otimes Q_2)F(x(t - \tau_1(t))) \\
&\quad + \tau_1^2 F^T(x(t))(U \otimes Q_2)F(x(t)) - \tau_1 \int_{t-\tau_1}^t F^T(x(s))(U \otimes Q_2)F(x(s))ds \\
&\leq (1 + \tau_1^2)F^T(x(t))(U \otimes Q_2)F(x(t)) - (1 - d_1)F^T(x(t - \tau_1(t)))(U \otimes Q_2)F(x(t - \tau_1(t))) \\
&\quad - \tau_1 \int_{t-\tau_1}^t F^T(x(s))(U \otimes Q_2)F(x(s))ds,
\end{aligned} \tag{10}$$

$$\begin{aligned}
\mathcal{L}V_4(x(t), i, t) &= \int_0^{\tau_2(t)} G^T(x(t))(U \otimes Q_3)G(x(t))ds - \int_0^{\tau_2(t)} G^T(x(t - \theta))(U \otimes Q_3)G(x(t - \theta))d\theta \\
&\quad + \int_0^{\tau_2} \int_0^u G^T(x(t))(U \otimes Q_3)G(x(t))d\theta du - \int_0^{\tau_2} \int_0^u G^T(x(t - \theta))(U \otimes Q_3)G(x(t - \theta))d\theta du \\
&= \tau_2(t)G^T(x(t))(U \otimes Q_3)G(x(t)) - \int_{t-\tau_2(t)}^t G^T(x(s))(U \otimes Q_3)G(x(s))ds \\
&\quad + \frac{\tau_2^2}{2}G^T(x(t))(U \otimes Q_3)G(x(t)) - \int_0^{\tau_2} \int_{t-u}^t G^T(x(s))(U \otimes Q_3)G(x(s))dsdu,
\end{aligned} \tag{11}$$

$$\mathcal{L}V_5(x(t), i, t) = \tau_1^2 \dot{x}^T(t)(U \otimes Q_4)\dot{x}(t) - \tau_1 \int_{t-\tau_1}^t \dot{x}^T(s)(U \otimes Q_4)\dot{x}(s)ds. \tag{12}$$

Base on lemma 1, it can be easy to know that

$$- \tau_1 \int_{t-\tau_1}^t F^T(x(s))(U \otimes Q_2)F(x(s))ds \leq - \int_{t-\tau_1}^t F^T(x(s))ds(U \otimes Q_2) \int_{t-\tau_1}^t F(x(s))ds, \tag{13}$$

$$- \int_{t-\tau_2(t)}^t G^T(x(s))(U \otimes Q_3)G(x(s))ds \leq - \frac{1}{\tau_2} \int_{t-\tau_2(t)}^t G^T(x(s))ds(U \otimes Q_3) \int_{t-\tau_2(t)}^t G(x(s))ds. \tag{14}$$

Also, from lemma 2, it is not difficult to verify that

$$- \tau_1 \int_{t-\tau_1}^t \dot{x}^T(s)(U \otimes Q_4)\dot{x}(s)ds \leq \begin{bmatrix} x^T(t) & x^T(t - \tau_1(t)) \end{bmatrix} \begin{bmatrix} -U \otimes Q_4 & U \otimes Q_4 \\ * & -U \otimes Q_4 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t - \tau_1(t)) \end{bmatrix} \tag{15}$$

Substituting (8)-(15) into (7) yields that

$$\begin{aligned}
\mathcal{LV}(x(t)) &\leq 2x^T(t)(U \otimes P_i(K - A))x(t) + 2x^T(t)(U \otimes P_i B)F(x(t)) + 2x^T(t)(U \otimes P_i C)F(x(t - \tau_1(t))) \\
&\quad + 2\sigma_1 x^T(t)(UV_i \otimes P_i \Gamma_1)x(t) + 2\sigma_2 x^T(t)(UL_i \otimes P_i \Gamma_2)x(t - \tau_1(t)) \\
&\quad + 2x^T(t)(U \otimes P_i M) \int_{t-\tau_2(t)}^t G(x(s))ds + 2x^T(t)(U \otimes P_i D)\omega(t) + x^T(t) \sum_{j=1}^s \pi_{ij}(U \otimes P_j)x(t) \\
&\quad + x^T(t)[(1 + \tau_1^2)(U \otimes Q_1)]x(t) - (1 - d_1)x^T(t - \tau_1(t))(U \otimes Q_1)x(t - \tau_1(t)) \\
&\quad - \tau_1 \int_{t-\tau_1}^t x^T(s)(U \otimes Q_1)x(s)ds + F^T(x(t))(1 + \tau_1^2)(U \otimes Q_2)F(x(t)) \\
&\quad - (1 - d_1)F^T(x(t - \tau_1(t)))(U \otimes Q_2)F(x(t - \tau_1(t))) \\
&\quad - \int_{t-\tau_1}^t F^T(x(s))ds(U \otimes Q_2) \int_{t-\tau_1}^t F(x(s))ds + \tau_2 G^T(x(t))(U \otimes Q_3)G(x(t)) \\
&\quad - \frac{1}{\tau_2} \int_{t-\tau_2(t)}^t G^T(x(s))ds(U \otimes Q_3) \int_{t-\tau_2(t)}^t G(x(s))ds + \frac{\tau_2^2}{2} G^T(x(t))(U \otimes Q_3)G(x(t)) \\
&\quad + \tau_1^2 \dot{x}^T(t)(U \otimes Q_4)\dot{x}(t) + \begin{bmatrix} x(t) & x(t - \tau_1(t)) \end{bmatrix}^T \begin{bmatrix} -U \otimes Q_4 & U \otimes Q_4 \\ * & -U \otimes Q_4 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t - \tau_1(t)) \end{bmatrix}.
\end{aligned} \tag{16}$$

With lemma 4 and (16), we have

$$\begin{aligned}
\mathcal{LV}(x(t)) &\leq \sum_{1 \leq k < l \leq N} [x_{kl}^T(t)(2P_i(K - A) + 2\sigma_1 NV_{kli}P_i \Gamma_1 + (1 + \tau_1^2)Q_1 - Q_4 + \sum_{j=1}^s \pi_{ij}P_j)x_{kl}(t) \\
&\quad + x_{kl}^T(t)(2\sigma_2 NL_{kli}P_i \Gamma_2 + 2Q_4)x_{kl}(t - \tau_1(t)) + 2x_{kl}^T(t)P_i Bf(x_{kl}(t)) \\
&\quad + 2x_{kl}^T(t)P_i C f(x_{kl}(t - \tau_1(t))) + 2x_{kl}^T(t)P_i M \int_{t-\tau_2(t)}^t g(x_{kl}(s))ds + 2x_{kl}^T(t)P_i D\omega_{kl}(t) \\
&\quad + x_{kl}^T(t - \tau_1(t))(-(1 - d_1)Q_1 - Q_4)x_{kl}(t - \tau_1(t)) \\
&\quad - \int_{t-\tau_1}^t x_{kl}^T(s)dsQ_1 \int_{t-\tau_1}^t x_{kl}(s)ds + f^T(x_{kl}(t))(1 + \tau_1^2)Q_2 f(x_{kl}(t)) \\
&\quad - (1 - d_1)f^T(x_{kl}(t - \tau_1(t)))Q_2 f(x_{kl}(t - \tau_1(t))) \\
&\quad - \int_{t-\tau_1}^t f^T(x_{kl}(s))dsQ_2 \int_{t-\tau_1}^t f(x_{kl}(s))ds + g^T(x_{kl}(t))(\tau_2 Q_3 + \frac{\tau_2^2}{2}Q_3)g(x_{kl}(t)) \\
&\quad - \frac{1}{\tau_2} \int_{t-\tau_2(t)}^t g^T(x_{kl}(s))dsQ_3 \int_{t-\tau_2(t)}^t g(x_{kl}(s))ds + \tau_1^2 \dot{x}_{kl}^T(t)(U \otimes Q_4)\dot{x}_{kl}(t)],
\end{aligned} \tag{17}$$

where

$$\begin{aligned}
x_{kl}(t) &= x_k(t) - x_l(t), \\
x_{kl}(t - \tau_1(t)) &= x_k(t - \tau_1(t)) - x_l(t - \tau_1(t)), \\
f(x_{kl}(t)) &= f(x_k(t)) - f(x_l(t)), \\
f(x_{kl}(t - \tau_1(t))) &= f(x_k(t - \tau_1(t))) - f(x_l(t - \tau_1(t))), \\
\int_{t-\tau_1}^t x_{kl}(s)ds &= \int_{t-\tau_1}^t x_k(s)ds - \int_{t-\tau_1}^t x_l(s)ds, \\
\int_{t-\tau_1}^t f(x_{kl}(s))ds &= \int_{t-\tau_1}^t f(x_k(s))ds - \int_{t-\tau_1}^t f(x_l(s))ds, \\
\int_{t-\tau_2(t)}^t g(x_{kl}(s))ds &= \int_{t-\tau_2(t)}^t g(x_k(s))ds - \int_{t-\tau_2(t)}^t g(x_l(s))ds, \\
\omega_{kl}(t) &= \omega_k(t) - \omega_l(t).
\end{aligned}$$

By noting that for any appropriately dimensioned matrices Φ_1 and Φ_2 , it is shown that

$$\begin{aligned}
0 = & 2[x^T(t)(U \otimes \Phi_1) + \dot{x}^T(t)(U \otimes \Phi_2)][-\dot{x}(t) + (I_N \otimes (K - A))x(t) + (I_N \otimes B)F(x(t)) \\
& + (I_N \otimes C)F(x(t - \tau_1(t))) + \sigma_1(V_i \otimes \Gamma_1)x(t) + \sigma_2(L_i \otimes \Gamma_2)x(t - \tau_1(t)) \\
& + (I_N \otimes M) \int_{t-\tau_2(t)}^t G(x(s))ds + (I_N \otimes D)\omega(t)].
\end{aligned} \tag{18}$$

It can be deduce from Assumption 1 that, for diagonal matrices S and T , the following inequalities hold:

$$\sum_{1 \leq k < l \leq N} [x_{kl}^T(t)\Lambda_1 S \Lambda_2 x_{kl}(t) - x_{kl}^T(t)S(\Lambda_1 + \Lambda_2)f(x_{kl}(t)) + f^T(x_{kl}(t))Sf(x_{kl}(t))] \leq 0, \tag{19}$$

$$\sum_{1 \leq k < l \leq N} [x_{kl}^T(t)\Lambda_1 T \Lambda_2 x_{kl}(t) - x_{kl}^T(t)T(\Lambda_1 + \Lambda_2)g(x_{kl}(t)) + g^T(x_{kl}(t))Tg(x_{kl}(t))] \leq 0. \tag{20}$$

The inequalities (4), (19) and (20) are established. And according to Lemma 3 and lemma 4, one can obtain

$$\begin{aligned}
\mathcal{L}V(x(t), i, t) + z^T(t)(U \otimes I)z(t) - \tilde{\gamma}^2 \omega^T(t)(U \otimes I)\omega(t) & \leq \mathcal{L}V(x(t), i, t) + z^T(t)(U \otimes I)z(t) \\
& - \tilde{\gamma}^2 \omega^T(t)(U \otimes I)\omega(t) + \rho x^T(t)(U \otimes P_i)x(t) \\
& - \rho x^T(t)(U \otimes P_i)x(t) \\
& = \sum_{1 \leq k < l \leq N} \zeta_{kl}^T \Theta(t) \zeta_{kl} + \rho x^T(t)(U \otimes P_i)x(t) \\
& < \rho x^T(t)(U \otimes P_i)x(t),
\end{aligned} \tag{21}$$

where

$$\begin{aligned}
\zeta_{kl} = & [x_{kl}^T(t) x_{kl}^T(t - \tau_1(t)) f^T(x_{kl}(t)) f^T(x_{kl}(t - \tau_1(t))) g^T(x_{kl}(t)) \int_{t-\tau_1}^t x_{kl}^T(s)ds \\
& \int_{t-\tau_1}^t f^T(x_{kl}(s))ds \int_{t-\tau_2(t)}^t g^T(x_{kl}(s))ds \dot{x}_{kl}^T(t) \omega_{kl}^T(t)]^T.
\end{aligned}$$

From (21), one can obtain that

$$\mathcal{L}V(x(t), i, t) < \rho V(x(t), i, t) - z^T(t)(U \otimes I)z(t) + \tilde{\gamma}^T \omega^T(t)(U \otimes I)\omega(t). \tag{22}$$

Multiplying the above inequality by $e^{-\rho t}$, we have

$$\begin{aligned}
\frac{d}{dt}(e^{-\rho t}V(x(t), i, t)) & \leq -e^{-\rho t}z^T(t)(U \otimes I)z(t) + e^{-\rho t}\tilde{\gamma}^2 \omega^T(t)(U \otimes I)\omega(t) \\
& \leq e^{-\rho t}\tilde{\gamma}^2 \omega^T(t)(U \otimes I)\omega(t).
\end{aligned} \tag{23}$$

By integrating the inequality in (23) between 0 and t_p and taking the expectation, one can obtain readily that

$$\begin{aligned}
\mathcal{E}\{V(x(t), i, t)\} & < e^{\rho t_p} \mathcal{E}\{V(x(0), i, 0)\} + \tilde{\gamma}^2 e^{\rho t_p} \mathcal{E}\left\{\int_0^{t_p} e^{-\rho t} \omega^T(t)(U \otimes I)\omega(t)dt\right\} \\
& \leq e^{\rho t_p} \mathcal{E}\{V(x(0), i, 0)\} + \tilde{\gamma}^2 e^{\rho t_p} \mathcal{W}.
\end{aligned} \tag{24}$$

On the other hand from (6) we get

$$\begin{aligned}
\mathcal{E}\{V(x(t), i, t)\} & \geq \mathcal{E}\{x^T(t)(U \otimes P_i)x(t)\} \\
& \geq \min\{\lambda_{\min}(P_i), i \in \mathcal{S}\} \sum_{1 \leq k < l \leq N} \mathcal{E}\{\|x_k(t) - x_l(t)\|^2\}.
\end{aligned} \tag{25}$$

And

$$\begin{aligned}
\mathcal{E} \{V(x(0), i, 0)\} &\leq x^T(0)(U \otimes P_i)x(0) + \int_{0-\tau_1(0)}^0 x^T(s)(U \otimes Q_1)x(s)ds \\
&\quad + \tau_1 \int_0^{\tau_1} \int_{-\theta}^0 x^T(s)(U \otimes Q_1)x(s)dsd\theta + \int_{-\tau_1(0)}^0 F^T(x(s))(U \otimes Q_2)F(x(s))ds \\
&\quad + \tau_1 \int_0^{\tau_1} \int_{-\theta}^0 F^T(x(s))(U \otimes Q_2)F(x(s))dsd\theta \\
&\quad + \int_0^{\tau_2(0)} \int_{-\theta}^0 G^T(x(s))(U \otimes Q_3)G(x(s))dsd\theta \\
&\quad + \int_0^{\tau_2} \int_0^u \int_{-\theta}^0 G^T(x(s))(U \otimes Q_3)G(x(s))dsd\theta du + \tau_1 \int_0^{\tau_1} \int_{0-\theta}^0 \dot{x}^T(s)(U \otimes Q_4)\dot{x}(s)dsd\theta \\
&\leq \max \{ \lambda_{\max}(P_i), i \in \mathcal{S} \} \sum_{1 \leq k < l \leq N} \mathcal{E} \left\{ \|x_k(0) - x_l(0)\|^2 \right\} \\
&\quad + \left(\tau_1 + \frac{\tau_1^3}{2} \right) \max \{ \lambda_{\max}(Q_1) \} \sum_{1 \leq k < l \leq N} \sup_{-\bar{\tau} \leq \theta \leq 0} \mathcal{E} \left\{ \|x_k(\theta) - x_l(\theta)\|^2 \right\} \\
&\quad + \left[\left(\tau_1 + \frac{\tau_1^3}{2} \right) \max \{ \lambda_{\max}(Q_2) \} \right] \times \lambda_{\max}(l^+ l^+) \sum_{1 \leq k < l \leq N} \sup_{-\bar{\tau} \leq \theta \leq 0} \mathcal{E} \left\{ \|x_k(\theta) - x_l(\theta)\|^2 \right\} \\
&\quad + \left[\left(\frac{\tau_2^2}{2} + \frac{\tau_1^3}{6} \right) \max \{ \lambda_{\max}(Q_3) \} \right] \times \lambda_{\max}(c^+ c^+) \sum_{1 \leq k < l \leq N} \sup_{-\bar{\tau} \leq \theta \leq 0} \mathcal{E} \left\{ \|x_k(\theta) - x_l(\theta)\|^2 \right\} \\
&\quad + 2\tau_1 \lambda_{\max}(Q_4) \sum_{1 \leq k < l \leq N} \mathcal{E} \left\{ \|x_k(0) - x_l(0)\|^2 \right\} \\
&\quad + 2\tau_1 \lambda_{\max}(Q_4) \sum_{1 \leq k < l \leq N} \sup_{-\bar{\tau} \leq \theta \leq 0} \mathcal{E} \left\{ \|x_k(\theta) - x_l(\theta)\|^2 \right\} \\
&\leq c_1 \left[\max \{ \lambda_{\max}(P_i), i \in \mathcal{S} \} + \left(\tau_1 + \frac{\tau_1^3}{2} \right) \max \{ \lambda_{\max}(Q_1) \} + 4\tau_1 \max \{ \lambda_{\max}(Q_4) \} \right] \\
&\quad + c_1 \left[\left(\tau_1 + \frac{\tau_1^3}{2} \right) \max \{ \lambda_{\max}(Q_2) \} \times \lambda_{\max}(l^+ l^+) \right] \\
&\quad + c_1 \left[\left(\frac{\tau_2^2}{2} + \frac{\tau_1^3}{6} \right) \max \{ \lambda_{\max}(Q_3) \} \times \lambda_{\max}(c^+ c^+) \right].
\end{aligned} \tag{26}$$

Then, we can obtain

$$\begin{aligned}
&\sum_{1 \leq k < l \leq N} \mathcal{E} \left\{ \|x_k(t) - x_l(t)\|^2 \right\} \\
&\leq \frac{c_1 \left[\max \{ \lambda_{\max}(P_i), i \in \mathcal{S} \} + \left(\tau_1 + \frac{\tau_1^3}{2} \right) \max \{ \lambda_{\max}(Q_1) \} + 4\tau_1 \max \{ \lambda_{\max}(Q_4) \} \right] + \tilde{\gamma}^2 e^{\rho t_p} \mathcal{W}}{\min \{ \lambda_{\min}(P_i), i \in \mathcal{S} \}} \\
&\quad + \frac{c_1 \left[\left[\left(\tau_1 + \frac{\tau_1^3}{2} \right) \max \{ \lambda_{\max}(Q_2) \} \right] \times \lambda_{\max}(l^+ l^+) + \left[\left(\frac{\tau_2^2}{2} + \frac{\tau_1^3}{6} \right) \max \{ \lambda_{\max}(Q_3) \} \right] \times \lambda_{\max}(c^+ c^+) \right]}{\min \{ \lambda_{\min}(P_i), i \in \mathcal{S} \}} \\
&< c_2.
\end{aligned} \tag{27}$$

Furthermore, under zero initial state, integrating the inequality in (24) between 0 and t_p and taking the expectation, yields

$$\begin{aligned}
0 &< \mathcal{E} \{V(x(t), i, t)\} \\
&< -e^{\rho t_p} \mathcal{E} \left\{ \int_0^{t_p} e^{-\rho t} z^T(t) (U \otimes I) z(t) dt \right\} + \tilde{\gamma}^2 e^{\rho t_p} \mathcal{E} \left\{ \int_0^{t_p} e^{-\rho t} \omega^T(t) (U \otimes I) \omega(t) dt \right\} \\
&\leq -\mathcal{E} \left\{ \int_0^{t_p} z^T(t) (U \otimes I) z(t) dt \right\} + \tilde{\gamma}^2 e^{\rho t_p} \mathcal{E} \left\{ \int_0^{t_p} e^{-\rho t} \omega^T(t) (U \otimes I) \omega(t) dt \right\},
\end{aligned} \tag{28}$$

which implies

$$\mathcal{E} \left\{ \int_0^{t_p} z^T(t) (U \otimes I) z(t) dt \right\} < \gamma^2 \mathcal{E} \left\{ \int_0^{t_p} \omega^T(t) (U \otimes I) \omega(t) dt \right\}, \quad \gamma = \tilde{\gamma} \sqrt{e^{\rho t_p}}.$$

It is clearly satisfied the conditions in the definition 2. Hence the complex dynamical networks with mixed time delays (3) is mean-square stochastically finite-time H_∞ synchronized according to Definition 2. The proof of this theorem is now complete.

Remark 3 Theorem 1 gives the sufficient conditions for the finite time H_∞ synchronization. But the conditions (4) and (5) are nonlinear matrix inequalities. Thus simplified into a linear matrix inequality (LMI) is very necessary in Theorem 2.

Theorem 2 Consider complex dynamical networks (3), let scalars $0 < c_1 < c_2$, $\rho > 0$, $t_p > 0$, $\tilde{\gamma} > 0$, $\mathcal{W} > 0$, matrix $R_0 > 0$ be given. Then there exists a properly controller, satisfying the aforementioned requirements if there exist scalar $\tilde{Q}_b > 0$, ($b = 1, 2, 3, 4$), $X_i > 0$, $Y_i > 0$ ($i = 1, 2$), constant $v > 0$, such that the following matrix inequalities hold for each $i \in \mathcal{S}$:

$$\tilde{\Theta}(t) < 0, \tag{29}$$

$$vR_0^{-1} < X < R_0^{-1}, \quad 0 < \tilde{Q}_1 < 2R_0^{-1}, \quad 0 < \tilde{Q}_2 < 2R_0^{-1}, \quad 0 < \tilde{Q}_3 < 2R_0^{-1}, \quad 0 < \tilde{Q}_4 < 2R_0^{-1}, \tag{30}$$

$$\varphi = \begin{bmatrix} c_1 + v(\tilde{\gamma}^2 e^{\rho t_p} \mathcal{W} - c_2) & \varphi_1 \\ * & -v \end{bmatrix} < 0, \tag{31}$$

where

$$\tilde{\Theta}(t) = \begin{bmatrix} \tilde{\Theta}_{11} & \tilde{\Theta}_{12} & \tilde{\Theta}_{13} & BX + \tilde{\Phi}_1 CX & \tilde{\Theta}_{15} & 0 & 0 & \tilde{\Theta}_{18} & \tilde{\Theta}_{19} & \tilde{\Theta}_{110} \\ * & \tilde{\Theta}_{22} & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{\Theta}_{29} & 0 \\ * & * & \tilde{\Theta}_{33} & 0 & 0 & 0 & 0 & 0 & (\tilde{\Phi}_2 B)^T X & 0 \\ * & * & * & -(1-d_1)\tilde{Q}_2 & 0 & 0 & 0 & 0 & (\tilde{\Phi}_2 C)^T X & 0 \\ * & * & * & * & \tilde{\Theta}_{55} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\tilde{Q}_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\tilde{Q}_2 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\frac{1}{\tau_1} \tilde{Q}_3 & (\tilde{\Phi}_2 M)^T X & 0 \\ * & * & * & * & * & * & * & * & \tilde{\Theta}_{99} & \tilde{\Phi}_2 DX \\ * & * & * & * & * & * & * & * & * & -\tilde{\gamma} N \end{bmatrix},$$

$$\tilde{\Theta}_{11} = (Y - AX) + (Y - AX)^T + 2\sigma_1 NV_{kli} \tilde{\Gamma}_1 + (1 + \tau_1^2) \tilde{Q}_1 - \tilde{Q}_4 - \Lambda_1 \tilde{S} \Lambda_2 - \Lambda_1 \tilde{T} \Lambda_2$$

$$+ 2\sigma_1 NV_{kli} \tilde{\Phi}_1 \tilde{\Gamma}_1 + 2\tilde{\Phi}_1(Y - AX) + \tilde{E} \tilde{E} - \rho X + \sum_{i=1}^S \pi_{ij} X,$$

$$\tilde{\Theta}_{12} = \sigma_1 NL_{kli} \tilde{\Gamma}_2 + \tilde{Q}_4 + \sigma_2 NL_{kli} \tilde{\Phi}_2 \tilde{\Gamma}_2,$$

$$\tilde{\Theta}_{13} = BX + \tilde{\Phi}_1 BX + \frac{1}{2} \tilde{S} (\Lambda_1 + \Lambda_2),$$

$$\tilde{\Theta}_{15} = \frac{1}{2} \tilde{T} (\Lambda_1 + \Lambda_2),$$

$$\tilde{\Theta}_{18} = MX + \tilde{\Phi}_1 MX,$$

$$\begin{aligned}
\tilde{\Theta}_{19} &= -\tilde{\Phi}_1 X + (\tilde{\Phi}_2(Y - AX))^T + (\sigma_1 N V_{kli} \tilde{\Phi}_2 \tilde{\Gamma}_2)^T, \\
\tilde{\Theta}_{110} &= DX + \tilde{\Phi}_1 DX, \\
\tilde{\Theta}_{22} &= -(1 - d_1)\tilde{Q}_1 - \tilde{Q}_4, \\
\tilde{\Theta}_{29} &= (\sigma_2 N L_{kli} \tilde{\Phi}_2 \tilde{\Gamma}_2)^T, \\
\tilde{\Theta}_{33} &= (1 + \tau_1^2)\tilde{Q}_2 - \tilde{S}, \\
\tilde{\Theta}_{55} &= d_2\tilde{Q}_3 + \frac{\tau_2^2}{2}\tilde{Q}_3 - \tilde{T}, \\
\tilde{\Theta}_{99} &= \tau_1^2\tilde{Q}_4 - \tilde{\Phi}_2 X, \\
\varphi_1 &= \left[8\tau_1 + \tau_1^3 + \lambda_7(\tau_1^3 + 2\tau_1^2) + \lambda_8(\frac{\tau_1^2}{3} + \tau_2^2) \right]^{1/2},
\end{aligned}$$

Then the complex dynamical networks (3) is finite time H_∞ synchronized with mixed mode-dependent delays, and the controller feedback gains are given as: $K = YX^{-1}$.

Proof: Let $X = P^{-1}$, pre- and post-multiply (29) by $\text{diag}\{X^{-1}, X^{-1}, \dots, X^{-1}\}$, and we denote

$$\begin{aligned}
Q_1 &= X^{-1}\tilde{Q}_1X^{-1}, Q_2 = X^{-1}\tilde{Q}_2X^{-1}, Q_3 = X^{-1}\tilde{Q}_3X^{-1}, Q_4 = X^{-1}\tilde{Q}_4X^{-1}, \\
S &= X^{-1}\tilde{S}X^{-1}, T = X^{-1}\tilde{T}X^{-1}, I = X^{-1}NX^{-1}, E = X^{-1}\tilde{E}, \\
\Gamma_1 &= \tilde{\Gamma}_1X^{-1}, \Gamma_2 = \tilde{\Gamma}_2X^{-1}, \Phi_1 = X^{-1}\tilde{\Phi}_1, \Phi_2 = X^{-1}\tilde{\Phi}_2.
\end{aligned}$$

It is clearly that (4) can be obtained easily.

On the other hand, by from $vR_0^{-1} < X < R_0^{-1}$, we can get $\lambda_1 > 1$, $\lambda_2 < 1/v$, from $0 < \tilde{Q}_1 < 2R_0^{-1}$, namely, $0 < Q_1 < 2PR_0^{-1}P$. Then $0 < R_0^{-1/2}Q_1R_0^{-1/2} < R_0^{-1/2}PR_0^{-1}PR_0^{-1/2} = R_0^{-1/2}PR_0^{-1/2}R_0^{-1/2}PR_0^{-1/2} \leq \frac{2}{v^2}I$, we can get $\lambda_3 < \frac{2}{v^2}$, and similarly, $\lambda_4 < \frac{2}{v^2}$, $\lambda_5 < \frac{2}{v^2}$ and $\lambda_6 < \frac{2}{v^2}$.

Based on above analysis, condition (5) can be guaranteed by

$$c_1\left(\frac{1}{v} + \frac{8\tau_1 + \tau_1^3}{v^2} + \frac{2\tau_1 + \tau_1^3}{v^2} \cdot \lambda_7 + \left(\frac{\tau_2^2}{2} + \frac{\tau_1^3}{6}\right) \cdot \frac{2}{v^2} \cdot \lambda_8\right) + \tilde{\gamma}^2 e^{\rho t_p} \mathcal{W} < c_2.$$

By using Lemma 5, (5) can be obtain from (31), which are LMIs. This completes the proof.

Remark 4 The model is involved Markovian jump. In this paper, the complex dynamical networks model (1) is more general then [48-50], which is not involved in Markovian jump. the complex dynamical networks system contains Markovian jumping parameters, the mixed time-delays and the external disturbance.

Remark 5 In Theorem 2, the criteria are established that ensures the finite-time H_∞ synchronization for complex dynamical networks with mixed mode-dependent time delays. Note that the LMIs which in our paper can be effectively solved and checked by algorithms such as the interior-point method from Matlab toolbox.

Remark 6 Given that $t_p > 0$, $c_1 > 0$ and $d_1 > 0$. . Then the condition (29) (30) and (31) in Theorem 2 are translated into strict LMIs. Therefore, we can obtain the optimal value over $\tilde{\gamma}^2 + c_2^2$, and the feasible conditions presented in Theorem 2 can be converted to the following convex optimization problems:

$$\begin{aligned}
&\min (\beta_1 + \beta_2) \\
&X, Y, \tilde{Q}_b (b = 1, 2, 3, 4), v, \rho \\
&\text{s.t. (29) (30) (31) with } \beta_1 = \tilde{\gamma}^2, \beta_2 = c_2^2.
\end{aligned}$$

4. Numerical examples

Example 1 In this section, we will give a simulation example to illustrate the usefulness of the provided methods. Consider the system (1) consisting of 5-nodes:

$$\begin{cases} \dot{x}_k(t) = -Ax_k(t) + Bf(x_k(t)) + Cf(x_k(t - \tau_1(t))) + \sigma_1 \sum_{j=1}^5 v_{kj}(r(t))\Gamma_1 x_j(t) \\ \quad + \sigma_2 \sum_{j=1}^5 l_{kj}(r(t))\Gamma_2 x_j(t - \tau_1(t)) + \int_{t-\tau_2(t)}^t Mg(x_i(s))ds + D\omega_k(t) + u_k(t) \\ z_k(t) = Ex_k(t) \end{cases}$$

The system has the following parameters:

$$\begin{aligned}
V_1 = v_{kl1} &= \begin{bmatrix} -4.2 & 0.9 & 0.5 & 2.5 & 0.3 \\ 0.9 & -2.3 & 0.1 & 1 & 0.3 \\ 0.5 & 0.1 & -1.8 & 0 & 1.2 \\ 2.5 & 1 & 0 & -3 & 0.4 \\ 0.3 & 0.3 & 1.2 & 0.4 & -2.2 \end{bmatrix}, V_2 = v_{kl2} = \begin{bmatrix} -3.4 & 1.2 & 1.9 & 0.2 & 0.1 \\ 1.2 & -2.7 & 0.8 & 0.1 & 0.6 \\ 1.9 & 0.8 & -6.5 & 1.5 & 2.3 \\ 0.2 & 0.1 & 1.5 & -2.2 & 0.4 \\ 0.1 & 0.6 & 2.3 & 0.4 & -3.4 \end{bmatrix}, \\
L_1 = l_{kl1} &= \begin{bmatrix} -2.3 & 0.7 & 0.3 & 1.1 & 0.2 \\ 0.7 & -2.6 & 0.1 & 0.6 & 1.2 \\ 0.3 & 0.1 & -2.2 & 1.4 & 0.4 \\ 1.1 & 0.6 & 1.4 & -3.7 & 0.7 \\ 0.2 & 1.2 & 0.4 & 0.7 & -3.5 \end{bmatrix}, L_2 = l_{kl2} = \begin{bmatrix} -2.9 & 0.4 & 0.4 & 1.8 & 0.3 \\ 0.4 & -2.3 & 0.7 & 1.2 & 0 \\ 0.4 & 0.7 & -3.1 & 0.2 & 1.8 \\ 1.8 & 1.2 & 0.2 & -3.7 & 0.5 \\ 0.3 & 0 & 1.8 & 0.5 & -2.6 \end{bmatrix}, \\
A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1.2 & -1.6 & 0 \\ -1.25 & -0.8 & 0.9 \\ 0 & 2.2 & 1.5 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1.2 & -0.5 \\ 0 & -2 & 1.2 \end{bmatrix}, \\
D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, M = \begin{bmatrix} -1 & 2 & 4 \\ -2 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}, \Phi_1 = -I_3, \Phi_2 = I_3, E = I_3, R_0 = I_3, \Lambda_1 = -I_3, \Lambda_2 = I_3.
\end{aligned}$$

And the nonlinear function, $f_1(x) = f_2(x) = f_3(x) = g_1(x) = g_2(x) = g_3(x) = \frac{1}{2}(|x+1| + |x-1|)$. It can be verified that Assumption 1 is satisfied. The time-varying delays are assumed to be $\tau_1(t) = 4 + \sin(0.2t)$, $\tau_2(t) = -0.5 + \sin(0.6t)$. It is easy to see that $\tau_1 = 4$, $\tau_2 = 0.5$, $d_1 = 0.2$, $d_2 = 0.6$. The disturbance input $\omega(t) = \frac{0.8}{\sqrt{6(t+1)}}$ which implies that $\mathcal{W} = 0.64$.

Suppose the transition probability matrix

$$\pi = \begin{bmatrix} -3 & 3 \\ 4 & -4 \end{bmatrix}.$$

Then, solving LMIs(29)-(31) in theorem 2 by using the matlab toolbox solving the LMIs, we can obtain the matrices gains are :

$$\begin{aligned}
X_1 &= \begin{bmatrix} 0.0921 & -0.0314 & -0.0154 \\ -0.0314 & 0.1753 & 0.0446 \\ -0.0154 & 0.0446 & 0.1072 \end{bmatrix}, Y_1 = \begin{bmatrix} -0.9209 & -0.0364 & -0.0140 \\ -0.0364 & -0.7567 & 0.0492 \\ -0.0140 & 0.0492 & -0.8926 \end{bmatrix}, \\
X_2 &= \begin{bmatrix} 0.0429 & -0.0149 & -0.0072 \\ -0.0149 & 0.0822 & 0.0211 \\ -0.0072 & 0.0211 & 0.0469 \end{bmatrix}, Y_2 = \begin{bmatrix} -0.4234 & -0.0163 & -0.0061 \\ -0.0163 & -0.3504 & 0.0211 \\ -0.0061 & 0.0211 & -0.4117 \end{bmatrix}, \\
X_3 &= \begin{bmatrix} 0.0733 & -0.0256 & -0.0123 \\ -0.0253 & 0.1401 & 0.0359 \\ -0.0123 & 0.0359 & 0.0804 \end{bmatrix}, Y_3 = \begin{bmatrix} -0.7301 & -0.0274 & -0.0103 \\ -0.0274 & -0.6057 & 0.0359 \\ -0.0103 & 0.0359 & -0.7094 \end{bmatrix},
\end{aligned}$$

$$\begin{aligned}
X_4 &= \begin{bmatrix} 0.2012 & -0.0657 & -0.0323 \\ -0.0657 & 0.3724 & 0.0932 \\ -0.0323 & 0.0932 & 0.2196 \end{bmatrix}, Y_4 = \begin{bmatrix} -2.0383 & -0.0774 & -0.0289 \\ -0.0774 & -1.6824 & 0.1021 \\ -0.0289 & 0.1021 & -1.9791 \end{bmatrix}, \\
X_5 &= \begin{bmatrix} 0.0032 & -0.0013 & -0.0005 \\ -0.0013 & 0.0062 & 0.0016 \\ -0.0005 & 0.0016 & 0.0033 \end{bmatrix}, Y_5 = \begin{bmatrix} -0.0419 & -0.0011 & -0.0004 \\ -0.0011 & -0.0384 & 0.0013 \\ -0.0004 & 0.0013 & -0.0416 \end{bmatrix}.
\end{aligned}$$

This means that the complex dynamical networks we choose is finite-time H_∞ synchronization. The corresponding gain matrix in theorem 2 is given by

$$\begin{aligned}
K_1 &= \begin{bmatrix} -10.8055 & -1.9030 & -0.9440 \\ -1.7703 & -5.2810 & 2.5442 \\ -0.9299 & 2.6965 & -10.1500 \end{bmatrix}, K_2 = \begin{bmatrix} -10.6829 & -1.8998 & -0.9154 \\ -1.7729 & -5.2342 & 2.5326 \\ -0.9071 & 2.6922 & -10.1287 \end{bmatrix}, \\
K_3 &= \begin{bmatrix} -10.7729 & -1.9037 & -0.9262 \\ -1.7753 & -5.2942 & 2.5376 \\ -0.9160 & 2.6962 & -10.1674 \end{bmatrix}, K_4 = \begin{bmatrix} -10.8997 & -1.8983 & -0.9292 \\ -1.7626 & -5.4601 & 2.5230 \\ -0.9178 & 2.6869 & -10.1674 \end{bmatrix}, \\
K_5 &= \begin{bmatrix} -14.4402 & -2.9825 & -0.8631 \\ -2.9173 & -7.7643 & 3.7164 \\ -0.8626 & 3.7899 & -14.5743 \end{bmatrix}.
\end{aligned}$$

Remark 7 In the process of mode jump P_i , V_i and L_i are in accordance with the same markov chain. In paper [47], the addressed complex networks stochastic variable V_i and L_i switch from one mode to another according to a Markovian chain with general unknown transition rates. In [51,52] the models consider Markovian jump stochastic variable P_i in complex networks according to a Markovian chain. In this paper, at the same time, considered the three random variables, and they are in accordance with the same markovian chain.

In the simulation, we choose the initial condition as $x_1(0) = [0.47 \ 0.2 \ -0.1]^T$, $x_2(0) = [-0.32 \ 1.5 \ 0.4]^T$, $x_3(0) = [-1.5 \ 0.9 \ 0]^T$, $x_4(0) = [-0.01 \ 0.5 \ 0.2]^T$, $x_5(0) = [0 \ 1.3 \ 0.1]^T$. The simulation results are shown in Fig.1-4, where the synchronization error trajectories ($x_{1l} - x_{kl}$, $l = 2, 3, 4, 5; k = 1, 2, 3, 4$) of the complex dynamical networks (3) is presented. Obviously, it is shown from the trajectories that H_∞ synchronization can be achieve over a finite-time interval.

5. Conclusion

This paper is concerned with finite time H_∞ synchronization for complex dynamical networks with mixed mode-dependent time delay. A new complex dynamical networks model is put up, and some assumptions, lemmas and definitions are given. The main contribute of this paper is to design a state feedback control such that the complex dynamical networks are finite time synchronization and finite time H_∞ synchronization by built properly Lyapunov-Krasovskii functions with triple integral terms. The sufficient criteria can convert to the feasible problems based on linear matrix inequalities. An example is also presented to illustrate the effectiveness of obtained results.

Fig.1. System trajectories $x_{11} - x_{k1}$, ($k = 2, 3, 4, 5$).

Fig.2. System trajectories $x_{12} - x_{k2}$, ($k = 2, 3, 4, 5$).

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Fig.3. System trajectories $x_{13} - x_{k3}$, ($k = 2, 3, 4, 5$).

Fig.4. System trajectories $x_{14} - x_{k4}$, ($k = 2, 3, 4, 5$).

References

- [1] S.H. Strogatz, Exploring complex networks, Nature 410 (2001) 268-276.
- [2] K. Park, Y.C. Lai, S. Gupte, J.W. Kim, Synchronization in complex networks with a modular structure, Chaos 16 (2006)

015105.

- [3] M.E.J. Newman, The structure and function of complex networks, *SIAM Review* 45 (2003) 167-256.
- [4] G.W. Wei, Y.Q. Jia, Synchronization-based image detection, *Europhy Lett.* 59 (6) (2002) 814-819.
- [5] F.C. Hoppensteadt, E.M. Lzhikevich, Pattern recognition via synchronization in phase-locked loop neural networks, *IEEE Trans, Neural Netw.* 11 (3) (2000) 734-738.
- [6] Q.X. Xie, G.R. Chen, E. Bollt, Hybrid chaos synchronization and its application in information processing, *Math. Comput. Model* 35 (1-2) (2002) 145-163.
- [7] B.C. Li, Pinning adaptive hybrid synchronization of two general complex dynamical networks with mixed coupling, *Applied Mathematical Modelling* 40 (4) (2016) 2983-2998.
- [8] J.W. Zhu, G.H. Yang, Robust H_∞ dynamic output feedback synchronization for complex dynamical networks with disturbances, *Neurocomputing* 175 (2016) 287-292.
- [9] G. He, J.A. Fang, W.B. Zhang, Z. Li, Synchronization of switched complex dynamical networks with non-synchronized subnetworks and stochastic disturbances, *Neurocomputing* 171 (2016) 39-47.
- [10] Z. Wu, J.H. Park, H. Su, B. Song, J. Chu, Exponential synchronization for complex dynamical networks with sampled-data, *J. Frankl. Inst.* 349 (9)(2012) 2735-2749.
- [11] T. Liu, J. Zhao, D.J. Hill, Synchronization of complex delayed dynamical networks with nonlinearly coupled nodes, *Chaos Solitons Fractals* 40 (3) (2009) 1506-1519.
- [12] P. He, C. Jing, T. Fan, C. Chen, Robust decentralized adaptive synchronization of general complex networks with coupling delayed and uncertainties, *Complexity* 19 (3) (2014) 10-26.
- [13] H. Shen, J.H. Park, Z.G. Wu, Finite-time synchronization control for uncertain Markov jump neural networks with input constraints, *Nonlinear Dyn.* 77 (2014) 1709-1720.
- [14] L. Wang, F. Xiao, Finite-time consensus problems for networks of dynamic agents, *IEEE Trans. Autom. Control* 55 (2010) 950-955.
- [15] W.X. Cui, S.Y. Sun, J.A. Fang, Y.L. Xu, L.D. Zhao, Finite-time synchronization of Markovian jump complex networks with partially unknown transition rates, *Journal of the Franklin Institute* 351 (2014) 2543-2561.
- [16] F. Amato, R. Ambrosino, C. Cosentino, G. Tommasi, Input-output finite-time stabilization of linear systems, *Automatica* 46 (2010) 1558-1562.
- [17] Q. Liu, C. Dang, J. Cao, A novel recurrent neural network with one neuron and finite-time convergence for k-winnerstake-all operation. *IEEE Trans. Neural Netw.* 21 (2010) 1140-1148.
- [18] Q. Ma, Z. Wang, J. Lu, Finite-time synchronization for complex dynamical networks with time-varying delays. *Nonlinear Dyn.* 70 (1) (2012) 841-848.
- [19] Y. Shen, C. Li, LMI-based finite-time boundedness analysis of neural networks with parametric uncertainties, *Neurocomputing* 71 (2008) 502-507.
- [20] X. Yang, J. Cao, Finite-time stochastic synchronization of complex networks, *Appl. Math. Model.* 34 (2010) 3631-3641.
- [21] Y. Sun, W. Li, D. Zhao, Finite-time stochastic outer synchronization between two complex dynamical networks with different topologies, *Chaos* 22 (2012) 023152.
- [22] N. Wang, S.L. Lv, Z.Z. Liu, Global finite-time heading control of surface vehicles, *Neurocomputing* 175 (2016) 662-666.
- [23] N. Wang, C.J. Qian, J.C. Sun, Y.C. Liu, Adaptive robust finite-time trajectory tracking control of fully actuated marine surface vehicles, *IEEE* 24 (2016) 1454-1462.
- [24] Y.H. Huang, S.Q. Fu, Y.J. Shen, Finite-time H_∞ control for one-sided Lipschitz systems with auxiliary matrices, *Neurocomputing* 194 (2016) 207-217.
- [25] Y.C. Ma, H. Chen, Reliable finite-time H_∞ filtering for discrete time-delay systems with Markovian jump and randomly occurring nonlinearities, *Applied Mathematics and Computation* 268 (2015) 897-915.
- [26] W.H. Chen, Z.Y. Jian, X.M. Lua, S.X. Luo, H_∞ synchronization for complex dynamical networks with coupling delays using distributed impulsive control, *Nonlinear Analysis: Hybrid Systems* 17 (2015) 111-127.
- [27] J. Lu, D.W.C. Ho, Stabilization of complex dynamical networks with noise disturbance under performance constraint, *Nonlinear Anal. Ser. B RWA* 12 (4) (2011) 1974-1984.
- [28] Z.K. Li, Z.S. Duan, G.R. Chen, Disturbance rejection and H_∞ pinning control of linear complex dynamical networks, *Chin. Phys. B* 18 (12) (2009) 5228-5234.
- [29] H.Q. Zheng, Y.W. Jing, Robust H_∞ observer-based control for synchronization of a class of complex dynamical networks, *Chin. Phys. B* 20 (6) (2011) 060504.

- [30] S. Li, J. Zhang, W. Tang, Robust H_∞ output feedback control for uncertain complex delayed dynamical networks, *Comput. Math. Appl.* 62 (1) (2011) 497-505.
- [31] H. Li, Cluster synchronization and state estimation for complex dynamical networks with mixed time delays, *Appl. Math. Model.* 37 (1) (2013) 7223-7244.
- [32] X.S. Yang, J.D. Cao, Finite-time stochastic synchronization of complex networks, *Applied Mathematical Modelling* 34 (2010) 3631-3641.
- [33] X.S. Yang, Z.Y. Wu, J.D. Cao, Finite-time synchronization of complex networks with nonidentical discontinuous nodes, *Nonlinear Dyn.* 73 (2013) 2313-2327.
- [34] W. X. Cui, S.Y. Sun, J.A. Fang, Y.L. Xu, L.D. Zhao, Finite-time synchronization of Markovian jump complex networks with partially unknown transition rates, *Journal of The Franklin Institute* 351 (2014) 2543-2561.
- [35] X.H. Liu, X.H. Yu, H.S. Xi, Finite-time synchronization of neutral complex networks with Markovian switching based on pinning controller, *Neurocomputing* 153 (2015) 148-158.
- [36] Q. Ma, Z. Wang, J.W. Lu, Finite-time synchronization for complex dynamical networks with time-varying delays, *Nonlinear Dyn.* 70 (2012) 841-848.
- [37] Y.H. Xu, W.N. Zhou, J.A. Fang, C.R. Xie, D.B. Tong, Finite-time synchronization of complex dynamical network with non-derivative and derivative coupling, *Neurocomputing* 173 (2016) 1356-1361.
- [38] F. Li, H. Shen, Finite-time H_∞ synchronization control for semi-Markov jump delayed neural networks with randomly occurring uncertainties, *neurocomputing* 166 (2015) 447-454.
- [39] Y. Liu, Z. Wang, X. Liu, Global exponential stability of generalized recurrent neural networks with discrete and distributed delays, *Neural Netw.* 19 (5) (2006) 667-675.
- [40] O.M. Kwon, S.M. Lee, J.H. Park, E.J. Cha, New approaches on stability criteria for neural networks with interval time-varying delays, *Appl. Math. Comput.* 218 (2012) 9953-9964.
- [41] W. Duan, C. Cai, Y. Zou, J. You, Synchronization criteria for singular complex dynamical networks with delayed coupling and non-delayed coupling, *Control Theory Appl.* 30 (8) (2013) 947-955.
- [42] A.N. Langville, W.J. Stewart, The Kronecker product and stochastic autonomous networks, *J. Comput. Appl. Math.* 167 (2004) 429-447.
- [43] Y. Liu, Z. Wang, X. Liu, Synchronization and state estimation for discrete-time complex networks with distributed delays, *IEEE Trans Syst. Man. Cyb.* 38 (5) (2008) 1314-1325.
- [44] S. Boyd, L.E. Ghaoui, E. Feron, V. Balakrishnan, *Linear matrix inequalities in system and control theory*, SIAM, Philadelphia, PA, 1994.
- [45] H. Shen, J.H. Park, Z.G. Wu, Z.Q. Zhang, Finite-time H_∞ synchronization for complex networks with semi-Markov jump topology, *Commun Nonlinear Sci Numer Simulat* 24 (2015) 40-51.
- [46] Z. Wu, P. Shi, H. Su, J. Chu, Asynchronous filtering for discrete-time stochastic Markov jump systems with randomly occurred sensor nonlinearities, *Automatica* 50 (2008) 180-186.
- [47] R.P. Xu, Y.G. Kao, C.C. Gao, Exponential synchronization of delayed Markovian jump complex networks with generally uncertain transition rates, *Applied Mathematics Computation* 271 (2015) 682-693.
- [48] L. Su, H. Shen, Mixed H_∞ /passive synchronization for complex dynamical networks with sampled-data control, *Applied Mathematics Computation* 259 (2015) 931-942.
- [49] S. Lakshmanan, K. Mathiyalagan, J. H. Park, R. Sakthivel, F. A. Rihan, Delay-dependent H_∞ state estimation of neural networks with mixed time-varying delays, *Neurocomputing* 129 (2014) 392-400.
- [50] D.B. Li, Z.C. Wang, G. F. Ma, Controlled synchronization for complex dynamical networks with random delayed information exchanges: A non-fragile approach, *Neurocomputing* 171 (2016) 1047-1052.
- [51] Y.C. Ma, Y.Q. Zheng, Synchronization of continuous-time Markovian jumping singular complex networks with mixed mode-dependent time delays, *Neurocomputing* 156 (2015) 52-59.
- [52] X.W. Liu, Y. Xu, Cluster synchronization in complex networks of non-identical dynamical systems via pinning control, *Neurocomputing* 168 (2015) 260-268.