Multi-Variable Intelligent Matching Pursuit Algorithm Using Prior Knowledge for Image Reconstruction by l_0 Minimization

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Abstract

Image reconstruction by l_0 minimization is an NP-hard problem that requires exhaustively listing all possibilities of the original signal with a very high computational complexity, which is difficult to be achieved by traditional algorithms. Although greedy algorithm aims at solving l_0 minimization, it is more likely to fall into a suboptimal solution. In this paper, we propose a multi-variable intelligent matching pursuit algorithm (MIMP), which can solve l_0 minimization problem essentially by taking the advantage of the intelligent optimization algorithm in solving combinatorial optimization problems and searching for the global optimal solution to improve the performance of image reconstruction. The updating mechanism of MIMP is designed by introducing the matching strategies of greedy algorithm to accelerate the reconstruction speed. Also, the multi-variable scheme is utilized to sample images and then the joint reconstruction is implemented to the measurements, which can not only improve the reconstruction accuracy but also reduce the computational complexity. Moreover, the edge saliency can be obtained as the prior knowledge to guide the compressive sensing reconstruction, which contributes a lot to reduce the computational complexity and accelerate the reconstruction speed. As the sparsity level of image is hard to be estimated, a new optimization function is proposed to solve this problem without knowing the sparsity level as a prior. Compared with other state-of-the-art algorithms, the proposed method MIMP can achieve

a better reconstruction accuracy by solving l_0 minimization essentially with intelligent optimization algorithms. Also, MIMP has a reasonable relatively faster reconstruction speed by introducing the matching strategies of greedy algorithm and using **the edge saliency** as a prior knowledge. Numerical experiments on several images demonstrate that the proposed method MIMP significantly outperforms the state-of-the-art algorithms and the structure based reconstruction algorithms in PSNR, SSIM and visual quality.

Keywords: Compressive sensing, l_0 minimization, Intelligent optimization algorithm, Prior knowledge, Multi-variable scheme.

1. Introduction

Compressive sensing (CS) [1, 2, 3] theory, proposed by Candes and Donoho in 2006, shows that signal can be reconstructed from many fewer measurements than suggested by Nyquist sampling theory if the signal is sparse or compressive
5 in some domains. If the original signal $x \in R^{N \times 1}$ is compressive, it can be represented sparsely in the sparse basis ψ . The measurement signal $y \in R^{M \times 1}$ ($M < N$) is obtained by $y = \phi * x$, where ϕ is uncorrelated to ψ and satisfies restrict isometry property (RIP). The relations can be represented by Equ.(1).

$$y = \phi * x = \phi * \psi * s \quad (1)$$

where, s is the sparse representation of x in the sparse basis ψ .

10 To simplify Equ.(1), x is assumed to be sparse in the following sections. As we all know, reconstruction is a process of recovering high-dimensional signal x from low-dimensional signal y , which can be obtained by optimizing a constrained optimization problem of sparsity. Then the constrained optimization problem of sparsity can be implemented by the l_0 minimization problem, which
15 can be described in Equ.(2).

$$x = \operatorname{argmin} \|x\|_0 \quad \text{subject to} \quad y = \phi * x, \quad (2)$$

where, x represents the l_0 norm of x , which is defined by the number of nonzero elements of x . Equ.(2) can be also written as the form of Equ.(3).

$$x = \operatorname{argmin} \|x\|_0 + \lambda \|y - \phi * x\|_2^2, \quad (3)$$

where λ is a positive constant between 0 and 1, which used to balance the relative weight between the sparsity and the data fidelity item.

20 However, l_0 minimization is an NP-hard problem that requires exhaustively listing all possibilities of the original signal and is difficult to be achieved by traditional algorithms. Since l_1 norm [4] is the optimal convex approximation of l_0 norm and it has been proved that l_1 norm is equivalent to the l_0 norm for many problems. It is widely adopted that the l_0 minimization problem
 25 can be solved by relaxing it to the l_1 minimization problem [5, 6, 7] which is shown in Equ.(4). Nevertheless, a fact that is often neglected is, the conditions guaranteeing the equivalence of l_0 minimization and l_1 minimization are not necessarily satisfied, which leads to a suboptimal solution. Many reconstruction methods have been already proposed recently, such as gradient projection sparse
 30 reconstruction [8, 9], non-linear filter [10] and iterative threshold [11], which also transform l_0 minimization to other mathematical models and are more likely to fall into suboptimal solutions.

$$x = \operatorname{argmin} \|x\|_1 + \lambda \|y - \phi * x\|_2^2. \quad (4)$$

The methods which aims at solving l_0 minimization problem available for compressive sensing reconstruction fall into two categories: greedy algorithm
 35 [12, 13, 14] and heuristic method. The objective of greedy algorithm is to estimate the support collection and its corresponding coefficients by iterating the coefficients based on the residual correlation. However, the residual correlation can only reflect the correlation degree between coefficients and the measurement signal, but can't indicate that the bigger the residual correlation is, the
 40 bigger the nonzero coefficient is. Also, greedy algorithm is more likely to find a suboptimal solution and can not solve l_0 minimization essentially. Intelligen-

t optimization algorithm [15, 16] is a class of heuristic optimization methods which is famous for its global searching ability and superior performance in solving combinatorial optimization problems. It is natural to consider that intelligent optimization algorithm can be used to solve l_0 minimization problem. Two methods [17, 18] based on simulated annealing algorithm are proposed for CS reconstruction by l_0 minimization. However, the single-cycle local searching scheme in [17, 18] is hard to find the global optimal solution. Also, the precondition of [17, 18] is that the sparsity level must be known, which is a very strong limitation as the sparsity level is always unknown in practice. In our precious work [19, 20], we use genetic algorithm and artificial immune algorithm [21, 22] to solve l_0 minimization directly. However, due to the randomness of intelligent optimization algorithm, the computational complexity of the proposed methods in [19, 20] is high and the reconstruction speed of them slow down.

Image reconstruction [23, 24, 25], which is a fundamental problem in the field of image processing, aims to reconstruct the original image with high quality from the degraded measurements. Prior knowledge [26, 27] plays an important role in image reconstruction and it has been proved that introducing prior knowledge can improve the efficiency of image reconstruction algorithms. One way of improving the reconstruction quality is to exploit the co-called structured sparsity. Some methods using particular structures such as tree structure [28, 29, 30] and dependence structure [31] of the wavelet coefficients have been applied in image reconstruction. Tree-structured correlations of wavelet coefficients is utilized to propose a group-sparsity regulation method in [32]. Bayesian CS reconstruction methods exploiting the prior of the wavelet tree structures have also been developed in [33, 34, 35]. Under the wavelet transformation, the main energy of image is concentrated on the wavelet coefficients of the low-frequency subspace and the wavelet coefficients of the high-frequency subspace show high sparsity, which means that the wavelet coefficients corresponding to the edge are significantly different from zero. Suppose the original image is locally smooth except edge area, the locations of the nonzero wavelet coefficients of the high-frequency parts happen to correspond with the edge of the original

image. So if the edge information of the image is extracted, it can be applied as prior knowledge to guide the process of image reconstruction, which can reduce the computational complexity. Several representative works based on the guidance of edge information have been proposed, such as edge guided MRI reconstruction [36] and edge based multiple pursuit algorithm (EMPA) [37]. Moreover, a multi-variable scheme using conventional linear sampling and compressive sampling for low frequency coefficients and high frequency coefficients respectively is developed for image reconstruction in [38].

In this paper, we propose a multi-variable intelligent pursuit algorithm (MIMP) using prior knowledge to solve l_0 minimization problem essentially. MIMP aims at searching the support collection and its corresponding coefficients of the original signal intelligently and accurately. As the sparsity level of image is hard to be estimated, a new optimization function is proposed when the sparsity level is unknown in advance. Due to the superior global optimizing strategy of intelligent optimization algorithm, artificial immune algorithm (AIA) is introduced to guide the process of support collection estimation. Artificial immune algorithm performs well in both global searching and local searching, which makes MIMP more likely to find the global optimal solution and have a fast reconstruction speed. The principle of the intelligent estimation is designed by the matching strategies of greedy algorithm, which is beneficial to reduce the computational complexity. Also, based on the high sparsity of the high-frequency parts of wavelet transformation, the multiple measurement scheme, which uses conventional linear sampling for the low-frequency coefficients and compressive sensing for the high-frequency coefficients, is introduced into MIMP to improve the reconstruction quality. In addition, as the edge saliency mostly happens to correspond with the locations of the nonzero wavelet coefficients in high-frequency subspace, it is introduced into MIMP to guide the process of global searching, which can reduce the computational complexity. As MIMP can solve l_0 minimization essentially and by means of the combination of edge saliency and multi-variable joint reconstruction, the proposed method MIMP significantly improves the reconstruction quality of those images with the ob-

various edges and high sparsity, such as computed tomography (CT) images and
 105 magnetic resonance (MR) images. Experiments on image reconstruction demon-
 strate that the proposed method MIMP produces more superior reconstruction
 performance than other state-of-the-art algorithms, such as multi-variable OM-
 P (M-OMP), multi-variable Polytope Faces Pursuit algorithm (M-PFP) [39],
 multi-variable backtracking-based matching pursuit (M-BAOMP) [40], multi-
 110 variable compressive sampling orthogonal matching pursuit (M-CoSaOMP) [41],
 multi-variable Sparsity Adaptive Matching Pursuit algorithm (M-SAMP) [42],
 tree-based CoSaMP (Tree-CoSaMP) [43], tree-structured wavelet compressive
 sensing (TSWCS) [33] and edge based multiple pursuit algorithm (EMPA), edge
 based compressive sensing algorithm (EdgeCS), multi-variable pursuit algorithm
 115 m using multi-variable K distribution (MPA-MK) and multi-variable pursuit
 algorithm using multi-variable Laplace distribution (MPA-ML).

The major contributions of this paper are fourfold:

1. MIMP improves the reconstruction quality significantly by taking the ad-
 vantage of intelligent optimization algorithm in solving l_0 minimization es-
 120 sentially and accelerates the reconstruction speed by introducing matching
 strategies of greedy algorithm.
2. As the sparsity level of image is hard to be estimated, a new optimiza-
 tion function is proposed for CS reconstruction when the sparsity level is
 unknown in advance.
- 125 3. A multi-variable scheme, including multiple measurement and multi-variable
 joint reconstruction, is introduced to improve the performance of image
 reconstruction significantly with fewer measurements.
4. **The edge saliency** is applied as prior knowledge to guide the process of
 image reconstruction, which can reduce the computational complexity and
 130 improve the reconstruction quality, especially for images with obvious
 edges and high sparsity in wavelet domain.

The layout of this paper is organized as follows: In section 2, the proposed

framework MIMP for image reconstruction based on l_0 minimization is provided. Experiment results are given in Section 3 to illustrate the performance of the proposed method MIMP. Section 4 concludes this paper.

The follow notations are used in the rest of this paper. The l_0 norm of signal x can be represented by $\|x\|_0$. Let ϕ indicate the measurement matrix and ϕ_i represents the i th column of ϕ . $S = 1, 2, \dots, N$ indicates the index of each entry of the original signal, where N represents the length of the original signal. Suppose m entries are selected from S to generate the support collection I , the vector x_I consists the entries of x indexed by $i \in I$ and the matrix ϕ_I is composed of the columns of ϕ with indexes $i \in I$. The pseudo-inverse operation of measurement matrix ϕ can be calculated by $\phi^\dagger = (\phi^\top * \phi)^{-1} * \phi^\top$, where $\top$ represents matrix transportation. The subscript $_{LL}$ represents the low-frequency part of wavelet transformation and the subscripts HL , LH and HH represent the high-frequency parts respectively.

2. Method MIMP

In this section, we first introduce the multi-variable scheme and the prior knowledge. Then the diagram of the proposed method MIMP is described in detail. In the end, we analysis the computational complexity.

2.1. Multi-variable scheme

The multi-variable scheme includes two parts, i.e. multiple measurement and multi-variable joint reconstruction. The schematic diagram of multi-variable scheme and an example to describe the process of multiple measurement is shown in Figure 1.

The multiple measurement scheme is based on the multiscale decomposition by using conventional linear sampling for low-frequency wavelet coefficients and compressive measurement on high-frequency wavelet coefficients. This framework of measurement can extract more useful prior knowledge for compressive sensing reconstruction. Suppose the original image is X . Firstly, the original

Figure 1: The schematic diagram of multi-variable scheme. The main step 3 is an example of multiple measurement for the high frequency coefficients. The size of the original matrix C is $12 * 12$ and the size of neighbour is set as $3 * 3$. Each neighbour can be reshaped to a signal with size of $1 * 9$ and C is represented as a $16 * 9$ multi-variable matrix C' . The $8 * 9$ measurement image Y can be obtained by a Gaussian random matrix Φ with size of $8 * 16$.

image is decomposed by one scale wavelet decomposition to obtain the wavelet coefficients in each scale, i.e. C_{LL} , C_{LH} , C_{HL} and C_{HH} . After the wavelet decomposition, low-frequency coefficients C_{LL} is measured using the conventional linear measurement to retain the whole C_{LL} in the encoding phase, and the high-frequency coefficients C_{LH} , C_{HL} and C_{HH} are down-sampled by a Gaussian random matrix. As we all know, the wavelet coefficients hold the property of aggregation, which indicates that if a wavelet coefficient is large, the coefficients adjacent to it are also very large, and vice versa. This property makes the neighbored wavelet coefficients much correlated and dependent. So when down-sample the high frequency coefficients, we sample the wavelet coefficients neighbors instead of all the wavelet coefficients with Gaussian random matrix, which transforms the unit-variable measurement to a multiple measurement.

Given the size of wavelet coefficients neighbors $d * d$, C_{LH} (C_{HL} , C_{HH}) is reshaped to a multi-variable form C'_{LH} (C'_{HL} , C'_{HH}) with size of $N' * d^2$, where $N' = N/d$. The multi-variable measurement matrix Y can be obtained by a measurement matrix with size of $M * N'$. In particularly, C'_{LH} (C'_{HL} , C'_{HH}) and Y will be reduced as a signal x with size of $N' * 1$ and y with size of $M' * 1$ when $d = 1$, which converts back to the unit-variable reconstruction.

After multiple measurement, X' is a joint sparse matrix, which can be jointly reconstructed from the measurement image Y and the measurement matrix Φ . The formulation of multi-variable joint reconstruction can be represented by Equ.(5).

$$\min_{X'} \text{supp}(X') \quad \text{subject to} \quad Y = \Phi * X', \quad (5)$$

where $\text{supp}(X') = \sum_j^N \text{supp}(X'(:, j))$, $X'(:, j)$ represents the j th column of X' and M represents the number of measurements.

2.2. Prior knowledge

Under the framework of multi-variable scheme, we can obtain the whole low-frequency wavelet coefficients and the down-sampled high-frequency wavelet coefficients in the encoding phase of compressive sensing framework. Since most energy of image is concentrated on the low-frequency wavelet coefficients, we can extract **the edge saliency** using the whole low-frequency coefficients as the prior knowledge. Firstly, set $C'_{HL} = 0$, $C'_{LH} = 0$, $C'_{HH} = 0$ and implement inverse wavelet transformation for C'_{LH} , C'_{HL} , C'_{HH} and C_{LL} to obtain a low-frequency image X_l . Secondly, extract the edge of the low-frequency image using edge detection algorithm, such as *canny* or *sobel*. Thirdly, implement wavelet decomposition to the extracted edge to obtain the wavelet coefficients $edge_{HL}$, $edge_{LH}$ and $edge_{HH}$ in high-frequency subspace which are mostly consistent with the locations of nonzero wavelet coefficients in C_{HL} , C_{LH} and C_{HH} . The $edge_{HL}$, $edge_{LH}$ and $edge_{HH}$ are sampled based on the multiple measurement scheme shown in Fig.1. The coefficients corresponding to the edge information

are named as edge coefficients and the others, which are very small or equal to zero, are named as non-edge coefficients. In the multi-variable form, neighbors which include edge coefficients are edge neighbors and that do not contain edge coefficients are non-edge neighbors. Let E represent the set of indexes of edge neighbors and EN represent the set of indexes of non-edge neighbors.

2.3. MIMP for image reconstruction

For the problem of multi-variable reconstruction, each column of X' can be represented as a signal x and reconstructed through the corresponding column of Y and the measurement matrix Φ . The proposed method MIMP aims at estimating the support collection I and its corresponding coefficients of the original signal intelligently and accurately. If the sparsity level K is known, for the support collection I , its corresponding coefficients can be calculated by Equ.(6) and the reconstructed signal $recx$ can be obtained by Equ.(7).

$$rec_I = \Phi_I^\dagger * y, \quad (6)$$

$$rec_I = \Phi_I^\dagger * y \quad \text{and} \quad rec_{S-I} = 0. \quad (7)$$

If the support collection I can be estimated, it must satisfy $\|\Phi * \Phi_I^\dagger * y - y\|_2^2 = 0$. As $\|\Phi * \Phi_I^\dagger * y - y\|_2^2 \geq 0$, the reconstruction problem can be transformed to optimize the optimization model Equ.(8).

$$\min_I f(I) = \|\Phi_I * \Phi_I^\dagger * y - y\|_2. \quad (8)$$

As mentioned above, the optimization model described in Equ.(8) requires a precondition that the sparsity level K is known in advance. However, the sparsity level of image is hard to be estimated in practice. Aiming at this problem, we propose a new optimization model when the sparsity level is unknown in advance. Define the measurement collection I_{mm} , which represents the collection including mm (where mm is larger than K and smaller than the measurement

number M) indexes selected in S . I_{mm} can also be used to reconstruct the original signal through Theorem 1 proposed in this paper.

Theorem 1: if the sparsity level is unknown in advance, the measurement
 225 collection I_{mm} , which contains the support collection of the original signal, can be estimated by optimizing the optimization model described in Equ.(9) and the reconstructed signal can be obtained by Equ.(10).

$$f(I_{mm}) = \|\Phi_{I_{mm}} * \Phi_{I_{mm}}^\dagger * y - y\|_2, \quad (9)$$

$$rec_{I_{mm}} = \Phi_{I_{mm}}^\dagger * y \quad \text{and} \quad rec_{S-I_{mm}} = 0. \quad (10)$$

Proof: Suppose Φ is the measurement matrix and the columns of Φ are in general position, I_0 denotes the support of the original signal x_0 and I_s denotes
 230 the support of x_s , if I_0 is a subset of I_s , then $x_s = x_0$. As the support collection I satisfies $\|\Phi_I * \Phi_I^\dagger * y - y\|_2 = 0$, if the measurement collection I_{mm} includes I , I_{mm} must satisfy the condition that $\|\Phi_{I_{mm}} * \Phi_{I_{mm}}^\dagger * y - y\|_2 = 0$. So Equ.(9) can be set as the objective function when the sparsity level is unknown and the reconstructed signal can be obtained by Equ.(10).

235 Artificial immune algorithm (AIA) is an intelligent optimization algorithm which is achieved by simulating the process of immune system. AIA performs better not only in global searching but also in local searching. In AIA, all the possible solutions consist of the searching area and each individual represents a possible solution I_{mm} . After the process of AIA, the obtained *memorycell*
 240 can be set as the estimated I_{mm} . The procedure of MIMP used for image reconstruction is shown in Algorithm 1. Moreover, we explain the details of MIMP clearly with the following steps.

(1) Fitness function. As shown in **Theorem 1**, the proposed optimization function $f(I_{mm}) = \|\Phi_{I_{mm}} * \Phi_{I_{mm}}^\dagger * y - y\|_2^2$ is set as the fitness function of AIA.

245 (2) Initialization. As mentioned in Section 2.2, the edge coefficients are mostly consistent with the locations of nonzero elements in C_{HL} , C_{LH} and C_{HH} , so we select mm indexes from E to generate the initial individuals.

Algorithm 1 MIMP for signal reconstruction

Require:

Measurement matrix Φ ;
 Measurement signal $C_{LL}, Y_{HL}, Y_{LH}, Y_{HH}$;
 Measurement number m ;
 Minimum error $Error$;
 Maximum iterations M_{max} and N_{max} ;
 The selected number q and parameter T_1 ;

Ensure:

- 1: Implement inverse wavelet transform to C_{LL} , $C_{HL} = 0$, $C_{LH} = 0$ and $C_{HH} = 0$ to obtain X_l
- 2: Implement edge extraction to X_l to obtain X_{edge}
- 3: Implement wavelet transform to X_{edge} to obtain the high-frequency parts $edge_{HL}$, $edge_{LH}$ and $edge_{HH}$
- 4: Reshape $edge_{HL}$, $edge_{LH}$ and $edge_{HH}$ to obtain the sets of indexes of edge neighbours E_{HL} , E_{LH} and E_{HH} .
- 5: Initialize $popsize$ individuals by selecting mm indexes from E_{HL} , E_{LH} and E_{HH} respectively
- 6: Calculate $f(I_{mm,0,HL})$, $f(I_{mm,0,LH})$, $f(I_{mm,0,HH})$ and take it as the initial error e_{HL} , e_{LH} and e_{HH} .
- 7: **for** $p = 1 : size(Y, 2)$ **do**
- 8: $y = Y(:, p)$
- 9: **while** $i \leq M_{max}$ or $e \leq Error$ **do**
- 10: implement clone operation to $pop(I_{mm,0}^i)$ to obtain $pop(I_{mm,0}^i)'$
- 11: **for** $j = 1 : N_{max}$ or $e \leq Error$ **do**
- 12: for each individual of $pop(I_{mm,0}^i)'$, select q elements from $E - I_{mm,j-1}^i$ to combine with $I_{mm,j-1}^i$ as $Q_{mm,j-1}^i$;
- 13: calculate $recx_{j-1} = \Phi_{Q_{mm,j-1}^i}^\dagger * y$
- 14: select the indexes of the mm bigger elements from $recx_i$ as $I_{mm,j}^i$
- 15: calculate $\delta = f(I_{mm,j}^i) - f(I_{mm,j-1}^i)$
- 16: **if** $p_j \geq \eta$ **then**
- 17: $I_{mm,j}^i = I_{mm,j}^i$
- 18: **else**
- 19: $I_{mm,j}^i = I_{mm,j-1}^i$
- 20: **end if**
- 21: **end for**
- 22: compare the individuals after the iterations and select the best one as *memorycell*
- 23: $I_{mm,0}^{i+1} = memorycell$
- 24: update $T_i = T_1 / (i + 1)$
- 25: calculate the error $f(I_{mm,0}^{i+1})$
- 26: **end while**
- 27: **if** $e > Error$ **then**
- 28: implement the iterations for the initial solution I_{mm} based on the feasible set S
- 29: **end if**
- 30: Calculate the reconstructed column $recx$ based on I_{mm}
- 31: $recC(:, p) = recx$
- 32: **end for**
- 33: After the iterations, the reconstructed images $recC_{HL}$, $recC_{LH}$ and $recC_{HH}$ can be obtained
- 34: reshape the multi-variable form $recC_{HL}$, $recC_{LH}$ and $recC_{HH}$ back to images $recX_{HL}$, $recX_{LH}$ and $recX_{HH}$
- 35: implement inverse wavelet transform to C_{LL} , $recX_{HL}$, $recX_{LH}$ and $recX_{HH}$ to obtain the reconstructed image $recx$

Output: $recx$

(3) Iteration. The iterations are implemented to reconstruct every variable x of the multi-variable from X' . As the reconstruction process of C_{HL} , C_{LH} and C_{HH} are all the same, we ignore the subscript in the following description. There are three main operations: clone, mutation and selection. Firstly, the clone operation is implemented to the initial individuals, which can generate more individuals to search simultaneously. Secondly, we implement mutation operation, namely the process of the inner loop, to all the individuals. The process of mutation operation for all individuals are the same, so we will take one for example. Firstly, select q elements from $E - I_{mm,j-1}^i$ randomly to combine with $I_{mm,j-1}^i$ as a new indexes collection $Q_{mm,j-1}^i$. Then, calculate $recx_{mm,j-1}$ by the equation $recx_{j-1} = \Phi_{mm,Q_{j-1}^i}^i * y$ and select the indexes of the larger mm elements from $recx_j$ as $I_{mm,j}^i$. Thirdly, we determine whether the $I_{mm,j}^i$ can be chosen as the updated individual based on Metropolis rules. The process of Metropolis rules is: if $p_j \geq \eta$, where p_j is calculated by $p_j = \min(1, \exp(-(f(I_{mm,j}^i) - f(I_{mm,j-1}^i))/T_i))$ and η is a random number chosen in $[0, 1]$, implement $I_{mm,j}^i = I_{mm,j}^i$, otherwise, implement $I_{mm,j}^i = I_{mm,j-1}^i$. The parameter T_i is updated by $T_i = T_1/\log(i+1)$ in every iteration. Finally, selection can retain the optimal solution to the next iteration.

(4) Terminal condition. The terminal condition of the iterations is that the number of iterations exceeds the maximum number or the error e is smaller than the minimum error $Error$. If the sets of edge neighbours can not includes all the indexes of the nonzero elements of E_{HL} , E_{LH} and E_{HH} , it will lead to a non-accurate reconstruction. However, if the error e can not satisfy the condition $e < Error$, we implement the above update process to estimate the I_{mm} based on the feasible set S .

Finally, the reconstructed multi-variable high-frequency images $recC_{HL}$, $recC_{LH}$ and $recC_{HH}$ can be obtained through the iterations. Reshape $recC_{HL}$, $recC_{LH}$ and $recC_{HH}$ to obtain the reconstructed high-frequency images $recX_{HL}$, $recX_{LH}$ and $recX_{HH}$, then implement inverse wavelet transform to C_{LL} , $recX_{HL}$, $recX_{LH}$ and $recX_{HH}$ to obtain the reconstructed image $recX$.

l_0 minimization is an NP-hard problem that requires exhaustively listing all

possibilities of the original signal and has a very high computational complexity.

280 If the sparsity level K is known as a prior, the purpose of signal reconstruction based on l_0 minimization is to find a global optimal solution in the feasible domain whose size can be calculated by C_N^K . In our proposed method MIMP, we use **the edge saliency** as the prior knowledge to guide the reconstruction, so the size of the feasible domain can be reduced to C_E^K , where E represents the size of edge collection.

285 of edge collection. Also, the use of multi-variable scheme is beneficial to increase the accuracy of the guidance of the prior knowledge. In addition, based on the optimization function proposed in **Theorem 1**, the size of the feasible domain can be calculated by C_E^{mm} , while there are C_{mm}^K optimal solutions, which not only can relax the precondition that the sparsity level needs to be known as a prior but also can increase the possibilities to find the optimal solutions.

290 So for many reconstruction problems, our proposed method MIMP has a lower computational complexity than that of solving the original NP-hard problem, and is more likely to find the global optimal solution.

The schematic diagram of MIMP is shown in Figure 2.

Figure 2: The schematic diagram of image reconstruction based on MIMP

295 2.4. Computational complexity analysis

As shown in Algorithm 1, the main computational complexity of MIMP lies in two parts: the initialization and the iteration. There are two algorithms, i.e. wavelet transform and edge extraction in the process of initialization. The computational complexity of the two algorithms are both upper bounded by $O(N^2)$, where N represents the size of the transformed image. So the computational complexity of initial solution is upper bounded by $4 * O(N^2)$ with 3 times of edge extraction and 1 time of wavelet transformation. In each iteration, the computational complexity lies in step 13 and step 14 shown in Algorithm 1, which both have matrix multiplication. The computational complexity of matrix multiplication is upper bounded by $O(N^3)$. Also, the maximum number of iterations can be calculated by $M_{max} * N_{max}$ and there are $popsiz$ e individuals in the group, so the computation complexity of the iteration is $popsiz$ e $* M_{max} N_{max} O(N^3)$. In conclusion, the computational complexity of MIMP is upper bounded by Equ.(11).

$$4 * O(N^2) + popsiz * M_{max} * N_{max} * O(N^3). \quad (11)$$

310 From Equ.(11), we can see that the computational complexity of MIMP is influenced by the number of iterations $M_{max} * N_{max}$ mostly. The clone operation that makes more individuals search simultaneously can reduce the number of iterations. Also, applying the set of edge neighbours as the initial solution is another method to reduce the iterations. Furthermore, some methods can be introduced to optimize the initial solution to reduce the iterations.

3. Experiment results

3.1. Experimental Configuration

To verify the validity of the proposed method MIMP, compressive sensing reconstruction experiments on several images are conducted. The well-known *haar* orthogonal wavelet is used for wavelet transformation in the implementation of our algorithm. Neighbourhood structure with the size of $4 * 4$ is used for

multiple measurement. In the following experiments, a block sampling scheme [44, 45, 46] is utilized to improve the reconstruction speed, that is, the original images are blocked by windows with the size of $32 * 32$. We obtain the mea-
 325 surements from the subblocks and reconstruct these subblocks respectively. The measurement matrix ϕ is obtained by extracting m rows of the $N * N$ orthogonal random matrix. We also set the main parameters $M_{max} = 200$, $N_{max} = 200$, $E = 10^{-5}$ in our algorithm for all test images.

All the simulations are implemented in Matlab 2011a on a PC with $3.2GHz$ Intel Core *i5* processor and $8.0GB$ memory running the Windows 7 system.
 330 Here, the compared algorithms OMP, PFP and BP are implemented using SolveOMP, SolvePFP and SolveBP in SparseLab toolbox [47] respectively.

Figure 3: The test images. (a). blobs image; (b). phantom image; (c). brain MR image; (d). brain CT image

3.2. Comparison to the state-of-the-art algorithms

In this experiment, the proposed method MIMP is compared with several
 335 state-of-the-art reconstruction algorithms, including multi-variable OMP (M-OMP), multi-variable BP (M-BP), multi-variable Polytope Faces Pursuit algorithm (M-PFP), multi-variable compressive sampling orthogonal matching pursuit (M-CoSaOMP) and multi-variable Sparsity Adaptive Matching Pursuit algorithm (M-SAMP), which aims to illustrate the superiority of MIMP by tak-
 340 ing advantage of intelligent optimization algorithm in solving l_0 minimization essentially and introducing the matching strategies of greedy algorithm which performs quite well in reconstruction speed. More specifically, all the compared

greedy algorithms and the proposed MIMP are based on the multi-variable measurement scheme. The test images are blobs image, phantom image, brain CT image and brain MR image respectively, which are presented in Fig.3. The test images are both famous for their high sparsity level in wavelet domain and obvious edge information. Two indexes, PSNR, which indicates the quality of reconstruction, and runtime, which indicates the reconstruction speed, are adopted to evaluate the proposed method MIMP. The reconstruction of the test images are processed with the measurement rate changing from 0.35 to 0.8. Experiment results shown in Fig.4, Fig.5, Fig.6 and Fig.7 demonstrate the reconstruction performance comparison between MIMP and the aforementioned algorithms in terms of reconstruction quality (PSNR) and reconstruction speed (Runtime).

From Fig. 4(a), Fig.5 (a), Fig. 6(a) and Fig.7 (a), we can see that the PSNR of image reconstruction gradually increase with the increasing measurement rate in all methods. Obviously, the proposed method MIMP obtains the highest PSNR, i.e., the best reconstruction accuracy, in all the measurement rates. In the case of low measurement rate, MIMP takes on the appearance that PSNR is several dB higher than that of the compared state-of-the-art algorithms. From some cases when the measurement rate is greater than a threshold, the PSNR of MIMP is significantly improved and higher than that of others. As we can see from the figures, the thresholds of blobs image, phantom image, brain CT image, brain MR image are 0.6, 0.5, 0.7 and 0.65 respectively.

From the runtime comparison shown in Fig.4 (b), Fig.5 (b), Fig.6 (b) and Fig.7 (b), we can see that the reconstruction speed of the compared state-of-the-art algorithms decrease with the increasing measurement rate but the reconstruction speed of MIMP increases with the increasing measurement rate. We can also see that M-OMP has the fastest reconstruction speed and MIMP is superior to M-BP in almost all the measurement rate. Also, MIMP is faster than M-PFP, M-SAMP and M-CoSaOMP with the measurement rate exceeding 0.75.

Through the experiment results mentioned above, the reconstruction accu-

Figure 4: Reconstruction performance of MIMP, M-OMP, M-PFP, M-SAMP, M-BP and M-CoSaOMP with the measurement rate changing from 0.35 to 0.8 for blobs image. (a). PSNR, (b). Runtime.

Figure 5: Reconstruction performance of MIMP, M-OMP, M-PFP, M-SAMP, M-BP and M-CoSaOMP with the measurement rate changing from 0.35 to 0.8 for phantom image. (a). PSNR, (b). Runtime.

Figure 6: Reconstruction performance of MIMP, M-OMP, M-PFP, M-SAMP, M-BP and M-CoSaOMP with the measurement rate changing from 0.35 to 0.8 for brain MR image. (a). PSNR, (b). Runtime.

Figure 7: Reconstruction performance of MIMP, M-OMP, M-PFP, M-SAMP, M-BP and M-CoSaOMP with the measurement rate changing from 0.35 to 0.8 for brain CT image. (a). PSNR, (b). Runtime.

racy of MIMP is superior to the compared state-of-the-art algorithms. As we
 375 use multi-variable measurement scheme in all the methods, the main reason of
 the better performance of MIMP is that we take the advantage of the intel-
 ligent optimization algorithm in solving combinatorial optimization problems
 and searching the global optimal solution. Also, we design novel and effective
 380 updating mechanism by introducing the matching strategies of greedy algorithm
 to make MIMP more likely to find the optimal solution. More specifically,
 MIMP improves the performance of image reconstruction by means of solving l_0
 minimization essentially and searching the global optimal solution more likely.

The experiment results also demonstrate that the reconstruction speed of
 MIMP is faster than M-BP but slower than the other compared algorithms in
 385 many cases. Compared with many state-of-the-art greedy algorithms, MIMP
 slows down due to the global searching, which needs more iterations to find
 the global optimal solution. However, with the measurement rate increasing,
 the performance of the updating mechanism of MIMP becomes more effective,
 which can reduce the number of iteration to accelerate the reconstruction speed.
 390 In addition, the using of prior knowledge can also improve the reconstruction
 speed through reducing the computational complexity of MIMP, which makes
 the reconstruction speed of MIMP more reasonable.

3.3. Comparison to algorithms using prior knowledge

In this experiment, the proposed method MIMP is compared with some
 395 state-of-the-art compressive sensing reconstruction methods which use the multi-
 variable scheme and set some structures of images as the prior knowledge for
 image reconstruction, which aims to illustrate the superiority of MIMP by using
 the multi-variable scheme and utilizing the edge saliency as the prior knowledge.
 The structured based reconstruction methods are well known Tree-CoSaMP,
 400 TSWCS, edge based multi-variable pursuit algorithm (EMPA), edge guided
 compressive sensing (EdgeCS), multi-variable pursuit algorithm using multi-
 variable K distribution (MPA-MK) and multi-variable pursuit algorithm using
 multi-variable Laplace distribution (MPA-ML) respectively. Note that the Tree-

Table 1: Reconstruction performance of Tree-CoSaMP, TSWCS, EMPA, EdgeCS, MPA-MK, MPA-ML and MIMP for blobs image and phantom image with the measurement rate fixed as 0.4. The evaluate indexes are PSNR (dB), SSIM and Time (Sec.).

Image	Algorithms	PSNR(dB)	SSIM	Time (Sec.)
blobs	Tree-CoSaMP	37.0963	1.0000	521.0282
	TSWCS	169.8774	1.0000	801.3333
	EMPA	41.8219	0.9997	3.8839
	EdgeCS	64.9615	1.0000	1.0586
	MPA-MK	33.4425	0.9983	0.5637
	MPA-ML	33.2983	0.9983	0.5924
	MIMP	48.5922	1.0000	30.0091
phantom	Tree-CoSaMP	41.7257	0.9965	843.1793
	TSWCS	126.9542	1.0000	1046.0712
	EMPA	39.2611	0.9867	3.8156
	EdgeCS	108.0631	1.0000	0.8732
	MPA-MK	31.4908	0.9870	0.5669
	MPA-ML	31.4896	0.9870	0.5693
	MIMP	254.9789	1.0000	16.2553

CoSaMP and TSWCS exploit the tree structure of wavelet coefficients and they are very effective in image reconstruction, especially in areas of image structures. EMPA and EdgeCS use the edge information as the prior knowledge and produce state-of-the-art reconstruction quality. MPA-MK and MPA-ML are multi-variable pursuit algorithms based on multi-variable measurement scheme using multi-variable scale mixture models as the prior distributions, which provide superior performance compared with many state-of-the-art compressive sensing reconstruction algorithms. The test images are blobs image, phantom image, brain CT image, brain MR image and liver CT image respectively, where the size of liver CT image is $512 * 512$ and that of others are $256 * 256$. Another quantitative measure, Structural Similarity (SSIM) index measurement [48], is adopted to the objective evaluation. A SSIM value much closer to 1 indicates the structure of the reconstructed image is more similar to the original image. The measurement rate is fixed as 0.4 for sparse images and 0.5 for medical images. Table 1 shows the PSNR, SSIM and Runtime comparison between the proposed method MIMP and Tree-CoSaMP, TSWCS, EMPA, EdgeCS, MPA-MK and MPA-ML for sparse image reconstruction. Experiment results shown in Table 2 demonstrate the superior reconstruction performance of our pro-

posed method MIMP by the comparison with Tree-CoSaMP, TSWCS, EMPA, EdgeCS, MPA-MK and MPA-ML for medical image reconstruction in terms of PSNR, SSIM and Runtime. The reconstructed images and their corresponding magnified parts based on the aforementioned methods are shown in Fig.8, Fig.9 and Fig.10, which aims to illustrate the superior performance of MIMP in reconstruction visual.

Table 2: Reconstruction performance of Tree-CoSaMP, TSWCS, EMPA, EdgeCS, MPA-MK, MPA-ML and MIMP for brain MR image, brain CT image and liver CT image with the measurement rate fixed as 0.5. The evaluate indexes are PSNR (dB), SSIM and Time (Sec.).

Image	Algorithms	PSNR(dB)	SSIM	Time (Sec.)
brain MR	Tree-CoSaMP	31.7627	0.8621	424.5496
	TSWCS	26.0477	0.7522	891.328
	EMPA	29.7933	0.8845	3.3043
	EdgeCS	36.0830	0.9750	6.0909
	MPA-MK	28.7693	0.9528	0.5800
	MPA-ML	27.9408	0.9392	0.6024
	MIMP	46.7829	0.9977	48.2934
brain CT	Tree-CoSaMP	28.0566	0.8621	649.3453
	TSWCS	27.2981	0.8561	874.0045
	EMPA	30.9356	0.9035	3.3236
	EdgeCS	35.7729	0.9701	5.9105
	MPA-MK	27.9925	0.9328	0.5922
	MPA-ML	27.3202	0.9229	0.6131
	MIMP	42.2817	0.9936	120.2910
liver CT	Tree-CoSaMP	28.0207	0.9655	1486.2143
	TSWCS	32.9495	0.9832	3428.6175
	EMPA	35.0608	1.0000	8.8397
	EdgeCS	36.6095	0.7308	26.8655
	MPA-MK	35.1983	0.9997	1.6344
	MPA-ML	34.2068	0.9997	1.5703
	MIMP	38.6541	1.0000	334.1973

From Table 1, we can see that the proposed method MIMP obtains the highest PSNR and SSIM in phantom image reconstruction. In the case of blobs image reconstruction, MIMP is superior to Tree-CoSaMP, EMPA, MPA-MK and MPA-ML on reconstruction quality. Table 2 shows that the proposed method MIMP outperforms other competing methods both on PSNR and SSIM for all the medical image reconstruction. As shown in Table 1 and Table 2, for all the test images, the reconstruction speed of MIMP is faster than that of Tree-

435 CoSaMP and TSWCS but slower than that of EMPA, EgdeCS, MPA-MK and
MPA-ML.

The experiment results demonstrate that MIMP improves the PSNR and SSIM of sparse image reconstruction significantly and the advantage PSNR gain of MIMP over the competing methods for medical image reconstruction is up
440 to several dB . The main reason is that phantom image has a higher sparsity in the wavelet domain and the edge saliency obtained from the low-frequency part of wavelet coefficients has a strong guidance for the process of reconstruction. Although the guidance of edge information of brain MR image, brain CT image and liver CT image are not that strong like that of phantom image, it can also
445 improve the reconstruction accuracy and reconstruction speed. Also, the multi-variable measurement scheme and multi-variable joint reconstruction can not only increase the guidance accuracy of the prior knowledge for reconstruction but also greatly accelerate the reconstruction speed.

To demonstrate the superiority of the proposed method MIMP over other
450 compared methods clearly, the visual quality of the reconstructed images based on the test methods are shown in Fig.8, Fig.9 and Fig.10. The odd rows present the reconstructed images and the even rows represent the local enlarged parts marked with the red blocks corresponding to the reconstructed images, in particular the areas of edges. From Fig.8, Fig.9 and Fig.10, we can observe that
455 the Tree-CoSaMP, EMPA, EdgeCS, MPA-MK and MPA-ML tend to blur the reconstructed images, especially in the edge region. Although TSWCS method has a high PSNR for blobs image and phantom image, it generates artefacts in the areas of edges. In contrast, the proposed method MIMP performs better on both the edge region and the smooth region of the reconstructed images.
460 Especially from the local enlarged parts of the reconstructed images, we can see that MIMP provides the reconstruction results with a best visual.

Figure 8: Visual performance of Tree-CoSaMP, TSWCS, EMPA, EdgeCS, MPA-MK, MPA-ML and MIMP for phantom image when the measurement rate is fixed as 0.4. The odd rows represent the global reconstructed images. The even rows represent the local enlarged parts marked by the red blocks.

Figure 9: Visual performance of Tree-CoSaMP, TSWCS, EMPA, EdgeCS, MPA-MK, MPA-ML and MIMP for brain MRI image when the measurement rate is fixed as 0.5. The odd rows represent the global reconstructed images. The even rows represent the local enlarged parts marked by the red blocks.

Figure 10: Visual performance of Tree-CoSaMP, TSWCS, EMPA, EdgeCS, MPA-MK, MPA-ML and MIMP for liver CT image when the measurement rate is fixed as 0.5. The odd rows represent the global reconstructed images. The even rows represent the local enlarged parts marked by the red blocks.

4. Conclusion

This paper proposes a novel method multi-variable intelligent matching pursuit algorithm (MIMP) using prior knowledge for image reconstruction with the sparsity level unknown as a prior. First of all, the proposed method MIMP can solve l_0 minimization problem essentially and find the global optimal solution by taking advantage of intelligent optimization algorithm which is famous for its superior performance in solving combinatorial optimization problems and searching the global optimal solution. MIMP has a reasonable relatively faster reconstruction speed by introducing the matching strategies of greedy algorithm to design the updating mechanism. Also, the multi-variable scheme is used to sample the original image and joint reconstruction is implemented for the measurements, which contribute a lot to enhancing the reconstruction performance. In addition, under the multi-variable measurement scheme, the edge saliency can be obtained from the low frequency wavelet coefficients and set as the prior knowledge to guide the high frequency wavelet coefficients reconstruction, which can not only reduce the computational complexity but also improve the reconstruction quality. Experiment results on several images demonstrate that MIMP outperforms the state-of-the-art algorithms such as multi-variable OMP (M-OMP), multi-variable PFP (M-PFP), multi-variable (SAMP), multi-variable BP (M-BP), multi-variable CoSaOMP (M-CoSaMP) and structure based algorithms such as tree-CoSaMP , TSWCS, EMPA EdgeCS, MPA-MK and MPA-ML, especially in the areas of edges. Through the experiment results, it can be demonstrated that our proposed method MIMP significantly improves the reconstruction quality of those images with the obvious edges and high sparsity in wavelet domain, such as CT images and MR images. Nevertheless, the proposed method MIMP also has room for improvement. One possible extension of our work would be to introduce more effective updating mechanism into MIMP to further reduce the computational complexity. Another interesting challenge would be to introduce more structures of image as the prior knowledge, such as texture, which can guide the process of MIMP for image reconstruction.

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