Time-varying group formation analysis and design for second-order multi-agent systems with directed topologies[☆]

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Abstract

Time-varying group formation control problems for second-order multi-agent systems with directed topologies are investigated. Firstly, a time-varying group formation control protocol is constructed using local relative positions and velocities of each agent and its neighbors. Then based on graph theory, nonsingular transformations are applied to the closed-loop multi-agent systems. Sufficient conditions for second-order multi-agent systems to achieve time-varying group formation are further presented together with the time-varying group formation feasibility constraints. Explicit expressions of the subgroup formation reference functions are derived to describe the macroscopic movement of the time-varying subgroup formations. Moreover, by solving an algebraic Riccati equation, an approach to design the time-varying group formation protocol is proposed. Finally, a numerical example with three subgroups is provided to demonstrate the effectiveness of the obtained results. In contrast to the traditional complete formation, where only one formation is realized by the multi-agent system, in the group formation discussed in the current paper, agents are classified into subgroups and each subgroup is required to form a specified time-varying sub-formation via inter-subgroup and intra-subgroup interactions.

Key words: Time-varying formation, group formation, second-order dynamics, multi-agent system, directed topology

1. Introduction

The research on formation control of multi-agent systems has attracted an increasing interest in recent years due to its broad potential applications in various fields, such as cooperative localization [1], load transportation [2], surveillance [3] and target enclosing [4]. Although in the past decades, several centralized and decentralized strategies have been proposed to deal with the formation control problems, how to realize the predefined formation via local neighboring interactions is still a current concern in both the control theory and robotics communities [5], [6].

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With the development of consensus theory, there is an emerging trend to study the formation control problem in the view of consensus control. Ren [7] proposed a general formation control framework for second-order multi-agent systems by extending the consensus protocols. Sufficient conditions for multi-agent systems with second-order dynamics to achieve time-invariant formations via local interactions were presented in [8]. Time-invariant formation control problems for second-order multi-agent systems with time delays were addressed in [9]. A distributed controller for first-order multi-agent systems to achieve rigid formations was constructed in [10]. Oh and Ahn [11] proposed a formation control strategy for first-order multi-agent systems using the distributed position estimation. Distributed formation control problems for first-order multi-agent systems were investigated in [12] by means of complex laplacian analysis. Sufficient conditions for first-order multi-agent systems to achieve circular formations in one-dimensional and three-dimensional spaces were presented in [13] and [14], respectively. Necessary and sufficient conditions for second-order multi-agent systems with fixed and switching topologies to achieve time-varying formations were proposed in [15] and [16], respectively.

It should be pointed out that in the aforementioned works [7]-[16], only complete formation control problems were considered, where all the agents form a single formation. However, in many practical circumstances, such as multi-target enclosing, obstacle avoidance and cooperative searching for multiple objects, agents in a multi-agent system may split into several subgroups to accomplish different distributed tasks. In such scenarios, group formation control problems arise, where there can be multiple different sub-formations in the multi-agent system and agents in the same subgroup achieve one predefined sub-formation. To the best of our knowledge, time-varying group formation control problems for second-order multi-agent systems have not been studied before. One relevant topic is group (or cluster) consensus control, where there exist multiple subgroups and agents in the same subgroup reach an agreement. Group consensus control problems for first-order multi-agent systems with switching topologies and communication delays were addressed in [17]. Necessary and sufficient conditions for first-order multi-agent systems to achieve the cluster or group consensus were provided in [18] and [19], respectively. Group consensus control problems for second-order multi-agent systems were studied in [20] through leader-following approach and pinning control. Necessary and sufficient conditions for second-order multi-agent systems to achieve group consensus were proposed in [21]. Although group/cluster consensus problems were studied in [17]-[21], the obtained results cannot be extended directly to deal with the time-varying group formation control problems for second-order multi-agent systems directly.

Motivated by the facts stated above, this paper investigates the time-varying group formation analysis and design problems for second-order multi-agent systems with directed topologies. Firstly, agents in the multi-agent systems are partitioned into subgroups. A time-varying sub-formation is predefined for each subgroup. Time-varying group formation control protocol is constructed based on the local inter-subgroup or intra-subgroup information. By assuming that the directed graph of the multi-agent system has an acyclic partition and each subgroup has a spanning tree, sufficient conditions for second-order multi-agent systems to achieve time-varying group formation are proposed. A time-varying group formation feasibility constraint is also given. Then explicit expressions of the subgroup formation reference functions are presented to describe the macroscopic movement

of the time-varying group formation. Finally, an approach to design the time-varying group formation protocol is given by solving an algebraic Riccati equation.

Compared with the previous results on formation control and group/cluster consensus control, the novel features of the current paper are threefold. Firstly, the agents in the current paper can be partitioned into multiple subgroups and multiple sub-formations can be realized by the whole multi-agent system. In [7]-[16], only complete formation control problems were studied, where all agents interact as a whole group and only one formation is realized for all the agents. Group formation control problems are much complicated and challenging than the complete formation since there exist both inter-subgroup interactions and intra-subgroup interactions, and multiple sub-formations. Moreover, the time-varying group formation is more general and these complete formations discussed in [7]-[16] can all be regarded as special cases of the one in the current paper where only one group exists. Secondly, the multiple sub-formations can be time-varying. In [17]-[21], group/cluster consensus control problems for first-order and second-order multi-agent systems were addressed. Although the results on group/cluster consensus are potential to solve some constant group formation control problems, they cannot be extended to solve the time-varying group formation control problems in the current paper due to that the time-varying formation will bring the derivative of the formation information to both the analysis and design of the group formation control law. By choosing the time-varying formation vectors to be zero, the time-varying group formation control problems become group/cluster consensus problems investigated in [17]-[21]. Thirdly, explicit expressions for the time-varying formation reference functions of all the sub-formations are derived to describe the macroscopic movements of all the subgroups. Although the macroscopic movement of the whole multi-agent system can be described by the explicit expression of the formation reference function in [15] and [16], they only considered the complete formation case and the approaches proposed in [15] and [16] cannot be directly used to solve the problems in the current paper.

The remainder of this paper is organized as follows. In Section 2, basic definitions and notations on graph theory are given and the problem description is introduced. Main results which include sufficient criteria and protocol design approaches are proposed in Section 3. In Section 4, simulation results are given to demonstrate the effectiveness of the results in this paper. Finally, conclusions are given in Section 5.

Some notations used in this paper are given as follows. Let 0_n and $\mathbf{1}_n$ be zero matrices and column vectors of ones with dimension n . Let I_N represent an identity matrix with dimension N , and $\otimes$ denote the Kronecker product.

2. Preliminaries and problem description

In this section, basic concepts and notations on graph theory are introduced and the problem description is presented.

2.1. Preliminaries

A weighted directed graph of order N can be denoted by $G = \{V, E, W\}$, where $V = \{v_1, v_2, \dots, v_N\}$ is the node set, $E \subseteq \{(v_i, v_j) : v_i, v_j \in V\}$ is the edge set and $W = [w_{ij}] \in \mathbb{R}^{N \times N}$ the weighted adjacency matrix. An edge in G is represented by $e_{ij} = (v_i, v_j)$ ($i \neq j$).

For any $i, j \in \{1, 2, \dots, N\}$, the adjacency element w_{ij} in W satisfies that $w_{ji} \neq 0$ if and only if $e_{ij} \in E$, and $w_{ij} = 0$ otherwise. Denote by $N_i = \{v_j \in V : e_{ji} \in E\}$ the neighbor set of node v_i . The in-degree of node v_i is defined as $\deg_{in}(v_i) = \sum_{j=1}^N w_{ij}$. Let $D = \text{diag}\{\deg_{in}(v_i), i = 1, 2, \dots, N\}$ be the degree matrix of G . The Laplacian matrix $L \in \mathbb{R}^{N \times N}$ of G is defined as $L = D - W$. A directed path from v_i to v_j is a finite ordered sequence edges with the form of $(v_i, v_{k_1}), (v_{k_1}, v_{k_2}), \dots, (v_{k_{l-1}}, v_{k_l}), (v_{k_l}, v_j)$. A directed graph with no cycles is called a directed acyclic graph. A directed graph is said to contain a spanning tree if there exist at least a node which has a directed path to any other nodes.

2.2. Problem description

Consider a second-order multi-agent system with N agents. The dynamics of agent i ($i \in \{1, 2, \dots, N\}$) is described by

$$\begin{cases} \dot{x}_i(t) = v_i(t), \\ \dot{v}_i(t) = \alpha_x x_i(t) + \alpha_v v_i(t) + u_i(t), \end{cases} \quad (1)$$

where $n \geq 1$ is the dimension of the space, $x_i(t) \in \mathbb{R}^n$, $v_i(t) \in \mathbb{R}^n$ and $u_i(t) \in \mathbb{R}^n$ denote, respectively, the position, velocity and control input vectors of agent i , and $\alpha_x \in \mathbb{R}$ and $\alpha_v \in \mathbb{R}$ are known damping constants. In the following, for simplicity of description, let $n = 1$ if not otherwise specified. However, by applying the Kronecker product, all the results hereafter can be directly extended to the higher dimensional case. The interaction topology among the N agents can be modeled by the directed graph G with node i representing the agent i in multi-agent system (1) and e_{ij} denoting the interaction from agent i to agent j with the associated interaction strength w_{ji} .

Remark 1. From (1), one sees that in the case where $\alpha_x = 0$ and $\alpha_v = 0$, the second-order model in the current paper becomes the classic double-integrator one. Therefore, the second-order model described by (1) can be treated as an extension to the double-integrator one and has more generality.

Suppose that multi-agent system (1) consists of $g \in \mathbb{N}$ ($g \geq 1$) subgroups, and the node set V can be partitioned into $V_1, V_2, \dots, V_g$, where $V_k \neq \emptyset$ ($k = 1, 2, \dots, g$), $\cup_{k=1}^g V_k = V$ and $V_k \cap V_l = \emptyset$ ($k, l \in \{1, 2, \dots, g\}; k \neq l$). For any $i, j \in \{1, 2, \dots, N\}$, denote by $\bar{i}$ and $\bar{j}$ the subscripts of the subsets to which agents i and j belong, namely, $i \in V_{\bar{i}}$ and $j \in V_{\bar{j}}$, and agents i and j are said to be in the same subgroup if $\bar{i} = \bar{j}$. In addition, let $q_{\bar{i}} \in \mathbb{N}$ ($q_{\bar{i}} \geq 1$) be the quantity of agents in the subgroup $V_{\bar{i}}$ and $G_{\bar{i}}$ be the underlying interaction topology associated with the subgroup $V_{\bar{i}}$, $\bar{i} \in \{1, 2, \dots, g\}$. Without loss of generality, the node set of the subgroup $\bar{i}$ ($\bar{i} \in \{1, 2, \dots, g\}$) is indexed as $V_{\bar{i}} = \{\Xi_{\bar{i}} + 1, \Xi_{\bar{i}} + 2, \dots, \Xi_{\bar{i}} + q_{\bar{i}}\}$ with $\Xi_{\bar{i}} = \sum_{k=0}^{\bar{i}-1} q_k$ and $q_0 = 0$.

Assumption 1. $\{V_1, V_2, \dots, V_g\}$ is an acyclic partition of the node set V , and for any $\bar{i} \in \{1, 2, \dots, g\}$, $G_{\bar{i}}$ contains a spanning tree.

Based on Assumption 1, as shown in [22] that the Laplacian matrix L associated with G has the following form

$$L = \begin{bmatrix} L_{11} & 0 & \cdots & 0 \\ L_{21} & L_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ L_{g1} & L_{g2} & \cdots & L_{gg} \end{bmatrix}, \quad (2)$$

where $L_{\bar{i}\bar{i}}$ is associated with $G_{\bar{i}}$, and $L_{\bar{i}\bar{j}}$ represents the interaction from groups $\bar{j}$ to $\bar{i}$, $\bar{i}, \bar{j} \in \{1, 2, \dots, g\}$. The following assumption is standard in investigating the group behavior of multi-agent systems (see, e.g., [17]-[21] and [22]).

Assumption 2. Each row sum of $L_{\bar{i}\bar{j}}$ ($\bar{i}, \bar{j} \in \{1, 2, \dots, g\}$) is zero.

Under Assumption 2 and (2), it holds that $L_{\bar{i}\bar{i}}$ is the Laplacian matrix of $G_{\bar{i}}$. Moreover, the following conclusion can be obtained.

Lemma 1 ([23]). If Assumptions 1 and 2 hold, then the Laplacian matrix L has g simple zero eigenvalues with associated g linearly independent right eigenvectors, i.e., $u_1 = [1_{q_1}^T, 0_{N-q_1}^T]^T$, $u_2 = [0_{q_1}^T, 1_{q_2}^T, 0_{N-q_1-q_2}^T]^T$, $\dots$, $u_g = [0_{N-q_g}^T, 1_{q_g}^T]^T$; and the real parts of all the other $N - g$ eigenvalues are positive.

For any subgroup $\bar{i} \in \{1, 2, \dots, g\}$, the desired time-varying sub-formation for it is described by the time-varying vector $\bar{h}_{\bar{i}}(t) = [h_{\Xi_{\bar{i}+1}}^T(t), h_{\Xi_{\bar{i}+2}}^T(t), \dots, h_{\Xi_{\bar{i}+q_{\bar{i}}}}^T(t)]^T \in \mathbb{R}^{2q_{\bar{i}}}$ with $h_j(t) = [h_{jx}(t), h_{jy}(t)]^T$ ($j \in \{\Xi_{\bar{i}} + 1, \Xi_{\bar{i}} + 2, \dots, \Xi_{\bar{i}} + q_{\bar{i}}\}$) piecewise continuously differentiable. Let $h(t) = [\bar{h}_1^T(t), \bar{h}_2^T(t), \dots, \bar{h}_g^T(t)]^T$ be the formation vector specifying the time-varying group formation for the whole multi-agent system (1), $\xi_j(t) = [x_j(t), v_j(t)]^T$ and $\bar{\xi}_{\bar{i}}(t) = [\xi_{\Xi_{\bar{i}+1}}^T(t), \xi_{\Xi_{\bar{i}+2}}^T(t), \dots, \xi_{\Xi_{\bar{i}+q_{\bar{i}}}}^T(t)]^T$.

Definition 1. Multi-agent system (1) is said to achieve the time-varying group formation specified by $h(t)$ if for any given bounded initial states and any subgroup $\bar{i} \in \{1, 2, \dots, g\}$, there exists a vector-valued function $r_{\bar{i}}(t) \in \mathbb{R}^2$ such that

$$\lim_{t \rightarrow \infty} (\bar{\xi}_{\bar{i}}(t) - \bar{h}_{\bar{i}}(t) - (I_{q_{\bar{i}}} \otimes r_{\bar{i}}(t))) = 0,$$

where $r_{\bar{i}}(t)$ is called the formation reference function of the subgroup $\bar{i}$.

From Definition 1, one gets that $\bar{h}_{\bar{i}}(t)$ is used to characterize the desired time-varying sub-formation for the $\bar{i}$ th subgroup and $r_{\bar{i}}(t)$ is a representation of the macroscopic movement of the $\bar{i}$ th sub-formation. To have an intuitive impression on the roles of $\bar{h}_{\bar{i}}(t)$, $r_{\bar{i}}(t)$ and Definition 1, consider the following illustrative example.

Illustrative example 1. Consider a multi-vehicle system with seven vehicles moving in the horizontal plane (XY plane). The dynamics of each vehicle is described by (1) with $n = 2$ and $\xi_i(t) = [x_{iX}(t), v_{iX}(t), x_{iY}(t), v_{iY}(t)]^T$ ($i = 1, 2, \dots, 7$), where $x_{iX}(t)$, $v_{iX}(t)$ and $x_{iY}(t)$, $v_{iY}(t)$ are the positions and velocities along the X and Y axes, respectively. The multi-vehicle system is classified into two subgroups; that is, vehicles 1-3 belong to the first subgroup and vehicles 4-7 belong to the second subgroup. The multi-vehicle system is required to realize

two time-invariant sub-formations that the positions of the first and second subgroups form a regular triangle formation and a square formation, respectively, while the velocities of the two subgroups reach the agreement. Since the sub-formations are time-invariant, $\bar{h}_i(t)$ ($\bar{i} = 1, 2$) can be rewritten as $\bar{h}_{\bar{i}}$. Let $h_i = [h_{ixX}, h_{ivX}, h_{ixY}, h_{ivY}]^T$, $h_{ix} = [h_{ixX}, h_{ixY}]^T$, $h_{iv} = [h_{ivX}, h_{ivY}]^T$ ($i = 1, 2, \dots, 7$), $r_i(t) = [r_{ixX}(t), r_{ivX}(t), r_{ixY}(t), r_{ivY}(t)]^T$, $r_{ix}(t) = [r_{ixX}(t), r_{ixY}(t)]^T$, $r_{iv}(t) = [r_{ivX}(t), r_{ivY}(t)]^T$, $x_i(t) = [x_{iX}(t), x_{iY}(t)]^T$ and $v_i(t) = [v_{iX}(t), v_{iY}(t)]^T$. Choose appropriate vectors h_{ix} ($i = 1, 2, 3$) and h_{jx} ($j = 4, 5, 6, 7$) to specify the regular triangle and square sub-formations, respectively. For any $i, j \in \{1, 2, \dots, 7\}$, $i \neq j$ and $\bar{i} = \bar{j}$, choose $h_{iv} = h_{jv}$. It should be pointed out that the vectors h_i ($i = 1, 2, \dots, 7$) to specify the desired sub-formations are not unique. If Definition 1 is satisfied, there exist $r_{ix}(t)$ and $r_{iv}(t)$ ($\bar{i} = 1, 2$) such that for any $i, j \in \{1, 2, 3\}$, $i \neq j$ and $i, j \in \{4, 5, 6, 7\}$, $i \neq j$, $\lim_{t \rightarrow \infty} ((x_i(t) - x_j(t)) - (h_{ix} - h_{jx})) = 0$ and $\lim_{t \rightarrow \infty} (v_i(t) - v_j(t)) = 0$ hold, respectively, which means that the desired two sub-formations are realized and the macroscopic movement of the $\bar{i}$ th sub-formation can be represented by $r_{\bar{i}}(t)$. Fig. 1 shows an illustration for the position components of the seven vehicles to achieve the regular triangle formation and square formation. From Fig. 1, one sees that if $\lim_{t \rightarrow \infty} (x_i(t) - h_{ix} - r_{1x}(t)) = 0$ ($i = 1, 2, 3$) and $\lim_{t \rightarrow \infty} (x_i(t) - h_{ix} - r_{2x}(t)) = 0$ ($i = 4, 5, 6, 7$), then the two triangles and two squares formed by h_{ix} and $x_i(t)$ ($i = 1, 2, \dots, 7$) are congruent with each other, which means that the position components of the first subgroup achieve the regular triangle formation and those of the second subgroup achieve the square formation. The conclusions for the velocity components can be illustrated directly in a similar way.

Fig. 1: Illustrative example for the positions of the seven vehicles to achieve two sub-formations.

Remark 2. Definition 1 presents a definition for achieving the time-varying group formation specified by the vector $h(t)$. If there is only one subgroup in the multi-agent system (1), namely, $g = 1$, then the definitions for complete formation in [7]-[9] and [15]-[16] become the special cases of Definition 1 in the current paper. By choosing $\bar{h}_i(t) \equiv 0$, $\bar{i} \in \{1, 2, \dots, g\}$, Definition 1 becomes the definitions for group consensus discussed in [20]-[21]. Moreover, By choosing $\bar{h}_i(t) \equiv 0$ and $r_i(t) \neq r_j(t)$, $\bar{i}, \bar{j} \in \{1, 2, \dots, g\}$, Definition 1 becomes the definition for cluster consensus.

Construct the following time-varying group formation control protocol

$$u_i(t) = K \sum_{j \in N_i} w_{ij} \left((\xi_i(t) - h_i(t)) - (\xi_j(t) - h_j(t)) \right) - \alpha h_i(t) + \dot{h}_{iv}(t), \quad (3)$$

where $i = 1, 2, \dots, N$, $\alpha = [\alpha_x, \alpha_v]$ and $K = [k_{11}, k_{12}]$ is a constant gain matrix to be determined later. Let $\xi(t) = [\xi_1^T(t), \xi_2^T(t), \dots, \xi_g^T(t)]^T$, $B_1 = [1, 0]^T$ and $B_2 = [0, 1]^T$. Under protocol (3), multi-agent system (1) can be written in a compact form as

$$\dot{\xi}(t) = (I_N \otimes (B_1 B_2^T + B_2 \alpha) + L \otimes B_2 K) \xi(t) - (I_N \otimes B_2 \alpha + L \otimes B_2 K) h(t) + (I_N \otimes B_2 B_2^T) \dot{h}(t). \quad (4)$$

For multi-agent system (4) with directed interaction topologies, the main concerns of this paper are twofold: (i) under what conditions the time-varying group formation $h(t)$ can be achieved; and (ii) how to design the group formation control protocol (3).

3. Main results

In this section, firstly, sufficient conditions for multi-agent system (4) with directed interaction topologies to achieve time-varying group formation specified by $h(t)$ are proposed. Then to describe the macroscopic movement of the group formation, explicit expressions of the formation reference functions for all the subgroups are given. Finally, an approach to design the group formation control protocol (3) is provided.

Define $\varphi_i(t) = \xi_i(t) - h_i(t)$ ($i = 1, 2, \dots, N$), $\bar{\varphi}_i(t) = [\varphi_{\Xi_i+1}^T(t), \varphi_{\Xi_i+2}^T(t), \dots, \varphi_{\Xi_i+q_i}^T(t)]^T$ ($\bar{i} = 1, 2, \dots, g$) and $\varphi(t) = [\bar{\varphi}_1^T(t), \bar{\varphi}_2^T(t), \dots, \bar{\varphi}_g^T(t)]^T$. Then multi-agent system (4) can be rewritten as

$$\dot{\varphi}(t) = (I_N \otimes (B_1 B_2^T + B_2 \alpha) + L \otimes B_2 K) \varphi(t) + (I_N \otimes B_1 B_2^T) h(t) - (I_N \otimes B_1 B_1^T) \dot{h}(t). \quad (5)$$

Let λ_i ($i = 1, 2, \dots, N$) be the eigenvalues of the Laplacian matrix L corresponding to the directed topology G . From Lemma 1, one gets that L has g simple zero eigenvalues. Without loss of generality, let $\lambda_{\bar{i}} = 0$ ($\bar{i} = 1, 2, \dots, g$) with the associated eigenvectors $u_1 = [1_{q_1}^T, 0_{N-q_1}^T]^T$, $u_2 = [0_{q_1}^T, 1_{q_2}^T, 0_{N-q_1-q_2}^T]^T$, $\dots$, $u_g = [0_{N-q_g}^T, 1_{q_g}^T]^T$, and $0 < \text{Re}(\lambda_{g+1}) \leq \dots \leq \text{Re}(\lambda_N)$. Let $U^{-1} L U = J$, where $U = [u_1, u_2, \dots, u_N]$, $U^{-1} = [\tilde{u}_1^H, \tilde{u}_2^H, \dots, \tilde{u}_N^H]^H$ and J is the Jordan canonical form of L . By Lemma 1, one has $J = \text{diag}\{0_g, \bar{J}\}$ with $\bar{J}$ consisting of the Jordan blocks corresponding to λ_i ($i = g+1, g+2, \dots, N$). Define $\vartheta(t) = (U^{-1} \otimes I_2) \varphi(t) = [\vartheta_1^H, \vartheta_2^H, \dots, \vartheta_N^H]^H$, $\varsigma(t) = [\vartheta_{g+1}^H, \vartheta_{g+2}^H, \dots, \vartheta_N^H]^H$, $\tilde{U} = [u_{g+1}, u_{g+2}, \dots, u_N]$ and $\tilde{U} = [\tilde{u}_{g+1}^H, \tilde{u}_{g+2}^H, \dots, \tilde{u}_N^H]^H$. Then multi-agent system (5) can be transformed into

$$\begin{cases} \dot{\vartheta}_1(t) = (B_1 B_2^T + B_2 \alpha) \vartheta_1(t) + (\tilde{u}_1 \otimes B_1)(h_v(t) - \dot{h}_x(t)), \\ \vdots \\ \dot{\vartheta}_g(t) = (B_1 B_2^T + B_2 \alpha) \vartheta_g(t) + (\tilde{u}_1 \otimes B_1)(h_v(t) - \dot{h}_x(t)), \end{cases} \quad (6)$$

$$\dot{\varsigma}(t) = (I_{N-g} \otimes (B_1 B_2^T + B_2 \alpha) + \bar{J} \otimes B K) \varsigma(t) + (\tilde{U} \otimes B_1)(h_v(t) - \dot{h}_x(t)). \quad (7)$$

Lemma 2 ([24]). Consider a linear time-invariant system $\dot{X} = MX$, where M is a 2×2 complex matrix with characteristic polynomial $f(s) = s^2 + a_1s + a_2$ where $a_1, a_2 \in \mathbb{C}$. The system is asymptotically stable if and only if $\text{Re}(a_1) > 0$ and $\text{Re}(a_1)\text{Re}(a_1\bar{a}_2) - \text{Im}(a_2)^2 > 0$, where $\bar{a}_2$ is the complex conjugate of a_2 .

Theorem 1. Suppose that Assumptions 1 and 2 hold. Multi-agent system (1) under protocol (3) achieves the time-varying group formation specified by $h(t)$ if for any $k \in \{g+1, g+2, \dots, N\}$,

$$\begin{cases} \alpha_2 + \text{Re}(\lambda_k)k_{12} < 0, \\ (\alpha_2 + \text{Re}(\lambda_k)k_{12})\Psi_i + \text{Im}(\lambda_k)^2k_{11}^2 < 0, \end{cases} \quad (8)$$

where

$$\Psi_i = \alpha_2\alpha_1 + \text{Re}(\lambda_k)(\alpha_2k_{11} + \alpha_1k_{12}) + (\text{Re}(\lambda_k)^2 + \text{Im}(\lambda_k)^2)k_{11}k_{12},$$

and for any $i \in \{1, 2, \dots, N\}$, the following group formation feasibility constraint is satisfied

$$\lim_{t \rightarrow \infty} \left((h_{ix}(t) - h_{jx}(t)) - (\dot{h}_{ix}(t) - \dot{h}_{jx}(t)) \right) = 0, \quad (j \in N_i, \bar{i} = \bar{j}) \quad (9)$$

Proof. Let

$$\begin{cases} \varphi_{F1}(t) = (U \otimes I_2)[\vartheta_1^H(t), 0_{2(N-1)}^H]^H, \\ \vdots \\ \varphi_{Fg}(t) = (U \otimes I_2)[0_{2(g-1)}^H, \vartheta_g^H(t), 0_{2(N-g)}^H]^H, \end{cases} \quad (10)$$

$$\varphi_{\bar{F}}(t) = (U \otimes I_2)[0_{2(N-g)}^H, \varsigma^H(t)]^H. \quad (11)$$

It can be shown that $[0_{2(\bar{i}-1)}^H, \vartheta_{\bar{i}}^H(t), 0_{2(N-\bar{i})}^H]^H = e_{\bar{i}} \otimes \vartheta_{\bar{i}}(t)$ ($\bar{i} \in \{1, 2, \dots, g\}$), where $e_{\bar{i}} \in \mathbb{R}^N$ has 1 as its $\bar{i}$ th component and 0 elsewhere. Therefore,

$$\begin{aligned} \varphi_{F\bar{i}}(t) &= (U \otimes I_2)(e_{\bar{i}} \otimes \vartheta_{\bar{i}}(t)) \\ &= Ue_{\bar{i}} \otimes \vartheta_{\bar{i}}(t) \\ &= [0_{\Xi_{\bar{i}}}^T, \mathbf{1}_{q_{\bar{i}}}^T, 0_{N-\Xi_{\bar{i}}-q_{\bar{i}}}^T]^H \otimes \vartheta_{\bar{i}}(t). \end{aligned} \quad (12)$$

Note that $\vartheta(t) = [\vartheta_1^H(t), \dots, \vartheta_g^H(t), \varsigma^H(t)]^H$ and $\varphi(t) = (U \otimes I_2)\vartheta(t)$. From (10) and (11), one has

$$\varphi(t) = \varphi_{F1}(t) + \dots + \varphi_{Fg}(t) + \varphi_{\bar{F}}(t). \quad (13)$$

Because $U \otimes I_2$ is nonsingular, it can be obtained from (10) and (11) that $\varphi_{F1}(t), \varphi_{F2}(t), \dots, \varphi_{Fg}(t)$ and $\varphi_{\bar{F}}(t)$ are linearly independent. From (10) and (11), one gets

$$\varphi_{\bar{F}}(t) = \varphi(t) - \left(\begin{bmatrix} \mathbf{1}_{q_1} \\ 0_{N-q_1} \end{bmatrix} \otimes \vartheta_1(t) + \dots + \begin{bmatrix} 0_{N-q_g} \\ \mathbf{1}_{q_g} \end{bmatrix} \otimes \vartheta_g(t) \right). \quad (14)$$

From (14), $\lim_{t \rightarrow \infty} \varphi_{\bar{F}}(t) = 0$ implies

$$\lim_{t \rightarrow \infty} \begin{bmatrix} \bar{\varphi}_1(t) - \mathbf{1}_{q_1} \otimes \vartheta_1(t) \\ \vdots \\ \bar{\varphi}_g(t) - \mathbf{1}_{q_g} \otimes \vartheta_g(t) \end{bmatrix} = 0, \quad (15)$$

that is, for any $\bar{i} \in \{1, 2, \dots, g\}$,

$$\lim_{t \rightarrow \infty} (\bar{\varphi}_{\bar{i}}(t) - \mathbf{1}_{q_{\bar{i}}} \otimes \vartheta_{\bar{i}}(t)) = 0. \quad (16)$$

Considering that $U \otimes I_2$ is invertible, from (11), (14) and (16), one gets that multi-agent system (1) under protocol (3) achieves the time-varying group formation specified by $h(t)$ if and only if

$$\lim_{t \rightarrow \infty} \varsigma(t) = 0. \quad (17)$$

Therefore, $\varsigma(t)$ describes the time-varying group formation error. It follows from (7) that for any given bounded initial states, $\lim_{t \rightarrow \infty} \varsigma(t) = 0$ if the system described by

$$\dot{\bar{\varsigma}}(t) = \left(I_{N-g} \otimes (B_1 B_2^T + B_2 \alpha) + \bar{J} \otimes B_2 K \right) \bar{\varsigma}(t), \quad (18)$$

is asymptotically stable, and

$$\lim_{t \rightarrow \infty} \left((\tilde{U} \otimes B_1)(h_v(t) - \dot{h}_x(t)) \right) = 0. \quad (19)$$

From the structure of $\bar{J}$, one gets that system (18) is asymptotically stable if and only if for any $k \in \{g+1, g+2, \dots, N\}$, the subsystem described by

$$\dot{\bar{\varsigma}}_i(t) = \left((B_1 B_2^T + B_2 \alpha) + \lambda_k B_2 K \right) \bar{\varsigma}_i(t), \quad (20)$$

is asymptotically. The characteristic polynomial of the subsystem (20) is $f_k(s) = s^2 - (\alpha_2 + \lambda_k k_{12})s - (\alpha_1 + \lambda_k k_{11})$ where s is a complex parameter. From Lemma 1, one gets that condition (8) can guarantee the asymptotic stability of the system (18).

If condition (9) holds, in virtue of Assumption 2, one can obtain

$$\lim_{t \rightarrow \infty} (L \otimes B_1)(h_v(t) - \dot{h}_x(t)) = 0. \quad (21)$$

Substituting $L = U \text{diag}\{0_g, \bar{J}\} U^{-1}$ into (21) and pre-multiplying the both sides of (21) by $U^{-1} \otimes I_2$ lead to

$$\lim_{t \rightarrow \infty} (\bar{J} \tilde{U} \otimes B_1)(h_v(t) - \dot{h}_x(t)) = 0. \quad (22)$$

Since $\bar{J}$ is invertible, one can pre-multiply the both sides of (22) by $\bar{J}^{-1} \otimes I_2$. Then it follows from (22) that

$$\lim_{t \rightarrow \infty} (\tilde{U} \otimes B_1)(h_v(t) - \dot{h}_x(t)) = 0,$$

that is, condition (19) holds. Since (18) and (19) are satisfied, it follows from (7) that $\lim_{t \rightarrow \infty} \varsigma(t) = 0$, which means that the group formation error converges to zero and multi-agent system (1) under protocol (3) achieves the time-varying group formation specified by $h(t)$. This completes the proof for Theorem 1.

Remark 3. From Theorem 1, one sees that not all the time-varying group formation can be realized by the multi-agent system (1) under protocol (3). It has been pointed out in [15]-[16] that even for complete formation control problems for second-order multi-agent systems, the feasible formation must be compatible with the dynamics of each agent. Condition (9) describes the feasibility constraint on the predefined time-varying group formation. It can be verified that in the case where there is only one subgroup, i.e., $g = 1$, the formation feasibility constraints in [15] and [16] can be treated as special cases of constraint (9) in Theorem 1.

Formation reference $r_{\bar{i}}(t)$ ($\bar{i} \in \{1, 2, \dots, g\}$) is a useful tool to describe the macroscopic movement of the subgroup formation specified by $\bar{h}_{\bar{i}}(t)$. The following theorem presents explicit expressions for the formation reference functions associated with the group formation.

Theorem 2. Suppose that Assumptions 1 and 2 hold. If multi-agent system (1) under protocol (3) achieves the time-varying group formation specified by $\bar{h}_{\bar{i}}(t)$ ($\bar{i} \in \{1, 2, \dots, g\}$), then the formation reference function $r_{\bar{i}}(t)$ for subgroup $\bar{i}$ satisfies

$$\lim_{t \rightarrow \infty} (r_{\bar{i}}(t) - r_{\bar{i}}^0(t) - r_{\bar{i}}^h(t)) = 0,$$

where

$$r_{\bar{i}}^0(t) = e^{(B_1 B_2^T + B_2 \alpha)t} (\tilde{u}_{\bar{i}} \otimes I_2) \xi(0),$$

$$r_{\bar{i}}^h(t) = -(\tilde{u}_{\bar{i}} \otimes I_2) h(t) + \int_0^t e^{(B_1 B_2^T + B_2 \alpha)(t-\tau)} ((\tilde{u}_{\bar{i}} \otimes B_2) \dot{h}_v(\tau) - (\tilde{u}_{\bar{i}} \otimes B_2 \alpha) h(\tau)) d\tau.$$

Proof. If multi-agent system (1) under protocol (3) achieves the time-varying group formation specified by $\bar{h}_{\bar{i}}(t)$ ($\bar{i} \in \{1, 2, \dots, g\}$), it follows from Theorem 1 that the group formation error $\varsigma(t)$ converges to zero as $t \rightarrow \infty$, which means that (16) holds. From Definition 1 and (16), one gets

$$\lim_{t \rightarrow \infty} (r_{\bar{i}}(t) - \vartheta_{\bar{i}}(t)) = 0 \quad (i = 1, 2, \dots, g). \quad (23)$$

It can be obtained that

$$\vartheta_{\bar{i}}(0) = (\tilde{u}_{\bar{i}} \otimes I_2) (\xi(0) - h(0)), \quad (24)$$

and

$$\begin{aligned} & \int_0^t e^{(B_1 B_2^T + B_2 \alpha)(t-\tau)} (\tilde{u}_{\bar{i}} \otimes I_2) \dot{h}(\tau) d\tau \\ &= e^{(B_1 B_2^T + B_2 \alpha)(t-\tau)} (\tilde{u}_{\bar{i}} \otimes I_2) h(\tau) \Big|_{\tau=0}^{\tau=t} - \int_0^t \frac{d}{d\tau} e^{(B_1 B_2^T + B_2 \alpha)(t-\tau)} (\tilde{u}_{\bar{i}} \otimes I_2) h(\tau) d\tau \\ &= (\tilde{u}_{\bar{i}} \otimes I_2) h(t) - e^{(B_1 B_2^T + B_2 \alpha)t} (\tilde{u}_{\bar{i}} \otimes I_n) h(0) + \int_0^t e^{(B_1 B_2^T + B_2 \alpha)(t-\tau)} (\tilde{u}_{\bar{i}} \otimes (B_1 B_2^T + B_2 \alpha)) h(\tau) d\tau. \end{aligned} \quad (25)$$

By solving the differential equation (6) and taking advantage of (23)-(25), the conclusion of Theorem 2 can be derived.

Remark 4. In Theorem 2, r_i^0 is said to be the nominal component of $r_i(t)$, which is determined by the dynamics of each agent and initial states of all agents in the same subgroup. $r_i^h(t)$ represents the contribution of $\bar{h}_i(t)$ to $r_i(t)$. If $\bar{h}_i(t) \equiv 0$, $r_i(t)$ becomes the explicit expression of the group consensus function for subgroup $\bar{i}$. Moreover, if $\bar{h}_i(t) \equiv 0$ and $r_i(t) \neq r_j(t)$ ($\bar{i}, \bar{j} \in \{1, 2, \dots, g\}$), then $r_i(t)$ becomes the explicit expression of the cluster consensus function for subgroup $\bar{i}$. It should be pointed that the interaction topology among agents also contributes to the group formation reference function due to that the inter-subgroup interactions and intra-subgroups interactions have effects on $\tilde{u}_i$.

Based on the previous results, an approach to design the time-varying group formation control protocol (3) is presented in the following theorem.

Theorem 3. Suppose that Assumptions 1 and 2 are satisfied. If condition (9) in Theorem 1 holds, multi-agent system (1) achieves time-varying group formation specified by $h(t)$ under protocol (3) with $K = -\delta[\text{Re}(\lambda_{g+1})]^{-1}R^{-1}B_2^T P$, where $\delta > 0.5$ and P is the positive solution to the following algebraic Riccati equation

$$P(B_1B_2^T + B_2\alpha) + (B_1B_2^T + B_2\alpha)^T P - PB_2R^{-1}B_2^T P + I_2 = 0, \quad (26)$$

where $R = R^T > 0$ is any given constant matrix.

Proof. Consider the stability of the subsystem (20). Construct the following Lyapunov function candidate

$$V_k(t) = \bar{\zeta}_k^H(t)P\bar{\zeta}_k(t) \quad (k \in \{g+1, g+2, \dots, N\}). \quad (27)$$

Taking the derivative of $V_k(t)$ along the trajectory of the subsystem (20) leads to

$$\dot{V}_k(t) = \bar{\zeta}_k^H(t) \left(P(B_1B_2^T + B_2\alpha) + (B_1B_2^T + B_2\alpha)^T P + \lambda_k^H(B_2K)^T P + \lambda_k PB_2K \right) \bar{\zeta}_k(t). \quad (28)$$

Substituting $K = -\delta[\text{Re}(\lambda_{g+1})]^{-1}R^{-1}B^T P$ and $P(B_1B_2^T + B_2\alpha) + (B_1B_2^T + B_2\alpha)^T P = PB_2R^{-1}B_2^T P - I$ into (28) one has

$$\dot{V}_k(t) = -\bar{\zeta}_k^H(t)\bar{\zeta}_k(t) + (1 - 2\delta[\text{Re}(\lambda_{g+1})]^{-1}\text{Re}(\lambda_k))\bar{\zeta}_k^H(t)(PB_2R^{-1}B_2^T P)\bar{\zeta}_k(t). \quad (29)$$

Due to that $1 - 2\delta[\text{Re}(\lambda_{g+1})]^{-1}\text{Re}(\lambda_k) < 0$, one gets

$$\dot{V}_k(t) < 0. \quad (30)$$

It follows from (30) that $\lim_{t \rightarrow \infty} \bar{\zeta}_k(t) = 0$; which means that the system described by (18) is asymptotically stable. Since condition (9) is satisfied, it holds from the proof of Theorem 1 that $\lim_{t \rightarrow \infty} \zeta(t) = 0$, which means that multi-agent system (1) achieves the time-varying group formation specified by $h(t)$ under the designed protocol (3). The proof for Theorem 3 is completed.

Remark 5. Theorem 3 presents an approach to design the time-varying group formation control protocol (3) by solving an algebraic Riccati equation described by (26). Since $(B_1B_2^T + B_2\alpha, B_2)$ is stabilizable, the existence of the solution to (26) can be guaranteed, which means that one can always obtain the desired gain matrix K . It should be pointed that the design approach in Theorem 3 requires the global information of the interaction topology to calculate the minimum nonzero eigenvalue of the Laplacian matrix. To reduce this constraint, one can consider introducing some adaptive approaches (see, e.g., [25]-[28]) into the protocol (3) to estimate the required global information on the interaction topology (see, e.g., [29] and [30]).

4. Numerical simulations

In this section, a numerical example is given to illustrate the effectiveness of the theoretical results obtained in the previous sections.

Consider a second-order multi-agent system with ten agents moving in the horizontal XY plane ($n = 2$). The dynamics of each agent is described by (1) with $\alpha_x = 0.04$, $\alpha_v = -1.98$, $\xi_i(t) = [x_{iX}(t), v_{iX}(t), x_{iY}(t), v_{iY}(t)]^T$ and $u_i(t) = [u_{iX}(t), u_{iY}(t)]^T$ ($i = 1, 2, \dots, 10$), where $x_{iX}(t)$, $v_{iX}(t)$, $u_{iX}(t)$ and $x_{iY}(t)$, $v_{iY}(t)$, $u_{iY}(t)$ are the positions, velocities and control inputs along the X axis and Y axis, respectively. These ten agents are partitioned into three subgroups, namely, $V_1 = \{1, 2, 3\}$, $V_2 = \{4, 5, 6, 7\}$ and $V_3 = \{8, 9, 10\}$. The interactions among the three subgroups and within each subgroup are shown in Fig. 2. The desired time-varying sub-formations are specified by

Fig. 2: Directed interaction topology G .

$$h_i(t) = \begin{bmatrix} 6 \sin(t + 2(i-1)\pi/3) \\ 6 \cos(t + 2(i-1)\pi/3) \\ 6 \cos(t + 2(i-1)\pi/3) \\ -6 \sin(t + 2(i-1)\pi/3) \end{bmatrix} \quad (i = 1, 2, 3),$$

$$h_i(t) = \begin{bmatrix} -8 \sin(2t + (i-4)\pi/2) \\ -16 \cos(2t + (i-4)\pi/2) \\ 8 \cos(2t + (i-4)\pi/2) \\ -16 \sin(2t + (i-4)\pi/2) \end{bmatrix} \quad (i = 4, 5, 6, 7),$$

$$h_i(t) = \begin{bmatrix} 10 \sin(3t + 2(i-8)\pi/3) \\ 30 \cos(3t + 2(i-8)\pi/3) \\ 10 \cos(3t + 2(i-8)\pi/3) \\ -30 \sin(3t + 2(i-8)\pi/3) \end{bmatrix} \quad (i = 8, 9, 10).$$

From $h_i(t)$ ($i = 1, 2, \dots, 10$), one gets that when the desired group formation is achieved, both the positions and velocities of the ten agents will form three different circular sub-formations.

It can be obtained that the time-varying group formation feasibility constraint (9) is satisfied. Choose $\delta = 0.6$. From Theorem 3, the gain matrix K can be derived as $K = I_2 \otimes [-0.5085, -0.3254]$. For simplicity, let the initial states of the ten agents be generated by $\xi_i(t) = 5\Theta$ ($i = 1, 2, \dots, 10$), where $\Theta \in \mathbb{R}^4$ is a random vector with each entry

Fig. 3: Position snapshots of the ten agents and $r_i(t)$ ($i = 1, 2, 3$).

Fig. 4: Velocity snapshots of the ten agents and $r_i(t)$ ($i = 1, 2, 3$).

Fig. 5: Curve of the time-varying group formation error $\varsigma(t)$.

between -0.5 and 0.5 . Figs. 3 and 4 show the position and velocity snapshots of the ten agents and the formation references at different time, respectively, where the states of the agents are marked by squares, diamonds, triangles, points, and the states of the formation references are denoted by pentagrams. Fig. 5 shows the curve of the time-varying group formation error $\varsigma(t)$. From Figs. 3-5, one sees that both the positions and velocities of

the three subgroups form the predefined time-varying circular formation, respectively, and the time-varying states of the formation references lie in the center of the sub-formations. Therefore, the desired time-varying group formation is realized by the multi-agent system.

5. Conclusions

Time-varying group formation control problems for second-order multi-agent systems with directed topologies were investigated. A time-varying group formation protocol was constructed by local relative information of each agent and its neighbors. Sufficient conditions for second-order multi-agent systems to achieve time-varying group formation were presented together with the time-varying group formation feasibility constraints. Explicit expressions of the formation reference functions for all the subgroups were derived to describe the macroscopic movement of the time-varying group formation. An approach to design the time-varying group formation protocol was proposed by solving an algebraic Riccati equation.

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Fig. 6: Photo of Xiwang Dong.

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Fig. 7: Photo of Qingdong Li.

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Fig. 8: Photo of Qilun Zhao.

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