Asynchronous H_∞ Fuzzy Control for A Class of Switched Nonlinear Systems via Switching Fuzzy Lyapunov Function Approach

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Abstract

This paper addresses the problem of H_∞ control for a class of continuous-time switched nonlinear systems via switching fuzzy Lyapunov functions (FLFs). The asynchronous switching is also considered in this paper, where “asynchronous” means that the switching of the controllers to be designed has a lag to the switching of the system models. The Takagi-Sugeno (T-S) fuzzy model is used here to approximate each nonlinear subsystem of the switched nonlinear systems. By using mode-dependent average dwell time (MDADT) techniques, we obtain stability conditions for the open-loop switched nonlinear systems and stabilization conditions with H_∞ performance for the closed-loop switched nonlinear systems. The desired H_∞ controller can be constructed by solving a set of linear matrix inequalities (LMIs). Finally, two numerical examples illustrate the utility and advantage of the proposed method.

Keyword: H_∞ control, switched nonlinear systems, Takagi-Sugeno (T-S) fuzzy model, mode-dependent average dwell time (MDADT), switching fuzzy Lyapunov functions (FLFs).

1 Introduction

Switched systems are an important class of hybrid systems. The last two decades have witnessed increasing research attention on switched systems, and many valuable stability and stabilization results have been obtained [1–5]. Up to now, two stability issues have been addressed in the literature, i.e., the stability under arbitrary switching and the stability under constrained switching. As one typical example of the constrained switching, the average dwell time (ADT) logic has been proposed in [6]. Stability problem and switching law design are addressed for switched systems with average dwell time [7–10]. However, most work assumed that the ADT is independent of the system models. By fully taking consideration of the characteristics of each subsystem, the mode-dependent average dwell time (MDADT) method is proposed in [11], which can further reduce the conservativeness of the results obtained by the ADT approach.

The considerations of the above work are focused on the switched linear systems. Recently, the control issue for the switched nonlinear systems has also been widely studied. Due to the nonlinearity, it is difficult to analyze the switched nonlinear systems directly. We used the well-known Takagi-Sugeno (T-S) fuzzy model to overcome such difficulties. The T-S fuzzy model is an effective tool in approximating most complex nonlinear systems [12], which utilizes local linear system description for each rule. The issue of stability and controller synthesis of T-S fuzzy systems has been studied extensively [13–20].

As we know that the results based on a common quadratic Lyapunov function (QLF) might be conservative. By taking into consideration the information of membership functions, the authors in [21,22] presented a stability result for the continuous-time fuzzy system using fuzzy Lyapunov functions. The FLFs approach only needs to search for a local common positive matrix in fuzzy model and also reduces the difficulty of solving LMIs. In this paper, we investigate switched nonlinear systems by employing T-S fuzzy models. The switching FLFs are proposed by using the switching information and structural information of membership function in the rule base.

Recently, the H_∞ control theory keeps attracting more and more attention because of its significant advances [23], and based on H_∞ performance, a number of methodologies have been developed for the H_∞ controller of filter design [24–30]. However, most of these

results focused on the synchronous switching between the system model and the matched controller, which is a quite ideal case. In practice, due to system identification and other reasons, the matched controller of each subsystem would not be operated immediately at each switching instant. To design more practical controller, the work [7] and [10] have studied the asynchronous control problem of discrete-time and continuous-time switched linear systems, respectively. As far as we know, the problem of H_∞ asynchronous control for continuous-time switched nonlinear systems remains to be unsolved, which motivates our research in this paper.

The main contributions of our work are as follows: (I) The previous work studied the H_∞ asynchronous control problem of switched systems mainly focused on the switched linear systems. We use the T-S fuzzy model to study the H_∞ asynchronous control problem of continuous-time switched nonlinear systems. Furthermore, our work can also reduce to study the H_∞ asynchronous control problem of continuous-time switched linear systems. (II) In order to further relax the conservativeness resulted from the nonlinearity and the common Lyapunov functions approach, we propose the switching fuzzy Lyapunov functions approach, and this approach can also apply to study other switched nonlinear systems.

The rest of this paper is organized as follows. Section II gives preliminaries and problem formulation. The main results are given in Section III. In Section IV, a simulation example is presented. Finally, some conclusions are included in Section V.

Notations: The notation used in this paper is fairly standard. The symbol “*” in a matrix stands for the transposed elements in the symmetric positions. The superscript “ T ” is the matrix transposition. N and N^+ denote the set of the natural numbers and the set of positive integers respectively. R^n denotes the n -dimensional Euclidean space. I and 0 represent the identity matrix and zero matrix respectively in the block matrix with appropriate dimensions. The notation $\| \cdot \|$ refers to the Euclidean vector norm. $L_2[0, \infty)$ is the space of square-integrable and for $v(t) \in L_2[0, \infty)$, and its norm is given by $\|v(t)\|_2 = \sqrt{\int_0^\infty v(t)^T v(t) dt}$. C^1 denotes the space of continuously differentiable function. A continuous function $\alpha : [0, \infty) \rightarrow [0, \infty)$ is said to be of class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. If α is also unbounded, then it is said to be of class $\mathcal{K}_\infty$. A function $\beta : [0, \infty) \times [0, \infty) \rightarrow [0, \infty)$ is said to be of class $\mathcal{KL}$ if $\beta(\cdot, t)$ is of class $\mathcal{K}$ for each fixed $t \geq 0$ and $\beta(s, t)$ decreases to 0

as $t \rightarrow \infty$ for each fixed $s \geq 0$. We use $P > 0 (\geq 0, < 0, \leq 0)$ to denote a positive definite (semi-positive definite, negative definite, semi-negative definite) matrix P . If not explicitly stated, matrices are assumed to have compatible dimensions.

2 Preliminaries and Problem Formulation

Let us consider the following switched nonlinear systems described by

$$\begin{cases} \dot{x}(t) = f_{\sigma(t)}(x(t), u(t), w(t)) \\ y(t) = g_{\sigma(t)}(x(t), u(t), w(t)) \end{cases} \quad (1)$$

where $x(t) \in R^{n_x}$, $u(t) \in R^{n_u}$, and $y(t) \in R^{n_y}$ denote the state vector, input vector and output vector respectively; $w(t) \in R^{n_w}$ is the disturbance that belongs to $L_2[0, \infty)$. $f_{\sigma(t)}$ and $g_{\sigma(t)}$ are nonlinear functions. $\sigma(t)$ is defined as a switching signal, which is a piecewise constant function of time and takes its values in the finite set $S = \{1, 2, \dots, N\}$, where N is the number of subsystems. For a switching sequence $0 < t_0 < t_1 < \dots < t_k < t_{k+1} < \dots$, $\sigma(t)$ is continuous from right everywhere. When $t \in [t_k, t_{k+1})$, we say that the $\sigma(t_k)$ subsystem is activated, and for $\sigma(t_k) = i$, $i \in S$.

The T-S fuzzy model which is described by fuzzy IF-THEN rules [12] is employed here to represent each subsystem of the switched nonlinear systems.

Rule n for subsystem i : IF $\nu_{i1}(t)$ is N_{i1n} and $\dots$ and $\nu_{ig}(t)$ is N_{ign} , THEN

$$\begin{cases} \dot{x}(t) = A_{in}x(t) + B_{in}u(t) + E_{in}w(t) \\ y(t) = C_{in}x(t) + D_{in}u(t) + F_{in}w(t) \end{cases} \quad (2)$$

where $\nu_i(t) = (\nu_{i1}(t), \nu_{i2}(t), \dots, \nu_{ig}(t))$ are some measurable premise variables and N_{ipn} ($p = 1, 2, \dots, g$) are fuzzy sets. $A_{in}, B_{in}, C_{in}, D_{in}, E_{in}$ and F_{in} are constant real matrices of the n th local model of the i th subsystem.

By using “fuzzy blending”, the final output of the i th subsystem is inferred as follows:

$$\begin{cases} \dot{x}(t) = \sum_{n=1}^{r_i} h_{in}(t) [A_{in}x(t) + B_{in}u(t) + E_{in}w(t)] \\ y(t) = \sum_{n=1}^{r_i} h_{in}(t) [C_{in}x(t) + D_{in}u(t) + F_{in}w(t)] \end{cases} \quad (3)$$

where $h_{in}(t) = l_{in}(t) / \sum_{n=1}^{r_i} l_{in}(t)$, $l_{in}(t) = \prod_{p=1}^g N_{ipn}(\nu_{ip}(t))$, r_i is the number of IF-THEN rules, and $N_{ipn}(\nu_{ip}(t))$ is the grade of the membership function of ν_{ip} in N_{ipn} . It is assumed

that $l_{in}(t) \geq 0$ for all t . Therefore, the normalized membership function $h_{in}(t)$ satisfies

$$h_{in}(t) \geq 0, \quad \sum_{n=1}^{r_i} h_{in}(t) = 1. \quad (4)$$

In view of the asynchronous behaviors, the controller $u(t)$ is divided into two parts $\bar{u}(t)$ and $\hat{u}(t)$, where $\bar{u}(t)$ denotes the unmatched controller, and $\hat{u}(t)$ represents the matched controller. The terms $\bar{u}(t)$ and $\hat{u}(t)$ have the following form:

$$\begin{cases} \bar{u}(t) = \sum_{m=1}^{r_i} h_{jm}(t) K_{jm} x(t), & t \in [t_k, \bar{t}_k) \\ \hat{u}(t) = \sum_{m=1}^{r_i} h_{im}(t) K_{im} x(t), & t \in [\bar{t}_k, t_{k+1}) \end{cases} \quad (5)$$

where notation $\bar{t}_k$ ($t_k \leq \bar{t}_k < t_{k+1}$) denotes the starting-operating instant of the matched controller, and K_{jm} and K_{im} are constant matrices. Substituting (5) into (3), we can obtain the following closed-loop system:

$$\begin{cases} \begin{cases} \dot{x}(t) = \bar{A}_i(t)x(t) + E_i(t)w(t) \\ y(t) = \bar{C}_i(t)x(t) + F_i(t)w(t) \end{cases}, & \forall t \in [t_k, \bar{t}_k), \\ \begin{cases} \dot{x}(t) = \hat{A}_i(t)x(t) + E_i(t)w(t) \\ y(t) = \hat{C}_i(t)x(t) + F_i(t)w(t) \end{cases}, & \forall t \in [\bar{t}_k, t_{k+1}) \end{cases} \quad (6)$$

where the $\bar{A}_i(t)$, $\bar{C}_i(t)$, $\hat{A}_i(t)$, $\hat{C}_i(t)$, $E_i(t)$ and $F_i(t)$ are defined as follows:

$$\begin{aligned} \bar{A}_i(t) &= \sum_{n=1}^{r_i} \sum_{m=1}^{r_i} h_{in}(t) h_{jm}(t) (A_{in} + B_{in} K_{jm}), & \bar{C}_i(t) &= \sum_{n=1}^{r_i} \sum_{m=1}^{r_i} h_{in}(t) h_{jm}(t) (C_{in} + D_{in} K_{jm}), \\ \hat{A}_i(t) &= \sum_{n=1}^{r_i} \sum_{m=1}^{r_i} h_{in}(t) h_{im}(t) (A_{in} + B_{in} K_{im}), & \hat{C}_i(t) &= \sum_{n=1}^{r_i} \sum_{m=1}^{r_i} h_{in}(t) h_{im}(t) (C_{in} + D_{in} K_{im}), \\ E_i(t) &= \sum_{n=1}^{r_i} h_{in}(t) E_{in}, & F_i(t) &= \sum_{n=1}^{r_i} h_{in}(t) F_{in}. \end{aligned}$$

To obtain the desired results, we introduce the following definitions.

Definition 1: [1] The switched system (1) with $w(t) \equiv 0$ is globally uniformly asymptotically stable (GUAS) if there exists a class $\mathcal{KL}$ function β such that for all switching signals $\sigma(t)$ and all initial condition $x(t_0)$, the solutions of (1) satisfy the following inequality

$$\|x(t)\| \leq \beta(\|x(t_0)\|, t), \quad \forall t \geq t_0. \quad (7)$$

Definition 2: [11] For switching signal $\sigma(t)$ and each $T \geq t \geq 0$. Let $N_{\sigma_i}(T, t)$ be the switching numbers such that the i th subsystem is activated over the interval $[t, T]$, and $T_i(T, t)$ denotes the total running time of the i th subsystem over the interval $[t, T]$, $\forall i \in S$.

We say that $\sigma(t)$ has a mode-dependent average dwell time (MDADT) T_{ai} if there exist positive numbers N_{0i} (N_{0i} denotes mode-dependent chatter bounds here) and T_{ai} such that

$$N_{\sigma_i}(T, t) \leq N_{0i} + T_i(T, t)/T_{ai}, \quad \forall T \geq t \geq 0. \quad (8)$$

Definition 3: [10] For $\alpha > 0$ and $\gamma > 0$, system (1) is said to have a weighted L_2 -gain, if under zero initial condition (i.e., $x(t_0) = 0$), it holds that

$$\int_0^\infty e^{-\alpha s} y^T(s) y(s) ds \leq \gamma^2 \int_0^\infty w^T(s) w(s) ds. \quad (9)$$

Remark 1: We can see from the above definition that if the system (1) has a L_2 -gain, then it must have a weighted L_2 -gain. However, the opposite is not true.

Definition 4: [21] Equation (10) is said to be a fuzzy Lyapunov function for the i th subsystem of T-S fuzzy system (3) if there exists a positive definite matrix P_{im} and the time derivative of $V_i(x(t))$ is always negative at $x(t) \neq 0$.

$$V_i(x(t)) = x^T(t) P_i(t) x(t) \quad (10)$$

where $P_i(t) = \sum_{m=1}^{r_i} h_{im}(t) P_{im}$ and $\dot{P}_i(t) = \sum_{m=1}^{r_i} \dot{h}_{im}(t) P_{im}$.

3 MAIN RESULTS

In this section, for simplicity, let $T(t_k, t_{k+1}) = t_{k+1} - t_k$ denote the total running time of each subsystem in each interval. $T_\uparrow(t_k, t_{k+1})$ denotes the running time of the unmatched controller in $[t_k, t_{k+1})$, and the Lyapunov function may increase or decrease during $T_\uparrow(t_k, t_{k+1})$. $T_\downarrow(t_k, t_{k+1})$ represents the running time of the matched controller, and the Lyapunov function is strictly decreasing during $T_\downarrow(t_k, t_{k+1})$.

3.1 Stability analysis

The open-loop system for (6) with $w(t) \equiv 0$ and $u(t) \equiv 0$ is listed as follow:

$$\begin{cases} \dot{x}(t) = \sum_{n=1}^{r_i} h_{in}(t) A_{in} x(t) \\ y(t) = \sum_{n=1}^{r_i} h_{in}(t) C_{in} x(t) \end{cases}, \quad \forall t \in [t_k, t_{k+1}). \quad (11)$$

We present Lemma 1 which is useful for the stability analysis of the open-loop system (11).

Lemma 1: Consider the system (11) and let $\alpha_i > 0$, $\mu_i \geq 1$ be given constants. If there exist positive definite C^1 function $V_{\sigma(t_k)} : R^n \rightarrow R$, with $V_{\sigma(t_0)}(x_{t_0}) \equiv 0$ satisfying

$$V_i(t_k) \leq \mu_i V_j(t_k^-) \quad (12)$$

and

$$\dot{V}_i(t) \leq -\alpha_i V_i(t), \quad t \in [t_k, t_{k+1}). \quad (13)$$

Then the system (11) is GUAS for any switching signal satisfying

$$T_{oi} \geq T_{oi}^* = \frac{\ln \mu_i}{\alpha_i}. \quad (14)$$

Proof: By (13), we have

$$V(t) \leq e^{-\alpha_i(t-t_k)} V(t_k). \quad (15)$$

By (12) and (15), we have

$$\begin{aligned} V(t) &\leq \mu_i e^{-\alpha_i(t-t_k)} V(t_k^-) \\ &\leq \mu_i e^{-\alpha_i(t-t_k) - \alpha_{i-1}(t_k - t_{k-1})} V(t_{k-1}) \\ &\leq \dots \\ &\leq \mu_i \mu_{i-1} \dots \mu_1 e^{-\alpha_i(t-t_k) - \dots - \alpha_0(t_1 - t_0)} V(t_0). \end{aligned} \quad (16)$$

Let $N_{\sigma i}$ denote $N_{\sigma i}(t, t_0)$ for simplicity. We get the following inequality

$$V(t) \leq \exp \left\{ \sum_{i=1}^N N_{\sigma i} (-\alpha_i T_{oi} + \ln \mu_i) \right\} V(t_0). \quad (17)$$

If supposing

$$-\alpha_i T_{oi} + \ln \mu_i \leq 0, \quad (18)$$

we obtain a sufficient condition that guarantees the GUAS of the system (11). The inequality (18) is equivalent to

$$T_{oi} \geq T_{oi}^* = \frac{\ln \mu_i}{\alpha_i}. \quad (19)$$

Therefore we can conclude that $V_i(t)$ converges to zero as $t \rightarrow \infty$ if the above condition is satisfied. This ends the proof.

The stability conditions for the system (11) can be summarized in the following theorem.

Theorem 1: Assume that

$$\left| \dot{h}_{is}(t) \right| \leq \varepsilon_{is}, \quad s = 1, 2, \dots, r_i \quad (20)$$

where $\varepsilon_{is} \geq 0$. Let $\alpha_i > 0$ and $\mu_i \geq 1$ be given constants. The system (11) is GUAS for any switching signal satisfying (14), if there exist matrices $P_{iu} > 0$ satisfying

$$P_{iu} \leq \mu_i P_{jv}, \quad P_{ik} \geq P_{ir_i}, \quad k = 1, 2, \dots, r_i - 1, \quad (21)$$

$$\Theta_{inu} + \Theta_{iun} < 0, \quad n \leq u \quad (22)$$

where

$$\Theta_{inu} = A_{in}^T P_{iu} + P_{iu} A_{in} + \sum_{s=1}^{r_i-1} \varepsilon_{is} (P_{is} - P_{ir_i}) + \alpha_i P_{iu}.$$

Proof: Differentiating (4) implies

$$\dot{h}_{ir_i}(t) = - \sum_{n=1}^{r_i-1} \dot{h}_{in}(t), \quad (23)$$

so we have

$$\dot{P}_i(t) = \sum_{n=1}^{r_i-1} \dot{h}_{in}(t) (P_{in} - P_{ir_i}). \quad (24)$$

Combining (20), (21) with (24) implies

$$\dot{P}_i(t) \leq \sum_{n=1}^{r_i-1} \varepsilon_{in} (P_{in} - P_{ir_i}). \quad (25)$$

Along the solution of the system (11), we have

$$\begin{aligned} \dot{V}_i(t) + \alpha_i V_i(t) &= x^T(t) \left[A_i^T(t) P_i(t) + P_i(t) A_i(t) + \dot{P}_i(t) + \alpha_i P_i(t) \right] x(t) \\ &\leq \sum_{n=1}^{r_i} h_{in}^2(t) x^T(t) \Theta_{inn} x(t) + \sum_{n < u}^{r_i} h_{in}(t) h_{iu}(t) x^T(t) [\Theta_{inu} + \Theta_{iun}] x(t). \end{aligned} \quad (26)$$

From (22) we can conclude that

$$\dot{V}_i(t) + \alpha_i V_i(t) \leq 0. \quad (27)$$

For $P_{iu} \leq \mu_i P_{jv}$, we can get

$$V_i(t_k) \leq \mu_i V_j(t_k^-). \quad (28)$$

Combining (27) with (28) we conclude that the system (11) is GUAS for any switching signal satisfying (14). This completes the proof.

Remark 2: By setting $P_{iu} = P_i$, we can get the corresponding stability conditions based on the QLFs method. So the results in Theorem 1 might have less conservativeness than those results based on the QLFs method.

3.2 H_∞ performance analysis and controller design

Lemma 2: Consider the closed-loop switched nonlinear system (6), and let $\alpha_i > 0$, $\beta_i > 0$, $\gamma > 0$ and $\mu_i \geq 1$ be given constants. If there exist positive definite C^1 function $V_{\sigma(t_k)} : R^n \rightarrow R$, with $V_{\sigma(t_0)}(x_{t_0}) \equiv 0$ satisfying

$$V_i(t_k) \leq \mu_i V_j(t_k^-) \quad (29)$$

and

$$\dot{V}_i(t) \leq \begin{cases} \beta_i V_i(t) - J(t), & t \in T_\uparrow(t_k, t_{k+1}) \\ -\alpha_i V_i(t) - J(t), & t \in T_\downarrow(t_k, t_{k+1}) \end{cases} \quad (30)$$

where $J(t) = y^T(t)y(t) - \gamma^2 w^T(t)w(t)$, then the system (6) is GUAS for any switching signal satisfying

$$T_{cai} \geq T_{cai}^* = \frac{T_M(\beta_i + \alpha_i) + \ln \mu_i}{\alpha_i} \quad (31)$$

and has a weighted H_∞ performance no greater than $\hat{\gamma} = \sqrt{\exp \left\{ \sum_{i=1}^N (T_{cai}^* \alpha_i) + \theta_{\max} T_M \right\}} \gamma$, where $T_M \triangleq \max \{T_\uparrow(t_k, t_{k+1}), \forall k = 0, 1, 2, \dots\}$.

Proof: Denote $\theta_i = \alpha_i + \beta_i$, $\theta_{\max} = \max\{\theta_i\}$ and $J(s) = y^T(s)y(s) - \gamma^2 w^T(s)w(s)$. Then by (30), we have

$$\begin{aligned} V(t) &\leq e^{-\alpha_i(t-t_k-T_M)+\beta_i T_M} V(t_k) - \int_{t_k+T_M}^t e^{-\alpha_i(t-s)} J(s) ds \\ &\quad - e^{-\alpha_i(t-t_k-T_M)} \int_{t_k}^{t_k+T_M} e^{\beta_i(t_k+T_M-s)} J(s) ds \\ &= e^{-\alpha_i(t-t_k)+\theta_i T_M} V(t_k) - \int_{t_k+T_M}^t e^{-\alpha_i(t-s)} J(s) ds \\ &\quad - \int_{t_k}^{t_k+T_M} e^{-\alpha_i(t-s)+\theta_i(t_k+T_M-s)} J(s) ds. \end{aligned} \quad (32)$$

By (29) and (32), we have

$$\begin{aligned}
V(t) &\leq \mu_i e^{-\alpha_i(t-t_k)+\theta_i T_M} V(t_k^-) - \int_{t_k+T_M}^t e^{-\alpha_i(t-s)} J(s) ds \\
&\quad - \int_{t_k}^{t_k+T_M} e^{-\alpha_i(t-s)+\theta_i(t_k+T_M-s)} J(s) ds \\
&\leq \mu_i e^{-\alpha_i(t-t_k)-\alpha_{i-1}(t_k-t_{k-1})+(\theta_i+\theta_{i-1})T_M} V(t_{k-1}) \\
&\quad - \mu_i e^{-\alpha_i(t-t_k)+\theta_i T_M} \int_{t_{k-1}+T_M}^{t_k} e^{-\alpha_{i-1}(t_k-s)} J(s) ds \\
&\quad - \mu_i e^{-\alpha_i(t-t_k)+\theta_i T_M} \int_{t_{k-1}}^{t_{k-1}+T_M} e^{-\alpha_{i-1}(t_k-s)+\theta_{i-1}(t_{k-1}+T_M-s)} J(s) ds \\
&\quad - \int_{t_k}^{t_k+T_M} e^{-\alpha_i(t-s)+\theta_i(t_k+T_M-s)} J(s) ds - \int_{t_k+T_M}^t e^{-\alpha_i(t-s)} J(s) ds \\
&\leq \dots \\
&\leq \mu_i \dots \mu_1 e^{-\alpha_i(t-t_k)-\dots-\alpha_0(t_1-t_0)+T_M(\theta_i+\dots+\theta_0)} V(t_0) \\
&\quad - \mu_i \dots \mu_1 e^{-\alpha_i(t-t_k)-\dots-\alpha_1(t_2-t_1)+T_M(\theta_i+\dots+\theta_1)} \times \left[\int_{t_0+T_M}^{t_1} e^{-\alpha_0(t_1-s)} J(s) ds \right. \\
&\quad \left. + \int_{t_0}^{t_0+T_M} e^{-\alpha_0(t_1-s)+\theta_0(t_0+T_M-s)} J(s) ds \right] \\
&\quad - \dots - \int_{t_k}^{t_k+T_M} e^{-\alpha_i(t-s)+\theta_i(t_k+T_M-s)} J(s) ds - \int_{t_k+T_M}^t e^{-\alpha_i(t-s)} J(s) ds \\
&= \Phi(t) - \Omega(s)
\end{aligned} \tag{33}$$

where

$$\begin{aligned}
\Phi(t) &= e^{\ln \mu_i + \dots + \ln \mu_1 - \alpha_i(t-t_k) - \dots - \alpha_0(t_1-t_0) + T_M(\theta_i + \dots + \theta_0)} V(t_0) \\
&= \exp \left\{ \sum_{i=1}^N N_{\sigma i} (-\alpha_i T_{cai} + \ln \mu_i + \theta_i T_M) \right\} V(t_0). \\
\Omega(s) &= \exp \left\{ \sum_{i=1}^N (\ln \mu_i + \theta_i T_M) - \alpha_i(t-t_k) - \dots - \alpha_1(t_2-t_1) \right\} \\
&\quad \times \left[\int_{t_0+T_M}^{t_1} e^{-\alpha_0(t_1-s)} J(s) ds + \int_{t_0}^{t_0+T_M} e^{-\alpha_0(t_1-s)+\theta_0(t_0+T_M-s)} J(s) ds \right] \\
&\quad + \dots + \int_{t_k}^{t_k+T_M} e^{-\alpha_i(t-s)+\theta_i(t_k+T_M-s)} J(s) ds + \int_{t_k+T_M}^t e^{-\alpha_i(t-s)} J(s) ds.
\end{aligned}$$

If supposing

$$-\alpha_i T_{cai} + \ln \mu_i + \theta_i T_M \leq 0, \tag{34}$$

we obtain a sufficient condition that guarantees the GUAS of the switched nonlinear system

(6). The inequality (34) can be rewritten as follows

$$T_{cai} \geq T_{cai}^* = \frac{T_M(\beta_i + \alpha_i) + \ln \mu_i}{\alpha_i}. \quad (35)$$

Therefore we can conclude that $V_i(t) \rightarrow 0$ as $t \rightarrow \infty$ if (35) is satisfied.

We define $\bar{\Omega}(s)$ and $\hat{\Omega}(s)$ with the $y^T(s)y(s)$ and $\gamma^2 w^T(s)w(s)$ terms, respectively. By defining $\alpha_{\max} \triangleq \max\{\alpha_i\}$ and letting all $\alpha_i = \alpha_{\max}$, we can obtain the following inequalities

$$\Omega(s) \geq \bar{\Omega}(s) - \hat{\Omega}(s), \quad (36)$$

where

$$\begin{aligned} \bar{\Omega}(s) = & \exp \left\{ \sum_{i=1}^N (\ln \mu_i + \theta_i T_M) \right\} \times \left[\int_{t_0+T_M}^{t_1} e^{-\alpha_{\max}(t-s)} y^T(s)y(s) ds \right. \\ & \left. + \int_{t_0}^{t_0+T_M} e^{-\alpha_{\max}(t-s)+\theta_0(t_0+T_M-s)} y^T(s)y(s) ds \right] \end{aligned} \quad (37)$$

+ ...

$$+ \int_{t_k}^{t_k+T_M} e^{-\alpha_{\max}(t-s)+\theta_i(t_k+T_M-s)} y^T(s)y(s) ds + \int_{t_k+T_M}^t e^{-\alpha_{\max}(t-s)} y^T(s)y(s) ds,$$

$$\begin{aligned} \hat{\Omega}(s) = & \exp \left\{ \sum_{i=1}^N (\ln \mu_i + \theta_i T_M) \right\} \times \left[\int_{t_0+T_M}^{t_1} e^{-\alpha_{\max}(t-s)} \gamma^2 w^T(s)w(s) ds \right. \\ & \left. + \int_{t_0}^{t_0+T_M} e^{-\alpha_{\max}(t-s)+\theta_0(t_0+T_M-s)} \gamma^2 w^T(s)w(s) ds \right] \end{aligned} \quad (38)$$

+ ...

$$+ \int_{t_k}^{t_k+T_M} e^{-\alpha_{\max}(t-s)+\theta_i(t_k+T_M-s)} \gamma^2 w^T(s)w(s) ds + \int_{t_k+T_M}^t e^{-\alpha_{\max}(t-s)} \gamma^2 w^T(s)w(s) ds.$$

Owing to $\exp \left\{ \sum_{i=1}^N (\ln \mu_i + \theta_i T_M) \right\} \geq 1$ and $\theta_i(t_k + T_M - s) \geq 0$, we can have

$$\begin{aligned} \bar{\Omega}(s) \geq & \int_{t_0+T_M}^{t_1} e^{-\alpha_{\max}(t-s)} y^T(s)y(s) ds + \int_{t_0}^{t_0+T_M} e^{-\alpha_{\max}(t-s)} y^T(s)y(s) ds \\ & + \dots + \int_{t_k}^{t_k+T_M} e^{-\alpha_{\max}(t-s)} y^T(s)y(s) ds + \int_{t_k+T_M}^t e^{-\alpha_{\max}(t-s)} y^T(s)y(s) ds \\ = & \int_{t_0}^t e^{-\alpha_{\max}t} y^T(s)y(s) ds \end{aligned} \quad (39)$$

Since $\theta_{\max}T_M \geq 0$ and $\theta_i(t_k + T_M - s) \leq \theta_{\max}T_M$, we can obtain the following inequality

$$\begin{aligned}
\hat{\Omega}(s) &\leq \exp \left\{ \sum_{i=1}^N (\ln \mu_i + \theta_i T_M) \right\} \times \left\{ \int_{t_0+T_M}^{t_1} e^{-\alpha_{\max}(t-s)+\theta_{\max}T_M} \gamma^2 w^T(s) w(s) ds \right. \\
&\quad + \int_{t_0}^{t_0+T_M} e^{-\alpha_{\max}(t-s)+\theta_{\max}T_M} \gamma^2 w^T(s) w(s) ds \\
&\quad + \cdots + \int_{t_k}^{t_k+T_M} e^{-\alpha_{\max}(t-s)+\theta_{\max}T_M} \gamma^2 w^T(s) w(s) ds \\
&\quad \left. + \int_{t_k+T_M}^t e^{-\alpha_{\max}(t-s)+\theta_{\max}T_M} \gamma^2 w^T(s) w(s) ds \right\} \\
&= \exp \left\{ \sum_{i=1}^N (T_{cai}^* \alpha_i) \right\} \times \int_{t_0}^t e^{-\alpha_{\max}(t-s)+\theta_{\max}T_M} \gamma^2 w^T(s) w(s) ds.
\end{aligned} \tag{40}$$

As for $\Omega(s)$, under zero initial condition, (33) gives

$$\Omega(s) \leq 0. \tag{41}$$

Then, combining (36), (39), (40) with (41), the following inequality is obtained

$$\int_{t_0}^t e^{-\alpha_{\max}t} y^T(s) y(s) ds \leq \hat{\gamma}^2 \int_{t_0}^t e^{-\alpha_{\max}(t-s)} w^T(s) w(s) ds \tag{42}$$

where $\hat{\gamma} = \sqrt{\exp \left\{ \sum_{i=1}^N (T_{cai}^* \alpha_i) + \theta_{\max}T_M \right\} \gamma}$. Let $t_0 = 0$, and integrating both sides of inequality (42) from $t = 0$ to ∞ leads to

$$\int_0^\infty e^{-\alpha_{\max}s} y^T(s) y(s) ds \leq \hat{\gamma}^2 \int_0^\infty w^T(s) w(s) ds. \tag{43}$$

This completes the proof.

Remark 3: In this proof, we consider the worst situation, that is the asynchronous time $T_{\uparrow}(t_k, t_{k+1})$ ($k = 0, 1, 2, \dots$) takes its maximal value T_M after every switching. Therefore, the results obtained in our work have more or less conservatism compared with the case that not all the asynchronous time $T_{\uparrow}(t_k, t_{k+1})$ takes its maximal value T_M . In practice, if not all the asynchronous time $T_{\uparrow}(t_k, t_{k+1})$ takes its maximal value T_M , the system can get GUAS with a relatively small T_{cai}^* and have a relatively better H_∞ performance.

Remark 4: For the fuzzy Lyapunov function $V_i(x(t)) = x^T(t) P_i(t) x(t)$, where $P_i(t) = \sum_{m=1}^{r_i} h_{im}(t) P_{im}$ and $\dot{P}_i(t) = \sum_{m=1}^{r_i} \dot{h}_{im}(t) P_{im}$, we need to consider the time derivative of $h_{im}(t)$, i.e., $\dot{h}_{im}(t)$. Similar as in [21, 22], we replace the time derivative $\dot{h}_{im}(t)$ with their upper bounds ε_{is} . Furthermore, the reference [21] gave a method to obtain the time derivative

upper bounds ϕ_{im} in the condition that there is no external disturbance. As we know, compared with the system state $x(t)$, the disturbance $w(t)$ is relatively small and norm bounded, so we can obtain the time derivative upper bounds ε_{is} by adding a relatively small number to the ϕ_{im} which is obtained by the method of [21].

Lemma 3: Let $\alpha_i > 0$, $\beta_i > 0$, $\gamma > 0$ and $\mu_i \geq 1$ be given constants. The closed-loop switched nonlinear system (6) is GUAS for any switching signal satisfying (31) and has a weighted H_∞ performance index $\hat{\gamma}$, if there exist matrices $P_i(t) > 0$ satisfying

$$P_i(t) \leq \mu_i P_j(t), \quad (44)$$

$$\begin{bmatrix} \Xi_{i1} & P_i(t)E_i(t) & \bar{C}_i^T(t) \\ * & -\gamma^2 I & F_i^T(t) \\ * & * & -I \end{bmatrix} < 0, \quad (45)$$

$$\begin{bmatrix} \Xi_{i2} & P_i(t)E_i(t) & \hat{C}_i^T(t) \\ * & -\gamma^2 I & F_i^T(t) \\ * & * & -I \end{bmatrix} < 0 \quad (46)$$

where

$$\Xi_{i1} = P_i(t)\bar{A}_i(t) + \bar{A}_i^T(t)P_i(t) + \dot{P}_i(t) - \beta_i P_i(t),$$

$$\Xi_{i2} = P_i(t)\hat{A}_i(t) + \hat{A}_i^T(t)P_i(t) + \dot{P}_i(t) + \alpha_i P_i(t).$$

Proof: Supposing i th subsystem is activated and the former one is j th subsystem. By the system (6), considering $t \in T_\uparrow(t_k, t_{k+1})$, from the Definition 4, we have

$$\dot{V}_i(t) - \beta_i V_i(t) + y^T(t)y(t) - \gamma^2 w^T(t)w(t) = \eta^T(t)(\zeta_{i1} + \zeta_{i2}^T \zeta_{i2})\eta(t) \quad (47)$$

where

$$\eta^T(t) = [x^T(t), w^T(t)], \quad \zeta_{i1} = \begin{bmatrix} \Xi_{i1} & P_i(t)E_i(t) \\ * & -\gamma^2 I \end{bmatrix}, \quad \zeta_{i2} = \begin{bmatrix} \bar{C}_i(t) & F_i(t) \end{bmatrix}.$$

Similarly, for $t \in T_\downarrow(t_k, t_{k+1})$, we obtain

$$\dot{V}_i(t) + \alpha_i V_i(t) + y^T(t)y(t) - \gamma^2 w^T(t)w(t) = \eta^T(t)(\zeta_{i3} + \zeta_{i4}^T \zeta_{i4})\eta(t) \quad (48)$$

where

$$\zeta_{i3} = \begin{bmatrix} \Xi_{i2} & P_i(t)E_i(t) \\ * & -\gamma^2 I \end{bmatrix}, \quad \zeta_{i4} = \begin{bmatrix} \hat{C}_i(t) & F_i(t) \end{bmatrix}.$$

Using Schurs complement, we know that the inequalities (45) and (46) imply $\zeta_{i1} + \zeta_{i2}^T \zeta_{i2} < 0$ and $\zeta_{i3} + \zeta_{i4}^T \zeta_{i4} < 0$, respectively. By letting $J(s) = y^T(s)y(s) - \gamma^2 w^T(s)w(s)$, we can obtain the following inequality:

$$\dot{V}_i(t) \leq \begin{cases} \beta_i V_i(t) - J(s), & t \in T_\uparrow(t_k, t_{k+1}) \\ -\alpha_i V_i(t) - J(s), & t \in T_\downarrow(t_k, t_{k+1}) \end{cases}. \quad (49)$$

Owing to $P_i(t) \leq \mu_i P_j(t)$, we can get

$$V_i(t_k) \leq \mu_i V_j(t_k^-). \quad (50)$$

According to Lemma 2, the system (6) is GUAS for any switching signal with MDADT satisfying (31) and has a weighted H_∞ performance index $\hat{\gamma}$. The proof is ended.

In Lemma 3 the matrix variables $P_i(t)$ are coupled with the system parameter matrices in (45) and (46). Thus it is difficult to design the controller directly. We introduce the following lemma to solve this problem.

Lemma 4: Let $\alpha_i > 0$, $\beta_i > 0$, $\gamma > 0$ and $\mu_i \geq 1$ be given constants. If there exist matrices $\tilde{P}_i(t) > 0$, $L_i(t)$, and matrix X , satisfying

$$\tilde{P}_i(t) \leq \mu_i \tilde{P}_j(t), \quad (51)$$

and

$$\begin{bmatrix} \Phi_{i1} & \Phi_{i2} & E_i(t) & \Phi_{i3} \\ * & -X - X^T & E_i(t) & 0 \\ * & * & -\gamma^2 I & F_i^T(t) \\ * & * & * & -I \end{bmatrix} < 0, \quad (52)$$

$$\begin{bmatrix} \Psi_{i1} & \Psi_{i2} & E_i(t) & \Psi_{i3} \\ * & -X - X^T & E_i(t) & 0 \\ * & * & -\gamma^2 I & F_i^T(t) \\ * & * & * & -I \end{bmatrix} < 0 \quad (53)$$

where

$$\begin{aligned} \Phi_{i1} &= A_i(t)X + X^T A_i^T(t) + B_i(t)L_j(t) + L_j^T(t)B_i^T(t) + \dot{\tilde{P}}_i(t) - \beta_i \tilde{P}_i(t), \\ \Phi_{i2} &= \tilde{P}_i(t) - X + X^T A_i^T(t) + L_j(t)B_i^T(t), \quad \Phi_{i3} = X^T C_i^T(t) + L_j^T(t)D_i^T(t), \\ \Psi_{i1} &= A_i(t)X + X^T A_i^T(t) + B_i(t)L_i(t) + L_i^T(t)B_i^T(t) + \dot{\tilde{P}}_i(t) + \alpha_i \tilde{P}_i(t), \\ \Psi_{i2} &= \tilde{P}_i(t) - X + X^T A_i^T(t) + L_i(t)B_i^T(t), \quad \Psi_{i3} = X^T C_i^T(t) + L_i^T(t)D_i^T(t). \end{aligned}$$

Then, we can conclude that the inequalities (44), (45) and (46) hold.

Proof: In order to decouple the matrix variables $P_i(t)$ and system parameter matrices, we introduce a slack matrix Q . By introducing a slack matrix Q , we now introducing the following inequalities:

$$\begin{bmatrix} \xi_{i1} & P_i(t) - Q^T + \bar{A}_i^T(t)Q & Q^T E_i(t) & \bar{C}_i^T(t) \\ * & -Q - Q^T & Q^T E_i(t) & 0 \\ * & * & -\gamma^2 I & F_i^T(t) \\ * & * & * & -I \end{bmatrix} < 0, \quad (54)$$

$$\begin{bmatrix} \xi_{i2} & P_i(t) - Q^T + \hat{A}_i^T(t)Q & Q^T E_i(t) & \hat{C}_i^T(t) \\ * & -Q - Q^T & Q^T E_i(t) & 0 \\ * & * & -\gamma^2 I & F_i^T(t) \\ * & * & * & -I \end{bmatrix} < 0 \quad (55)$$

where

$$\xi_{i1} = Q^T \bar{A}_i(t) + \bar{A}_i^T(t)Q + \dot{P}_i(t) - \beta_i P_i(t), \quad \xi_{i2} = Q^T \hat{A}_i(t) + \hat{A}_i^T(t)Q + \dot{P}_i(t) + \alpha_i P_i(t).$$

Multiplying (54) from the left and right, respectively, by $\bar{\Lambda}_i(t)$ and its transpose, and multiplying (55) from the left and right, respectively, by $\hat{\Lambda}_i(t)$ and its transpose, where

$$\bar{\Lambda}_i = \begin{bmatrix} I & \bar{A}_i^T(t) & 0 & 0 \\ 0 & E_i^T(t) & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}, \quad \hat{\Lambda}_i = \begin{bmatrix} I & \hat{A}_i^T(t) & 0 & 0 \\ 0 & E_i^T(t) & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}.$$

Then we conclude that (45) and (46) hold. Notice that if the conditions in (54) and (55) hold, the matrix Q is nonsingular. We define the following matrices:

$$X = Q^{-1}, \quad L_i(t) = K_i(t)X, \quad \tilde{P}_i(t) = X^T P_i(t)X. \quad (56)$$

Multiplying (54) from the left and right by diagonal matrix $\text{diag}(Q^{-T}, Q^{-T}, I, I)$ and its transpose respectively. We can obtain

$$\begin{bmatrix} \xi_{i3} & \xi_{i4} & E_i(t) & Q^{-T} \bar{C}_i^T(t) \\ * & -Q^{-1} - Q^{-T} & E_i(t) & 0 \\ * & * & -\gamma^2 I & F_i^T(t) \\ * & * & * & -I \end{bmatrix} < 0 \quad (57)$$

where

$$\begin{aligned}\xi_{i3} &= \bar{A}_i(t)Q^{-1} + Q^{-T}\bar{A}_i^T(t) + Q^{-T}\dot{P}_i(t)Q^{-1} - \beta_i Q^{-T}P_i(t)Q^{-1}, \\ \xi_{i4} &= Q^{-T}P_i(t)Q^{-1} - Q^{-1} + Q^{-T}\bar{A}_i^T(t).\end{aligned}$$

By the similar operation to (55), we can get

$$\begin{bmatrix} \xi_{i5} & \xi_{i6} & E_i(t) & Q^{-T}\hat{C}_i^T(t) \\ * & -Q^{-1} - Q^{-T} & E_i(t) & 0 \\ * & * & -\gamma^2 I & F_i^T(t) \\ * & * & * & -I \end{bmatrix} < 0 \quad (58)$$

where

$$\begin{aligned}\xi_{i5} &= \hat{A}_i(t)Q^{-1} + Q^{-T}\hat{A}_i^T(t) + Q^{-T}\dot{P}_i(t)Q^{-1} + \alpha_i Q^{-T}P_i(t)Q^{-1}, \\ \xi_{i6} &= Q^{-T}P_i(t)Q^{-1} - Q^{-1} + Q^{-T}\hat{A}_i^T(t).\end{aligned}$$

By considering (54)-(56) and the definition of matrices $\bar{A}_i(t)$, $\bar{C}_i(t)$, $\hat{A}_i(t)$ and $\hat{C}_i(t)$ in (6), we conclude that LMIs (57) and (58) imply (52) and (53) respectively. Moreover, by performing a congruence transformation to $P_i(t) \leq \mu_i P_j(t)$, we obtain

$$Q^{-T}P_i(t)Q^{-1} \leq \mu_i Q^{-T}P_j(t)Q^{-1} \Leftrightarrow \tilde{P}_i(t) \leq \mu_i \tilde{P}_j(t). \quad (59)$$

Thus, if (51)-(53) hold, we can conclude that (44)-(46) hold. This completes the proof.

Remark 5: Due to the existence of $\dot{h}_{im}(t)$, the conditions in lemma 2 and lemma 3 can not be solved analytically or even numerically. Similar as [21, 22], we transform the conditions of lemma 2 and lemma 3 into the conditions which can be solved numerically by replacing the time derivative $\dot{h}_{im}(t)$ with their upper bounds ε_{im} . Then, a reasonable estimation of $\dot{P}_i(t)$ is give as $\dot{P}_i(t) = \sum_{m=1}^{r_i} \dot{h}_{im}(t)P_{im} \leq \sum_{m=1}^{r_i} \varepsilon_{im}P_{im}$.

The following theorem provides the stabilization conditions with H_∞ performance for the closed-loop switched nonlinear system (6).

Theorem 2: Assume that

$$\left| \dot{h}_{is}(t) \right| \leq \varepsilon_{is}, \quad s = 1, 2, \dots, r_i, \quad (60)$$

where $\varepsilon_{is} \geq 0$. Let $\alpha_i > 0$, $\beta_i > 0$, $\gamma > 0$ and $\mu_i \geq 1$ be given constants. If there exist matrices $\tilde{P}_{iu} > 0$, L_{in} and matrix X satisfying

$$\tilde{P}_{iu} \leq \mu_i \tilde{P}_{jv}, \quad \tilde{P}_{ik} \geq \tilde{P}_{ir_i}, \quad k = 1, 2, \dots, r_i - 1, \quad (61)$$

and

$$\bar{\Theta}_{imnu} + \bar{\Theta}_{iunm} < 0, \quad n \leq u \quad (62)$$

$$\begin{cases} \hat{S}_{immm} = \hat{\Theta}_{immm} < 0, \\ \hat{S}_{immn} = \frac{1}{3} [\hat{\Theta}_{immn} + \hat{\Theta}_{imnm} + \hat{\Theta}_{inmm}] < 0, m \neq n, \\ \hat{S}_{imnu} = \frac{1}{6} [\hat{\Theta}_{imnu} + \hat{\Theta}_{imun} + \hat{\Theta}_{inmu} + \hat{\Theta}_{inum} + \hat{\Theta}_{iunm} + \hat{\Theta}_{iumn}] < 0, m < n < u \end{cases} \quad (63)$$

for all $m, n, u, v \in \{1, 2, \dots, r_i\}$, where

$$\bar{\Theta}_{imnu} = \begin{bmatrix} \Phi_{i1} & \Phi_{i2} & E_{in} & \Phi_{i3} \\ * & -X - X^T & E_{in} & 0 \\ * & * & -\gamma^2 I & F_{in}^T \\ * & * & * & -I \end{bmatrix}, \hat{\Theta}_{imnu} = \begin{bmatrix} \Psi_{i1} & \Psi_{i2} & E_{in} & \Psi_{i3} \\ * & -X - X^T & E_{in} & 0 \\ * & * & -\gamma^2 I & F_{in}^T \\ * & * & * & -I \end{bmatrix},$$

and

$$\begin{aligned} \Phi_{i1} &= A_{in}X + X^T A_{in}^T + B_{in}L_{jm} + L_{jm}^T B_{in}^T + \sum_{s=1}^{r_i-1} \varepsilon_{is}(\tilde{P}_{is} - \tilde{P}_{ir_i}) - \beta_i \tilde{P}_{iu}, \\ \Phi_{i2} &= \tilde{P}_{iu} - X + X^T A_{in}^T + L_{jm}^T B_{in}^T, \quad \Phi_{i3} = X^T C_{in}^T + L_{jm}^T D_{in}^T, \\ \Psi_{i1} &= A_{in}X + X^T A_{in}^T + B_{in}L_{im} + L_{im}^T B_{in}^T + \sum_{s=1}^{r_i} \varepsilon_{is}(\tilde{P}_{is} - \tilde{P}_{ir_i}) + \alpha_i \tilde{P}_{iu}, \\ \Psi_{i2} &= \tilde{P}_{iu} - X + X^T A_{in}^T + L_{im}^T B_{in}^T, \quad \Psi_{i3} = X^T C_{in}^T + L_{im}^T D_{in}^T, \end{aligned}$$

then the system (6) is GUAS for any switching signal with MDADT satisfying (31) and has a weighted H_∞ performance index $\hat{\gamma}$. Moreover, if a feasible solution exists, the admissible controller gains can be given by

$$\begin{cases} K_{jm} = L_{jm}X^{-1}, & t \in [t_k, \bar{t}_k) \\ K_{im} = L_{im}X^{-1}, & t \in [\bar{t}_k, t_{k+1}). \end{cases} \quad (64)$$

Proof: Denote the left side of (52) and (53) as $\bar{\Theta}_i(t)$ and $\hat{\Theta}_i(t)$, respectively. If the conditions in Theorem 2 are satisfied, we can obtain

$$\begin{aligned} \bar{\Theta}_i(t) &\leq \sum_{m=1}^{r_i} \sum_{n=1}^{r_i} \sum_{u=1}^{r_i} h_{in}(t)h_{jm}(t)h_{iu}(t)\bar{\Theta}_{imnu} \\ &= \sum_{m=1}^{r_i} h_{jm}(t) \left[\sum_{n=1}^{r_i} h_{in}^2(t)\bar{\Theta}_{imnn} + \sum_{n=1}^{r_i} \sum_{n < u}^{r_i} h_{in}(t)h_{iu}(t)(\bar{\Theta}_{inmu} + \bar{\Theta}_{iumn}) \right] < 0 \end{aligned} \quad (65)$$

and

$$\begin{aligned} \hat{\Theta}_i(t) &\leq \sum_{m=1}^{r_i} \sum_{n=1}^{r_i} \sum_{u=1}^{r_i} h_{im}(t)h_{in}(t)h_{iu}(t)\hat{\Theta}_{imnu} \\ &= \sum_{m=1}^{r_i} h_{im}^3(t)\hat{S}_{immm} + 3 \sum_{m=1}^{r_i} \sum_{\substack{n=1 \\ n \neq m}}^{r_i} h_{im}(t)h_{in}(t)\hat{S}_{immn} \\ &\quad + 6 \sum_{m=1}^{r_i} \sum_{n > m} \sum_{u > n} h_{im}(t)h_{in}(t)h_{iu}(t)\hat{S}_{imnu} < 0. \end{aligned} \quad (66)$$

From Lemmas 2, 3 and 4, we know the H_∞ controller design problem is solved. By (56), we can conclude that the gains in (5) can be constructed by (64). This completes the proof.

Remark 6: Theorem 2 can also be applicable to relevant problems for the switched nonlinear system without asynchronous behaviors. For this case, if (62) in Theorem 2 is removed, we can conclude that the system is GUAS and has a weighted H_∞ performance index $\hat{\gamma}_s = \sqrt{\exp \left\{ \sum_{i=1}^N (T_{ai}^* \alpha_i) \right\} \gamma}$ for any switching signal satisfies $T_{ai} \geq T_{ai}^* = \frac{\ln \mu_i}{\alpha_i}$.

Remark 7: All the parameters (α_i , β_i , μ_i and γ) of Theorem 2 have their physical meaning, and these parameters can be designed according to the system performance requirements. For example, the parameter α_i denotes the decline rate of the Lyapunov function, which corresponds to the convergence rate of the system in synchronous state. And the parameter β_i denotes the increasing rate of the Lyapunov function when the system is running in the asynchronous state. In practice, we can design the values of these parameter to make the systems reach the desired performance.

Remark 8: In our method, in order to reduce the fuzzy approximation error, much more fuzzy rules should be used, which will cause to greater computational complexity.

4 Numerical Example

In this section, two numerical examples are given to illustrate the effectiveness and advantage of our results.

Example 1: Let $x(t) = [x_1^T(t) \ x_2^T(t)]^T$, and consider the open-loop switched fuzzy system (11) consisting of two subsystems, and each subsystem has two fuzzy rules, where

$$A_{11} = \begin{bmatrix} a & -4 \\ -1 & -2 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} -2 & -4 \\ 20 & -2 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} -5 & -2 \\ -6 & -3 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} b & -2 \\ 15 & -3 \end{bmatrix}.$$

The fuzzy membership functions are taken as

$$h_{11} = 1 - \frac{2}{1 + \exp(-x_1)} \times \frac{2}{1 + \exp(x_1)}, \quad h_{12} = 1 - h_{11}, \\ h_{21} = \sin^2(x_2 + 0.5), \quad h_{22} = 1 - h_{21}.$$

By the method of [21], we can obtain the parameters ε_{is} as $\varepsilon_{11} = \varepsilon_{12} = 1$ and $\varepsilon_{21} = \varepsilon_{22} = 0.5$.

In order to show the merit of the FLFs method, we compare the feasible area for the stability conditions based on the FLFs method and QLFs method by changing a and b .

The parameters used in these two methods have the same values, that is $\mu_1 = 2$, $\mu_2 = 2$, $\alpha_1 = 0.1$ and $\alpha_2 = 0.15$. The simulation results are shown in Figure 1. It can be seen from Figure 1 that the feasible area for FLFs method is larger than that for QLFs method, so the conditions of QLFs method lead to conservative results compared with FLFs method.

Figure 1: Feasible area for the stability conditions of FLFs method and QLFs method 1, where a and b take a step of 0.2.

Example 2: Consider the closed-loop switched fuzzy system (6) consisting of two subsystems, and each subsystem has two fuzzy rules, where

$$\begin{aligned}
 A_{11} &= \begin{bmatrix} -0.15 & -0.03 \\ 0 & 0.15 \end{bmatrix}, A_{12} = \begin{bmatrix} -0.1 & -0.05 \\ 0 & 0.1 \end{bmatrix}, A_{21} = \begin{bmatrix} 0.1 & 0 \\ 0.15 & -0.05 \end{bmatrix}, \\
 A_{22} &= \begin{bmatrix} 0.4 & 0 \\ 0.1 & -0.15 \end{bmatrix}, B_{11} = \begin{bmatrix} 0.25 & -0.08 \\ 0.1 & 0.16 \end{bmatrix}, B_{12} = \begin{bmatrix} 0.2 & 0.01 \\ 0.1 & 0.15 \end{bmatrix}, \\
 B_{21} &= \begin{bmatrix} 0.65 & -0.01 \\ 0.02 & -0.5 \end{bmatrix}, B_{22} = \begin{bmatrix} 0.58 & -0.01 \\ 0.01 & -0.4 \end{bmatrix}, \\
 C_{11} &= \begin{bmatrix} 0.3 & 0.5 \end{bmatrix}, C_{12} = \begin{bmatrix} 0.2 & 0.2 \end{bmatrix}, C_{21} = \begin{bmatrix} 0.15 & 0.25 \end{bmatrix}, C_{22} = \begin{bmatrix} 0.2 & 0.2 \end{bmatrix}, \\
 D_{11} &= \begin{bmatrix} 0.25 & 0.25 \end{bmatrix}, D_{12} = \begin{bmatrix} 0.25 & 0.25 \end{bmatrix}, D_{21} = \begin{bmatrix} 0.2 & 0.2 \end{bmatrix}, D_{22} = \begin{bmatrix} 0.18 & 0.22 \end{bmatrix}, \\
 E_{11} &= \begin{bmatrix} 0.35 \\ 0.22 \end{bmatrix}, E_{12} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, E_{21} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, E_{22} = \begin{bmatrix} 0.12 \\ 0.08 \end{bmatrix}, \\
 F_{11} &= 0.1, F_{12} = 0.05, F_{21} = 0.1, F_{22} = 0.1.
 \end{aligned}$$

The fuzzy membership functions used here are the same as in Example 1. The initial

conditions are assumed to be $x(t_0) = [1, 1.5]^T$, and the disturbance input is assumed to be $w(t) = \sin(\pi t) \exp(-0.15t)$. We choose $\gamma = 2$, $N_{0i} = 0$, $T_M = 1$, $\mu_1 = 1.1$, $\mu_2 = 1.2$, $\alpha_1 = 0.15$, $\alpha_2 = 0.2$, $\beta_1 = 0.05$ and $\beta_2 = 0.1$. So by Theorem 2, we obtain $T_{ca1}^* = 1.9687$, $T_{ca2}^* = 2.4116$ and $\hat{\gamma} = 3.4280$.

It is clear that the uncontrolled system is unstable. We will use our results to design a set of mode-dependent controllers such that the closed-loop switched system (6) is GUAS and the H_∞ performance is achieved. In practice, the asynchronous switching between the systems and the controllers generally exists. First, we study the asynchronous switching, while the controllers are designed by only considering the synchronous conditions. By solving (61) and (63) with the above parameters, the controller gains can be obtained as follows:

$$K_{11} = \begin{bmatrix} -1.4709 & 1.0249 \\ 0.4338 & -3.5225 \end{bmatrix}, K_{12} = \begin{bmatrix} -1.5182 & 2.2835 \\ 0.7339 & -3.6958 \end{bmatrix},$$

$$K_{21} = \begin{bmatrix} -0.8744 & -1.0862 \\ -0.2575 & 1.6865 \end{bmatrix}, K_{22} = \begin{bmatrix} -1.4171 & -0.7935 \\ -0.1907 & 1.4885 \end{bmatrix}.$$

The state response for this case is shown in Figure 2. As shown in Figure 2 the systems become unstable if we ignore the effects of asynchronous behavior on the controllers design.

Figure 2: State response of the closed-loop asynchronous switched systems with controllers designed by synchronous conditions.

Then, we investigate the asynchronous switching and the controllers are designed by considering the synchronous and asynchronous conditions. By solving (61), (62) and (63)

with the above parameters, the controller gains are listed as follows:

$$K_{11} = \begin{bmatrix} 0.7701 & -18.5215 \\ -0.0831 & 3.4435 \end{bmatrix}, K_{12} = \begin{bmatrix} 1.1679 & -19.0306 \\ -0.2474 & 4.3554 \end{bmatrix},$$

$$K_{21} = \begin{bmatrix} 1.2707 & -19.9038 \\ -0.1819 & 4.4981 \end{bmatrix}, K_{22} = \begin{bmatrix} 0.9029 & -16.9572 \\ -0.2061 & 3.6144 \end{bmatrix}.$$

The state response for this situation is shown in Figure 3. As is shown in Figure 3 that the states of the closed-loop system converge to zero. It means that the designed controllers by considering asynchronous conditions are effective. Since the asynchronous switching is very common in real world, it is necessary for us to consider the effects of asynchronous behavior on the controllers design. Through comparing Figure 2 with Figure 3, we can conclude if we ignore the effects of asynchronous behavior on the controllers design, the controllers designed can not meet the actual requirements.

Figure 3: State response of the closed-loop asynchronous switched systems with controllers designed by synchronous and asynchronous conditions.

5 Conclusion

In this study, the switching FLFs are used to investigate the H_∞ control problem for the switched nonlinear systems with asynchronous behaviors. By using MDADT techniques, we obtain stability conditions for the open-loop switched nonlinear systems and stabilization conditions with H_∞ performance for the closed-loop switched nonlinear systems. A

state-feedback fuzzy controller is designed, which can ensure the stability of the closed-loop switched system with a weighted H_∞ performance. We remark that the obtained results can also apply to the switched nonlinear system without asynchronous behaviors. Finally, two numerical examples are given to demonstrate the effectiveness and advantage of our method. The obtained results might be used in the controller design for the network control system. Then potential extensions of our current results to the stability analysis and control synthesis for the systems under the network-based environment [31] or Markovian jump systems with time delay [32] need further study, which will be our future work. On the other hand, it has been shown that the piecewise Lyapunov functions method is another way to relax the conservativeness inherent in common Lyapunov function method [25,26], and a T-S fuzzy affine dynamic model has much improved function approximation capabilities [25,26]. Therefore it would be our another future work to investigate the H_∞ control problem for the T-S fuzzy affine systems via the piecewise Lyapunov functions method.

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