Online algorithms for 2D bin packing with advice

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Abstract

In advice complexity model of online bin packing, suppose there is an offline oracle with infinite computational power. The oracle can provide some information to the online calculation about future items after scanning the whole list of items. This approach can loosen online constraints. The online algorithm receives an advice bit upon the arrival of each item and makes the packing decision. In 2-dimensional online bin packing with advice problem, each item is a rectangle of side lengths less than or equal to 1. The items are to be packed into square bins of size 1×1 , without overlapping, allowing 90 degrees rotations. The goal is to pack the items into the minimum number of square bins. In this paper, a 2.25-competitive 2-dimensional online rectangular packing algorithm with three bits of advice per item is presented. Furthermore, an online strategy with competitive ratio 2 requiring three bits of advice per item is given for square packing.

Keywords: Online 2D packing, rotatable, advice string, asymptotic worst case ratio

1. Introduction

The bin packing problem has been studied extensively and is well-known to be NP-hard [1]. It has numerous applications in the real world such as memory

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allocation and resources scheduling [2, 3, 4, 5].

5 The one-dimensional (1D) online bin packing problem can be defined as follows: Given a list L of items $l_1, l_2, \dots$, arriving in an online manner, each of which has size $s(l_i) \in (0, 1]$, and an infinite number of unit-capacity bins. The problem is to irrevocably pack the current item into a bin without any information on the next items. The total size of packed items in each bin
10 cannot exceed the bin capacity. The goal is to use as few bins as possible.

 This paper considers the two-dimensional (2D) version of the problem that each item in the list L is a rectangle with width and height no larger than 1. Each bin is a square of unit side length. The packing of all items is orthogonal. Any two items in the same bin never overlap. 90 degrees rotations of each item
15 are allowed. The one-dimensional online bin packing is NP-hard, so does its 2D variant[6].

 The 2D online bin packing with rotation problem was first studied by Fujita and Hada [7]. They presented two online algorithms and claimed that the asymptotic competitive ratio of the second algorithm is at most 2.565. Leah
20 Epstein [8] presented an online algorithm of competitive ratio slightly below 2.45 through using a more advanced analysis and a better classification of items. For square packing, Coppersmith and Raghavan [9] showed an upper bound of $43/16 \approx 2.6875$ for online square packing. The upper bound for square packing was improved to $395/162 < 2.43828$ by Seiden and van Stee [10]. Epstein and
25 van Stee [11, 12] gave an upper bound of 2.24437 for square packing. This result is improved to 2.1187 by Han et al. [13].

 Recently there is a rising study of the advice complexity model for online algorithms [14, 15, 16]. In the real world application, the total lack of information about the future is unrealistic [17]. The advice complexity model is
30 proposed to loosen the online constraint. In the online bin packing problem, suppose that there is an offline oracle with infinite computational power which can quickly skim the whole list L and generate some useful information. The online algorithm receives those helpful information upon the arrival of each item and makes the packing decision according to the advice string and items arrived

35 previously. The advice string helps the online algorithm to achieve an improved packing ratio.

Boyar et al. [18] first considered the online 1D bin packing with advice problem. They designed an algorithm with $2n + o(n)$ bits of advice and achieved a competitive ratio of $\frac{4}{3} + \varepsilon$. The idea is to classify items into different classes
40 based on the final optimal packing and pack different classes of items by using distinct algorithms. After that, Renault et al. [19] gave an online algorithm with advice which is $(1 + \varepsilon)$ -competitive and uses $O(\frac{1}{\varepsilon} \log \frac{1}{\varepsilon})$ bits of advice per request, for $0 < \varepsilon < \frac{1}{2}$.

This paper presented an online algorithm for the rotatable 2D bin packing
45 with advice problem, which receives three bits of advice per item and analyzes its asymptotic competitive ratio. Furthermore, we give an online strategy with competitive ratio 2 for square packing with three bits of advice per item. The basic idea is to extend the one-dimensional bin packing algorithm into two dimensions and combine large items into one bin as many as possible. By far,
50 it is the first study of the advice complexity model of 2D packing problem. In this paper, each item has an advice bit and the 2D bin packing problem has been changed. Since this paper is the first time to study the 2D bin packing with advice problem, only the result of one-dimensional packing with advice is known. The previous results are inappropriate to be compared with our results.
55 The reasonable way to evaluate the advantage of our algorithm is to compare the gap between 2D bin packing with and without advice and the gap between 1D bin packing with and without advice. Boyar et al's work [18] gave a 4/3-competitive algorithm for 1D online bin packing with advice problem. The best result of 1D bin packing without advice is 1.58889 [20] and the best lower bounds
60 is 1.54014 [21]. The result of bin packing with advice complexity model is better than the lower bound. The best result of 2D online bin packing without advice is 2.5545 [22] and the best lower bound is 1.907 [23]. Our result of 2D bin packing with advice is better than the upper bound of 2D bin packing without advice, while this result has not achieved the corresponding lower bound. For
65 square packing without advice, the best upper bound is 2.1187 [13], while our

result is 2 for square packing with advice. Both of our results are better than the upper bound of the same problem without advice.

The idea of the presented algorithm is based on the work of Boyar et al. [18] who first studied the 1D online bin packing with advice problem. They
70 classify items into different types by size and use advice bits to differentiate the packing methods and the types of bins into which the item should be packed. In this paper, we extend the 1D packing problem into two dimensions. The incoming item with advice bits is packed into 2D bins according to their types. The resource allocation problem in data center networks can be seen as an
75 application of the 2D bin packing with advice. Each computer in data center has CPUs and memory resources which can be seen as 2D bins. Each request can be seen as an item. The CPUs and memory that each request needed can be seen as the size of each item. The total number of requests is known at first. Given this information, the allocation of requests into computers may be more
80 reasonable.

The remainder of this paper is organized as follows. Section 2 introduces the basic definitions of asymptotic worst case ratio and the advice complexity model. Section 3 gives the presented online algorithm for rotated rectangle packing with advice and its upper bound analysis. Section 4 describes the
85 online square packing algorithm with advice and its competitive ratio analysis. The concluding remarks and future work are given in Section 5.

2. Basic definitions

The standard measurement of performance (output quality) of an online approximation algorithm is called asymptotic competitive ratio, which is the
90 ratio of the online algorithm's performance versus that of the optimal offline algorithm on any given input when the problem size goes to infinity. For the bin packing problem, denoted by $OPT(L)$ the performance of an optimal offline bin packing algorithm on a sequence of items L , by $A(L)$ the performance of an online approximation algorithm A , where performance refers to the number of

95 used bins. The asymptotic worst case ratio of an online approximation algorithm A is defined as

$$R_A^\infty = \lim_{n \rightarrow \infty} \sup_L \left\{ \frac{A(L)}{OPT(L)} \mid OPT(L) = n \right\}.$$

Let $\mathcal{O}$ be a set of all online bin packing algorithms. The optimal asymptotic worst case ratio is defined as

$$R_{OPT}^\infty = \inf_{A \in \mathcal{O}} R_A^\infty.$$

In this paper, we analyze the least occupation ratio of all closed bins and
100 compute the worst-case ratio by its reciprocal.

The definition of the advice complexity model that we use in this paper is described below.

Definition 1. *In the online 2D bin packing problem with advice, the input is a sequence of rectangles, $\sigma = \langle (x_1, y_1), \dots, (x_n, y_n) \rangle$ revealed to the algorithm in an online manner ($0 < x_i, y_i \leq 1$). The goal is to pack these rectangles in
105 the minimum number of bins of unit square size. At time step t , an online algorithm should pack item (x_t, y_t) into a bin. The decision of the algorithm to select the target bin is a function of $\phi, (x_1, y_1), \dots, (x_{t-1}, y_{t-1})$, where ϕ is the content of the advice tape. An algorithm A with advice tape ϕ is c -competitive with advice complexity $s(n)$ if there exists a constant c_0 such that, for every
110 n and for each input sequence σ of length no larger than n , there exists some ϕ such that $A(\sigma) \leq cOPT(\sigma) + c_0$, and at most the first $s(n)$ bits of advice of the tape have been accessed by the algorithm. An algorithm A is optimal if $A(\sigma) = OPT(\sigma)$.*

115 3. 2.25-competitive rectangle packing algorithm with three bits of advice

3.1. The classification of items

Throughout the paper, we denote a rectangular item with width x and height y by (x, y) . Since exactly 90 degrees rotations are allowed, we can assume that

Table 1: The partition of type A

$\begin{array}{c} x \\ \backslash \\ y \end{array}$	$(\frac{2}{3}, 1]$	$(\frac{3}{5}, \frac{2}{3}]$	$(\frac{1}{2}, \frac{3}{5}]$	$(\frac{2}{5}, \frac{1}{2}]$	$(\frac{1}{3}, \frac{2}{5}]$	$(\frac{1}{4}, \frac{1}{3}]$
$(\frac{2}{3}, 1]$	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}	A_{15}
$(\frac{3}{5}, \frac{2}{3}]$	-	A_{21}	A_{22}	A_{23}	A_{24}	A_{25}
$(\frac{1}{2}, \frac{3}{5}]$	-	-	A_{32}	A_{33}	A_{34}	A_{35}
$(\frac{2}{5}, \frac{1}{2}]$	-	-	-	A_{43}	A_{44}	A_{45}
$(\frac{1}{4}, \frac{2}{5}]$	-	-	-	-	A_{54}	A_{55}

Table 2: The partition of type B

$\begin{array}{c} x \\ \backslash \\ y \end{array}$	$(\frac{2}{3}, 1]$	$(\frac{3}{5}, \frac{2}{3}]$	$(\frac{1}{2}, \frac{3}{5}]$	$(\frac{2}{5}, \frac{1}{2}]$	$(\frac{1}{3}, \frac{2}{5}]$	$(\frac{1}{4}, \frac{1}{3}]$
$(0, 1/4]$	B_0	B_1	B_2	B_3	B_4	B_5

120 $x \geq y$ without loss of generality. The sequence of items L is first partitioned into three types A , B , C as

$$A = \{(x, y) | 1/4 < x \leq 1 \text{ and } 1/4 < y \leq 1\}$$

$$B = \{(x, y) | 1/4 < x \leq 1 \text{ and } 0 < y \leq 1/4\}$$

$$C = \{(x, y) | 0 < x \leq 1/4 \text{ and } 0 < y \leq 1/4\}$$

Assume that items from type A are large, those from type B are medium and those from type C are small. For brevity, we call them A item, B item and C item, respectively. In the presented algorithm, a bin that only accepts
 125 type α items will be referred to as an α -bin. Actually, our algorithm packs each type of items into bins by different packing strategy. Next, we partition the interval $(1/4, 1]$ into six segments: $(1/4, 1/3]$, $(1/3, 2/5]$, $(2/5, 1/2]$, $(1/2, 3/5]$, $(3/5, 2/3]$, $(2/3, 1]$. Then type A and B are partitioned into subclasses as in Table 1 and Table 2.

130 Note that in our presented algorithm, items from different types are packed by different algorithms. Here, items in $A_{11} \cup A_{12} \cup A_{21} \cup A_{22} \cup A_{24} \cup A_{32} \cup A_{33}$

are packed by the improved packing which is not the existing algorithms.

In addition, we classify some types of A items into three groups.

Group 1 is the set of two large items that can be packed together into the
135 same bin. This set includes type A_{11} with type A_{14} or type A_{24} or A_{34} or A_{44} ,
type A_{12} with type A_{14} or type A_{24} or A_{34} or A_{44} or A_{54} , type A_{21} with type
 A_{14} or type A_{24} or A_{34} , type A_{22} with type A_{14} or type A_{34} , type A_{32} with
type A_{14} or type A_{24} .

Group 2 is the set of three large items that can be packed together into one
140 bin. It includes type A_{22} with two type A_{34} items, type A_{32} combined with two
type A_{34} items, two type A_{24} items with one type A_{14} or one type A_{34} or one
type A_{54} , three type A_{24} items, two type A_{33} with one type A_{14} or A_{24} or A_{34}
or A_{54} .

Group 3 is the set of four large items that can be combined together into
145 one bin. This set contains type A_{22} with type A_{21} and A_{34} and A_{44} , type A_{32}
with two type A_{33} and one type A_{43} , type A_{21} with two type A_{24} and one type
 A_{54} .

In the presented algorithm, items from the same group are packed into one
bin as much as possible. Note that not all items in the above group are packed
150 together. The grouping step is to show the possibility of combination of different
types of items which can be packed into one bin.

3.2. Advice bit

In the presented algorithm, each item comes with an advice which has three
bits. This advice string is generated by an offline oracle, which gives helpful
155 information to the online algorithm. Let M denote the set of advice. Three bits
of advice provide eight different advices. The meaning of each advice indicating
how the incoming item should be packed is shown in Table 3.

As mentioned before, some types of large and medium items cannot be
packed by using the existing algorithms, but improved packing. The types of
160 items which the existing algorithms do not pack well are chosen to be packed by
improved algorithms. These types of large items are A_{ij} -items that: 1). $i = 1$

Table 3: The meaning of advice string M

AB	# of items	IT	Open or not	Operations
000	≥ 1	$!C$	Y	Two-dimensional Harmonic
000	≥ 5	C	Y	Round
001	≥ 5	B	Y	Rotated two-dimensional Next Fit
001	3 or 4	A_{34}	Y	Rotated Harmonic
010	2	A	N	Open a bin and pack it in the bottom-left corner
011	2	A	Y	Pack the item by Match Two in an opened bin
100	3	A	N	Open a bin and pack it in the bottom-left corner
101	3	A	Y	Pack the item by MatchThree in an opened bin
110	4	A	N	Open a bin and pack it in the bottom-left corner
111	4	A	Y	Pack the item by MatchFour in an opened bin

and $j = 1, 2; 2)$. $i = 2$ and $j = 1, 2, 4; 3)$. $i = 3$ and $j = 2, 3$. Except the harmonic and round packing in Table 3, all other packing strategies belong to the improved packing.

165 In Table 3, the column of name “AB” refers to the advice bits. The second column shows the total number of items contained in one bin with the corresponding advice. The next column of name “IT” refers to the item type. The fourth column indicates the bin which accommodates the incoming item is now opened or not. The last column gives the packing operations under the
170 corresponding advice.

3.3. Algorithm description

This section presents the online rotatable 2D rectangle packing with advice algorithm (RA). The online algorithm receives the advice bits of the item when
175 it comes and packs the item into an open bin or a newly created one. The outline of our online algorithm is described as follows.

Algorithm RA

Let $L = (l_1, l_2, \dots, l_n)$ be the sequence of rectangular items that appears online.
Use $M = (m_1, m_2, \dots, m_n)$ to denote the list of advice strings that is given with
180 items one at a time. Once an item comes, the presented algorithm checks its
advice bits and selects a bin to pack the item according to table 3. Open a new
bin if necessary. The packing of item l_i is determined by its size and advice bits
 m_i in the following manner:

Case 1: $m_i = 000$, if $l_i \in C$

185 **then** Pack l_i using Round algorithm described in subsection 3.3.1.
 else $l_i \notin C$
 then Pack l_i using Two-dimensional Harmonic algorithm [24].

Case 2: $m_i = 001$, if $l_i \in B$

then pack l_i using Rotated two-dimensional Next Fit algorithm shown in
190 Subsection 3.3.2.
 else $l_i \in A$, pack l_i using Rotated Harmonic algorithm shown in Subsec-
 tion 3.3.3.

Case 3: If $m_i = 010$ or 100 or 110 , then open a new bin and pack l_i into the
bottom-left corner. Assign the bin type m_i .

195 **Case 4:** If $m_i = 011$, then pack l_i using MatchTwo algorithm shown in Sub-
section 3.3.4.

Case 5: If $m_i = 101$, then pack l_i using MatchThree algorithm shown in Sub-
section 3.3.5.

Case 6: If $m_i = 111$, then pack l_i using MatchFour algorithm shown in Sub-
200 section 3.3.6.

3.3.1. Round packing of type C

Items of type C are packed in a similar way to Round [25]. Let X_1, X_2, X_3 and X_4 be four sets of reals in $(0, 1]$ defined as follows:

$$\begin{aligned} X_1 &= \{1/2^i | i = 2, 3, 4, \dots\} \\ X_2 &= \{1/(5 \times 2^i) | i = 0, 1, 2, \dots\} \\ X_3 &= \{1/(3 \times 2^i) | i = 1, 2, 3, \dots\} \\ X_4 &= \{1/(7 \times 2^i) | i = 0, 1, 2, \dots\} \end{aligned}$$

The width x_i of each incoming C item l_i is rounded up to a value $\bar{x}_i$ which
 205 is the smallest real in $\cup_{i=1}^4 X_i$, which is not smaller than x_i . For example, $1/11$ is rounded up to $1/10$ ($\in X_2$) and $15/112$ is rounded up to $1/7$ ($\in X_4$). In the following, a C item (x, y) is said to be of type i if $\bar{x} \in X_i$ for $i = 1, 2, 3, 4$. Note that $X_i \cap X_j = \emptyset$ for $i \neq j$.

In algorithm RA, each C -bin is split into strips of height 1 and with a width
 210 that is drawn from set $\cup_{i=1}^4 X_i$. Items belong to different types cannot be packed into the same bin. Then there are four active bins, each of which is devoted for the packing of one type of items. There are also active strips in each C -bin. Once a strip accommodates its first item, it becomes active. When it is closed, it cannot be active again. Below, we show a packing processing of a C item
 215 (x, y) of type 1, i.e., $\bar{x} = 2^{-k}$ for some $k > 1$. The packing of items of other types can be done in a similar way.

Packing of C item of type 1

Step 1: Find an active strip of width $\bar{x}$ that can accommodate the incoming
 item $l_i = (x, y)$. If it is impossible, then close all active strips of width $\bar{x}$
 220 and execute the following operations:

1. If there exists an empty strip of width $\bar{x} = 2^{-k}$, then make it a new active strip of width $\bar{x}$.
2. Else, find an empty strip of width $2^{-k'}$ such that $k' < k$ and k' is the largest value that is smaller than k . Bisect this strip continuously into

Figure 1: An example of the packing of rotated 2DNF.

225 $k - k' + 1$ empty strips of width $2^{-(k'+1)}$, $2^{-(k'+2)}$, $\dots$, $2^{-(k-1)}$, 2^{-k} and 2^{-k} , respectively. Select one of the two strips of width 2^{-k} to be an active one.

Step 2: Pack the item into the active strip in a bottom-left manner.

3.3.2. Rotated 2D Next Fit packing of type B

230 In algorithm rotated two-dimensional Next Fit (2DNF), each B -bin is divided into several parts by vertical lines according to the width x of items contained in the bin. The algorithm keeps one active bin for the packing of each subclasses of type B item as shown in Figure 1. When a B item comes, the algorithm packs it into the bin with the same type of items as follows:

235 Packing of B item

1. Pack the item into one part of the bin partitioned by x coordinate using Next Fit [26] in a bottom-up manner.
2. Else if there are remaining free space, rotate the item and pack it into the remaining part of the bin from right to left by using Next Fit [26].
- 240 3. Otherwise, close this bin and open a new bin. Divide the bin into two vertical parts by the largest width of this type of item and do step 1.

3.3.3. Rotated harmonic packing of type A_{34}

First, the type A_{34} items are further divided into two sub types according to the y coordinate. The sub types are type $A_{34}^1 = \{(x, y) | 1/2 < x \leq 3/5, 1/4 <$

Figure 2: Rotated harmonic packing.

245 $y \leq 1/3\}$ and type $A_{34}^2 = \{(x, y) | 1/2 < x \leq 3/5, 1/3 < y \leq 2/5\}$. Each A_{34} -bin is also partitioned into two parts just like the partition of each B -bin. The A_{34}^1 and A_{34}^2 -items are packed separately into A_{34}^1 -bin and A_{34}^2 -bin. The rotated harmonic packing keeps two active bins for each type of A_{34} items. An example of the closed bin which is filled by rotated harmonic packing is shown in Figure 250 2.

Next, the $3/5$ part of each A_{34}^1 -bin is divided into three equal horizontal strips. The $3/5$ part of each A_{34}^2 -bin is divided into three different sized horizontal strips, where two strips are with height $2/5$ and one strip is with height $1/5$. Now we can describe the packing procedure which packs an A_{34} item into 255 an active bin.

Packing of A_{34} items

1. Pack the incoming item into one of the horizontal strips in the $3/5$ part of current active bin.
2. Else, rotate the item by 90 degrees and pack it into the $2/5$ part
- 260 3. Otherwise, close this bin and open a new bin with the same type. Partition the bin into two parts from $3/5$. Divide the bigger one into three horizontal strips (equal for A_{34}^1 and different for A_{34}^2). Do step 1.

3.3.4. The MatchTwo algorithm

As mentioned before, some large items are packed two per bin. The MatchTwo 265 algorithm is used to pack these items whose advice are 011. The description of this algorithm is as follows.

When an item l_i with advice bit $m_i = 011$ comes,
 Rotate the item l_i by 90 degrees and pack it into one corner of an open bin
 with type 010 by using Best Fit [26]. Close this bin after packing this item.

270 3.3.5. The MatchThree algorithm

The MatchThree algorithm addresses items that are packed exactly three
 items per bin. The outline of the algorithm is described as follows.

When an item l_i with advice string $m_i = 101$ comes, the algorithm MatchThree
 runs in the following steps:

- 275 1. Rotate the item.
2. If $l_i \in A_{32}$ or A_{22} , find open bins which contain two A_{34} items. Pack it
 into one of these bins by Best Fit [26]. Close this bin. Otherwise, find
 open bins with one A_{34} item and pack the item into one of these bins by
 Best Fit.
- 280 3. If $l_i \in A_{33}$, find open bins which contain one A_{33} item. Pack the new item
 in the same way as the first packed item into one of these bins by Best Fit
 [26].
4. Else, find open bins which contain two items and pack this item into one
 of these bins by Best Fit [26]. Close this bin.
- 285 5. Else, find open bins which contain one item and pack the item into one of
 these bins by Worst Fit [26].

3.3.6. The MatchFour algorithm

When an item l_i with advice string $m_i = 111$ comes, the algorithm Match-
 Four finds the bin in the following steps and packs the item in one free corner
 290 of this bin:

1. If $l_i \in A_{24}$, find opened bins which contain one or two or three types of
 items in $A_{24} \cup A_{21} \cup A_{54}$. Pack l_i into one of these bins by Best Fit [26].
 Rotate it if necessary.

2. If $l_i \in A_{33}$, find opened bins which contain one or two or three types of
 295 items in $A_{32} \cup A_{33} \cup A_{43}$. Pack l_i into one of these bins by Best Fit [26].
 Rotate it if necessary.
3. If $l_i \in A_{54}$ (A_{21}), find opened bins which contain two A_{24} items. Pack
 l_i into one of these bins by Best Fit. Otherwise, find opened bins which
 contain one or two types of items in $A_{24} \cup A_{21}$ ($A_{24} \cup A_{54}$). Pack it into
 300 one bin by Best Fit [26].
4. If $l_i \in A_{43}$ (A_{32}), find opened bins which contain two A_{33} items. Pack
 l_i into one of these bins by Best Fit. Otherwise, find opened bins which
 contain one or two types of items in $A_{32} \cup A_{33}$ ($A_{43} \cup A_{33}$). Pack it into
 one bin by Best Fit [26].
- 305 5. If $l_i \in A_{22}$ (or A_{23} or A_{34} or A_{44}), find opened bins which contain one
 or two or three types of items in $A_{23} \cup A_{34} \cup A_{44}$ (or $A_{22} \cup A_{34} \cup A_{44}$ or
 $A_{22} \cup A_{23} \cup A_{44}$ or $A_{22} \cup A_{23} \cup A_{34}$). Pack l_i into one of these bins by Best
 Fit [26]. Rotate the item if necessary.
6. Close the bins which have exactly four items.

310 3.4. Analysis of upper bound

In this section, we give the analysis of the least occupation ratio of all closed bins which are packed by algorithm RA.

Lemma 1. *When the packing finished, the closed bins which are packed by harmonic algorithm have a least occupation ratio larger than $4/9$.*

315 *Proof.* From Table 3 and the classification of type A items described in section 3.1, items which are packed by harmonic are type A items which are in $A_{10} \cup A_{13} \cup A_{14} \cup A_{23} \cup A_{43} \cup A_{44} \cup A_{54}$.

These items are packed based on their types with one active bin per type. When the packing of list L completed, except seven bins, the least occupation
 320 ratio in these classes is no less than $4/9$. $\square$

Lemma 2. *When the packing finished, the occupation ratio of each closed C -bin is at least $21/32$.*

Proof. The occupancy of each C -bin can be evaluated as follows. After all the items being packed, the total wasted space in all C -bins which only contains type 1 items will not exceed two bins. This can be proved through repacking the empty strips and active strips to latter bins. Consider all the non-wasted strips in the same C -bin, the total height is at least $3/4$. In addition, our round packing guarantees that every C item is rounded up its width by no more than a factor of $8/7$. Then we can infer that the packing ratio of non-wasted strips in one bin is at least $21/32$. $\square$

Lemma 3. *When the packing finished, the occupation ratio of each closed B -bin is at least $9/20$.*

Proof. Based on the size of each B type items we can compute the occupation ratio of each closed B -bin. This can be done by further classifying B -items into subclasses according to both x and y coordinate. For example, we now compute the least occupation ratio of closed type B_2 -bins. According to the algorithm rotated 2DNF packing, each B_2 -bin is partitioned into two horizontal parts division from $3/5$. The B_2 items are first packed into $3/5$ part by Next Fit. The occupied height accounts for at least $3/4$. Then B_2 items that are coming later will be rotated by 90 degrees and packed into the other $2/5$ part by Next Fit. The packed height of this part is at least $\frac{2}{5} - \frac{1}{4} = \frac{3}{20}$. In total, the occupied area is at least $\frac{1}{2} \times (\frac{3}{4} + \frac{3}{20}) = \frac{9}{20} > \frac{4}{9}$. $\square$

Lemma 4. *When the packing finished, the least occupation ratio of closed A_{34} -bins which are packed by rotated harmonic algorithm is $1/2$.*

Proof. According to the rotated harmonic algorithm, each A_{34} -bin is partitioned into two parts divided from $3/5$. For the packing of A_{34}^1 item, the part of width $3/5$ is further divided into three equal strips with height $1/3$. We pack one A_{34}^1 item per strip. The fourth A_{34}^1 item is rotated by 90 degrees and packed into the other $2/5$ part of the bin. Totally, there are four A_{34}^1 items in one bin. The least occupation ratio is $1/2$. The packing of A_{34}^2 item is the same as the packing of A_{34}^1 item. The $3/5$ part of each A_{34}^2 -bin is partitioned into two strips

with height $2/5$. With the rotated A_{34}^2 item, we have three items in one bin in all. The occupation ratio is at least $1/2$. $\square$

Lemma 5. *When the packing finished, the bins which are packed by MatchTwo algorithm have an occupation ratio of at least $4/9$.*

Proof. Group 1 described in section 3.1 is packed by MatchTwo algorithm. Items in this group are exactly packed two per bin. The lemma is proven through computing the joint area of all two items in group 1. For example, the joint of one item from type A_{21} and one item from type A_{34} belongs to group 1.
 360 This can be done by packing A_{21} item in the bottom-left corner and rotated A_{34} item in the bottom right corner of the bin. The total area is at least $97/200 = 0.485 > 4/9$. $\square$

Lemma 6. *When the packing finished, the occupation ratios of the bins which are packed by MatchThree algorithm are all larger than $4/9$.*

365 *Proof.* Items of group 2 are packed by MatchThree algorithm. We verify three cases of combination in group 2. The first case is two A_{33} items with one A_{54} item. The packing method is to pack two A_{33} items in the left from bottom to up and pack rotated A_{54} item in the right. The total area is at least 0.4625. The second case is two A_{24} items with one A_{54} item. The last case is two A_{34}
 370 items with one A_{32} item. The packing methods of both two cases are the same as the first case. The total area is at least 0.4833 and $7/12$ respectively. $\square$

Lemma 7. *When the packing finished, the occupation ratios of the bins which are packed by MatchFour algorithm are all larger than $4/9$.*

Proof. Group 3 which is packed by MatchFour has four items in each case .
 375 There are three cases. The first case is two A_{24} items with one A_{54} item and one A_{21} item. Actually the total area of the first three items is already larger than $4/9$ as proved in lemma 6. Adding one more item increases the total area. The second and third cases are also similar. We can verify that their total area are all larger than $4/9$ in both cases. $\square$

380 From the above lemmas, we have the following theorem.

Theorem 1. *For given list of rectangular items, the asymptotic competitive ratio of the online algorithm RA is at most $9/4$.*

4. 2-competitive square packing algorithm with three bits of advice

Let L be a sequence of square items $l_1, l_2, \dots$. Denoted by s_i the side length
 385 of l_i and a_i the advice string of l_i . Let M be an integer that is greater than or
 equal to 6. As will be shown later, we will fix M to 6, as in [27]. Let T be the
 set of items that can be packed into a unit-square bin; i.e., $T \stackrel{def}{=} (0, 1] \times (0, 1]$.
 In the presented online algorithm for square packing with advice problem, the
 set of unit-square bins is also classified into several categories according to the
 390 items that it contains.

4.1. Classification of items

In this section, the incoming items are classified into types according to
 the side length. A square item l_i is tiny provided $s_i \leq \frac{1}{M}$; small provided
 $s_i \in (\frac{1}{6}, \frac{1}{5}]$ or $s_i \in (\frac{1}{5}, \frac{1}{4}]$; medium provided $s_i \in (\frac{1}{4}, \frac{3}{4}]$ and otherwise it is big
 395 when $\frac{3}{4} < s_i \leq 1$. The medium items are partitioned into four subsets $A, B,$
 C, D as

$$\begin{aligned} A &= \{(s_i, s_i) | 1/4 < s_i \leq 1/3\} \\ B &= \{(s_i, s_i) | 1/3 < s_i \leq 1/2\} \\ C &= \{(s_i, s_i) | 1/2 < s_i \leq 2/3\} \\ D &= \{(s_i, s_i) | 2/3 < s_i \leq 3/4\} \end{aligned}$$

For an abuse of conception, the side length s_i also denote item s_i itself. An
 item from type A is called an A item, and accordingly the B item, C item and
 D item.

400 4.2. Advice bit

In the presented algorithm, each item comes with an advice which has three bits. Each square bin is assigned with a type once it is opened. In other words, we can detect the subset of T to which the incoming item belongs through the three bits of advice. Items in different subsets will be packed by different
405 packing strategies to achieve a larger packing ratio.

The three bits of advice is from 000 to 111. The bin types are 000, 010, 100, 110 and 111. To be more precise, we give an explanation of the using of advice bits in Table 4. The first column named “AB” gives the advice bits. The second column “BT” gives the corresponding bin type. The third column “#” shows
410 how many items are contained in this type of bin after the bin is closed. The fourth column “IT” shows the item types that are contained in the bin. The fifth column indicates whether the bin is open or not when the item comes. The sixth column gives the various meanings of advice bits which can differentiate the subsets of T .

415 The packing of medium items is the main focus of the presented algorithm. Let $\alpha = 0.1041$. The medium items are packed by three different methods in our algorithm. At first, we try to pack two medium items into one bin, and the sum of two side lengths are larger than or equal to $1 - \alpha$. These two items are packed into two corners of the bin on the left. If it is possible, the algorithm
420 fills the other two empty corners by two A items. Otherwise, we only have two items with sum side lengths smaller than $1 - \alpha$. The algorithm combines these two items with two B items one per corner if possible. Otherwise, the algorithm packs the medium items on the left of the bin as high as possible and fills the empty space by small items according to the harmonic algorithm. Here
425 we call the items filled up the free space of the bin except the left-top corner, the complement item. Based on the types of items, we call them A complement, B complement and small complements as in Table 4.

(a) The bin with type 010 or 100. (b) The bin with type 000. (c) The bin with type 110.

Figure 3: The packing of medium items.

Table 4: The using of advice bits.

AB	BT	#	IT	Open or Not	Meaning
000	000	> 4	medium and small	not	one medium item with small complements
001	000 or 110	> 4	medium and small	open	medium items and small items
010	010	4	medium	not	$s_i + s_j \geq 1 - \alpha$ with A complement
011	010	4	medium	open	$s_i + s_j \geq 1 - \alpha$ with A complement
100	100	4	medium	not	$s_i + s_j < 1 - \alpha$ with B complement
101	100	4	medium	open	$s_i + s_j < 1 - \alpha$ with B complement
110	110	> 4	medium and small	not	two medium items with small complements
111	111	any	big or medium or small or tiny	open	items in the same type

4.3. Algorithm description

This section describes how to pack square items into bins. Once an item
430 comes, the presented algorithm checks its advice bits and selects a bin to pack
it according to table 4. Open a new bin if necessary.

When an item l_i with an advice m_i comes, the 2D online square packing
with advice algorithm (SA) runs as follows:

1. If $m_i = 000$ (or 110 or 010 or 100), open a bin with type 000 (or 110 or
435 010 or 100). If l_i is a medium item, then pack l_i into the left-top corner
of the newly created bin. Else, pack l_i into the right-top corner by two-
dimensional harmonic algorithm and reserve the space of $\frac{3}{4} \times \frac{3}{4}$ ($\frac{1}{2} \times 1$ for
type 110) on the left-top corner of the bin. (See Figure 3.)
2. If $m_i = 111$ and l_i is tiny, pack l_i by Round [24] into one of the bins
440 with type 111. Otherwise, if $m_i = 111$ and l_i is not tiny, pack l_i by two-
dimensional harmonic into one of the bins with type 111.
3. If $m_i = 011$ (or 101), pack l_i into one empty corner of the bins with type
010 by Best Fit. Close the bin if it has exactly four items.
4. If $m_i = 001$ and l_i belongs to C or D type, pack l_i into the reserved space
445 of the left-top corner of the bin with type 000 by First Fit. Adjust the
area of the reserved space to reflect the real size of l_i .
5. If $m_i = 001$ and l_i belongs to type B , pack l_i into the left-top corner of the
bin with type 000 as high as possible by First Fit.
6. If $m_i = 001$ and l_i is small, select bins with type 000 or 110 which con-
450 tain medium item. Pack l_i into the first one which can hold it by two-
dimensional harmonic. If there is not enough space in such bins, close
these bins. Find the bins with type 000 or 110 which has a reserved space
in the left of the bin for medium item. Pack l_i into the one which can
hold it by two-dimensional harmonic. Adjust the area of reserved space if
455 necessary.

4.4. Analysis of upper bound

After the packing has finished, each used bin has a type in (000, 010, 100, 110, 111). This section analyzes the least occupancy ratio of each type of closed bins. Among all these five values, the reciprocal of the smallest one is the competitive ratio of the presented online algorithm. Now the packing ratio of
460 each closed bins classified by the bin type can be computed.

Lemma 8. *When all items have been packed, the least packing ratio of all closed bins with type 111 is 9/16.*

Proof. According to the algorithm, items with type 111 are packed into these
465 bins. They are big or medium or small or tiny. Any one of these four types is possible. The items of the first three types are packed by harmonic algorithm. The last type of items is packed by Round. As for big and medium items of type A, harmonic algorithm packs them one per bin (nine per bin for medium items) and the occupied area is at least 9/16. There are two classes of small items.
470 Through harmonic packing, the occupied area are 16/25 and 25/36, respectively. The side length of each tiny item is rounded up by the smallest value that is larger than it as in paper [24]. The least packing ratio of the bins which only contain tiny items is 25/36. Then the smallest packing ratio is 9/16. $\square$

Lemma 9. *When all items have been packed, the least packing ratio of all closed
475 bins with type 010 is 0.5263.*

Proof. This type of bin contains four medium items as show in Figure 3a. Two medium items subject to $s_i + s_j \geq 1 - \alpha$ and the other two items are type A items. For the former case, we have $s_i^2 + s_j^2 \geq 2 \times (\frac{1-\alpha}{2})^2$. Then the occupied area of a closed bin with type 010 is at least $s_i^2 + s_j^2 + \frac{1}{8} \geq 2 \times (\frac{1-\alpha}{2})^2 + \frac{1}{8} \approx 0.5263$. $\square$

Lemma 10. *When all items have been packed, the least packing ratio of all
480 closed bins with type 100 is 0.5347.*

Proof. This type of bin contains four medium items too. There is an item with side length bigger than 1/2 out of the first two items and the other item satisfies

that $s_i + s_j \geq 1 - \alpha$. Then the inequation $s_i + s_j < 1 - \alpha$ is true. Another two
485 items are both complimentary items. Assume that s_i is the item that has side
length bigger than $1/2$. The complimentary items are s_k and s_l . They are both
type B items. The worst case happens when s_i is a C -item and s_j is an A -item.
The total occupied area is at least $\frac{1}{4} + \frac{1}{16} + \frac{2}{9} \approx 0.5347$. $\square$

Lemma 11. *For the closed type 000 bins, the least occupied ratio is 0.5 when
490 all items have been packed.*

Proof. The packing is as shown in Figure 3b. The medium item has side length
bigger than $1/2$ and is packed on the left-top corner of the bin. The remaining
free space is all packed by small items with the same type according to harmonic
algorithm. According to the classification of items, there are six cases to be
495 considered.

Case 1. The medium item s_i is a B item and all small items belong to $(\frac{1}{5}, \frac{1}{4}]$.
The reserved space for the medium item is $\frac{1}{2} \times \frac{1}{2}$ in the left-top corner. The
remaining free space is partitioned by the size of small items according to the
harmonic algorithm. So totally the bin can hold 12 small items. Then, the
500 occupied ratio is at least $\frac{1}{9} + \frac{12}{25} \approx 0.5911$.

Case 2. The medium item s_i is a B item and all small items belong to
 $(\frac{1}{6}, \frac{1}{5}]$. The reserved space for the medium item is $\frac{2}{3} \times \frac{2}{3}$ in the left-top corner.
The remaining free space is partitioned by the size of small items according to
the harmonic algorithm. Totally, the bin can hold 16 small items. Then, the
505 occupied ratio is at least $\frac{1}{9} + \frac{16}{36} \approx 0.5556$.

Case 3. The medium item s_i is a C item and all small items belong to $(\frac{1}{5}, \frac{1}{4}]$.
The reserved space for the medium item is $\frac{2}{3} \times \frac{2}{3}$ in the left-top corner. The
remaining free space is partitioned by the size of small items according to the
harmonic algorithm. Totally, the bin can hold 7 small items. Thus, the occupied
510 ratio is at least $\frac{1}{4} + \frac{7}{25} = 0.53$.

Case 4. The medium item s_i is a C item and all small items belong to
 $(\frac{1}{6}, \frac{1}{5}]$. The reserved space for the medium item is $\frac{2}{3} \times \frac{2}{3}$ in the left-top corner.
The remaining free space is partitioned by the size of small items according to

the harmonic algorithm. Totally, the bin can hold 9 small items. Then, the
 515 occupied ratio is at least $\frac{1}{4} + \frac{9}{36} = 0.5$.

Case 5. The medium item s_i is a D item and all small items belong to
 $(\frac{1}{5}, \frac{1}{4}]$. The reserved space for the medium item is $\frac{2}{3} \times \frac{2}{3}$ in the left-top corner.
 The remaining free space is partitioned by the size of small items according to
 the harmonic algorithm. Totally, the bin can hold 7 small items. Then, the
 520 occupied ratio is at least $\frac{4}{9} + \frac{7}{25} \approx 0.7244$.

Case 6. The medium item s_i is a D item and all small items belong to
 $(\frac{1}{6}, \frac{1}{5}]$. The reserved space for the medium item is $\frac{2}{3} \times \frac{2}{3}$ in the left-top corner.
 The remaining free space is partitioned by the size of small items according to
 the harmonic algorithm. Totally, the bin can hold 9 small items. Then, the
 525 occupied ratio is at least $\frac{4}{9} + \frac{9}{36} \approx 0.6944$. $\square$

Lemma 12. *For all the closed bins with type 110, the least occupied ratio is 0.5 when all items have been packed.*

Proof. The packing scenario is as shown in Figure 3c. The medium items are
 two items of type B . According to the type of small items, there are two possible
 530 cases.

Case 1. The size of small item is in interval $(\frac{1}{5}, \frac{1}{4}]$. The reserved space for the
 medium item is the left-half of the bin. The remaining free space is partitioned
 by the size of small items according to the harmonic algorithm. Totally, the bin
 can hold 8 small items. The occupied ratio is at least $\frac{2}{9} + \frac{8}{25} \approx 0.5422$ when the
 535 bin is closed.

Case 2. The size of small item is in interval $(\frac{1}{6}, \frac{1}{5}]$. The reserved space for the
 medium item is the left-half of the bin. The remaining free space is partitioned
 by the size of small items according to the harmonic algorithm. Totally, the bin
 can hold 10 small items. Then, the occupied ratio is at least $\frac{2}{9} + \frac{10}{36} = 0.5$ when
 540 the bin is closed. $\square$

Theorem 2. *The asymptotic competitive ratio of the proposed online two-dimensional square packing with advice algorithm in this paper is at most 2.*

Proof. From the above lemmas, it is true that the least occupancy of all closed bins is 0.5. Then, the theorem 2 is true. $\square$

545 5. Conclusion

This paper presented two online algorithms for rotated 2D rectangle and square packing problem under the advice complexity model. In this model, each item comes with an advice bit, assigned by an offline oracle, used to differentiate the several packing methods. To the best of our knowledge, this is the first study
550 of the problem of 2D online bin packing with advice. Compared to the results of two-dimensional online packing without advice, our results are better. A question is left open, which is how we can improve the two algorithms, allowing higher occupancy with advice bits less than three.

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