Coupling influences of nonlocal stress and strain gradients on dynamic pull-in of functionally graded nanotubes reinforced nano-actuator with damping effects

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Abstract The paper reports a result of investigation on the dynamic pull-in instability of functionally graded carbon nanotubes (FGCNTs) reinforced nano-actuator considering damping behavior. Here, the nonlocal stress gradient and strain gradient theories are jointly utilized to capture the size effects of nanoscale structures. The material properties of FGCNTs reinforced nano-actuator are temperature-dependent, and the influences of geometrical nonlinearity and intermolecular forces such as van der Waals interaction and Casimir force are also considered. The results indicate that the fundamental frequency decreases with the increase of initial amplitude and electrostatic voltage until drops to zero when the dynamic pull-in occurs. It is shown that the frequencies of nano-actuator decline with the increment of nonlocal stress gradient parameter whereas increasing the strain gradient parameters enlarges the frequencies. In addition, it is observed from the time history and phase portrait that the dynamic pull-in voltage of system with damping behavior is larger than that without damping system, since the dissipative effect of damping requires more energy injected into the system.

Keywords: dynamic pull-in, carbon nanotube reinforced nano-actuator, nonlocal stress gradient, nonlocal strain gradient, Casimir force

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1. Introduction

Electrostatically actuated nano-device is widely applied in nano-switches [1], nano-tweezers [2] and nano-gyroscopes [3]. However, the dynamic pull-in phenomenon as one critical instability might lead the nano-actuators to fail, such as stiction, adhesion, and breakdown [4]. Therefore, investigations on dynamic pull-in instability of electrostatic nano-actuators have been reported in recent years.

With the continuing decrease of devices' characteristic size, some new phenomenon at nanoscale becomes significant, such as van der Waals interaction and Casimir force. Van der Waals force as the essential interaction among molecules, atoms and induced dipole particles, becomes dominant when the initial gap of two electrodes is less than characteristic wave length of material [5, 6]. When the separation of macroscopic bodies is larger than 20 nm, the interaction of multiple dipole particles within two macroscopic solids incorporating retarded effect is described as Casimir force [7-9]. For the electrostatic devices based on MEMS and NEMS (micro-electro-mechanical system and nano-electro-mechanical system), van der Waals interaction and Casimir force should be considered in the study of dynamic pull-in instability of nano-actuators.

Experimental observations indicate the size-dependent properties of nanoscale materials and structures, but the conventional continuum mechanics have no ability to embody the specific feature of size dependency. In recent years, some non-classical elasticity theories considering the effects of material length scales were proposed to investigate the size-dependent mechanical properties of sub-micron and nanoscale structures. These nonlocal elasticity theories are divided into two categories: nonlocal stress gradient model, for instance, Eringen's nonlocal stress gradient theory [10, 11], and nonlocal strain gradient model, such as (modified) couple stress theory [12-14] and strain gradient theory [15, 16].

Within the domain of pull-in instability of electrostatic devices, based on the nonlocal stress gradient theory, some investigations on the size-dependent electromechanical coupling influences of nano-actuators are presented in literatures

[16-24]. From the observation of results, nonlocal stress gradient theory can well explain the softening stiffness with the increment of characteristic length parameter, but the main contribution of strain gradient is the stiffening of material [13,16,17]. Many works indicated the effects of stiffness softening by nonlocal stress gradient model and stiffness enhancement by nonlocal strain gradient model as the internal length scale parameters increase, respectively [22-25]. Thus, the aim of this work is to bring both of length scale parameters together into one unified constitutive equation to investigate the size effect of dynamic pull-in instability of nano-actuators. The exhibiting mechanical responses depend on the coupling influences of nonlocal stress gradient and strain gradient parameters for a specific materials and structures at nanoscale.

Moreover, functionally graded materials (FGMs) incorporating two or more constituent phases vary spatially in terms of non-uniform distribution of the reinforcement phase [26, 27]. Due to the high strength ratio and high stiffness ratio, carbon nanotubes (CNTs) as one kind of promising reinforcement have been widely utilized in the field of nano-composite structures. There have been literatures that studied the nonlinear bending[28, 29], free vibration [30], buckling [31], thermal bucking and post-buckling [32-34] problems of functionally graded CNTs (FGCNTs) reinforced composites.

Furthermore, energy dissipation mechanism of electromechanical structure element plays a critical role in the performance of nano-actuators. Ghayesh et al. [35] investigated the effects of viscous damping and length-scale parameter on nonlinear dynamic behaviors of electrically actuated MEMS resonator based on modified couple stress theory. Azizi et al. [36] studied the thermoelastic damping of transversal vibration in a functionally graded piezoelectric microbeam resonator. And Liu and Wang [37] used hybrid differential transformation method to analyze the squeeze-film damping effect on nonlinear dynamic responses of microbeam actuator. Squeeze-film damping as a main source of dissipation in MEMS/NEMS are extrinsic damping, which is equivalently considered as viscous damping [38]. In addition, Tajaddodianfar et al. [39] investigated viscous and structural damping effects on size-dependent

nonlinear dynamics of shallow nano-arch actuated by electrostatic force.

The present work focuses on the dynamic pull-in instability of FGCNTs reinforced nano-actuator with damping effects, and investigates the coupling effects of nonlocal stress gradient and strain gradient parameters by using the homotopy perturbation method (HPM). Results show that the damping effects (viscous and structural damping) lead to the systematical energy dissipation, thus alter the dynamic characteristics of nano-actuator subjected to electrostatic loads and intermolecular forces. In addition, the influences of temperature-dependent material properties and geometrical distribution of CNTs on fundamental frequency and dynamic pull-in are also described and discussed. Finally, the time story and phase portrait with damping effects are presented to analyze the coupling impacts of nonlocal stress gradient and strain gradient on the nonlinear dynamics characteristics of the size-dependent thermo-electro-mechanical system.

2. Theoretical model and formulae

Fig.1 shows a typical FGCNTs reinforced nano-actuator subjected to electrostatic force and dispersion force, such as van der Waals and Casimir force. The FGCNTs reinforced nano-actuator is considered as the long and thin nanobeam with the length L and width b , where the thickness is h_e . The initial gap between the electrodes is g . The coordinate system is located to the neutral axis at the middle-left end of FGCNTs reinforced nano-actuator, where x and z present the horizontal and perpendicular direction, respectively.

2.1 Constitutive relation based on unified stress and strain gradients

Generally, the mechanical behavior of nano-materials appears in the characteristics of size-dependency, thus the investigation on size effect of nano-structure is important. Based on the Eringen's nonlocal stress gradient theory, the stress at a reference point in a body not only depends on the strain of this point, but also is influenced by all

other points in the body, thus the nonlocal internal energy density potential is expressed as [18]

$$U_0(\varepsilon_{ij}, \varepsilon'_{ij}, \alpha_0) = \frac{1}{2} \varepsilon_{ij} C_{ijkl} \int_V \alpha_0(|x - x'|, e_0 a) \varepsilon'_{kl} dV \quad (1)$$

where ε_{ij} and ε'_{ij} represent the Cartesian components of the strain tensors at points x and x' within the domain V ; C_{ijkl} is the elastic modulus tensor of classic elasticity and α_0 is the attenuation kernel function referred to scale effect with Euclidean distance between two points; e_0 and a are the nonlocal material constant and internal characteristic length, respectively.

To overcome the shortcoming of stiffness softening of nonlocal elasticity different from the experimental observation of nanoscale material, Lim *et al* established the unified stress and strain gradient theory by incorporating nonlocal elasticity and strain gradient theory from two different physical characteristics of nanoscale material and structure [18]. Except for nonlocal effects of the classical strain field, the influence of higher-order strain gradients is considered. The extended internal energy density potential is described as [18]

$$U_0(\varepsilon_{ij}, \varepsilon'_{ij}, \alpha_0; \varepsilon_{ij,m}, \varepsilon'_{ij,m}, \alpha_1) = \frac{1}{2} \varepsilon_{ij} C_{ijkl} \int_V \alpha_0(|x - x'|, e_0 a) \varepsilon'_{kl} dV \\ + \frac{1}{2} l^2 \varepsilon_{ij,m} C_{ijkl} \int_V \alpha_1(|x - x'|, e_1 a) \varepsilon'_{kl,m} dV \quad (2)$$

where $\varepsilon_{ij,m}$ and $\varepsilon'_{ij,m}$ are higher-order counterparts of strain tensors at point x and x' ; α_1 is another attenuation kernel function representing the nonlocal effect of the first-order strain gradient field; l and e_1 are the material length scale originating from higher-order strain gradient field and related material constant, respectively.

If the improved higher-order nonlocal strain gradient theory is further simplified by assuming $e_1 = e_0$ and keeping related terms of order $O(\nabla^2)$, the reduced model incorporating nonlocal elasticity and strain gradients is expressed as

$$\left[1 - (e_0 a)^2 \nabla^2\right] \sigma_{xx} = E_{eff} (1 - l^2 \nabla^2) \varepsilon_{xx} \quad (3)$$

where E_{eff} represents the effective bulk modulus of solids. Note that the nonlocal elastic stress field can be obtained by setting material length scale $l = 0$ as

$$\left[1 - (e_0 a)^2 \nabla^2\right] \sigma_{xx} = E_{eff} \varepsilon_{xx} \quad (4)$$

And the pure strain gradient constitutive is given by setting nonlocal material constant $e_0 = 0$ as

$$\sigma_{xx} = E_{eff} (1 - l^2 \nabla^2) \varepsilon_{xx} \quad (5)$$

2.2 Material properties of FGCNTs reinforced composite

According to the extensive rule of mixture [40] and considering that material properties of CNTs and silicon are temperature-dependent, the effective material properties of CNTs reinforced composite can be expressed

$$E_{eff}(z, T) = \eta V_{cnt}(z) E_{cnt}(T) + V_m(z) E_m(T) \quad (6a)$$

$$\alpha_{eff}(z, T) = V_{cnt}(z) \alpha_{cnt}(T) + V_m(z) \alpha_m(T) \quad (6b)$$

$$\rho_{eff}(z, T) = V_{cnt}(z) \rho_{cnt}(T) + V_m(z) \rho_m(T) \quad (6c)$$

where E_{eff} , α_{eff} and ρ_{eff} are the effective Young's modulus, thermal expansion coefficient and mass density of CNTs reinforced composite, E_{cnt} , α_{cnt} and ρ_{cnt} are the Young's modulus, thermal expansion coefficient and mass density of CNTs reinforcement, and E_m , α_m and ρ_m are the Young's modulus, thermal expansion coefficient and mass density of matrix, respectively. η is the efficiency parameter of CNTs induced by the size-dependent property of CNTs, and V_{cnt} and V_m are the volume fractions of CNTs and matrix.

Here, the distribution of CNTs is either uniformly distributed (UD-CNTRC) or functionally graded in thickness direction of the reinforced elastic layer, such as O-type (FGO-CNTRC) and X-type (FGX-CNTRC), shown as in **Fig.1** [41]. All the geometrical distribution of CNTs considered in this work are expressed as

$$V_{cnt}(z) = V_{cnt}^*, \quad (\text{UD-CNTRC}) \quad (7a)$$

$$V_{cnt}(z) = 2(1 - 2|z|/h_e)V_{cnt}^*, \quad (\text{FGO-CNTRC}) \quad (7b)$$

$$V_{cnt}(z) = 2(2|z|/h_e)V_{cnt}^*, \quad (\text{FGX-CNTRC}) \quad (7c)$$

where

$$V_{cnt}^* = \frac{m_{cnt}}{m_{cnt} + \rho_{cnt} / \rho_m - (\rho_{cnt} / \rho_m)m_{cnt}} \quad (8)$$

and m_{cnt} is the mass fraction of CNTs reinforcement. Therefore, the effective material properties of FGCNTs reinforced composite, such as E_{eff} , α_{eff} and ρ_{eff} , depend on environmental temperature and distribution type of CNTs reinforcement.

2.3 Governing equation of FGCNTs reinforced nano-actuator

Based on Euler-Bernoulli beam theory, the displacement at any point of the FGCNTs reinforced nano-actuator is defined as

$$u_x(x, z, t) = u(x, t) - z \frac{\partial w(x, t)}{\partial x}, \quad u_y(x, z, t) = 0, \quad u_z(x, z, t) = w(x, t) \quad (9)$$

Where $u(x, t)$ and $w(x, t)$ are the displacement components of the mid-plane at time t . And the von Kármán type strain-displacement relation is expressed as

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x} + \frac{1}{2} \left(\frac{\partial u_z}{\partial x} \right)^2 = \frac{\partial u}{\partial x} - z \frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \quad (10)$$

Assuming that the FGCNTs reinforced nano-actuator is subjected to a thermal loading (the change of environmental temperature), the additional strains induced by the thermal loading are considered. Based on coupling nonlocal stress gradient and strain gradient theories [18], the constitutive relation of FGCNTs reinforced nano-actuator is defined as

$$\left[1 - (e_0 a)^2 \nabla^2 \right] \sigma_{xx}(x, z) = E_{eff}(z, T) \left(1 - l^2 \nabla^2 \right) \left[\frac{\partial u}{\partial x} - z \frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 - \alpha_{eff}(z, T) \Delta T \right] \quad (11)$$

On the other hand, the axial force N_x and bending moment M_x induced by the

stress fields are

$$N_x = \int_{-1/2h_e}^{1/2h_e} \sigma_{xx}(x, z) b dz \quad (12)$$

$$M_x = \int_{-1/2h_e}^{1/2h_e} \sigma_{xx}(x, z) z b dz \quad (13)$$

Substituting Eq. (11) into Eqs. (12) and (13) leads to

$$\left[1 - (e_0 a)^2 \frac{\partial^2}{\partial x^2} \right] N_x = \tilde{A} \left(1 - l^2 \frac{\partial^2}{\partial x^2} \right) \left[\frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \right] - N^T \quad (14a)$$

$$\left[1 - (e_0 a)^2 \frac{\partial^2}{\partial x^2} \right] M_x = -\tilde{D} \left(1 - l^2 \frac{\partial^2}{\partial x^2} \right) \frac{\partial^2 w}{\partial x^2} - M^T \quad (14b)$$

where $\{\tilde{A}, \tilde{D}\} = \int_{-1/2h_e}^{1/2h_e} E_{eff} b \{1, z^2\} dz$ are the stretching stiffness and bending stiffness, respectively, and N^T and M^T are the axial membrane force and bending moment induced by the thermal effect, as follows:

$$N^T = \int_{-1/2h_e}^{1/2h_e} E_{eff}(z, T) \alpha_{eff}(z, T) \Delta T b dz, \quad M^T = \int_{-1/2h_e}^{1/2h_e} E_{eff}(z, T) \alpha_{eff}(z, T) \Delta T z b dz \quad (15)$$

By employing Hamilton's principle and no-considering the external axial force, the dynamic equilibrium equations of electrical actuator with damping systems are expressed as

$$\frac{\partial N_x}{\partial x} = I_1 \frac{\partial^2 u}{\partial t^2} \quad (16a)$$

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial}{\partial x} \left(N_x \frac{\partial w}{\partial x} \right) + f_z - C_{vd} \frac{\partial w}{\partial t} - C_{sd} \frac{\partial^2 w}{\partial t \partial x} = I_1 \frac{\partial^2 w}{\partial t^2} \quad (16b)$$

where $I_1 = \int_{-1/2h_e}^{1/2h_e} \rho_{eff}(z, T) b dz$ is the inertia of nano-actuator.

In Eq.(16b), C_{vd} and C_{sd} are, respectively, defined as the viscous and structural damping coefficients based on a generalized Kelvin model, in which the viscous damping rises from the transverse deformation velocity of beam and the structural damping rises from the rotation velocity of the cross section of beam [39].

In the above formula, f_z is the transversely distributed force which is composed of the electrostatic force with first-order corrected fringing field effect, van der Waals force per unit length and Casimir force, as follows:

$$f_z = F_e + F_{vdW} + F_{Cas} \quad (17)$$

where, F_e is the electrostatic force per unit length exerted on the nano-actuator as follows [42]

$$F_e = \frac{\varepsilon_0 \bar{V}_e^2 b}{2(g-w)^2} \left(1 + 0.65 \frac{g-w}{b} \right) \quad (18)$$

here $\varepsilon_0 = 8.854 \times 10^{-12} C^2 N^{-1} m^{-2}$ is the permittivity of vacuum, $\bar{V}_e$ is the applied electrostatic voltage and g is the initial gap between the bottom surface of nano-device and the substrate.

In Eq.(17), F_{vdW} is van der Waals force per unit length induced by the intermolecular attraction between nano-actuator and the substrate, as follows[43]

$$F_{vdW} = \frac{\bar{A}b}{6\pi(g-w)^3} \quad (19)$$

where $\bar{A}$ is the Hamaker constant.

In Eq.(17), F_{Cas} is the Casimir force as the macro effect of quantum field fluctuation per unit length, as follows [44]

$$F_{Cas} = \frac{\pi^2 \hbar c_0 b}{240(g-w)^4} \quad (20)$$

where $\hbar = 1.055 \times 10^{-34} Js$ is the Planck's constant divided by 2π and $c = 3 \times 10^8 m \cdot s^{-1}$ is the speed of light.

Substituting Eqs. (14a) and (14b) into Eq. (16a) and neglecting the axial accelerator of nano-actuator give

$$\begin{aligned} \frac{\partial N_x}{\partial x} = 0 &\Leftrightarrow \tilde{A} \left[\frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \right] = C_1(t) \\ \Rightarrow \tilde{A} \left[u + \int_0^x \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 dx \right] &= xC_1(t) + C_2(t) \end{aligned} \quad (21)$$

where C_1 and C_2 are integration of constant based on time. After some

mathematical manipulations, one has $C_1(t) = \frac{\tilde{A}}{L} \int_0^L \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 dx$ and $C_2(t) = 0$. Thus, the term of axial force is summarized to

$$N_x = \frac{\tilde{A}}{L} \int_0^L \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 dx - N^T \quad (22)$$

Substituting Eqs.(14b) and (22) into Eq.(16b), yields

$$\begin{aligned} M_x &= (e_0 a)^2 \left[I_1 \frac{\partial^2 w}{\partial t^2} - \frac{\partial}{\partial x} \left(N_x \frac{\partial w}{\partial x} \right) - f_z + C_{vd} \frac{\partial w}{\partial t} \right. \\ &\quad \left. + C_{sd} \frac{\partial^2 w}{\partial t \partial x} \right] - \tilde{D} \left(1 - l^2 \frac{\partial^2}{\partial x^2} \right) \frac{\partial^2 w}{\partial x^2} - M^T \\ &= (e_0 a)^2 \left\{ I_1 \frac{\partial^2 w}{\partial t^2} - \left[\frac{\tilde{A}}{L} \int_0^L \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 dx - N^T \right] \frac{\partial^2 w}{\partial x^2} - f_z \right. \\ &\quad \left. + C_{vd} \frac{\partial w}{\partial t} + C_{sd} \frac{\partial^2 w}{\partial t \partial x} \right\} - \tilde{D} \left(1 - l^2 \frac{\partial^2}{\partial x^2} \right) \frac{\partial^2 w}{\partial x^2} - M^T \end{aligned} \quad (23)$$

Then substituting Eq.(22) and Eq.(23) into Eq.(14b) generates the governing equation based on nonlocal elasticity and strain gradients, as follows:

$$\begin{aligned} &\left\{ \tilde{D} + (e_0 a)^2 \left[\frac{\tilde{A}}{L} \int_0^L \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 dx - N^T \right] \right\} \frac{\partial^4 w}{\partial x^4} - \tilde{D} l^2 \frac{\partial^6 w}{\partial x^6} + I_1 \frac{\partial^2 w}{\partial t^2} \\ &- (e_0 a)^2 I_1 \frac{\partial^4 w}{\partial t^2 \partial x^2} - \left[\frac{\tilde{A}}{L} \int_0^L \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 dx - N^T \right] \frac{\partial^2 w}{\partial x^2} \\ &- \left[1 - (e_0 a)^2 \frac{\partial^2}{\partial x^2} \right] \left(f_z - C_{vd} \frac{\partial w}{\partial t} - C_{sd} \frac{\partial^2 w}{\partial t \partial x} \right) = 0 \end{aligned} \quad (24)$$

where the nonlocal governing equation can be obtained by setting $l=0$ and the pure strain gradient governing equation can be given by setting $e_0=0$.

Substituting the transversely distributed forces f_z in Eq.(17) which represents the sum of all dispersion forces from different physical origins, including electrostatic force incorporating fringing field effect f_{elec} , van der Waals force f_{vdW} ($n=0$) and Casimir force f_{Cas} ($n=1$) into Eq. (24) leads to the dimensionless governing equation based on nonlocal elasticity and strain gradients, as follows:

$$\begin{aligned}
& K_3 \frac{\partial^4 W}{\partial \xi^4} + \mu \left[K_1 \int_0^1 \left(\frac{\partial W}{\partial \xi} \right)^2 d\xi - K_2 N^T \right] \frac{\partial^4 W}{\partial \xi^4} - \eta K_3 \frac{\partial^6 W}{\partial \xi^6} + K_4 \frac{\partial^2 W}{\partial \tau^2} - \mu K_4 \frac{\partial^4 W}{\partial \xi^2 \partial \tau^2} \\
& - \left[K_1 \int_0^1 \left(\frac{\partial W}{\partial \xi} \right)^2 d\xi - K_2 N^T \right] \frac{\partial^2 W}{\partial \xi^2} + \tilde{C}_{sd} \left(\frac{\partial^2 W}{\partial \tau \partial \xi} - \mu \frac{\partial^4 W}{\partial \tau \partial \xi^3} \right) + \tilde{C}_{vd} \left(\frac{\partial W}{\partial \tau} - \mu \frac{\partial^3 W}{\partial \tau \partial \xi^2} \right) \\
& - \left(1 - \mu \frac{\partial^2}{\partial \xi^2} \right) \left[\frac{V_e^2 \gamma}{1-W} + \frac{V_e^2}{(1-W)^2} + \frac{(1-n)\alpha_3}{(1-W)^3} + \frac{n\alpha_4}{(1-W)^4} \right] = 0
\end{aligned} \tag{25}$$

In the above formula, the dimensionless variables are given by

$$\xi = \frac{x}{L}, \quad W = \frac{w}{g}, \quad \tau = t \sqrt{\frac{D_m}{I_m L^4}} \tag{26a}$$

$$\begin{aligned}
K_1 &= \frac{\tilde{A} g^2}{2D_m}, \quad K_2 = \frac{L^2}{D_m}, \quad K_3 = \frac{\tilde{D}}{D_m}, \quad K_4 = \frac{I_1}{I_m}, \\
\mu &= \left(\frac{e_0 a}{L} \right)^2, \quad \eta = \left(\frac{l}{L} \right)^2, \quad \tilde{C}_{sd} = \frac{L}{\sqrt{D_m I_m}} C_{sd}, \quad \tilde{C}_{vd} = \frac{L^2}{\sqrt{D_m I_m}} C_{vd}, \\
V_e &= \bar{V}_e \sqrt{\frac{\varepsilon_0 L^4}{2g^3 D_m}}, \quad \gamma = 0.65 \frac{g}{b}, \quad \alpha_3 = \frac{\bar{A} L^4}{6\pi g^4 D_m}, \quad \alpha_4 = \frac{\pi^2 \hbar c_0 L^4}{240 g^5 D_m}
\end{aligned} \tag{26b}$$

where D_m and I_m represent the bending stiffness and moment of inertia of nano-actuator without CNTs reinforcement, respectively. In order to solve 6th order differential equation (25) based on nonlocal stress-strain gradients, for the clamped-clamped nano-actuator, four nondimensional classical boundary conditions and two additional unclassical boundary conditions originate from strain gradients are expressed as [45]:

$$\begin{aligned}
& W(\xi, \tau)|_{\xi=0} = 0, \quad W(\xi, \tau)|_{\xi=1} = 0, \\
& \frac{\partial W(\xi, \tau)}{\partial \xi}|_{\xi=0} = 0, \quad \frac{\partial W(\xi, \tau)}{\partial \xi}|_{\xi=1} = 0, \\
& \frac{\partial^2 W(\xi, \tau)}{\partial \xi^2}|_{\xi=0} = 0, \quad \frac{\partial^2 W(\xi, \tau)}{\partial \xi^2}|_{\xi=1} = 0
\end{aligned} \tag{27}$$

For the nonlinear dynamic system, applying the Galekin's decomposition method to Eq. (25) yields to a second-order nonlinear ordinary differential equation [46]. Assume the deflection function has the following form as

$$W(\xi) = q(\tau)\phi(\xi) \quad (28)$$

where $q(\tau)$ represents an undetermined time-dependent function and $\phi(\xi)$ is the linear mode shape function satisfying the boundary condition (27), which is given by

$$\phi(\xi) = \cosh \lambda_m \xi - \cos \lambda_m \xi - \frac{\cosh \lambda_m - \cos \lambda_m}{\sinh \lambda_m - \sin \lambda_m} (\sinh \lambda_m \xi - \sin \lambda_m \xi) \quad (29)$$

where λ_m is the m -th root of characteristic equation of clamped-clamped nanobeam.

Substituting Eqs. (28) and (29) into Eq. (25), using Taylor's series expansion for dimensionless transversely distributed forces (i.e. four-order expansion of deflection having high enough accuracy), and multiplying by the linear mode shape function $\phi(\xi)$ and then integrating along the nano-actuator span, one can obtain

$$\ddot{q}(\tau) + \psi_5 \dot{q}(\tau) + \psi_4 (q(\tau))^4 + \psi_3 (q(\tau))^3 + \psi_2 (q(\tau))^2 + \psi_1 q(\tau) + \psi_0 = 0 \quad (30)$$

where the parameters $\psi_i, i \in [0, 5]$ are list in **Appendix A**.

3. Closed-form equations of frequency and amplitude

The capacitive-type MEMS and NEMS devices are generally actuated by a step voltage. When an external voltage is not applied, the system is at rest and has no stored energy; however, when a step voltage is applied, the energy of the system is stored as kinetic and potential energy. Over time, the stored energy before related to the equilibrium position is dissipated though damping effects (i.e. viscous and structural damping). According to energy balance of the system at any time, one obtains [47]

$$E_{system} = E_{kinetic} + E_{potential} + E_{dissipated} \quad (31)$$

where E_{system} is the energy inputting into the system, $E_{kinetic}$ is the kinetic energy of the system, $E_{potential}$ is the elastic potential energy stored in the system and $E_{dissipated}$ is the dissipated energy from damping effects of the system.

To achieve the lowest value of the dynamic pull-in voltage, the damping effects are

generally suppressed. The minimum dynamic pull-in voltage can be obtained by neglecting the damping effect, thus Eq. (30) can be rewritten to further calculate the dynamic pull-in voltage without damping effect, as follows:

$$\ddot{q}(\tau) + \psi_4(q(\tau))^4 + \psi_3(q(\tau))^3 + \psi_2(q(\tau))^2 + \psi_1 q(\tau) + \psi_0 = 0 \quad (32)$$

The basic character of the HPM is to employ homotopy technique and embed a parameter which typically ranges from zero to one. When the embedding parameter is zero, the equation is one of a linear system; when it is one, the equation is the same as the original one[48].

To illustrate the basic idea of the HPM, consider the following nonlinear differential equation,

$$\mathcal{A}(a) - f(r) = 0, \quad r \in \Omega \quad (33)$$

with associated boundary conditions as

$$\mathcal{B}\left(a, \frac{\partial a}{\partial s}\right) = 0, \quad r \in \Gamma \quad (34)$$

where $\mathcal{A}$ is a general differential operator, $\mathcal{B}$ is a boundary operator, $f(r)$ is a known analytical function, and Γ is the boundary of the domain Ω . In general, the operator $\mathcal{A}$ consists of linear part $\mathcal{L}$ and nonlinear $\mathcal{N}$. Thus, Eq. (33) is expressed as

$$\mathcal{L}(a) + \mathcal{N}(a) - f(r) = 0 \quad (35)$$

Based on homotopy technique, a homotopy is constructed as [49]

$$\mathcal{H}(v, p) = (1 - p)[\mathcal{L}(v) - \mathcal{L}(a_0)] + p[\mathcal{A}(a) - f(r)] = 0 \quad (36)$$

where $p \in [0, 1]$ denotes the embedding parameter and a_0 is an initial value for the solution of Eq. (33). Note that the homotopy $\mathcal{H}(v, p)$ depends on not only the undetermined variable a , but also the embedding parameter $p \in [0, 1]$. The solution of Eq. (36) can be described as a power series of p as

$$v = v_0 + p v_1 + p^2 v_2 + p^3 v_3 + \dots \quad (37)$$

and the embedding parameter p is used to expand the square of the undetermined oscillation fundamental frequency as

$$\omega_0 = \omega^2 - p\omega_1 - p^2\omega_2 - L \quad (38)$$

where ω_0 is the coefficient of $a(r)$ in Eq. (36), satisfying kinetic boundary conditions, and the coefficients $\omega_j (j=1,2,\dots)$ are undetermined parameters.

When $p \rightarrow 1$, the approximated solution of Eq. (35) and the fundamental frequency ω , respectively, are expressed as

$$a = \lim_{p \rightarrow 1} v = v_0 + v_1 + v_2 + v_3 + L \quad (39)$$

$$\omega^2 = \omega_0 + \omega_1 + \omega_2 + L \quad (40)$$

Utilizing HPM to solve Eq. (32), the homotopy equation can be constructed in the following form as

$$\mathcal{H}(q, p) = (1-p)[\ddot{q} + \psi_1 q] + p[\ddot{q} + \psi_0 + \psi_1 q + \psi_2 q^2 + \psi_3 q^3 + \psi_4 q^4] = 0 \quad (41)$$

Suppose the solution of Eq.(32) has the following form as

$$q(\tau) = q_0(\tau) + pq_1(\tau) + p^2 q_2(\tau) + L \quad (42)$$

And the coefficient of q is expanded into a series of p as

$$\psi_1 = \omega^2 - p\omega_1 - p^2\omega_2 - L \quad (43)$$

Substituting Eqs. (42) and (43) into Eq. (41), and equating the terms with the identical powers of p ,

$$p^0: \ddot{q}_0(\tau) + \omega^2 q_0(\tau) = 0, \quad q_0(0) = A, \quad \dot{q}_0(0) = 0, \quad (44a)$$

$$p^1: \ddot{q}_1(\tau) + \omega^2 q_1(\tau) = \omega_1 q_0(\tau) - [\psi_0 + \psi_2 (q_0(\tau))^2 + \psi_3 (q_0(\tau))^3 + \psi_4 (q_0(\tau))^4], \\ q_1(0) = 0, \quad \dot{q}_1(0) = 0, \quad (44b)$$

$$p^2: \ddot{q}_2(\tau) + \omega^2 q_2(\tau) = \omega_1 q_1(\tau) + \omega_2 q_0(\tau) - [2\psi_2 q_0(\tau) q_1(\tau) + 3\psi_3 (q_0(\tau))^2 q_1(\tau) \\ + 4\psi_4 (q_0(\tau))^3 q_1(\tau)], \quad q_2(0) = 0, \quad \dot{q}_2(0) = 0 \quad (44c)$$

It is easy to obtain the solution of Eq. (44a) as

$$q_0(\tau) = A \cos(\omega \tau) \quad (45)$$

Because the solution of Eq. (44b) should not incorporate the so-called secular term $\cos(\omega \tau)$, substituting Eq.(45) into Eq. (44b) generates

$$\begin{aligned} \frac{8\omega}{4\omega}(\tau) + \omega^2 q_1(\tau) = & \left(\omega_1 A - \frac{3}{4} \psi_3 A^3 \right) \cos(\omega \tau) - \left(\frac{1}{2} \psi_2 A^2 + \frac{1}{2} \psi_4 A^4 \right) \cos(2\omega \tau) \\ & - \frac{1}{2} \psi_2 A^2 - \frac{3}{8} \psi_4 A^4 - \psi_0 - \frac{1}{4} \psi_3 A^3 \cos(3\omega \tau) - \frac{1}{8} \psi_4 A^4 \cos(4\omega \tau) \end{aligned} \quad (46)$$

Note the secular terms of Eq. (46) should be zero, thus one can obtain

$$\omega_1 = \frac{3}{4} \psi_3 A^2 \quad (47)$$

Solving the updated Eq. (46) can give rise to one second-order approximated expression of $q_1(\tau)$ as

$$\begin{aligned} q_1(\tau) = & \frac{1}{480\omega^2} \left[(96\psi_4 A^4 + 160\psi_2 A^2 - 15\psi_3 A^3 + 480\psi_0) \cos(\omega \tau) \right. \\ & + (80\psi_4 A^4 + 80\psi_2 A^2) \cos(2\omega \tau) + 15\psi_3 A^3 \cos(3\omega \tau) \\ & \left. + 4\psi_4 A^4 \cos(4\omega \tau) - (480\psi_0 + 180\psi_4 A^4 + 240\psi_2 A^2) \right] \end{aligned} \quad (48)$$

Furthermore, Eq. (43) with two terms approximation of power series when $p \rightarrow 1$,

$$\omega_2 = \omega^2 - \psi_1 - \omega_1 \quad (49)$$

Substituting Eq. (49) into Eq. (44c) for $q_2(\tau)$ and requiring the secular terms to be zero lead to

$$\begin{aligned} S(\omega) = & -\omega^4 A + \psi_1 \omega^2 A + \frac{3}{4} \psi_3 \omega^2 A^3 - 2\psi_0 \psi_2 A - \frac{5}{6} \psi_2^2 A^3 \\ & + \frac{3}{2} \psi_0 \psi_3 A^2 + \frac{1}{2} \psi_2 \psi_3 A^4 - \frac{3}{128} \psi_3^2 A^5 - 3\psi_0 \psi_4 A^3 \\ & - \frac{7}{4} \psi_2 \psi_4 A^5 + \frac{3}{10} \psi_3 \psi_4 A^6 - \frac{63}{80} \psi_4^2 A^7 = 0 \end{aligned} \quad (50)$$

By solving Eq. (50) respect to the fundamental frequency ω , the analytical function of second-order frequency-amplitude relationship is obtained as

$$\omega(A) = \left[\frac{1}{2}\psi_1 + \frac{3}{8}\psi_3 A^2 + \left(\frac{1}{4}\psi_1^2 - 2\psi_0\psi_2 + \frac{3}{2}\psi_0\psi_3 A + \frac{3}{8}\psi_1\psi_3 A^2 - 3\psi_0\psi_4 A^2 - \frac{5}{6}\psi_2^2 A^2 \right. \right. \\ \left. \left. + \frac{1}{2}\psi_2\psi_3 A^3 + \frac{15}{128}\psi_3^2 A^4 - \frac{7}{4}\psi_2\psi_4 A^4 + \frac{3}{10}\psi_3\psi_4 A^5 - \frac{63}{80}\psi_4^2 A^6 \right)^{1/2} \right]^{1/2} \quad (51)$$

4. Results and discussion

4.1 Validation of analytical method

To check the validity and accuracy of the HPM used in the paper, the results calculated by the HPM are compared with the experimental data and numerical methods. Gupta *et al* [50] fabricated one doubly-clamped polysilicon microbeam and the parameters of the microbeam are listed in **Table 1**. The comparisons of dynamic pull-in voltage obtained from different methods are listed in **Table 2**. σ_0 is the gap-dependent residual stress induced by thermal mismatch and fabrication of micromachining. It is seen from Table 2 that the present results appear in good agreement with the experimental data, and the dynamic pull-in voltage calculated in present model is also well agree with those by other methods, such as Displacement Iteration Pull-In Extraction (DIPIE algorithm) [51] and Energy Balance Method (EBM)[52]. The accuracy of HPM in the paper is also verified by comparing with those by the Runge-Kutta method, shown as in **Fig.2**, where UD-CNTRC type nano-actuator with Casimir force is considered, and $V_e = 8$, $V_{cnt} = 0.12$, $\mu = 0.01$, $\Delta T = 200K$ and $\alpha_4 = 10$. It can be seen from **Fig.2** that the non-dimensional amplitudes by the second-order HPM have enough high accuracy with the results by numerical method. In addition, **Table 3** describes the convergence of HPM when van der Waals force and Casimir force are, respectively, considered. The absolute error decreases with the increase of initial amplitude, but the highest error are only 0.48% for van der Waals force and 0.65% for Casimir force, respectively. Therefore the HPM used in the paper exhibits good accuracy and convergence in investigating dynamic

pull-in of FGCNTs reinforced nano-actuator.

To further verify present analytical model, a comparison of the present method with nonlocal stress and strain gradient theory from references [1, 53, 54] has been given in **Table 4**, in which the comparison shows that the present HPM has good accuracy to compute the pull-in voltage of nano-actuator with size-dependency.

4.2 Coupling effects of stress and strain gradients

Table 5 gives the temperature-dependent material properties of CNTs and silicon matrix under the temperature conditions of 300K (room temperature), 400K, 500K, 600K and 700K [28, 55, 56], where we select the elastic modulus $[1\bar{1}0]$ direction silicon and perform data fitting to obtain the values for corresponding temperature points. The resultant temperature-dependent elastic modulus of silicon satisfy the relationship of $E_m = (175.639 - 0.01987T)\text{MPa}$, where $T = T_0 + \Delta T$ and $T_0 = 300\text{K}$ (room temperature). **Table 6** lists the scale-dependent efficiency parameter of CNTs for volume fraction of 0.12, 0.17 and 0.28 which are determined by MD simulation and the rule of mixture incorporating scale-dependent effect [40]. The physical and geometrical parameters of FGCNTs reinforced nano-actuator for latter numerical analysis are taken as $h_e = 10\text{nm}$, $L = 25h_e$, $g = 0.3h_e$ and $b = 4.5h_e$, respectively.

The respective influences of nonlocal stress gradient and strain gradient on relation of fundamental frequency and applied voltage are shown in **Fig.3**. When the voltage increases to a certain critical value, the dynamic pull-in phenomenon happens; this critical voltage is called dynamic pull-in voltage. It is found that the frequency of system declines with the increase of applied voltage, and the frequency drops to zero at the pull-in state. Furthermore, the magnitude of frequency decreases with the increase of nonlocal parameter, but with the decrease of strain gradient parameter. Moreover, the influence of nonlocal stress field theory decreases the stiffness of nano-actuator as the increase of the nonlocal parameter, whereas the increase of the

high-order strain gradient induces the stiffness of nano-actuator to increase.

Fig.4 depicts the influences of geometrical distribution of CNTs on the vibration frequency of nano-actuator for different nonlocal stress gradient and strain gradient models. It is shown that based on the nonlocal stress gradient model, the FGX-CNTRC nano-actuator undergoes the wider range of variation of frequency as the nonlocal parameter increases by comparing among the UD-CNTRC and FGO-CNTRC type nano-actuator. Based on the nonlocal strain gradient model, the effect of distribution form of CNTs in nano-actuator does not change the variable range of frequency of the nano-actuator.

Fig.5 demonstrates the effects of volume fraction of CNTs in nano-actuator on the frequency of the nano-actuator based on different nonlocal stress and strain gradient models. It is concluded that to enhance the volume fraction of CNTs leads to the increase of fundamental frequency of nano-actuator, since the increment of CNTs in nano-actuator improves the stiffness of the nano-actuator. Moreover, the frequency of nano-actuator declines with the increase of stress gradient parameter but increases with the increase of strain gradient parameter.

The influences of environmental temperature change on the frequency of nano-actuator are illustrated in **Fig.6**. It is seen that the effect of temperature change on the frequency of nano-actuator based on different nonlocal models are significant, and the increase of nonlocal stress gradient parameter decreases the frequency of nano-actuator for different temperature changes. On the other hand, the influence of strain gradient parameter on the differences of frequency at different temperature change is small, since the influence of nonlocal stress gradient is larger than that of higher-order strain gradient.

Fig.7 describes the effects of van der Waals force and Casimir force on the frequency of nano-actuator for two nonlocal models. It is seen that as the nonlocal stress gradient parameter increases the effects of van der Waals force and Casimir force on the frequency of nano-actuator gradually increase, but as the strain gradient parameter increases the effects of van der Waals force and Casimir force on the frequency of nano-actuator gradually decrease. Also, the frequency of nano-actuator

decrease with the increment of van der Waals force and Casimir force until the pull-in instability occurs.

Fig.8 depicts the nonlinear coupling effects of nonlocal stress gradient and strain gradient on the fundamental frequency of nano-actuator. It is found that the influences of stress and strain gradient parameters on the frequency nano-actuator are contrary and the relation of frequency and stress/strain gradient parameters shows significantly nonlinear. If the material constant, characteristic length parameter and strain gradient parameter of nanoscale materials and structures are determined, the unified stress and strain gradient model is able to accurately predict the dynamic behaviors and dynamic pull-in instability of nano-actuator incorporating size effect.

Furthermore, the influences of stress gradient and strain gradient parameters on the frequencies of first three modes of nano-actuator have also been described. The first three frequencies as functions of applied voltage are discussed, and the new results are shown in **Fig.9**. It is seen from **Fig.9** that the nonlocal stress gradient parameter significantly decreases the frequencies of modes before the pull-in voltage; whereas, the frequencies of modes increase as the strain gradient parameter increases. In addition, because the pull-in instability firstly occurs at the primary frequency, thus the frequencies of higher modes might still exist after pull-in state of nano-actuator, which is similar to the result from Ref.[57].

4.3 Dynamic pull-in and bifurcation analysis

To investigate the dynamic characteristics of nano-actuator near pull-in state, the curves of time history and phase portrait of the nano-actuator under different applied voltages are illustrated in **Fig.10**. The nonlocal stress and strain gradient parameters are taken as $\mu = 0.1$ and $\eta = 0.1$; the non-dimensional structural and viscous damping parameters are $\tilde{C}_{sd} = 1$ and $\tilde{C}_{vd} = 1$, respectively. It is from **Fig.10(a)** that when applied a lower voltage (i.e. $V_e = 17.78$ for UD-CNTRC type nano-actuator), the non-dimension deflection decreases as the non-dimension time increases due to the structural and viscous damping effects; whereas if increasing the applied voltage to

the dynamic pull-in value (i.e. $V_e=17.79$), the system becomes unstable and the nano-actuator collapses onto the substrate. Moreover, it is found from **Fig.10(b)** that before the pull-in state, the phase portrait of system has one stable center, and as the time history progresses, the phase portrait converges into the center point. However, the system becomes dynamically unstable at the pull-in state and appears bifurcation from the unstable saddle node. When the voltage beyond the pull-in voltage (i.e. $V_e=17.78$), the stable center vanishes and the nano-actuator sticks onto the substrate.

In order to observe the influences of damping effects, **Fig.11** depicts the curves of time history and phase portrait of nano-actuator without damping effects ($\tilde{C}_{sd}=0$ and $\tilde{C}_{vd}=0$). It is obviously seen that compared to the dynamic pull-in voltage with damping, the dynamic pull-in voltage of system without damping effects decreases to $V_e=17.72$ and the maximum velocity of system is slightly smaller, since the dissipation of energy injected into system increases the dynamic pull-in voltage of the damping nano-actuator. The non-dimensional deflection of nano-actuator without damping keep the identical amplitude, and the system without damping has one stable center and there exists periodic orbits around the center. When the value of voltage surpasses the pull-in voltage, the stable center disappears and the saddle bifurcation occurs, thus the nano-actuator drops onto the substrate. It is concluded that the damping effects increase the dynamic pull-in voltage of system because higher initial energy are required in order to cause dynamic instability of the nano-actuator.

Fig.12 shows the coupling influences of nonlocal stress and strain gradient parameters on time history and phase portrait of the nano-actuator at the dynamic pull-in state. It is seen from **Fig.12(a)** that as the nonlocal stress gradient parameter increases, the dynamic pull-in time increases while the dynamic pull-in deflection of system decreases. On the other hand, as the nonlocal strain gradient parameter increases, the dynamic pull-in time decreases while the dynamic pull-in deflection increases. **Fig.12(b)** shows that the maximum velocity of system decreases with the increase of nonlocal stress gradient parameter, while increases with the increment of strain gradient parameter. The coupling influences of nonlocal stress and strain

gradient capture the significant size-dependence of dynamic pull-in instability by unifying stress and strain gradients from different physical characteristics of nanoscale materials and structures.

5. Conclusion

An analytical model of FGCNTs reinforced nano-actuator with damping effect, subjected to electrostatic loads are presented to study the dynamic pull-in instability, and the size effects of electrostatically actuated nano-device are characterized by coupling impacts of nonlocal elasticity and strain gradient theory in this work. Main conclusions are given by

- 1) The frequency of nano-actuator decreases with the increment of initial amplitude of the nano-actuator and applied voltage, then the frequency drops to zero at dynamic pull-in instability.
- 2) The frequency of nano-actuator decreases with the increase of nonlocal stress gradient parameter while increases with the increment of nonlocal strain gradient parameter.
- 3) The frequency of FGX-CNTRC type system is higher than that of UD-CNTRC and FGO-CNTRC due to the distinct stiffness enhancement of CNTs in nano-actuator.
- 4) The magnitude of non-dimensional deflection gradually declines due to the dissipative effects of damping and thus the dynamic pull-in voltage is larger than that of undamped system.
- 5) The maximum velocity of system decreases as the nonlocal stress gradient parameter increases whereas increase with the increment of strain gradient parameters.

Appendix A

The associated coefficients of the dimensionless nonlocal governing equation of motion are expressed as follows

$$\psi_i = \frac{H_{i+1}}{H_0}, \quad i \in [0, 5],$$

$$H_0 = K_4 \int_0^1 \phi \phi d\xi - \mu K_4 \int_0^1 \phi'' \phi d\xi$$

$$H_1 = -\left[V_e^2(1+\gamma) + (1-n)\alpha_3 + n\alpha_4 \right] \int_0^1 \phi d\xi$$

$$H_2 = \left(k_3 - \mu k_2 N^T \right) \int_0^1 \phi^{(4)} \phi d\xi - \eta k_3 \int_0^1 \phi^{(6)} \phi d\xi + k_2 N^T \int_0^1 \phi'' \phi d\xi \\ - \left[V_e^2(2+\gamma) + 3(1-n)\alpha_3 + 4n\alpha_4 \right] \int_0^1 \phi(\phi - \mu\phi'') d\xi$$

$$H_3 = -\left[V_e^2(3+\gamma) + 6(1-n)\alpha_3 + 10n\alpha_4 \right] \int_0^1 \phi \left[\phi^2 - 2\mu(\phi')^2 - 2\mu\phi\phi'' \right] d\xi$$

$$H_4 = k_1 \left[\int_0^1 (\phi')^2 d\xi \right] \left(\mu \int_0^1 \phi^{(4)} \phi d\xi - k_1 \int_0^1 \phi\phi'' d\xi \right) \\ - \left(V_e^2(4+\gamma) + 10(1-n)\alpha_3 + 20n\alpha_4 \right) \int_0^1 \phi^2 \left[\phi^2 - 6\mu(\phi')^2 - 3\mu\phi\phi'' \right] d\xi$$

$$H_5 = -\left(V_e^2(5+\gamma) + 15(1-n)\alpha_3 + 35n\alpha_4 \right) \int_0^1 \phi^3 \left[\phi^2 - 12\mu(\phi')^2 - 4\mu\phi\phi'' \right] d\xi$$

$$H_6 = \hat{C}_{sd} \left(\int_0^1 \phi' \phi d\xi - \mu \int_0^1 \phi^{(3)} \phi d\xi \right) + \hat{C}_{vd} \left(\int_0^1 \phi \phi d\xi - \mu \int_0^1 \phi'' \phi d\xi \right)$$

Acknowledge

The authors acknowledge financial support from National Science Foundation of China under Number 11172165.

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Capital of Figures

Fig.1. (a) Schema of FGCNTs reinforced nano-actuator,
(b) Geometrical distribution of CNTs in the matrix.

Fig. 2. Validation of time history by HPM and numerical method.

Fig.3. (a) Effects of nonlocal stress gradient parameter,
(b) Effects of strain gradient parameter,
where UD-CNTRC, $V_{cnt} = 0.12$, $A = 0.2$, $\Delta T = 200K$, $\gamma = 0.65$, $\alpha_4 = 1$.

Fig.4. Effects of geometrical distribution of CNTs on the frequency of nano-actuator
for two nonlocal models ,where (a) stress gradient model, (b) strain gradient model,

$$V_{cnt} = 0.12, V_e = 5, A = 0.2, \Delta T = 200K, \gamma = 0.65, \alpha_4 = 1.$$

Fig.5. Effects of on volume fraction of CNTs on the frequency of FGX-CNTRC
nano-actuator for two nonlocal models, where (a) stress gradient model,
(b) strain gradient model, and $V_e = 5$, $A = 0.2$, $\Delta T = 200K$, $\gamma = 0.65$, $\alpha_4 = 1$.

Fig.6. Effects of temperature change on the frequency of FGX-CNTRC nano-actuator
for two nonlocal models, where (a) stress gradient model, (b) strain gradient model,

$$\text{and } V_{cnt} = 0.12, V_e = 5, A = 0.2, \gamma = 0.65, \alpha_4 = 1.$$

Fig.7. Effects of dispersion forces on the frequency of FGX-CNTRC actuator
for two nonlocal models, where (a) stress gradient model, (b) strain gradient model,

$$\text{and } V_{cnt} = 0.12, V_e = 5, A = 0.2, \Delta T = 200K, \gamma = 0.65.$$

Fig.8. Coupling effects of stress and strain gradient parameters on frequency of
UD-CNTRC nano-actuator, where, $V_{cnt} = 0.12$, $V_e = 10$, $A = 0.2$,
 $\Delta T = 200K$, $\gamma = 0.65$, $\alpha_4 = 1$.

Fig.9. Effects of applied voltage on the frequency of high modes,
where (a) nonlocal stress gradient parameter, (b) strain gradient parameter,
and FGO-CNTRC, $V_{cnt} = 0.12$, $A = 0.2$, $\Delta T = 200K$, $\gamma = 0.65$, $\alpha_4 = 1$.

Fig. 10. The time history (a) and the phase portrait (b) of actuator near dynamic pull-in state, where $\mu=0.1$ and $\eta=0.1$, and the damping effects are taken as

$$\tilde{C}_{sd}=1 \text{ and } \tilde{C}_{vd}=1 \text{ .}$$

Fig.11. The time history (a) and the phase portrait (b) of actuator near dynamic pull-in state, where $\mu=0.1$ and $\eta=0.1$, and without damping

$$\text{system } \tilde{C}_{sd}=0 \text{ and } \tilde{C}_{vd}=0 \text{ .}$$

Fig.12. Comparison of stress gradient and strain gradient parameters on the time history (a) and the phase portrait (b) of actuator with damping system at dynamic pull-in state.

Table 1. Referent parameters of dynamic pull-in experiment for verification of HPM.

Table 2. Comparison of dynamic pull-in voltage by different methods.

Table 3. Comparison of vibrational amplitudes by HPM and numerical method.

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Table 5. Temperature-dependent material property of CNTs [28] and silicon [55, 56].

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Fig.6. Effects of temperature change on the frequency of FGX-CNTRC nano-actuator for two nonlocal models, where (a) stress gradient model, (b) strain gradient model, and $V_{cnt} = 0.12, V_e = 5, A = 0.2, \gamma = 0.65, \alpha_4 = 1$.

Fig.7. Effects of dispersion forces on the frequency of FGX-CNTRC actuator for two nonlocal models, where (a) stress gradient model, (b) strain gradient model,

and $V_{cnt} = 0.12, V_e = 5, A = 0.2, \Delta T = 200\text{K}, \gamma = 0.65$.

Fig.8.Coupling effects of stress and strain gradient parameters on frequency of UD-CNTRC nano-actuator, where $V_{cnt} = 0.12$, $V_e = 10$, $A = 0.2$,

$$\Delta T = 200\text{K}, \gamma = 0.65, \alpha_4 = 1.$$

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(a)

(b)

Fig. 10. The time history (a) and the phase portrait (b) of actuator near dynamic pull-in state, where $\mu = 0.1$ and $\eta = 0.1$, and the damping effects are taken as

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Fig.11. The time history (a) and the phase portrait (b) of actuator near dynamic pull-in state, where $\mu = 0.1$ and $\eta = 0.1$, and without damping

system $\tilde{C}_{sd} = 0$ and $\tilde{C}_{vd} = 0$.

(a)

(b)

Fig. 12. Comparison of stress gradient and strain gradient parameters on the time history (a) and the phase portrait (b) of actuator with damping system at dynamic pull-in state.

Table 1

Referent parameters of dynamic pull-in experiment for verification of HPM.

Parameters	Symbol	Value/unit
Length	L	610 μm
Width	b	40 μm
Height	h	2.12 μm
Initial gap	g_0	2.07 μm
Young's modulus	E	164 GPa
Poisson's ratio	ν	0.28
Residual stress	σ_0	-3.5 MPa

Table 2

Comparison of dynamic pull-in voltage by different methods.

Method	Dynamic pull-in voltage (V)		Reference
	$\sigma_0 = -3.5\text{MPa}$	$\sigma_0 = 0$	
Experiment	8.10	—	[50]
DIPIE algorithm	—	10.752	[51]
EBM	8.027	10.974	[52]
HPM	7.985	11.108	Present

Table 3

Comparison of vibrational amplitudes by HPM and numerical method.

Initial amplitude	Van der Waals force/ α_3		Casimir force/ α_4	
	Numerical	2 nd HPM	Numerical	2 nd HPM
0	0.0039738	0.0039745	0.0039450	0.0039457
0.1	0.0950536	0.0951715	0.0950612	0.0951844
0.2	0.186546	0.186825	0.186686	0.186986
0.3	0.278728	0.279294	0.279223	0.279872
0.4	0.372013	0.373142	0.373271	0.374671
0.5	0.466961	0.469236	0.469634	0.472728

Table 4

Comparison of pull-in voltage with other size-dependent analytical models.

Model	Method	Classical	Unclassical
Nonlocal stress	LDL [53]	1.616	0.452
gradient theory	DQM [1]	1.680	0.497
($\mu = 0.1$)	Present	1.680	0.498
Strain gradient	MVIM [54]	9.87	10.10
theory ($\eta = 0.1$)	Present	9.853	10.082

Table 5

Temperature-dependent material property of CNTs [28] and silicon [55, 56].

Temperature (K)	E_{cnt} (TPa)	α_{cnt} ($\times 10^{-6} / K$)	E_m (MPa)	α_m ($\times 10^{-6} / K$)
300	5.6466	3.4584	169.68	2.57
400	5.5813	4.0919	167.69	3.14
500	5.5308	4.5361	165.71	3.59
600	5.4952	4.7438	163.72	3.88
700	5.4744	4.6677	161.73	3.99

Table 6

CNTs efficient parameter of scale-dependent property for different volume fraction [40].

V_{cnt}^*	0	0.12	0.17	0.28
η	—	1.2833	1.3414	1.3238

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