Gyroscope Sensing and Self-Calibration Architecture Based on Signal Phase Shift

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1. Introduction

Thanks to advances in MEMS technology, the use of inertial microsensors is rapidly expanding into an ever increasing number of applications, ranging from game controllers and mobile phones to digital cameras and tablet computers [1, 2]. In turn, the spreading commercial use of MEMS inertial sensors is spurring research into the development of ever more accurate devices that could be used in high-performance applications, such as inertial measurement units for dead reckoning. In particular, a significant amount of research is currently devoted to the development of navigation-grade MEMS gyroscopes.

The sensing element in all current MEMS gyroscopes is a vibrating structure of varying complexity, such as beams [3], shells [4], rings [5] or spring-suspended proof masses [6]. More recently, gyroscopes that rely on bulk acoustic wave (BAW) vibration modes have been introduced [7]. These devices operate in the megahertz frequency range, and can attain very high shock resistance and large bandwidths [8]. Regardless of the specific type of vibrating structure used, rotation is sensed by detecting the effect that the Coriolis force has on two natural resonance modes of the device [9]. Ideally, the resonance frequencies of the two modes are equal when the gyroscope is stationary in an inertial frame of reference (i.e. the device is mode-matched), as this improves the sensitivity of the gyroscope by the quality factor of the resonance modes [9, 10], which can be in the order of 10^5 [7].

The effect of the Coriolis force can be detected in a number of ways. For example, it can be shown that when a mode-matched gyroscope rotates, the Coriolis force causes the two modal frequencies to split by an amount equal to $2\Omega_z$, where Ω_z is the angular velocity of the device around its sensitive axis. A gyroscope based on this principle has been demonstrated recently [11]. In a

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sense, a device of this type can be thought of as a frequency-modulated (FM) gyroscope, in that the rate of rotation is measured from the shift in the resonance frequency of the vibrating structure.

A more common method to measure the rotation rate is based on the transfer of energy between the two vibration modes of the device that is caused by the Coriolis force. In this scheme, one of the two modes (usually referred to as the drive mode) is excited by an external stimulus. When the gyroscope rotates, the Coriolis force causes energy to be transferred to the other mode (the sense mode). The rotation rate is measured based on the detected vibration amplitude of the sense mode. Many implementations of this technique have been published (see [5, 12, 13] for a few representative examples). Devices based on this principle of operation can be thought of as amplitude-modulated (AM) gyroscopes. In a variant of this method, a feedback control loop is used to apply a variable excitation to the sense mode, so that the sense mode output remains at zero even in the presence of a Coriolis force [14]. This mode of operation, usually referred to as force-feedback or force-to-rebalance control, has also been implemented in a variety of MEMS gyroscopes (e.g. [15, 16]).

This paper describes and analyzes in depth a new method to measure the rotation rate of vibratory gyroscopes. This method, previously introduced in [17], is based on the phase shift caused by the Coriolis force in the gyroscope outputs. Unlike in the case of AM gyroscopes, in this scheme both vibration modes of the devices are excited by externally applied stimuli that are of equal amplitude but 90° out of phase. It can be shown that under these conditions the Coriolis force induces a phase shift between the input and the output of each mode, and that this phase shift is, to first order, proportional to the angular rotation rate of the gyroscope. Therefore this particular architecture can be thought of as a phase-modulated (or PM) gyroscope.

Excitation of both resonance modes is found in other gyroscope control schemes: for example, in the force-feedback configuration mentioned above. Another example is mode reversal, in which the roles of the drive and sense mode are switched periodically in order to minimize the error due to gyro bias [18, 19]. However, the novel aspect of the architecture described in this paper is that it relies on the phase shift of the output signal (as opposed to the amplitude of the output signal or of the applied feedback excitation) to measure the rotation rate.

A significant advantage of this phase-based sensor method is that it can be used with only minor modifications to calibrate the device by mimicking the effect of the Coriolis force through the application of suitable amplitude-modulated excitations. This is based on the fact that the phase response of an ideal gyroscope to the Coriolis force is mathematically related to the phase response of the gyroscope to an AM excitation [20, 21]. A similar observation was made in [22]. However, a second innovative aspect of the architecture proposed herein is that it relies on the gyroscope phase response for both sensing and calibration, thereby making it possible to use much of the control and signal processing electronics both for sensing and for calibration. This shared functionality pays off in multiple way, the most obvious of which is a reduction

in size and cost of the control and sensing components of the system. Even more important, however, is the fact that deviations from ideality in those components affect both sensing and calibration in equal manner, thereby further improving the accuracy and stability of calibration.

In summary, the innovative aspects of the proposed sensing and self-calibration architecture are the use of signal phase shift to measure the rotation rate; the mimicking of the Coriolis force through the application of AM excitations for calibration purposes; and the ability to use much of the signal post-processing electronics both for sensing and for calibration, which minimizes errors in the calibration of the gyroscope potentially introduced by those components. Furthermore, this self-calibration technique would completely eliminate the need for any mechanical calibration of MEMS gyroscopes, with clear advantages in terms of system stability, reliability and cost-effectiveness.

The theoretical basis for the phase-based gyroscope sensor architecture is presented in Section 2, and the related calibration architecture in Section 3. The effects of deviations from the ideal mode-matched gyroscope model on both architectures are described in Section 4. Finally, the results of tests conducted on circuit-board implementations of the sensor and calibration architectures are presented and discussed in Section 5.

2. Sensor Mode Operation

As mentioned earlier, the operation of MEMS vibratory gyroscopes relies on the effect of the Coriolis force on two natural resonance modes of the device. In this paper, those modes will be designated as I and Q , respectively. At the functional level, the dynamics of such gyroscopes can be modeled by a simple two-degree-of-freedom mass-spring system [10]. The equations describing the dynamics of this system can be written as follows

$$\begin{aligned} \ddot{x}_1 + \frac{\omega_1}{Q_1}\dot{x}_1 + d_{12}\dot{x}_2 - 2\lambda\Omega_z\dot{x}_2 + \omega_1^2x_1 + \omega_{12}^2x_2 &= f_1(t) \\ \ddot{x}_2 + \frac{\omega_2}{Q_2}\dot{x}_2 + d_{12}\dot{x}_1 + 2\lambda\Omega_z\dot{x}_1 + \omega_2^2x_2 + \omega_{12}^2x_1 &= f_2(t) \end{aligned} \quad (1)$$

where x_1 and x_2 are generalized coordinates [23], ω_1 and ω_2 the resonance frequencies of the two resonance modes of the system, Q_1 and Q_2 their respective quality factors, and d_{12} and ω_{12} are coefficients that model the coupling between the two resonance modes caused by non-ideal behavior of the physical device. The right-hand side of (1) represents the external excitations applied to each resonance mode, while the effect of the Coriolis force is modeled by terms $2\lambda\Omega_z\dot{x}_1$ and $2\lambda\Omega_z\dot{x}_2$, where Ω_z is the angular rotation velocity around the sensitive axis of the gyroscope, and λ a constant that depends on the gyroscope type and on the index of the resonance modes of the device [23]. If the system is in sinusoidal steady-state at a given frequency ω , then (1) is equivalent to the following set of equations

$$\begin{bmatrix} H_1(j\omega) & j\omega(-2\lambda\Omega_z + d_{12}) + \omega_{12}^2 \\ j\omega(2\lambda\Omega_z + d_{12}) + \omega_{12}^2 & H_2(j\omega) \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} F_I \\ F_Q \end{bmatrix} \quad (2)$$

Figure 1: Gyroscope sensor architecture based on output signal amplitude

Figure 2: Gyroscope sensor architecture based on output signal phase shift

where F_I , F_Q and X_1 , X_2 are the phasors of the applied excitations and of the generalized coordinates, respectively, and

$$\begin{aligned} H_1(j\omega) &= \omega_1^2 - \omega^2 + j\omega(\omega_1/Q_1) \\ H_2(j\omega) &= \omega_2^2 - \omega^2 + j\omega(\omega_2/Q_2) \end{aligned}$$

Using matrix notation, the system of equations in (2) can be written more compactly as

$$\mathbf{A}\mathbf{X} = \mathbf{F} \quad (3)$$

Figures 1 and 2 show high-level functional schematic diagrams of two architectures capable of generating output signals that are related to Ω_z , the rotation rate of the gyroscope. Specifically, Fig. 1 illustrates the familiar amplitude-based sensor scheme, in which only one of the resonance modes (commonly referred to as the drive mode) is excited. The value of Ω_z is determined from the amplitude of the output corresponding to the other mode (the so-called sense mode).

Consider first a gyroscope in which $\omega_1 = \omega_2 = \omega_0$ and $d_{12} = \omega_{12} = 0$. In the remainder of this paper, a device of this type will be referred to as an *ideal gyroscope* (note that it is not assumed that $Q_1 = Q_2$). Furthermore, assume that the frequency of the external excitation is equal to the resonance frequency ω_0 . In such case (2) simplifies to

$$j\omega_0 \begin{bmatrix} \omega_0/Q_1 & -2\lambda\Omega_z \\ 2\lambda\Omega_z & \omega_0/Q_2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}. \quad (4)$$

In the amplitude-based sensor scheme $F_2 = 0$, and in this case the solution of (4) is

$$\begin{aligned} X_1 &= -j \frac{F_1/Q_2}{(2\lambda\Omega_z)^2 + (\omega_0/Q)^2} \\ X_2 &= j \frac{2\lambda\Omega_z(F_1/\omega_0)}{(2\lambda\Omega_z)^2 + (\omega_0/Q)^2} \end{aligned} \quad (5)$$

where $Q = \sqrt{Q_1 Q_2}$.

Figure 3: Functional-level diagram of a mixer/low-pass filter circuit

To compute the signal value at the output of the low-pass filter, consider the simple circuit shown in Fig. 3, which consists of a mixer followed by a low-pass filter. Assume that the input signals to the mixer are

$$\begin{aligned} w_1(t) &= |W_1| \cos(\omega t + \angle W_1) \\ w_2(t) &= |W_2| \cos(\omega t + \angle W_2) \end{aligned}$$

where W_1 , W_2 are the phasors representing $w_1(t)$ and $w_2(t)$, respectively. Then the input signal to the low-pass filter is

$$w_1(t)w_2(t) = \frac{|W_1||W_2|}{2} [\cos(2\omega t + \angle W_1 + \angle W_2) + \cos(\angle W_1 - \angle W_2)]$$

Assuming that the sinusoidal component at frequency 2ω is eliminated completely by the low-pass filter, the filter output can be expressed as

$$z(t) = \frac{|W_1||W_2|}{2} \cos(\angle W_1 - \angle W_2) = \frac{1}{2} \Re[W_1 W_2^*] \quad (6)$$

where $\Re[\cdot]$ denotes the real part of its argument, and W_2^* is the complex conjugate of W_2 .

Hence, using (6), the signal at the output of the low-pass filter in Fig. 1 is

$$z_2 = \frac{1}{2} \Re[-jF_1 X_2^*] = -\frac{|F_1|^2}{2\omega_0} \frac{2\lambda\Omega_z}{(2\lambda\Omega_z)^2 + (\omega_0/Q)^2} = -\frac{|F_1|^2}{2\omega_0} G(2\lambda\Omega_z, \omega_0/Q) \quad (7)$$

where

$$G(x, a) = \frac{x}{x^2 + a^2} \quad (8)$$

In the phase-based sensor scheme both gyroscope resonance modes are driven simultaneously by excitations that are 90° out of phase, as shown in Fig. 2. In such case the solution of (4) is

$$\begin{aligned} X_1 &= -\frac{j}{\omega_0} \frac{(F_1/Q_2)\omega_0 + 2\lambda\Omega_z F_2}{(2\lambda\Omega_z)^2 + (\omega_0/Q)^2} \\ X_2 &= \frac{j}{\omega_0} \frac{2\lambda\Omega_z F_1 - \omega_0(F_2/Q_1)}{(2\lambda\Omega_z)^2 + (\omega_0/Q)^2} \end{aligned} \quad (9)$$

Using again (6), the outputs of the phase-based sensor architecture are

$$\begin{aligned} z_1 &= \frac{1}{2} \Re[F_1 X_1^*] = \frac{\Re[jF_1 F_2^*]}{2\omega_0} \frac{2\lambda\Omega_z}{(2\lambda\Omega_z)^2 + (\omega_0/Q)^2} \\ &= -\frac{\Im[F_1 F_2^*]}{2\omega_0} G(2\lambda\Omega_z, \omega_0/Q) \\ z_2 &= \frac{1}{2} \Re[F_2 X_2^*] = \frac{\Re[-jF_2 F_1^*]}{2\omega_0} \frac{2\lambda\Omega_z}{(2\lambda\Omega_z)^2 + (\omega_0/Q)^2} \\ &= -\frac{\Im[F_1 F_2^*]}{2\omega_0} G(2\lambda\Omega_z, \omega_0/Q) \end{aligned}$$

where $\Im[\cdot]$ denotes the imaginary part of its argument. Therefore

$$z_1 = z_2 = -\frac{|F_1||F_2|}{2\omega_0} \sin(\angle F_1 - \angle F_2) G(2\lambda\Omega_z, \omega_0/Q) \quad (10)$$

A comparison of (10) with (7) shows that in the case of an ideal gyroscope, the output of either sensor scheme is the same function of Ω_z . Note that for fixed

values of $|F_1|$ and $|F_2|$ — that is to say, for fixed values of the amplitudes of the excitation signals — the sensitivity of the output signals to Ω_z is maximum when $\angle F_1 - \angle F_2 = \pm 90^\circ$. This is the reason why Fig. 2 shows a 90° phase shift between the excitation signals of the two modes, even if in principle the architecture would still operate correctly with different phase shift values, albeit at the price of a lower sensitivity.

If $\Omega_z \ll \omega_0/Q$ then $G(x, a) \approx x/a^2$. In this case, assuming that $\sin(\angle F_1 - \angle F_2) = 1$, the relationship in (10) becomes

$$z_1 = z_2 \approx -\frac{\lambda Q^2 |F_1| |F_2|}{\omega_0^3} \Omega_z = -\frac{\lambda Q_1 Q_2 |F_1| |F_2|}{\omega_0^3} \Omega_z \quad (11)$$

Thus if $\Omega_z \ll \omega_0/Q$ the output signal levels are proportional to Ω_z through a scale factor S_r given by

$$S_r = -\frac{\lambda Q_1 Q_2 |F_1| |F_2|}{\omega_0^3} \quad (12)$$

3. Gyroscope Self-Calibration

Calibration of the gyroscope is performed by applying amplitude-modulated excitations to both resonance modes, as shown in Fig. 4. The cutoff frequency of the low-pass filters is assumed to be low enough to eliminate components at the lowest value of Ω_z used in calibration. As will be shown next, the gyroscope response to this particular type of excitation is closely related to its response to the Coriolis force, and it is this relationship that is exploited to calibrate the gyroscope.

Figure 4: Gyroscope calibration architecture based on amplitude-modulated excitations

If the gyroscope is ideal and it is excited at the resonance frequency of its modes, its response is given by the steady-state solution of the following set of

equations

$$\begin{aligned}
\ddot{x}_1 + \frac{\omega_0}{Q_1} \dot{x}_1 + \omega_0^2 x_1 &= F_1 \cos \omega_0 t \cos \Omega_z t \\
&= \frac{F_1}{2} (\cos \omega_- t + \cos \omega_+ t) \\
\ddot{x}_2 + \frac{\omega_0}{Q_2} \dot{x}_2 + \omega_0^2 x_2 &= F_2 \sin \omega_0 t \sin \Omega_z t \\
&= \frac{F_2}{2} (\cos \omega_- t - \cos \omega_+ t)
\end{aligned} \tag{13}$$

where $\omega_- = \omega_0 - \Omega_z$, $\omega_+ = \omega_0 + \Omega_z$.

Consider first the components of the excitations at frequency ω_- . The phasors of the corresponding components of the gyroscope outputs are

$$\begin{aligned}
X_1^- &= \frac{F_1}{2} \frac{1}{H_1(j\omega_-)} \\
X_2^- &= \frac{F_2}{2} \frac{1}{H_2(j\omega_-)}
\end{aligned}$$

Let $x_1^-(t)$ and $x_2^-(t)$ be the sinusoids at frequency ω_- represented by phasors X_1^- and X_2^- , respectively. When $x_1^-(t)$ is mixed with the excitation of the other mode, as shown in Fig. 4, the resulting mixing products are

$$x_1^-(t) F_2 \sin \omega_0 t \sin \Omega_z t = \frac{F_2}{2} [x_1^-(t) \cos \omega_- t - x_1^-(t) \cos \omega_+ t]$$

The components of the second product term, $x_1^-(t) \cos \omega_+ t$, are sinusoids at frequencies $\omega_+ + \omega_- = 2\omega_0$ and $\omega_+ - \omega_- = 2\Omega_z$. If the cutoff frequency of the low-pass filter is sufficiently low, these components are completely eliminated by the filter. Therefore only the first product term, $x_1^-(t) \cos \omega_- t$, contributes to the final output. Based on (6), this contribution is

$$\begin{aligned}
z_1^- &= \frac{1}{2} \Re \left[\frac{F_2}{2} (X_1^-)^* \right] = \frac{F_1 F_2}{8} \Re \left[\frac{1}{H_1^*(j\omega_-)} \right] \\
&= \frac{F_1 F_2}{8} \Re \left[\frac{H_1(j\omega_-)}{|H_1(j\omega_-)|^2} \right] = \frac{F_1 F_2}{8} \frac{(\omega_0 + \omega_-) \Omega_z}{|H_1(j\omega_-)|^2}
\end{aligned} \tag{14}$$

Similarly, the contribution of $x_2^-(t)$ to the final output is

$$z_2^- = \frac{1}{2} \Re \left[\frac{F_1}{2} (X_2^-)^* \right] = \frac{F_1 F_2}{8} \frac{(\omega_0 + \omega_-) \Omega_z}{|H_2(j\omega_-)|^2} \tag{15}$$

The same type of analysis shows that the phasors representing the components of the gyroscope outputs at frequency ω_+ are

$$\begin{aligned}
X_1^+ &= \frac{F_1}{2} \frac{1}{H_1(j\omega_+)} \\
X_2^+ &= -\frac{F_2}{2} \frac{1}{H_2(j\omega_+)}
\end{aligned}$$

and their contributions to the final outputs are

$$\begin{aligned} z_1^+ &= -\frac{1}{2}\Re\left[\frac{F_2}{2}(X_1^+)^*\right] = \frac{F_1 F_2}{8} \frac{(\omega_0 + \omega_+)\Omega_z}{|H_1(j\omega_+)|^2} \\ z_2^+ &= \frac{1}{2}\Re\left[\frac{F_1}{2}(X_2^+)^*\right] = \frac{F_1 F_2}{8} \frac{(\omega_0 + \omega_+)\Omega_z}{|H_2(j\omega_+)|^2} \end{aligned}$$

Therefore the signals at the output of the low-pass filters are

$$\begin{aligned} z_1 &= z_1^- + z_1^+ = \frac{F_1 F_2}{8} \left[\frac{(\omega_0 + \omega_-)\Omega_z}{|H_1(j\omega_-)|^2} + \frac{(\omega_0 + \omega_+)\Omega_z}{|H_1(j\omega_+)|^2} \right] \\ z_2 &= z_2^- + z_2^+ = \frac{F_1 F_2}{8} \left[\frac{(\omega_0 + \omega_-)\Omega_z}{|H_2(j\omega_-)|^2} + \frac{(\omega_0 + \omega_+)\Omega_z}{|H_2(j\omega_+)|^2} \right] \end{aligned} \quad (16)$$

Note that

$$\begin{aligned} |H_1(j\omega_-)|^2 &= \Omega_z^2(\omega_0 + \omega_-)^2 + (\omega_0\omega_-/Q_1)^2 \\ |H_1(j\omega_+)|^2 &= \Omega_z^2(\omega_0 + \omega_+)^2 + (\omega_0\omega_+/Q_1)^2 \end{aligned}$$

The approximations $\omega_+ \approx \omega_0$ and $\omega_- \approx \omega_0$ then lead to the following approximate equality

$$|H_1(j\omega_-)|^2 \approx |H_1(j\omega_+)|^2 \approx (2\omega_0\Omega_z)^2 + (\omega_0^2/Q_1)^2$$

Using similar approximations for $|H_2(j\omega)|^2$, the expressions for $z_1(t)$ and $z_2(t)$ in (16) can be rewritten as

$$\begin{aligned} z_1 &\approx \frac{F_1 F_2}{2\omega_0} \frac{\Omega_z}{(2\Omega_z)^2 + (\omega_0/Q_1)^2} = \frac{F_1 F_2}{4\omega_0} G(2\Omega_z, \omega_0/Q_1) \\ z_2 &\approx \frac{F_1 F_2}{2\omega_0} \frac{\Omega_z}{(2\Omega_z)^2 + (\omega_0/Q_2)^2} = \frac{F_1 F_2}{4\omega_0} G(2\Omega_z, \omega_0/Q_2) \end{aligned} \quad (17)$$

A comparison of (17) with (10) shows clearly that the gyroscope response to amplitude-modulated excitations is closely related to its response to rotation around its sensitive axis. In this respect, the modulating frequency Ω_z in (13) can be thought of as a virtual rotation rate being applied to the gyroscope. In particular, it follows from (17) and (10) that if $Q_1 = Q_2$ the values of the output signals when the gyroscope is operating in sensor mode and is rotating at angular velocity Ω_z are twice the output values generated when the gyroscope is operating in calibration mode with a “virtual rotation rate” $\lambda\Omega_z$. As stated at the beginning of this section, it is this relationship that makes it possible to calibrate the gyroscope through the application of amplitude-modulated excitations.

Even if $Q_1 \neq Q_2$ it is still possible to establish a quantitative relationship between the gyroscope responses in sensor and calibration modes. Assuming $\Omega_z \ll \omega_0/Q_1$ and $\Omega_z \ll \omega_0/Q_2$, the approximation $G(x, a) \approx x/a^2$ applied to

the expressions in (17) yields

$$\begin{aligned} z_1 &\approx \frac{Q_1^2 F_1 F_2}{2\omega_0^3} \Omega_z = S_{c1} \Omega_z \\ z_2 &\approx \frac{Q_2^2 F_1 F_2}{2\omega_0^3} \Omega_z = S_{c2} \Omega_z \end{aligned} \quad (18)$$

It follows from (18) and (12) that

$$|S_r| = 2\lambda \sqrt{S_{c1} S_{c2}} \quad (19)$$

This equation shows that for an ideal gyroscope and for small values of Ω_z the scale factor of the sensor architecture of Fig. 2 can be derived from the scale factors of the calibration architecture of Fig. 4.

4. Non-Ideal Gyroscopes

The results obtained in the previous section show that the responses of the phase-based sensor and calibration architectures can be quantitatively related when the gyroscope is ideal. Both architectures, however, are necessarily affected by deviations from the ideal gyroscope behavior assumed in the previous section.

In vibratory MEMS gyroscopes the most important causes of deviations from ideal behavior are mismatches in the frequencies of the two resonance modes of the device ($\omega_1 \neq \omega_2$), and physical coupling effects (both dissipative and non-dissipative) between the modes. These effects can be modeled by assuming $d_{12} \neq 0$ and $\omega_{12} \neq 0$ in (1). This section presents a quantitative analysis of the effects of such deviations on the responses of both the sensor and the calibration architectures.

Starting from the sensor architecture, let $\mathbf{A}_0$ be the coefficient matrix of the system of equations in (2) when $\Omega_z = 0$, that is

$$\mathbf{A}_0(j\omega) = \begin{bmatrix} H_1(j\omega) & j\omega d_{12} + \omega_{12}^2 \\ j\omega d_{12} + \omega_{12}^2 & H_2(j\omega) \end{bmatrix}$$

where ω is the excitation frequency. Let $\mathbf{X}_0 = \mathbf{A}_0^{-1} \mathbf{F}$ be the corresponding solution of (2). Note that

$$\mathbf{A}_0^{-1} = \frac{1}{|\mathbf{A}_0|} \begin{bmatrix} H_2(j\omega) & -(j\omega d_{12} + \omega_{12}^2) \\ -(j\omega d_{12} + \omega_{12}^2) & H_1(j\omega) \end{bmatrix} = -\frac{1}{|\mathbf{A}_0|} \mathbf{J} \mathbf{A}_0 \mathbf{J} \quad (20)$$

where $|\mathbf{A}_0|$ denotes the determinant of $\mathbf{A}_0$ and

$$\mathbf{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Based on (6), the corresponding values of the output signals are

$$\begin{aligned} z_{1,0} &= \frac{1}{2} \Re[F_1 X_{1,0}^*] \\ z_{2,0} &= \frac{1}{2} \Re[F_2 X_{2,0}^*] \end{aligned}$$

Since $\mathbf{X}_0$ is the solution of (2) when $\Omega_z = 0$, $z_{1,0}$ and $z_{2,0}$ give the bias values of the sensor architecture outputs in the case of a non-ideal gyroscope.

When $\Omega_z \neq 0$, the coefficient matrix of (2) becomes $\mathbf{A} = \mathbf{A}_0 + \Delta\mathbf{A}$, where $\Delta\mathbf{A} = j2\lambda\omega\Omega_z\mathbf{J}$. The corresponding change $\Delta\mathbf{X}$ in the solution of (2) can be computed by solving the modified system of equations

$$(\mathbf{A}_0 + \Delta\mathbf{A})(\mathbf{X}_0 + \Delta\mathbf{X}) = \mathbf{F}$$

For sufficiently small values of Ω_z , the second-order term $\Delta\mathbf{A}\Delta\mathbf{X}$ can be neglected: this is equivalent to retaining only those terms in $\Delta\mathbf{X}$ that depend linearly on Ω_z , and neglecting any terms that depend on higher-order powers of Ω_z . Under this assumption $\Delta\mathbf{X}$ is the solution of

$$\mathbf{A}_0\Delta\mathbf{X} = -\Delta\mathbf{A}\mathbf{X}_0 \quad (21)$$

Hence

$$\Delta\mathbf{X} = -\mathbf{A}_0^{-1}\Delta\mathbf{A}\mathbf{A}_0^{-1}\mathbf{F} = -j2\lambda\omega\Omega_z\mathbf{A}_0^{-1}\mathbf{J}\mathbf{A}_0^{-1}\mathbf{F}$$

Using (20) and the fact that $\mathbf{J}^2 = -\mathbf{I}$, where $\mathbf{I}$ is the identity matrix, the expression above can be rewritten as follows

$$\Delta\mathbf{X} = \frac{j2\lambda\omega\Omega_z}{|\mathbf{A}_0|} \mathbf{A}_0^{-1} \mathbf{J}^2 \mathbf{A}_0 \mathbf{J} \mathbf{F} = -\frac{j2\lambda\omega\Omega_z}{|\mathbf{A}_0|} \mathbf{J} \mathbf{F} \quad (22)$$

The corresponding changes in the outputs of the sensor architecture are

$$\begin{aligned} \Delta z_1 &= \frac{1}{2} \Re[F_1 \Delta X_1^*] = -\lambda\omega\Omega_z \Re \left[\frac{jF_1 F_2^*}{|\mathbf{A}_0|^*} \right] \\ \Delta z_2 &= \frac{1}{2} \Re[F_2 \Delta X_2^*] = -\lambda\omega\Omega_z \Re \left[\frac{jF_1 F_2^*}{|\mathbf{A}_0|} \right] \end{aligned} \quad (23)$$

Note that in the case of an ideal gyroscope $|\mathbf{A}_0| = -\omega^4/Q_1Q_2$ and $\Re[jF_1 F_2^*] = -|F_1||F_2|\sin(\angle F_1 - \angle F_2)$, so that the expressions in (23) coincide with those previously obtained in (11). The equations in (23) also show that in the case of a non-ideal gyroscope the scale factors of the two outputs are generally different. However, if $\angle F_1 - \angle F_2 = \pm 90^\circ$ then $jF_1 F_2^* = \pm |F_1||F_2|$, and in this case the two scale factors are the same because $\Re[1/|\mathbf{A}_0|] = \Re[1/|\mathbf{A}_0|^*]$.

In some applications where high accuracy is required, it may be useful to have a quantitative estimate of how much deviations of the behavior of a physical gyroscope from its ideal model affect the sensitivity of the outputs. Based on

(11) and (23) and assuming $\angle F_1 - \angle F_2 = \pm 90^\circ$, the relative difference between the scale factors in the two cases is

$$E_{rel} = \frac{|\Re[1/|\mathbf{A}_0|] + Q_1 Q_2 / \omega^4|}{Q_1 Q_2 / \omega^4}$$

A quantitative upper bound on E_{rel} can be obtained using lengthy but fairly straightforward algebra, the details of which are given in the Appendix. The final result is

$$\begin{aligned} E_{rel} \leq & \left(\frac{2\Delta\omega_1}{\omega_1/Q_1} + \frac{2\Delta\omega_2}{\omega_2/Q_2} \right)^2 + \left(\frac{2|\Delta\omega_1|}{\omega_1/Q_1} \right) \left(\frac{2|\Delta\omega_2|}{\omega_2/Q_2} \right) \\ & + \frac{|\Delta\omega_1 + \Delta\omega_2|}{\omega} + \left| \frac{\omega^2 d_{12}^2 - \omega_{12}^4}{\omega^4 / Q_1 Q_2} \right| \end{aligned} \quad (24)$$

where $\Delta\omega_1 = \omega_1 - \omega$ and $\Delta\omega_2 = \omega_2 - \omega$.

A similar analysis can be performed to evaluate the effects of non-ideal gyroscope behavior on the calibration architecture and, in particular, on the scale factors given in (18). As in the case of an ideal gyroscope, the outputs of the calibration architecture are

$$\begin{aligned} z_1 &= \frac{1}{2} \Re \left[\frac{F_2}{2} (X_1^-)^* \right] - \frac{1}{2} \Re \left[\frac{F_2}{2} (X_1^+)^* \right] \\ z_2 &= \frac{1}{2} \Re \left[\frac{F_1}{2} (X_2^-)^* \right] + \frac{1}{2} \Re \left[\frac{F_1}{2} (X_2^+)^* \right] \end{aligned}$$

It is shown in the Appendix that the equalities above yield the following approximate expressions for the outputs of the calibration architecture

$$\begin{aligned} z_1 &= \frac{F_1 F_2}{8\omega_0} (G[2(\Omega_z + \Delta\omega_1), \omega_1/Q_1] + G[2(\Omega_z - \Delta\omega_1), \omega_1/Q_1]) \\ &\quad - \frac{F_2^2}{8} \Re \left[\frac{j\omega_- d_{12} + \omega_{12}^2}{|\mathbf{A}_0(j\omega_-)|} + \frac{j\omega_+ d_{12} + \omega_{12}^2}{|\mathbf{A}_0(j\omega_+)|} \right] \end{aligned} \quad (25)$$

$$\begin{aligned} z_2 &= \frac{F_1 F_2}{8\omega_0} (G[2(\Omega_z + \Delta\omega_2), \omega_2/Q_2] + G[2(\Omega_z - \Delta\omega_2), \omega_2/Q_2]) \\ &\quad - \frac{F_1^2}{8} \Re \left[\frac{j\omega_- d_{12} + \omega_{12}^2}{|\mathbf{A}_0(j\omega_-)|} + \frac{j\omega_+ d_{12} + \omega_{12}^2}{|\mathbf{A}_0(j\omega_+)|} \right] \end{aligned} \quad (26)$$

The corresponding calibration scale factors for small values of Ω_z are

$$\begin{aligned} S_{c1} &\approx \frac{F_1 F_2}{2\omega_0} \frac{(\omega_1/Q_1)^2 - (2\Delta\omega_1)^2}{[(2\Delta\omega_1)^2 + (\omega_1/Q_1)^2]^2} \\ S_{c2} &\approx \frac{F_1 F_2}{2\omega_0} \frac{(\omega_2/Q_2)^2 - (2\Delta\omega_2)^2}{[(2\Delta\omega_2)^2 + (\omega_2/Q_2)^2]^2} \end{aligned} \quad (27)$$

Therefore the relative error in the calibration scale factor due to non-ideal gyroscope behavior is

$$\begin{aligned} E_{rel1} &\approx \left(\frac{2\Delta\omega_1}{\omega_1/Q_1} \right)^2 \\ E_{rel2} &\approx \left(\frac{2\Delta\omega_2}{\omega_2/Q_2} \right)^2 \end{aligned} \quad (28)$$

having assumed $|\Delta\omega_{1,2}| \ll \omega_{1,2}/Q_{1,2}$.

The expressions for the relative errors derived in this section make it possible to obtain quantitative bounds on the accuracy of calibration of a physical gyroscope whose behavior is not ideal. In an ideal gyroscope, the sensor scale factor can be computed exactly from the calibration scale factors through (19). In practice, the sensor and calibration scale factors of a physical gyroscope will deviate from their ideal values by relative amounts that are bounded by (24) and (28). It follows that the calibration of the scale factor of a physical gyroscope is accurate within a relative error no larger than the sum of the bounds in (24) and (28).

Conversely, those bounds can be used to determine performance requirements on a physical gyroscope that are needed to ensure that scale factor calibration is accurate to specified tolerances. Specifically, let ε be the target calibration accuracy. This can be attained if $E_{rel} \leq \varepsilon/2$ in (24) and (28). To derive corresponding bounds in gyroscope performance, note that

$$\frac{|\Delta\omega_1 + \Delta\omega_2|}{\omega_0} \ll \left(\frac{2|\Delta\omega_{1,2}|}{\omega_{1,2}/Q_{1,2}} \right)$$

since $Q_{1,2} \gg 1$. Therefore the term $|\Delta\omega_1 + \Delta\omega_2|/\omega_0$ can be neglected in (24). Moreover

$$\left(\frac{2\Delta\omega_1}{\omega_1/Q_1} + \frac{2\Delta\omega_2}{\omega_2/Q_2} \right)^2 + \left(\frac{2|\Delta\omega_1|}{\omega_1/Q_1} \right) \left(\frac{2|\Delta\omega_2|}{\omega_2/Q_2} \right) \approx 5 \left(\frac{2|\Delta\omega|}{\omega/Q} \right)^2$$

having assumed $\omega_1 \approx \omega_2 \approx \omega$ and $Q_1 \approx Q_2 = Q$. Therefore $E_{rel} \leq \varepsilon/2$ if

$$\begin{aligned} 5 \left(\frac{2|\Delta\omega|}{\omega/Q} \right)^2 &\leq \frac{\varepsilon}{4} \\ \left| \frac{\omega^2 d_{12}^2 - \omega_{12}^4}{\omega^4/Q_1 Q_2} \right| &\leq \frac{\varepsilon}{4} \end{aligned} \quad (29)$$

The first inequality in (29) places a bound on the frequency split between the two resonance modes of the gyroscope. The second involves model parameters ω_{12} and d_{12} , which are related to the degree of cross-coupling between the modes. Although it is not straightforward to measure the individual values of these parameters, it is possible to relate the left-hand side of the second inequality in (29) to a readily measurable quantity. For this purpose, consider the response

of a stationary gyroscope to the excitation of a single mode, which is described by the following set of equations

$$\begin{bmatrix} H_1(j\omega) & \omega_{12}^2 + j\omega d_{12} \\ \omega_{12}^2 + j\omega d_{12} & H_2(j\omega) \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ 0 \end{bmatrix}$$

Solving this system yields

$$\left| \frac{X_2}{X_1} \right|^2 = \left| \frac{\omega_{12}^2 + j\omega d_{12}}{H_2(j\omega)} \right|^2 \approx \frac{\omega_{12}^4 + \omega^2 d_{12}^2}{\omega^4 / Q_1 Q_2}$$

having assumed $H_2(j\omega) \approx j\omega\omega_2/Q_2$, $\omega \approx \omega_2$ and $Q_1 \approx Q_2$. Since $|\omega^2 d_{12}^2 - \omega_{12}^4| \leq \omega^2 d_{12}^2 + \omega_{12}^4$, the second inequality in (29) is satisfied if $|X_2/X_1| < \sqrt{\varepsilon}/4$. In other words, when only one mode is excited the ratio of the output amplitudes of the two modes must be less than approximately the square root of the desired relative calibration accuracy. For example, for a 1 ppm (10^{-6}) accuracy, the ratio $|X_2/X_1|$ should be less than 10^{-3} , or -60 dB. Of course, it should be kept in mind that the inequalities in (29) are worst case estimates of the inaccuracies due to deviations from ideal gyroscope behavior. In practice, it is quite possible that the desired accuracy in the scale factor of a physical gyroscope may be achieved by less stringent requirements than those obtained from (29).

Besides deviations from ideal gyroscope behavior, calibration errors can also be introduced by the circuits used to process the gyroscope outputs. The possible sources of calibration error due to signal processing are numerous and an exhaustive quantitative analysis of their impact on the overall calibration accuracy is beyond the scope of this paper. Moreover, the impact of most such sources of error can be easily controlled using proven techniques (e.g. digital signal processing) to the point where their contribution to the overall calibration error is negligible. One particular source of error, however, merits further discussion, namely, the error due to insufficient attenuation by the low-pass filters of unwanted signal components.

It was mentioned in Section 3 that the cut-off frequency of the low-pass filters should be sufficiently low to eliminate signal components at frequencies as low as $2\Omega_z$. Since the low end of the range of measurable rotation rates can potentially be quite low (of the order of millihertz, or even lower), this requirement would seem to impose unrealistic limits on the cutoff frequency of the low-pass filters. A solution to this potential problem comes from the observation that it is not necessary to calibrate the gyroscope at the very low end of the expected range of measurable rotation rates. The reason is that both in readout and in calibration mode the input-output relationship of the system is determined by function $G(x, a)$ defined in (8). In particular, in calibration mode $x = 2\Omega_z$ and $a = \omega_0/Q$. Let z_1 and z_2 be the system outputs measured at two different virtual rotation rates, and x_1 and x_2 the corresponding values of x . Since $z_1 = AG(x_1, a)$ and $z_2 = AG(x_2, a)$, the values of a and A can be computed by solving the following system of equations

$$\begin{aligned} x_1 A - z_1 a^2 &= z_1 x_1^2 \\ x_2 A - z_2 a^2 &= z_2 x_2^2 \end{aligned}$$

The virtual rotation rates chosen above can be any two values in the expected range of operation of the gyroscope and don't have to be at the very low end of that range. In fact, calibration measurements can be taken at more than two virtual rotation rates, using then curve fitting techniques to extract the values of a and A with better accuracy. Once those values are known, calibration outputs at lower rotation rates can be extrapolated from the equation $z = AG(x, a)$. Therefore, it is not necessary to perform calibration measurements through the entire range of measurable rotation rates of the gyroscope.

Figure 5: (Left) SEM image of a silicon disk bulk acoustic wave gyroscope [7]; (right) the gyroscope drive mode shows a quality factor of 56,000 at 9.665 MHz resonance frequency.

5. Numerical and Experimental Results

The results from the theoretical analysis presented in the previous sections were verified by numerical simulations and validated by experimental measurements. The numerical simulations were performed on an ANSYS model of a bulk acoustic wave (BAW) disk gyroscope of the same type as the one shown in Fig. 5. The BAW gyroscope consists of a center-supported disk structure fabricated on a [100] silicon wafer with capacitively coupled drive, sense, and control electrodes. Through its bulk acoustic actuation, the device can achieve mode-matched operation at frequencies much higher than its counterparts. The gyroscope operates in the in-plane resonance mode of index $n = 3$, thus ensuring that both secondary degenerate modes have the same resonance frequency (approximately 10.41 MHz simulated, 9.66 MHz measured), despite the anisotropic nature of single-crystal silicon [8]. The main physical parameters of the gyroscope are listed in Table 1.

A first set of simulations was used to evaluate the phase shift in the gyroscope response caused by the Coriolis force. In these simulations sinusoidal excitations were applied to a pair of electrodes aligned with one of the degenerate resonance modes. Similar excitations of equal amplitude but 90° out of phase were applied to the pair of electrodes perpendicular to the first pair, which is aligned with

Table 1: Disk BAW gyroscope parameters

Disk diameter	600 μm
Disk thickness	40 μm
Number of electrodes	12
Electrode angular width	25°
Capacitive gap width	200 nm
Polarization voltage	18 V
Modal resonance freq. ($n = 3$)	9.66 MHz
Quality factor	56,000

the other degenerate mode. Finite element analysis was used to compute the phases of the electrode output currents at various rotation rates.

The results of the simulations are plotted in Fig. 6, which shows the phase difference between the input voltage and the output current of each mode, evaluated at the modal resonance frequency and computed at six different rotation rates ranging from zero to 100 °/s. It can be seen that the relationship between phase shift θ and rotation rate Ω_z is linear in that range, as predicted by the theoretical analysis in Section 2. In fact, assuming $F_2 = jF_1$ and $Q_1 = Q_2 = Q$, it follows from (9) that

$$\theta = \tan^{-1} \frac{2\lambda Q \Omega_z}{\omega_0} \approx \frac{2\lambda Q \Omega_z}{\omega_0},$$

the approximate equality being valid for small values of Ω_z . This means that the slope of the line in Fig. 6 is equal to $2\lambda Q/\omega_0$. This relationship can be used to extract the value of λ from the simulation results, which in this case yield $\lambda \approx 0.6174$ (using $f_0 = 10.41$ MHz, $Q = 56,000$). This is close to the value $\lambda = 2n/(n^2 + 1) = 0.6$ obtained from the theoretical analysis of the resonance modes of an ideal ring gyroscope [23].

Figure 7 plots the simulated outputs of the sensor and calibration architectures of Figs. 2 and 4, respectively, as functions of Ω_z . In the case of the sensor architecture, Ω_z represents the rotation rate used in the ANSYS simulations. In the case of the calibration architecture, Ω_z is the frequency of the signal modulating the gyroscope excitation signals. It can be seen that in both cases the output is directly proportional to Ω_z , and that the ratio of the slopes of the two lines is $344.3/278.9 = 1.2345$, in excellent agreement with the value of $2\lambda = 1.2348$ obtained by taking the ratio of (12) and (18).

The theoretical results were also verified experimentally on a printed circuit board (PCB) implementation of the phase-based sensor and calibration architectures using a disk BAW gyroscope (Fig. 5) as the sensor. Figure 8 shows the block diagram of a reconfigurable system architecture that combines the phase-based sensor and self-calibration techniques. The input excitations defined in Section 2 and Section 3 are provided by an AM I/Q signal generator consisting of two mixers and two 90° phase shifters. On each mixer, one in-

Figure 6: Phase shift θ between gyroscope input voltage and output current as a function of gyroscope rotation rate, as computed by finite-element ANSYS simulations.

Figure 7: Simulated output signals, taken to be currents, of the sensor (Fig. 2) and calibration (Fig. 4) architectures as functions of Ω_z .

Figure 8: Block diagram of a system that combines phase-based sensor and self-calibration architectures

put is at carrier frequency ω_0 , while the other input is either a DC signal in sensor mode ($A(t) = DC$), or a sinusoid at frequency Ω_z for self-calibration ($A(t) = \cos \Omega_z t$). It must be noted that in this scheme, the carrier frequency excitation is provided externally from a 4395A network analyzer output. At the output of the gyroscope, two transimpedance amplifiers translate the output currents into voltages, which are also used by the network analyzer for real-time monitoring of the gyroscope frequency response.

Two analog multiplexers are used to connect the right combination of drive signals to the demodulating mixers. In sensor mode $SELECT$ is set to logic zero, and therefore the output of each mode is mixed to its respective input, e.g. I_{SENSE} is mixed with I_{DRIVE} . In self-calibration mode $SELECT$ is set to logic one, and therefore the output of each mode is mixed with the input of the other mode, e.g. I_{SENSE} is mixed with Q_{DRIVE} . The higher-frequency components of mixer output signals $w_1(t)$ and $w_2(t)$ are then eliminated by low-pass filters to generate outputs $z_1(t)$ and $z_2(t)$. It should be noted that the circuits that process the gyroscope output signals are the same in sensor and calibration mode of operation. Therefore any changes in the behavior of those circuits due to environmental and other factors (e.g. temperature, aging, etc.) will equally affect both modes of operation, thereby improving self-calibration accuracy.

The operation of the architecture in sensor mode ($A(t) = 2VDC$) was tested under sinusoidal rotation rates, applied on an Ideal Aerosmith rate table. Sinusoidal waveforms were observed at output signals $z_1(t)$ and $z_2(t)$ with amplitudes proportional to the applied rate, as shown in Fig. 9. The BAW gyroscope was excited with an input power of -10 dBm applied to the drive electrodes of the device. The measured response of the system to sinusoidal rotation rates ranging from 5 to $100^\circ/s$ is shown in Fig. 10. The values are closely aligned along a straight line with a slope of approximately $330 \mu V/^\circ/s$, confirming the linear response predicted by the theoretical analysis. It is noteworthy that because of

the relatively low mode frequency mismatch of 45 Hz, the input excitation is reduced to -10 dBm, to avoid saturation of the amplifier outputs.

In calibration mode, the calibration scale factor was measured on the BAW disk gyroscope at -10 dBm input power, while the frequency of the modulating signal, $A(t) = 2 \cos \Omega_z t$ V, was changed from 10 Hz to 15 Hz in 1 Hz steps every few seconds, to mimic rotation rates ranging from 3600 $^\circ$ /s to 5400 $^\circ$ /s in 360 $^\circ$ /s steps, as shown in Fig. 11 (a). The corresponding measured output values, shown in Fig. 11 (b), indicate a sensitivity of 60.4 mV/Hz, which translates to a 167.7 μ V/ $^\circ$ /s calibration scale factor.

The ratio between the experimentally derived sensor and calibration scale factors is $330/167.7 \approx 1.968$, which is somewhat different from the theoretical value of $2\lambda = 1.235$. According to this theoretical ratio, the calibration scale factor should be 267.2 μ V/ $^\circ$ /s. However, this assumes that the gyroscope is ideal and, in particular, that the frequencies of its two resonance modes are exactly identical. Based on (27), it can be calculated that a difference in the modal frequencies of just 34 Hz is sufficient to reduce the calibration scale factor to approximately 168 μ V/ $^\circ$ /s, thus accounting for the higher ratio between the measured sensor and calibration scale factors.

6. Conclusion

The capability of MEMS vibratory gyroscopes to measure rotation rates is based on the effect of the Coriolis force on the gyroscope dynamics. This effect can be detected in a number of different ways. This paper has presented a method to sense the rotation rate that relies on the phase shift induced by the Coriolis force in the gyroscope outputs. In principle, this method is applicable to any gyroscope that can be modeled at the functional level as a 2-DOF mass-spring system, including bulk acoustic wave gyroscopes. A system-level analysis of the phase-based sensor architecture indicates that its performance is theoretically equivalent to that of the more commonly used amplitude-based sensor scheme, even when deviation from ideal behavior of the device — such as coupling between the two resonance modes, or mismatches in their resonance frequencies — are taken into account. On the downside, the circuits needed for readout and for local generation of the excitation signals are more complex than in the case of rotation rate sensing based on signal amplitude. The advantage of the phase-based sensor method, however, is that it can be used also for gyroscope calibration with only minor modifications to the control electronics. In fact, the phase-based sensor and calibration functions can be integrated in a reconfigurable fashion, so that processing of the gyroscope outputs is performed by the same electronics in both cases [21]. This means that any changes in those electronics due to aging or other environmental factors will affect sensor and calibration outputs equally, thus increasing the accuracy of the calibration of the device. This is a significant benefit in high-performance applications that require frequent in-situ gyroscope calibration.

A final remark should be made about the way in which the gyroscope excitation signal is generated. The analysis presented in this paper assumes that

Figure 9: Output waveforms showing the BAW gyroscope response to a $10^\circ/\text{s}$ rotation at 0 dBm input power

Figure 10: Measured response of the phase-based sensor architecture to physical rotation. The output levels represent changes relative to values at zero rotation rate (bias). The solid line shows the best linear fit to the data

Figure 11: (a) Output signal of the system operating in calibration mode, as the modulation frequency increases from 10 to 15 Hz in 1 Hz steps, mimicking rotation rates from 3600 to 5400 $^{\circ}/s$ (b) Measured output values of a BAW gyroscope operating in calibration mode, indicating a scale factor of $167.7 \mu V/^{\circ}/s$

such signal is provided by an external source. However, this would not be a practical way to implement the proposed sensing and calibration architecture, due to the fact that the resonance frequencies of the gyroscope modes are bound to fluctuate in response to changes in environmental factors. The simplest way to ensure that the frequency of the excitation signal tracks such changes is to generate the signal in an oscillator loop that includes the gyroscope itself as the frequency setting element. This does not alter the basic functionality of phase-based sensing and calibration, although some aspects of the analysis presented in this paper must be modified. A closed-loop architecture of this type will be examined in detail in an upcoming paper.

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8. Appendix

A quantitative upper bound on E_{rel} , the magnitude of the difference between the sensor scale factors in (11) and (23), can be obtained in the following way.

Let R_0 and I_0 be the real and imaginary parts of $|\mathbf{A}_0|$, that is $|\mathbf{A}_0| = R_0 + jI_0$. Then $\Re[1/|\mathbf{A}_0|] = R_0/(R_0^2 + I_0^2)$ and

$$\begin{aligned} |\Re[1/|\mathbf{A}_0|] + Q_1 Q_2 / \omega^4| &\leq |\Re[1/|\mathbf{A}_0|] - 1/R_0| + |1/R_0 + Q_1 Q_2 / \omega^4| \\ &= \frac{I_0^2}{|R_0|(R_0^2 + I_0^2)} + \left| \frac{R_0 + \omega^4/Q_1 Q_2}{R_0 \omega^4 / Q_1 Q_2} \right| \end{aligned}$$

Note that

$$\begin{aligned} R_0 + \omega^4/Q_1 Q_2 &\approx 4\omega^2 \Delta\omega_1 \Delta\omega_2 + \omega^2(\omega^2 - \omega_1 \omega_2)/Q_1 Q_2 + \omega^2 d_{12}^2 - \omega_{12}^4 \\ &= 4\omega^2 \Delta\omega_1 \Delta\omega_2 - \omega^2(\omega \Delta\omega_1 + \omega \Delta\omega_2 + \Delta\omega_1 \Delta\omega_2)/Q_1 Q_2 \\ &\quad + \omega^2 d_{12}^2 - \omega_{12}^4 \end{aligned}$$

Neglecting the second-order term $\Delta\omega_1 \Delta\omega_2$ with respect to the first-order term $\omega(\Delta\omega_1 + \Delta\omega_2)$ in the expression above leads to the following inequality

$$\begin{aligned} \left| \frac{R_0 + \omega^4/Q_1 Q_2}{R_0 \omega^4 / Q_1 Q_2} \right| &\leq \left| \frac{4\omega^2 \Delta\omega_1 \Delta\omega_2}{R_0 \omega^4 / Q_1 Q_2} \right| + \left| \frac{\omega^3(\Delta\omega_1 + \Delta\omega_2)/Q_1 Q_2}{R_0 \omega^4 / Q_1 Q_2} \right| \\ &\quad + \left| \frac{\omega^2 d_{12}^2 - \omega_{12}^4}{R_0 \omega^4 / Q_1 Q_2} \right| \end{aligned}$$

Then the following approximation

$$R_0 \approx -\Im[H_1(j\omega)]\Im[H_2(j\omega)] = -\omega^2 \omega_1 \omega_2 / Q_1 Q_2 \quad (30)$$

makes it possible to rewrite the inequality above as

$$\begin{aligned} \left| \frac{R_0 + \omega^4/Q_1 Q_2}{R_0 \omega^4 / Q_1 Q_2} \right| &\leq (Q_1 Q_2 / \omega^4) \left(\frac{2|\Delta\omega_1|}{\omega_1/Q_1} \right) \left(\frac{2|\Delta\omega_2|}{\omega_2/Q_2} \right) + \frac{|\Delta\omega_1 + \Delta\omega_2|}{|R_0|\omega} \\ &\quad + \left| \frac{\omega^2 d_{12}^2 - \omega_{12}^4}{R_0 \omega^4 / Q_1 Q_2} \right| \end{aligned} \quad (31)$$

Furthermore, assuming that $|I_0| \ll |R_0|$, then

$$\frac{I_0^2}{|R_0|(R_0^2 + I_0^2)} \approx \frac{1}{|R_0|} \left(\frac{I_0}{R_0} \right)^2 \approx \frac{1}{|R_0|} \left(\frac{2\Delta\omega_1}{\omega_1/Q_1} + \frac{2\Delta\omega_2}{\omega_2/Q_2} \right)^2$$

where (30) has been used together with the additional approximate equality

$$\begin{aligned} I_0 &\approx \Re[H_1(j\omega)]\Im[H_2(j\omega)] + \Im[H_1(j\omega)]\Re[H_2(j\omega)] \\ &= (\omega_1^2 - \omega^2)\omega\omega_2/Q_2 + (\omega_2^2 - \omega^2)\omega\omega_1/Q_1 \\ &\approx 2\omega^2 \Delta\omega_1 \omega_2 / Q_2 + 2\omega^2 \Delta\omega_2 \omega_1 / Q_1 \end{aligned}$$

Finally, the further approximation $|R_0| \approx \omega^4/Q_1 Q_2$ together with the expressions in (31) yield the desired upper bound on E_{rel}

$$E_{rel} \leq \left(\frac{2\Delta\omega_1}{\omega_1/Q_1} + \frac{2\Delta\omega_2}{\omega_2/Q_2} \right)^2 + \left(\frac{2|\Delta\omega_1|}{\omega_1/Q_1} \right) \left(\frac{2|\Delta\omega_2|}{\omega_2/Q_2} \right) + \frac{|\Delta\omega_1 + \Delta\omega_2|}{\omega} + \left| \frac{\omega^2 d_{12}^2 - \omega_{12}^4}{\omega^4 / Q_1 Q_2} \right| \quad (32)$$

A similar analysis can be performed to evaluate the effects of non-ideal gyroscope behavior on the calibration architecture and, in particular, on the scale factors given in (18). As in the case of an ideal gyroscope, the outputs of the calibration architecture are

$$\begin{aligned} z_1 &= \frac{1}{2} \Re \left[\frac{F_2}{2} (X_1^-)^* \right] - \frac{1}{2} \Re \left[\frac{F_2}{2} (X_1^+)^* \right] \\ z_2 &= \frac{1}{2} \Re \left[\frac{F_1}{2} (X_2^-)^* \right] + \frac{1}{2} \Re \left[\frac{F_1}{2} (X_2^+)^* \right] \end{aligned}$$

From (20)

$$X_1^- = \frac{1}{2|\mathbf{A}_0|} [H_2(j\omega_-)F_1 - (j\omega_-d_{12} + \omega_{12}^2)F_2]$$

where $\omega_- = \omega - \Omega_z$. Note that

$$\frac{H_2(j\omega_-)}{|\mathbf{A}_0(j\omega_-)|} = \frac{1}{H_1(j\omega_-)} \left[1 + \frac{(j\omega_-d_{12} + \omega_{12}^2)^2}{|\mathbf{A}_0(j\omega_-)|^2} \right] \approx \frac{1}{H_1(j\omega_-)}$$

where the last approximation is justified if

$$\left| \frac{(j\omega_-d_{12} + \omega_{12}^2)^2}{|\mathbf{A}_0(j\omega_-)|^2} \right| \ll 1$$

that is to say, if the deviation from the ideal gyroscope model is sufficiently small. Therefore

$$X_1^- \approx \frac{1}{2} \left[\frac{F_1}{H_1(j\omega_-)} - \frac{(j\omega_-d_{12} + \omega_{12}^2)F_2}{|\mathbf{A}_0(j\omega_-)|} \right]$$

Similarly

$$\begin{aligned} X_1^+ &= \frac{1}{2|\mathbf{A}_0(j\omega_+)|} [H_2(j\omega_+)F_1 + (j\omega_+d_{12} + \omega_{12}^2)F_2] \\ &\approx \frac{1}{2} \left[\frac{F_1}{H_1(j\omega_+)} + \frac{(j\omega_+d_{12} + \omega_{12}^2)F_2}{|\mathbf{A}_0(j\omega_+)|} \right] \end{aligned}$$

where $\omega_+ = \omega + \Omega_z$. Consequently

$$\begin{aligned} z_1 &= \frac{F_1F_2}{8} \Re \left[\frac{1}{H_1(j\omega_-)} - \frac{1}{H_1(j\omega_+)} \right] \\ &\quad - \frac{F_2^2}{8} \Re \left[\frac{j\omega_-d_{12} + \omega_{12}^2}{|\mathbf{A}_0(j\omega_-)|} + \frac{j\omega_+d_{12} + \omega_{12}^2}{|\mathbf{A}_0(j\omega_+)|} \right] \end{aligned} \quad (33)$$

Following the same steps used to derive (16) it can be seen that

$$\begin{aligned} \Re \left[\frac{1}{H_1(j\omega_-)} \right] &= \frac{\Re[H_1(j\omega_-)]}{|H_1(j\omega_-)|^2} = \frac{(\Omega_z + \Delta\omega_1)(\omega_1 + \omega_-)}{|H_1(j\omega_-)|^2} \\ \Re \left[\frac{1}{H_1(j\omega_+)} \right] &= \frac{\Re[H_1(j\omega_+)]}{|H_1(j\omega_+)|^2} = -\frac{(\Omega_z - \Delta\omega_1)(\omega_1 + \omega_+)}{|H_1(j\omega_+)|^2} \end{aligned}$$

Note also that

$$\begin{aligned} |H_1(j\omega_-)|^2 &= (\Omega_z + \Delta\omega_1)^2(\omega_1 + \omega_-)^2 + (\omega_1\omega_-/Q_1)^2 \\ |H_1(j\omega_+)|^2 &= (\Omega_z - \Delta\omega_1)^2(\omega_1 + \omega_+)^2 + (\omega_1\omega_+/Q_1)^2 \end{aligned}$$

Using the approximations $\omega_+ \approx \omega$, $\omega_- \approx \omega$ and $\omega_1 + \omega_- \approx \omega_1 + \omega_+ \approx 2\omega$, the expressions above can be simplified as follows

$$\begin{aligned} |H_1(j\omega_-)|^2 &\approx \omega^2[(2\Omega_z + 2\Delta\omega_1)^2 + (\omega_1/Q_1)^2] \\ |H_1(j\omega_+)|^2 &\approx \omega^2[(2\Omega_z - 2\Delta\omega_1)^2 + (\omega_1/Q_1)^2] \end{aligned}$$

Then

$$\begin{aligned} &\Re \left[\frac{1}{H_1(j\omega_-)} - \frac{1}{H_1(j\omega_+)} \right] \\ &\approx \frac{1}{\omega} \left(\frac{2(\Omega_z + \Delta\omega_1)}{[2(\Omega_z + \Delta\omega_1)]^2 + (\omega_1/Q_1)^2} + \frac{2(\Omega_z - \Delta\omega_1)}{[2(\Omega_z - \Delta\omega_1)]^2 + (\omega_1/Q_1)^2} \right) \\ &= \frac{G[2(\Omega_z + \Delta\omega_1), \omega_1/Q_1] + G[2(\Omega_z - \Delta\omega_1), \omega_1/Q_1]}{\omega} \end{aligned} \quad (34)$$

and (33) can be rewritten as

$$\begin{aligned} z_1 &= \frac{F_1 F_2}{8\omega} (G[2(\Omega_z + \Delta\omega_1), \omega_1/Q_1] + G[2(\Omega_z - \Delta\omega_1), \omega_1/Q_1]) \\ &\quad - \frac{F_2^2}{8} \Re \left[\frac{j\omega_- d_{12} + \omega_{12}^2}{|\mathbf{A}_0(j\omega_-)|} + \frac{j\omega_+ d_{12} + \omega_{12}^2}{|\mathbf{A}_0(j\omega_+)|} \right] \end{aligned} \quad (35)$$

The corresponding calibration scale factor for small values of Ω_z is equal to $(dz_1/d\Omega_z)_{\Omega_z=0}$. Note that

$$\begin{aligned} \frac{d}{d\Omega_z} \left(\frac{j\omega_- d_{12} + \omega_{12}^2}{|\mathbf{A}_0(j\omega_-)|} \right)_{\Omega_z=0} &= -\frac{d}{d\omega} \left(\frac{j\omega d_{12} + \omega_{12}^2}{|\mathbf{A}_0(j\omega)|} \right) \\ \frac{d}{d\Omega_z} \left(\frac{j\omega_+ d_{12} + \omega_{12}^2}{|\mathbf{A}_0(j\omega_+)|} \right)_{\Omega_z=0} &= \frac{d}{d\omega} \left(\frac{j\omega d_{12} + \omega_{12}^2}{|\mathbf{A}_0(j\omega)|} \right) \end{aligned}$$

Consequently the last term in (35) does not contribute to the scale factor, which is therefore given by

$$\begin{aligned} S_{c1} &= \left(\frac{dz_1}{d\Omega_z} \right)_{\Omega_z=0} = \frac{F_1 F_2}{4\omega} \left[\frac{\partial G}{\partial x}(2\Delta\omega_1, \omega_1/Q_1) + \frac{\partial G}{\partial x}(-2\Delta\omega_1, \omega_1/Q_1) \right] \\ &= \frac{F_1 F_2}{2\omega} \frac{(\omega_1/Q_1)^2 - (2\Delta\omega_1)^2}{[(2\Delta\omega_1)^2 + (\omega_1/Q_1)^2]^2} \end{aligned} \quad (36)$$

A similar procedure can be used to derive expressions for z_2 and the corresponding calibration scale factor.

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