Linear Stiffness Compensation Using Magnetic Effect to Improve Electro-Mechanical Coupling for Piezoelectric Energy Harvesting

J. Xu

Graduate Research Assistant and Ph.D. Candidate

Department of Mechanical Engineering

The University of Connecticut

Storrs, CT 06269

USA

J. Tang[†]

Professor

Department of Mechanical Engineering

The University of Connecticut

Storrs, CT 06269

USA

Phone: (860) 486-5911, Email: jtang@engr.uconn.edu

**Submitted to Sensors and Actuators A: Physical
(Revised -2nd)**

[†] Corresponding author

Linear Stiffness Compensation Using Magnetic Effect to Improve Electro-Mechanical Coupling for Piezoelectric Energy Harvesting

J. Xu and J. Tang[†]

Department of Mechanical Engineering

The University of Connecticut

Storrs, CT 06269

USA

Phone: (860) 486-5911, Email: jtang@engr.uconn.edu

ABSTRACT

Piezoelectric transducers have been widely used in vibration-based energy harvesting. The efficiency of piezoelectric energy harvesting depends to a large extent on the electro-mechanical coupling of the harvester. Improvement of the coupling effect is possible through a variety of means from the transducer material-level to the device-level. At the device-level, the coupling effect of a cantilever-type energy harvester is inversely proportional to its mechanical stiffness which consists of the stiffness of the cantilever beam and that of the piezoelectric transducer. The mechanical stiffness, however, cannot be easily reduced because of the necessary inclusion of the cantilever beam and the inherent stiffness of the transducer. Both the beam and the transducer cannot be made arbitrarily thin due to typical design considerations. This paper reports a device-level approach for improvement of energy conversion efficiency by directly compensating the effective stiffness of a cantilever-type vibration energy harvester through magnetic effect. Specifically, two sheet-type permanent magnets are placed in the vicinity of the piezoelectric composite beam, and another slender magnetic bar is attached onto the beam together with an iron proof mass where one of its poles is located between the two magnetic sheets. It is shown analytically and experimentally that, under such configuration, a linear magnetic field yielding linear force pointing towards the magnetic sheets can be produced, which results in the reduction of the effective stiffness of the energy harvester to improve the electro-mechanical coupling. The experimental case studies demonstrate that the electro-mechanical coupling coefficient can be increased by 65% with 44.1% stiffness compensated. Both the open-circuit voltage and the power output are enhanced.

Keywords: piezoelectric energy harvesting, vibration energy, electro-mechanical coupling, magnetic effect, stiffness compensation.

[†] Corresponding author

1. INTRODUCTION

Due to the rapid advancement in portable and wireless devices, harvesting energy from ambient sources to prolong power supply to these devices has received significant attention (Paradiso and Starner, 2005; Roundy et al, 2005; Mitcheson et al, 2008). As vibration energy is virtually ubiquitous, utilizing piezoelectric transducers for energy harvesting, i.e., converting vibration energy into electrical energy, has emerged as one of the primary methods (Anton and Sodano, 2007; Priya, 2007; Khaligh et al, 2010). Piezoelectric transducers are compact, have high bandwidth, and maintain good linearity within functional range. Design and optimization of piezoelectric transducer-based energy harvesters have been addressed in several aspects (Liang and Liao, 2011): 1) absorbing mechanical energy from vibration source (Shenck and Paradiso, 2001; Shahruz, 2006; Ji et al, 2010); 2) converting mechanical energy into electrical energy (Baker et al, 2005; Xu et al, 2012); and 3) storing electrical energy (Guyomar et al, 2005). Amongst these aspects, the conversion of mechanical energy into electrical energy is generally characterized by the electro-mechanical coupling coefficient of the device, i.e., the ratio of the energy converted to that imposed (Tang and Wang, 2001; Roundy, 2005; Lefeuvre et al, 2006).

The effect of electro-mechanical coupling on energy harvesting has been analyzed in a number of studies. Roundy et al (2005) suggested that the power generated by a piezoelectric harvester was in general proportional to the electro-mechanical coupling. Lefeuvre et al (2006) indicated that the power output at resonance was proportional to the coupling in low coupling condition. Shu and Lien (2006) modeled and analyzed the power output of a piezoelectric energy harvesting system, and concluded that better performance could be achieved with larger coupling and quality factor. Meanwhile, Kim et al (2012) derived an optimal value for maximum power when the coupling coefficient was greater than a certain threshold value. While saturation of power output may exist (Renno et al, 2009), the maximum power output envelop could be increased with the increase of electro-mechanical coupling (Guyomar et al, 2005; Shu and Lien, 2006). Tang and Yang (2011) further pointed out that a large coupling coefficient boosts power output of a system with standard circuit even more than that with synchronized charge extraction technique (SCE). The electro-mechanical coupling coefficient is directly related to the transducer material property, predominantly the piezoelectric coupling constant at the material-level. For example, one of the most commonly used piezoelectric transducers, PZT5H, has limited piezoelectric coupling constant of 0.44 in the 31 direction and 0.75 in the 33 direction, respectively (when attached to a beam, the 1 coordinate of the transducer is along the length direction, the 2 coordinate the width direction and the 3 coordinate the transversal direction) (IEEE Standard). Choosing transducers with larger piezoelectric coupling constant may certainly help. Rakkamrungsri et al (2010) compared the performance of two energy harvesters with the same configuration but two different transducers, PMN-PT and PZT. The one with PMN-PT showed higher power output, but this type of material is more expensive.

At the device-level, a number of studies have defined an electro-mechanical coupling coefficient of an energy harvester, and indicated that this coefficient is related to not only the material property of the transducer, but also the specific design features of the harvester. For example, the harvester in general stores a significant portion of energy through its mechanical stiffness throughout the energy harvesting process, which reduces the energy conversion efficiency (Roundy, 2005; Kenji and Takaaki, 2010; Liang and Liao, 2011). It is worth emphasizing that the mechanical stiffness of the energy harvester includes contributions from both the host beam and the piezoelectric transducer. A significant portion of the mechanical stiffness in the harvester comes from the piezoelectric bending stiffness, which cannot be easily reduced. In the unimorph case, a host beam is required, which shifts the beam-shaped transducer's neutral line so the charges generated by the top and bottom halves of the transducer during bending vibration will not offset. The host beam thus has to possess certain thickness, yielding considerable mechanical stiffness. Many investigations have been carried out to explore manipulating the mechanical behavior as well as the circuitry dynamics of the harvester in order to enhance the electro-mechanical coupling. Chen and Wang (2005) investigated the increase of the electro-mechanical coupling of a piezoelectric cantilever by thickness ratio optimization, and identified a maximum coupling under a proper thickness ratio. Alberto and Paolo (2007) studied the optimal positioning of piezoelectric transducer. These investigations are in the category of mechanical tailoring, and therefore are subject to the limitation of the material-level coupling constant.

It is worth noting that there actually has been a stream of efforts to improve the electro-mechanical coupling coefficient and to tune the system dynamics in a device by resorting to external elements. For example, Tang and Wang (2001) indicated that adding an operational amplifier-based negative capacitance, which offsets the inherent capacitance of the piezoelectric transducer, could increase the electro-mechanical coupling. Since the negative capacitance element itself consumes power, such an approach is more suitable for control/damping applications. For another example, it was found that compressive axial forces could reduce the mechanical bending stiffness of the cantilever (Lesieutre and Davis, 1997; Leland and Wright, 2006; Masana and Daqaq, 2011; Niri and Salamone, 2012). This type of methods is generally effective when the vibration amplitudes are small. To introduce favorable dynamic effects to energy harvesters, conservative forces due to permanent magnets, because of the non-contact nature, recently emerged as an attractive option. Challa et al (2008) attached magnetic blocks on the top and bottom of the cantilever, and created attractive and repulsive forces to adjust the harvester's resonant frequency to match the frequency of the ambient vibration to improve the bandwidth of energy harvesting. As the force by the magnetic blocks exhibits nonlinearity, the system is essentially nonlinear. Cottone et al (2009) placed magnets on the cantilever tip and in the vicinity to generate nonlinearity which was then used to improve the bandwidth of energy harvesting. One challenge in these nonlinear

methods is that the system behavior is usually amplitude-dependent, which may limit the effectiveness of the respective methods to a certain operating range.

One ideal solution would be to add a negative spring element to a harvester to reduce its bending stiffness while maintaining the linearity. As mentioned, reducing the mechanical stiffness can increase the electro-mechanical coupling at the system level, and maintaining linearity can ensure that the performance enhancement can be realized within the entire operating range. Such a negative spring element should be non-contact with no additional damping. Inspired by the recent explorations in incorporating magnets into energy harvester design (Challa et al, 2008; Cottone et al, 2009), in this research we develop a design scheme that enables the compensation of the effective stiffness of the energy harvester by utilizing magnetic effects. In particular, a linear magnetic field effect is synthesized by placing properly configured magnetic elements in the vicinity of the piezoelectric cantilever. The magnetic field produced by these elements yields a linear force with respect to the cantilever displacement that is equivalent to the negative spring element effect, thereby reducing the portion of energy stored in transducer and host structure and improving the electro-mechanical coupling of the system. As the additional force induced is linear, the electro-mechanical coupling improvement is effective throughout the entire operating range of the harvester, and is not amplitude-dependent. Another potential advantage of this design scheme is that adding the aforementioned magnetic elements does not exclude the usage of other design optimization methods proposed previously (Shenck and Paradiso 2001; Baker et al, 2005; Guyomar et al, 2005; Shahruz, 2006; Challa et al, 2008; Cottone et al, 2009; Xu et al, 2012).

The rest of this paper is organized as follows. In Section 2, the concept of the new design is outlined first, followed by the analysis of the magnetic field to demonstrate the benefits to electro-mechanical coupling improvement and energy harvesting performance. In Section 3, experimental studies are reported that validate the analytical predictions. Finally, concluding remarks are provided.

2. DESIGN CONCEPT AND SYSTEM ANALYSIS

2.1 Schematic of design

Figure 1a illustrates the concept of stiffness compensation, where a fictitious negative spring element is included. Figure 1b shows the schematic diagram of the new piezoelectric energy harvester system proposed, which is a conventional piezoelectric cantilever incorporated with specially arranged magnetic elements to improve the electro-mechanical coupling coefficient. The piezoelectric cantilever consists of a cantilevered steel beam bonded with a piezoelectric transducer near the fixed-end. The poling axis of the piezoelectric transducer is perpendicular to the beam. Both the transducer and the beam are geometrically uniform. A sphere-shaped iron mass is attached to the free end of the beam as a proof-mass.

A thin, slender magnetic bar is bonded on the top of the beam near the free end. Two additional magnetic sheets are mounted above and underneath the proof-mass, respectively. Figure 2 shows more specifically the arrangement of the two magnetic sheets. These magnetic sheets are placed with the same poles facing each other, and are bonded to non-magnetic backsheets that are fixed in space. They are placed symmetrically with respect to the proof-mass. The relation between the magnetic field of a magnet and the distance is fourth-order in general (Moon, 1978). As stated, our goal here is to create a linear force applied to the beam as it vibrates, that is equivalent to the effect of a negative spring element so the electro-mechanical coupling can be increased. The challenges and the rationale of the new design are presented as follows.

One challenge here is to identify an arrangement of the magnetic elements that can indeed yield a linear magnetic field. By using the two magnetic sheets shown in Figure 2, our hypothesis is that under certain design, the overall effect of the two opposing magnetic fields could lead to a linear field. For example, as the beam vibrates towards the top magnetic sheet, the effects of the two fields due to the top magnetic sheet (from its top and bottom surfaces) increase at a fourth-order relation with respect to beam displacement; meanwhile, the effects of the two fields due to the bottom magnetic sheet are in the opposite directions and also at a fourth-order relation. As the distances between the magnetic sheet surfaces and the beam are in a three-dimensional space, it is possible that, by properly choosing the dimensions of the magnetic sheets as well as the gap between the sheets and the beam, the nonlinearity in the field effects may offset. A similar idea has been explored in a recent study on vibration isolation (Wu et al, 2014).

Another challenge is to utilize the linear magnetic field to create a linear force. One cannot create a linear force by simply placing soft magnetic material or permanent magnet in a linear magnetic field. Indeed, the force applied onto the soft magnet material (such as iron) is proportional to the gradient of the magnetic field, which will not be linearly proportional to the displacement if the magnetic field is linear. If a permanent magnet is employed, as its poles are both subject to the linear magnetic field, the combinatory effect leads to a nonlinear force. In this research, we propose to synthesize a magnetic monopole effect by using a thin, slender magnetic bar bonded on the top of the beam. One pole of this magnetic bar is placed inside the linear magnetic field, while the other pole is outside. As the other pole is outside of the aforementioned magnetic field, such a placement forms the basis of a magnetic monopole. Moreover, at the end of the slender magnetic bar a sphere-shaped proof-mass is attached, which will regulate the magnetic induction to induce the magnetic monopole effect. As will be shown later, with this configuration, the softening/hardening effect due to the nonlinear magnetic fields can be eliminated, leading to a linear force equivalent to a negative spring element.

2.2 Magnetic force analysis

In this section we analyze the magnetic force generated under the design scheme described in Section 2.1. Let l_x, l_z and t_m represent the width, length and thickness of each magnetic sheet, and let h represent the distance between the two magnetic sheets, as shown in Figure 3. Let r represent the radius of the iron ball (i.e., the proof-mass). Furthermore, let B_1 and B_2 represent the magnetic flux density at the surfaces of the magnetic sheets and that of the sphere-shaped iron element, respectively. Since the sphere-shaped iron element is directly connected to the magnetic bar ($1 \times 1 \times 20$ mm), it is regarded as a permanent magnet as it maintains magnetization. The parameters of the prototype system analyzed in this research are tabulated in Table 1.

The analysis of magnetic force involves two parts, the magnetic field from magnetic sheets and the magnetic property of the proof-mass. We first consider the magnetic field from the two magnetic sheets. In general, the magnetic force between two arbitrary points is related to the magnetic flux density at each point and the distance between the two points (Moon, 1978). Without loss of generality, we can write the magnetic force between an arbitrary point on the surface of the magnetic sheet and the proof-mass as

$$F = q_{\text{ball}} H = \frac{q_{\text{ball}} \sigma_s}{4\pi\mu_0} \int \frac{1}{l^2} dS \quad (1)$$

where q_{ball} is the magnetic charge on the iron ball, H denotes the magnetic H -field, σ_s is the magnetic charge density on the surfaces of the magnetic sheets, μ_0 is the magnetic permeability in a classical vacuum, l is the distance between the iron ball and an arbitrary magnetic charge on the surface of the magnetic sheet, and dS represents an infinitesimal area on the surface of the magnetic sheet. Recall that the two magnetic sheets are symmetrically arranged with respect to the beam neutral plane. We specifically consider a line that connects the center points of the two sheets. The magnetic force at an arbitrary point on this line results from the magnetic forces from all four surfaces of the two magnetic sheets, and the y-direction component is

$$F_y = \frac{q_{\text{ball}}}{4\pi\mu_0} \sigma_s \int_S \left(\frac{1}{s_1^2} \cos \theta_1 - \frac{1}{s_2^2} \cos \theta_2 - \frac{1}{s_3^2} \cos \theta_3 + \frac{1}{s_4^2} \cos \theta_4 \right) dS \quad (2)$$

where S stands for the four surfaces mentioned above, s_1, s_2, s_3 and s_4 denote the distances from those surfaces to the proof-mass, and $\theta_1, \theta_2, \theta_3$ and θ_4 denote the angles between the lines of s_1, s_2, s_3, s_4 and the y-axis, respectively,

$$s_1 = \sqrt{x^2 + z^2 + \left(\frac{1}{2}h - w(l_b, t) \right)^2}, \quad s_2 = \sqrt{x^2 + z^2 + \left(\frac{1}{2}h + t_m - w(l_b, t) \right)^2}$$

$$s_3 = \sqrt{x^2 + z^2 + \left(\frac{1}{2}h + w(l_b, t)\right)^2}, \quad s_4 = \sqrt{x^2 + z^2 + \left(\frac{1}{2}h + t_m + w(l_b, t)\right)^2} \quad (3a-d)$$

As the beam vibrates during energy harvesting, we let $w(x, t)$ denote the displacement of the beam in the y -direction at location x and time t (to be further analyzed in the subsequent section). $w(l_b, t)$ in Equation (3) then denotes the displacement of the proof-mass, where l_b is the length of the host beam. It is worth noting that, owing to the symmetrical arrangements, the horizontal components of the magnetic forces offset. We can further derive

$$F_y = q_{\text{ball}} \frac{\sigma_s}{4\pi\mu_0} \int_{-\frac{l_x}{2}}^{\frac{l_x}{2}} \int_{-\frac{l_z}{2}}^{\frac{l_z}{2}} \left(\frac{1}{s_1^2} \frac{\frac{1}{2}h - w(l_b, t)}{s_1} - \frac{1}{s_2^2} \frac{\frac{1}{2}h + t_m - w(l_b, t)}{s_2} - \frac{1}{s_3^2} \frac{\frac{1}{2}h + w(l_b, t)}{s_3} + \frac{1}{s_4^2} \frac{\frac{1}{2}h + t_m + w(l_b, t)}{s_4} \right) dx dz \quad (4)$$

At this point we may evaluate the linearity of F_y with respect to the beam displacement/vibration. Since the magnetic charge on the iron ball keeps constant, the magnetic force depends upon the magnetic flux density from the two sheets. As shown in Equation (1), the force is the product of the magnetic charge and the flux density. We perform numerical analysis of the magnetic flux density by using the parameters given in Table 1, and the results are shown in Figure 4. While the individual magnetic flux density of each sheet is obviously nonlinear, the combined flux density shows good linearity. The linearity of the combined one is due to that the high-order terms in the magnetic flux density of the two sheets offset. We derive the Taylor series expansion of F_y with respect to the beam displacement/vibration (within the range of 0.01 m), and obtain

$$F_y = 1.67 \times 10^{-2} w(l_b, t) + 1.37 \times 10^{-5} w(l_b, t)^2 + 2.14 \times 10^{-6} w(l_b, t)^3 + o(w(l_b, t)^4) \quad (5)$$

The above equation shows a nearly linear relation. Moreover, the tangential stiffness due to the magnetic force is given as

$$k_y = 1.67 \times 10^{-2} + 2.74 \times 10^{-5} w(l_b, t) + 6.42 \times 10^{-6} w(l_b, t)^2 + o(w(l_b, t)^3) \quad (6)$$

It should be noted that for the beam considered, the range within which the magnetic force shows good linearity, 0.01 m, is actually quite large. The high-order terms in the tangential stiffness expression are much smaller (i.e., three orders of magnitude smaller) than the constant term.

We analyze the magnetic flux density using the equations provided above. Indeed, the magnetic flux density between the two magnetic sheets is proportional to F_y shown in Equation (4). We calculate the magnetic flux density on the neutral yz -plane under given parameters, and the result is shown in Figure 5. Along the central line that passes through the proof mass, very good linearity can be observed. Figure 6 plots the difference of the magnetic flux density calculated and its linear approximation, which is very

small as expected.

We carry out some further analyses to elucidate the parametric influence. The linearity of the magnetic field depends on a number of parameters, for instance, the distance between the two sheets, h , the length, l_x , and the width, l_z , of the sheets. Figures 7 and 8 illustrate the influences of h and width l_z under constant l_x . From Figure 7, it can be seen that good linearity can be achieved with properly chosen h . Although decreasing h will increase the magnetic flux density, more significant nonlinearity may also arise. Similar conclusion can be drawn from Figure 8.

We now consider the effect of the magnetic property of the proof-mass. As indicated in Section 2.1, ideally, one would like to have a mono-pole like proof-mass to make utilization of the magnetic field. Here we resort to 3-D finite element analysis using ANSYS, as shown in Figure 9. A sphere-shaped iron mass with radius 2 mm is attached at the end of the slender magnetic bar ($1 \times 1 \times 20$ mm). In ANSYS analysis, the domain is defined as air with permeability of 1, and Solid236 element is used to model the air and Solid111 element is used to model the infinite boundary condition. The static magnetic flux density obtained from ANSYS is shown in Figure 10a. It can be observed that magnetic induction lines spread in a spherically symmetric manner from the proof-mass, which indeed yields the form of magnetic mono-pole. In other words, this particular proof-mass selected can be regarded as a point-like magnetic source with constant magnetic flux density. By this design, the linearity of the magnetic force derived above can be realized.

In comparison, we also analyze the influence of the size of the sphere-shaped proof-mass to the distribution of the magnetic flux density. Figures 10b and 10c plot the results corresponding to different radii. In Figure 10b, when the radius is 4 mm, one can observe an under-magnetization situation which is due to the ineffective magnetization of the proof-mass. In Figure 10c, when the radius is 1 mm, one can observe an over-magnetization situation. It should be mentioned that in general the magnetic bar should be sufficiently long in order to avoid the magnetic confinement fusion between its two ends.

In summary, the analysis in this sub-section shows that it is feasible to produce a linear negative force that can compensate the mechanical stiffness of the piezoelectric cantilever by properly configured magnetic fields. Hereafter, the equivalent negative stiffness is expressed as

$$k_{\text{mag}} = \left. \frac{dF_y}{dw(l_b, t)} \right|_{w(l_b, t)=0} \quad (7)$$

2.3 Modeling and analysis of the harvester

In this section, we investigate the characteristics of the piezoelectric cantilever incorporated with the proposed stiffness compensation scheme. Assumptions are made as follows: (1) this prototype is

considered as a composite Euler–Bernoulli beam; therefore, the effects of shear deformation and rotary inertia of the beams are neglected; (2) the piezoelectric layer is perfectly bonded to the beam; thus, there is no sheer strain between the layers; (3) only the damping of the harvester structure is considered; and (4) the system is excited by base-movement $u(t)$.

The equations of motion of the system can be derived by using the assumed mode method. Here we let the beam displacement be expressed as

$$w(x,t) = \phi(x)q(t) \quad (8)$$

where we use one-mode assumption and, without loss of generality, $\phi(x)$ is the first bending mode of the beam. q is the generalized displacement. Based on the extended Hamilton's principle, we can derive (please see Appendix)

$$\begin{cases} M\ddot{q} + c\dot{q} + (K - k_m)q + k_1Q = F_a \\ R\dot{Q} + k_2Q + k_1q = 0 \end{cases} \quad (9)$$

where M , c , and K are respectively the modal mass, damping, and open-circuit stiffness of the piezoelectric cantilever, k_1 is the electro-mechanical coupling term, R is the resistance load in the system, and k_2 is the inverse of the capacitance of the piezoelectric transducer. F_a is the modal force due to the based excitation. Details of these terms can be found in Appendix. In Equation (9), the negative stiffness effect due to the magnetic effect is denoted as

$$k_m = k_{\text{mag}}\phi^2(l_b) \quad (10)$$

where k_{mag} is given by Equation (7). Essentially, k_m represents the stiffness reduction to the mode of interest.

We now analyze the frequency response characteristics of the harvester. Hereafter we use the tilde notation to indicate the magnitude of the related quantity. The frequency response of displacement amplitude $\tilde{q}$ with respect to base excitation amplitude $\tilde{F}_a$ is given as

$$\frac{\tilde{q}}{\tilde{F}_a} = \frac{Ri\omega + k_2}{(Ri\omega + k_2)(-\omega^2 M + ci\omega + K - k_m) - k_1^2} \quad (11)$$

We carry out non-dimensionalization, and define,

$$\omega_n = \sqrt{\frac{K - k_m}{M}}, \quad \Omega = \frac{\omega}{\omega_n}, \quad \varsigma = \frac{c}{\sqrt{(K - k_m)M}}, \quad r = \frac{R}{k_2} \omega_n \quad (12a-d)$$

which represent, respectively, natural frequency of the harvester beam, non-dimensional excitation frequency, mechanical damping ratio, and resistance load. We further introduce the definition of the electro-mechanical coupling coefficient of the harvester (Guyomar et al, 2005; Shu and Lien, 2006),

$$k_e^2 = \frac{k_1^2}{(K - k_m)k_2} \quad (13)$$

As pointed out in previous studies (Tang and Wang, 2001; Guyomar et al, 2005; Shu and Lien, 2006), at the device level, this electro-mechanical coupling coefficient explicitly includes the mechanical stiffness into the analysis of energy conversion, i.e., the ratio of the energy converted to that imposed. Indeed, the mechanical stiffness of a system integrated with a piezoelectric transducer is dependent on many factors, e.g., the geometry of the transducer and the configuration of the host structure, etc. Using the same transducer material, the equivalent mechanical stiffness of the actual system may vary drastically, causing significant difference in system-level energy conversion capability. For the proposed harvester with magnetic effect, the relation between the electro-mechanical coupling coefficient and the negative stiffness due to the magnetic effects, k_m , is shown in Figure 11. As can be observed, as the stiffness compensation effect increases and less potential energy is stored in the system, the electro-mechanical coupling generally increases.

The normalized displacement of the beam can be obtained as

$$\bar{q} = \frac{\tilde{q}}{\tilde{F}_a / (K - k_m)} = \frac{ir\Omega + 1}{(ir\Omega + 1)(-\Omega^2 + i\zeta\Omega + 1) - k_e^2} \quad (14)$$

Under open-circuit and short-circuit conditions, we have $r \rightarrow \infty$ and $r \rightarrow 0$, respectively, and the normalized frequency responses are

$$\bar{q}_{\text{open}} = \frac{\tilde{q}}{\tilde{F}_a / (K - k_m)} = \frac{1}{-\Omega^2 + i\zeta\Omega + 1} \quad (15a)$$

$$\bar{q}_{\text{short}} = \frac{\tilde{q}}{\tilde{F}_a / (K - k_m)} = \frac{1}{-\Omega^2 + i\zeta\Omega + 1 - k_e^2} \quad (15b)$$

Therefore, under open-circuit condition, the frequency response of displacement is not related to the electro-mechanical coupling coefficient k_e . It is worth noting that the open-circuit resonant frequency ω_o and the short-circuit resonant frequency ω_s are related by the aforementioned electro-mechanical coupling coefficient in the following manner (Tang and Wang, 2001),

$$\omega_s = \sqrt{1 - k_e^2} \omega_o \quad (16)$$

which can be used subsequently in the experimental analysis. Similarly, the normalized frequency response of voltage $\bar{V}$ output of the harvester can be derived as

$$\bar{V} = \frac{\tilde{V}}{\tilde{F}_a M k_2} = \frac{i\Omega k_e r}{(ir\Omega + 1)(-\Omega^2 + i\zeta\Omega + 1) - k_e^2} \quad (17)$$

Under open-circuit condition, we have

$$\bar{V}_{\text{open}} = \frac{\tilde{V}}{\tilde{F}_a M k_2} = \frac{i\Omega k_e}{-\Omega^2 + i\zeta\Omega + 1} \quad (18)$$

We can readily observe that the open-circuit voltage response is proportional to the electro-mechanical coupling coefficient. The frequency responses of open-circuit voltage under increased electro-mechanical coupling coefficients (due to stiffness compensation) are shown in Figure 12. It can be seen that the normalized frequency response of voltage is increased in the entire frequency range with the increase of the electro-mechanical coupling coefficient.

3. EXPERIMENTAL VALIDATION AND DISCUSSION

Experiments are carried out to verify the proposed stiffness compensating scheme as well as the new harvester performance. The parameters involved are given in Table 1, which are consistent with those used in the preceding analysis. The experimental setup is shown in Figure 13. The transducer, PZT-5H, was purchased from Piezo Systems Inc. The magnetic sheets were purchased from K&J Magnet Inc. The PZT transducer was bonded to the steel beam by using epoxy glue (DP100) and underwent curing for 24 hours at room temperature. The two magnetic sheets were bonded to non-magnetic backsheets, and the magnetic bar and iron ball were bonded to the cantilever beam in the same way.

Figure 14 shows the experimental data of magnetic flux density along the central line that passes through the proof-mass measured by a Hall sensor. As can be observed, very good linearity of the magnetic flux density is achieved, and the numerical and experimental results match very well.

Recall the relation between the electro-mechanical coupling coefficient and the resonant frequencies shown in Equation (16). The coupling coefficient k_e^2 can be calculated through measured open-circuit and short-circuit resonant frequencies. Here we record the frequency responses of displacements at the middle of cantilever under sinusoidal base movement with constant amplitude acceleration of 0.1 g measured by an eddy current displacement sensor (3300XL/8mm). Figures 15a and 15b show the frequency responses comparison. For the piezoelectric cantilever without stiffness compensation, the open-circuit and short-circuit resonant frequencies are, 48.29 Hz and 47.44 Hz, respectively. The coupling coefficient is 0.0349. For the piezoelectric cantilever with stiffness compensation, the open-circuit and short-circuit resonant frequencies are, 36.1 Hz and 35.04 Hz, respectively. Thus, the stiffness of the piezoelectric cantilever is reduced by 44.1%, and the coupling coefficient is increased to 0.0576. The coupling coefficient is increased by 65%. It is worth commenting that under open-circuit condition, the peak responses are actually smaller, possibly caused by the additional energy dissipation due to leakage resistance of the piezoelectric transducer.

Figure 16 shows the experimental data of open-circuit voltage frequency response of the piezoelectric cantilever without and with the magnetic effects under 0.2 g base excitation. To validate the performance

improvement, Figure 16 also includes the numerical prediction results obtained based on the model presented in the preceding section. As can be observed, the numerical and experimental results match very well. The peak voltage output of the piezoelectric cantilever measured without the magnetic effects is 8.6 V at 48.29 Hz. With the magnetic effects, the resonant frequency is reduced to 36.1 Hz and the peak voltage output is increased to 10.9 V. In other words, the peak voltage output is increased by 27%. It is worth commenting that, while the proposed scheme of stiffness compensation reduces the resonant frequency in a way similar to adding a larger proof-mass, it has a unique benefit that the open-circuit voltage is increased in the entire frequency range (Equation (18)) since the electro-mechanical coupling coefficient of the harvester is increased. This method can be combined with other techniques to optimize the harvester performance.

We also analyze the frequency responses of voltage output for the system with stiffness compensation under different excitation levels, and the measurement results are plotted in Figure 17. The peak voltage output is measured as 6.2 V, 10.9 V, 15.8 V and 19.2 V at resonance under excitation levels of 0.1 g, 0.2 g, 0.3 g, and 0.4 g, respectively. The relation between excitation level and peak voltage output is plotted in Figure 18, where we can observe that the voltage output maintains proportionality with respect to the excitation levels, which indicates the amplitude-independent feature of this scheme. In other words, the effectiveness of the enhanced harvester is not limited to a certain operating range. It should be noted that, the peak-to-peak displacement of the tip of the cantilever under 0.4 g excitation is 5.6 mm, which is within the space limitation between the two magnetic sheets. Therefore, the effective operating range of the proposed scheme is quite large.

Finally we examine the power output performance. For easy comparison, we simply apply the optimal resistance load (Shu and Lien, 2006; Ji et al, 2010)

$$R_{\text{opt}} \approx \frac{\pi}{2\omega C_p} \quad (19)$$

where $C_p = 1/k_2$ is the piezoelectric capacitance (k_2 is shown in Equation (A12b)), and ω is the excitation frequency. For the system without and with stiffness compensation, the optimal resistance loads measured under 0.015 g base acceleration for maximum power output are plotted in Figure 19. The best matching loads are chosen as 158 k Ω and 189 k Ω for the system without and with stiffness compensation, respectively. Since the best matching load is a function of the resonant frequency, for the respective system the best matching loads are different although the piezoelectric transducers have the same size. The resulting peak-to-peak voltage across the load resistance was measured by an oscilloscope. The output power was calculated as (Lefeuvre et al, 2006; Challa et al, 2008)

$$P = \frac{V_{p-p}^2}{8R_{\text{opt}}} \quad (20)$$

Figure 20 shows the power outputs of the original system without stiffness compensation and the proposed system with stiffness compensation. The base excitation level is 0.2 g. Without stiffness compensation, the maximum power output is 14.2 μW . With the stiffness compensation scheme proposed in this research, the maximum power output is increased to 16.05 μW . Figure 20b highlights the off-resonance performance comparison. It can be observed that the stiffness compensation scheme can also increase the power output at off-resonance frequencies. This is due to that the stiffness compensation can increase the electro-mechanical coupling coefficient at the device level.

4. CONCLUDING REMARKS

In this research, a scheme to improve the energy conversion capability of vibration-based piezoelectric energy harvester is developed. It is shown theoretically and experimentally that properly configured and positioned magnet elements can induce a linear magnetic field that yields negative stiffness. This scheme can effectively compensate the mechanical stiffness of the harvester and increase the electro-mechanical coupling at the device-level. This can improve the energy harvesting efficiency at both resonance and off-resonance situations. Experimental studies on a prototype show that electro-mechanical coupling factor can be increased by 65% with 44.1% stiffness compensated. Both the open-circuit voltage and the power output are enhanced.

ACKNOWLEDGMENT

This research is supported in part by NSF under grant CMMI-1130724.

APPENDIX: EQUATIONS OF MOTION

Let l_p , l_b , b , m_{pm} , t_p , t_b , ρ_p and ρ_b represent, respectively, the length of the piezoelectric transducer and that of the beam, the width of the piezoelectric composite beam, the proof mass, the thickness of the transducer and that of the beam, and the mass density of the transducer and that of the beam. The equations of motion of the system can be derived by using the extended Hamilton's principle,

$$\int_{t_1}^{t_2} (\delta T - \delta U + \delta W_c + \delta W_f) dt = 0 \quad (A1)$$

where T is the kinetic energy, U is the potential energy, W_c is the virtual work of damping force, and W_f is the virtual work due to the magnetic effects. As mentioned, the beam displacement is discretized as shown in Equation (8), in which the assumed mode is chosen as the first bending mode of the cantilever beam (Ertuck and Inman, 2010),

$$\phi(x) = \cos \frac{\lambda}{l_b} x - \cosh \frac{\lambda}{l_b} x + \varsigma \left(\sin \frac{\lambda}{l_b} x - \sinh \frac{\lambda}{l_b} x \right) \quad (\text{A2})$$

where $\varsigma = \frac{\sin \lambda - \sinh \lambda}{\cos \lambda + \cosh \lambda}$, and the eigenvalue λ is the smallest one solved from $\cos(\lambda l_b) \cosh(\lambda l_b) = -1$.

Let $A_b = b t_b$ and $A_p = b t_p$ represent, respectively, the cross sectional area of the beam and that of the piezoelectric transducer.

The kinetic energy and its variation can be derived as

$$T = \frac{1}{2} \rho_b A_b \int_0^{l_b} \left(\frac{\partial w(x,t)}{\partial t} + \frac{\partial u(t)}{\partial t} \right)^2 dx + \frac{1}{2} \rho_p A_p \int_0^{l_p} \left(\frac{\partial w(x,t)}{\partial t} + \frac{\partial u(t)}{\partial t} \right)^2 dx + \frac{1}{2} m_{pm} \left(\frac{\partial w(l_b,t)}{\partial t} + \frac{\partial u(t)}{\partial t} \right)^2 \quad (\text{A3})$$

and

$$\begin{aligned} \delta T = & [\rho_b A_b \int_0^{l_b} \phi^2(x) \ddot{q}(t) dx + \rho_p A_p \int_0^{l_p} \phi^2(x) \ddot{q}(t) dx + m_{pm} \phi^2(l_b) \ddot{q}(t) + \rho_b A_b \int_0^{l_b} \phi(x) \ddot{u}(t) dx \\ & + \rho_p A_p \int_0^{l_p} \phi(x) \ddot{u}(t) dx + m_{pm} \phi(l_b) \ddot{u}(t)] \delta q + [\rho_b A_b \int_0^{l_b} \phi(x) \ddot{q}(t) dx + \rho_p A_p \int_0^{l_p} \phi(x) \ddot{q}(t) dx + m_{pm} \phi(l_b) \ddot{q}(t) \\ & + \rho_b A_b \int_0^{l_b} \ddot{u}(t) dx + \rho_p A_p \int_0^{l_p} \ddot{u}(t) dx + m_{pm} \ddot{u}(t)] \delta u \end{aligned} \quad (\text{A4})$$

The coefficient of the inertia term, i.e., the modal mass, is

$$M = \rho_b A_b \int_0^{l_b} \phi^2(x) dx + \rho_p A_p \int_0^{l_p} \phi^2(x) dx + m_{pm} \phi^2(l_b) \quad (\text{A5})$$

As the base movement/excitation effect is included in the kinetic energy, the modal force due to base movement/excitation can be written as

$$F_a = - \left[\rho_b A_b \int_0^{l_b} \phi(x) dx + \rho_p A_p \int_0^{l_p} \phi(x) dx + m_{pm} \phi(l_b) \right] \ddot{u}(t) \quad (\text{A6})$$

The linear constitutive relation of the piezoelectric transducer is (IEEE, 1987)

$$\begin{cases} \tau = E_p \varepsilon - h_{31} D \\ E = -h_{31} \varepsilon + \beta_{33} D \end{cases} \quad (\text{A7})$$

where τ , ε , D and E represent, respectively, the stress, strain, electrical displacement (charge/area) and electrical field (voltage/length) of the piezoelectric transducer, and E_p , h_{31} and β_{33} are the Young's modulus, piezoelectric constant and dielectric constant of the transducer. We let E_b be the Young's modulus of the host beam, and let I_b and I_p represent the moments of inertia of the beam and the transducer, respectively. The elastic and electrical energy of the harvester comes from the host beam and the transducer. The variation of the elastic and electrical energy can be derived as (Tang and Wang, 2001)

$$\begin{aligned}
\delta U &= \delta U_b + \delta U_p = \int_{V_b} \tau \delta \varepsilon dV_b + \int_{V_p} (\tau \delta \varepsilon + E \delta D) dV_p \\
&= \int_{V_b} E_b \varepsilon \delta \varepsilon dV_b + \int_{V_p} E_p \varepsilon \delta \varepsilon dV_p - \int_{V_p} h_{31} D \delta \varepsilon dV_p - \int_{V_p} h_{31} \varepsilon \delta D dV_p + \int_{V_p} \beta_{33} D \delta D dV_p \\
&= \int_0^{l_b} EI_b (\phi''(x))^2 q(t) \delta q(t) dx + \int_0^{l_p} EI_p (\phi''(x))^2 q(t) \delta q(t) dx \\
&\quad - \int_0^{l_p} h_{31} D \int_0^{l_p} y \phi''(x) \delta q(t) dy dx - \int_0^{l_p} h_{31} \int_0^{l_p} y \phi''(x) q(t) \delta D dy dx + \int_0^{l_p} \int_0^{l_p} \beta_{33} D \delta D dy dx \\
&= (k_b + k_p) q \delta q + k_{pq} D \delta q + k_{pq} q \delta D + k_{pp} D \delta D
\end{aligned} \tag{A8}$$

where

$$K = k_b + k_p = \int_0^{l_b} EI_b (\phi'')^2 dx + \int_0^{l_p} EI_p (\phi'')^2 dx, \quad k_{pq} = \int_0^{l_p} F_p h_{31} \phi'' dx, \quad k_{pp} = \int_0^{l_p} A_p \beta_{33} dx \tag{A9a-c}$$

F_p is the first moment of area of the transducer. K is the modal mass of the piezoelectric composite beam.

The virtual work terms due to the mechanical damping and the magnetic force can be written as

$$\delta W = \delta W_c + \delta W_F = - \int_0^{l_b} c \frac{\partial w(x,t)}{\partial t} \delta q dx + k_{\text{mag}} \phi^2(l_b) q \delta q \tag{A10}$$

In this research we assume that the charge Q is uniformly distributed, i.e.,

$$D = \frac{Q}{bl_p} \tag{A11}$$

We further define

$$k_1 = \frac{k_{pq}}{bl_p}, \quad k_2 = \frac{k_{pp}}{(bl_p)^2} \tag{A12a, b}$$

Finally, substitution of the above expressions into Equation (A1) yields the equations of motion of the entire system as shown in Equation (9).

REFERENCES

- Anton, S. R., and Sodano, H. A., 2007, “A review of power harvesting using piezoelectric materials (2003–2006),” *Smart Materials and Structures*, V16, R1.
- Alberto, B., and Paolo, E., 2007, “Optimum placement of piezoelectric ceramic modules for vibration suppression of highly constrained structures”, *Smart Materials and Structures*, V16, 1662–1671.
- Baker, J., Roundy, S., and Wright, P., 2005, “Alternatives geometries for increasing power density in vibration energy scavenging for wireless sensor networks,” *Proceedings of the 3rd International Energy Conversion Engineering Conference* (San Francisco, CA, August 15–18, August), 959–70.
- Challa, V. R., Prasad, M. G., Shi, Y., and Fisher, F. T., 2008, “A vibration energy harvesting device with bi-directional resonance frequency tunability,” *Smart Materials and Structures*, V17, 015035.

Chen, Q., and Wang, Q., 2005, "The effective electromechanical coupling coefficient of piezoelectric thin-film resonators", *Applied Physics Letter*, V86, 022904.

Cottone, F., Vocca, H., and Gammaitoni, L., 2009, "Nonlinear energy harvesting," *Physical Review Letters*, V102, 080601.

Guyomar, D., Badel, A., Lefeuvre, E., and Richard, C., Daniel, G., Adrien, B., Lefeuvre, E., and Richard, C., 2005, "Toward energy harvesting using active materials and conversion improvement by nonlinear processing," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, V52, 584 - 595.

Henck, N. S., and Paradiso, J. A., 2001, "Energy scavenging with shoe-mounted piezoelectrics," *IEEE Micro Magazine*, V21, 30–42.

IEEE Standard on Piezoelectricity, ANSI/IEEE Std 176-1987.

Ji, J., Kong, F., He, Guan, Q., and Feng, Z., 2010, "Piezoelectric wind-energy-harvesting device with reed and resonant cavity," *Japanese Journal of Applied Physics*, V49, 050204.

Kenji, U., and Takaaki, I., 2010, "Energy Flow Analysis in Piezoelectric Energy Harvesting Systems," *Ferroelectrics*, V400, 305-320.

Khaligh, A., Zeng, P., and Zheng, C., 2010, "Kinetic energy harvesting using piezoelectric and electromagnetic technologies—state of the art," *IEEE Transactions on Industrial Electronics*, V57, 850–860.

Kim, D., Hewa-Kasakarage, N. N., Yoon, S., and Hall, N. A., 2012, "On the minimum coupling required for maximum theoretical power capture from vibration energy harvesters," *Applied Physics Letters*, V101, 103904.

Lefeuvre, E., Badel, A., Richard, C., Petit, L., and Guyomar, D., 2006, "A comparison between several vibration-powered piezoelectric generators for standalone systems," *Sensors and Actuators A: Physical*, V126, 405-416.

Lesieutre, G. A., and Davis, C. L., 1997, "Can a Coupling Coefficient of a Piezoelectric Device be Higher Than Those of Its Active Material?", *Journal of Intelligent Material Systems and Structures*, V8, 859-867.

Leland, E. S., and Wright, P. K., 2006, "Resonance tuning of piezoelectric vibration energy scavenging generators using compressive axial preload," *Smart Materials and Structures*, V15, 051413.

Liang, J. R., and Liao, W. H., 2011, "Energy flow in piezoelectric energy harvesting systems," *Smart Materials and Structures*, V20, 015005.

Mitcheson, P. D., Yeatman, E. M., Kondala, R. G., Holmes, A. S., and Green, T. C., 2008, "Energy harvesting from human and machine motion for wireless electronic devices," *Proceedings of the IEEE*, V96(9), 1457 - 1486.

Moon, F. C., 1978, "Problems in magneto-solid mechanics", *Mechanics Today*, V4, Edited by Nemat-Nasser, S. (New York: Pergamon).

Niri, E. D., and Salamone, S., 2012, "A passively tunable mechanism for a dual bimorph energy harvester with variable tip stiffness and axial load," *Smart Materials and Structures*, V21, 125025.

Paradiso, J. A., and Starner, T., 2005, "Energy scavenging for mobile and wireless electronics," *IEEE Pervasive Computing Magazine*, V4, 18–27.

Priya, S., 2007, "Advances in energy harvesting using low profile piezoelectric transducers," *Journal of Electroceramics*, V19, 165–182.

Rakbamrung, P., Lallart, M., Guyomar, D., Muensit, N., Thanachayanont, C., Lucat, C., Guiffard, B., Petit, L., and Sukwisut, P., Prissana, R., Mickaël, L., Daniel, G., Nantakan, M., Chanchana, T., Claude, L., Benoît, G., Lionel, P., and Pisan, S., 2010, "Performance comparison of PZT and PMN-PT piezoceramics for vibration energy harvesting using standard or nonlinear approach," *Sensors and Actuators A: Physical*, V163, 493–500.

Roundy, S., 2005, "On the effectiveness of vibration-based energy harvesting," *Journal of Intelligent Material Systems and Structures*, V16, 809-823.

Roundy, S., Leland, E. S., Baker, J., Carleton, E., Reilly, E. K., Lai, E., Otis, B. P., Rabaey, J. M. M., Wright, P. K., and Sundararajan, V., 2005, "Improving power output for vibration-based energy scavengers," *IEEE Pervasive Computing Magazine*, IEEE, V4, 28–36.

Renno, J. M., Daqaq, M. F., and Inman, D. J., 2009, "On the optimal energy harvesting from a vibration source," *Journal of Sound and Vibration*, V320, 386-405.

Shahruz, S. M., 2006. "Design of mechanical band-pass filters for energy scavenging," *Journal of Sound and Vibration*, V292, 987-998.

Shenck, N. S., and Paradiso, J. A., 2001, "Energy scavenging with shoe-mounted piezoelectrics", *IEEE Micro*, V21, 30–42.

Shu, Y. C., and Lien, I. C., 2006, "Efficiency of energy conversion for a piezoelectric power harvesting system," *Journal of Micromechanics and Micro-engineering*, V16, 2429–2438.

Tang, L., and Yang, Y., 2011, "Analysis of synchronized charge extraction for piezoelectric energy harvesting," *Smart Materials and Structures*, V20, 085022.

Tang, J., and Wang, K. W., 2001, "Active-passive hybrid piezoelectric networks for vibration control: comparisons and improvement," *Smart Materials and Structures*, V10, 794-806.

Wu, W., Chen, X., and Shan, Y., 2014, "Analysis and experiment of a vibration isolator using a novel magnetic spring with negative stiffness," *Journal of Sound and Vibration*, V333, 2958-2970.

Xu, J., Liu, Y., Shao, W., and Feng, Z., 2012, "Optimization of a right-angle piezoelectric cantilever using auxiliary beams with different stiffness levels for vibration energy harvesting," *Smart Materials and Structures*, V21, 065017.

Zhu, D., Roberts, S., Tudor, M. J., and Beeby, S. P., 2010, "Design and experimental characterization of a tunable vibration-based electromagnetic micro-generator," *Sensor and Actuators A: Physical*, V158, 284-293.

Table 1. Parameters of prototype energy harvester.

$l_x = 25.8 \text{ mm}$	$l_z = 9.9 \text{ mm}$	$t_m = 1.5 \text{ mm}$
$h = 11.9 \text{ mm}$	$r = 2 \text{ mm}$	$l_b = 94 \text{ mm}$
$l_p = 27.8 \text{ mm}$	$b = 11 \text{ mm}$	$h_p = 0.25 \text{ mm}$
$h_b = 0.45 \text{ mm}$	$B_I = 1876 \text{ Gauss}$	$B_2 = 552 \text{ Gauss}$
$\rho_p = 7500 \text{ kg/m}^3$	$\rho_p = 7900 \text{ kg/m}^3$	$E_b = 109 \text{ GPa}$
$E_p = 62 \text{ GPa}$	$d_{31} = -320 \text{ pC/N}$	$\beta_{33} = 2.92 \times 10^8 \text{ Vm/C}$
$h_{31} = 5.9 \times 10^8 \text{ N/C}$	$m_{pm} = 0.32 \text{ g}$

Figure 1. (a) Schematic of stiffness compensation; (b) Schematic diagram of the proposed new piezoelectric energy harvester.

Figure 2. Arrangements of magnetic elements.

Figure 3. Schematic for magnetic flux density calculation.

Figure 4. Magnetic flux densities calculated. -*-: combined magnetic flux density, -x-: magnetic flux density from top sheet, -. -: magnetic flux density from bottom sheet.

Figure 5. Magnetic flux density on neutral plane.
 (Neutral plane refers to the yz -plane connecting the middle lines of two plates.
 Central line refers to the middle line on the neutral plane.
 The tip of the cantilever moves along the central line)

Figure 6. Percentage difference between magnetic flux density calculated and the linear approximation.

Figure 7. Influences of h under constant l_x and l_z . -*-: $h = 10$ mm, -x-: $h = 12$ mm, -Δ-: $h = 14$ mm, -+-: $h = 16$ mm.

Figure 8. Influences of l_z under constant l_x and h . -x-: $l_z = 6$ mm, -Δ-: $l_z = 8$ mm, -*-: $l_z = 9.9$ mm, -+-: $l_z = 12$ mm, --: $l_z = 14$ mm.

Figure 9. Meshed 3-D scheme of proof-mass and magnetic bar.

Figure 10. Static magnetic flux density computed: (a) $r = 2$ mm; (b) $r = 4$ mm; (c) $r = 1$ mm.

Figure 11. Electro-mechanical coupling coefficient versus compensating stiffness ratio.

Figure 12. Normalized voltage frequency responses under increased coupling coefficient.

Figure 13. Experimental setup. a) schematic of the experiment, b) photograph of the prototype.

Figure 14. Measured magnetic flux density and linear fit.

—◆— measured data; — linear fit.

(a)

(b)

Figure 15. Measured displacement frequency responses (a) without stiffness compensation, $+$: short-circuit, Δ : open-circuit; (b) with stiffness compensation, $*$: short-circuit, x : open-circuit.

Figure 16. Measured and computed frequency responses of voltage output. -x-: response with stiffness compensation, -*: response without stiffness compensation.

Figure 17. Measured frequency responses of voltage output under different excitation amplitudes with stiffness compensation. -x-: amplitude of 0.1 g, -*: amplitude of 0.2 g, -Δ-: amplitude of 0.3 g, -+-: amplitude of 0.4 g.

Figure 18. Relation between excitation level and peak-to-peak voltage output.

(a)

(b)

Figure 19. Optimal resistance load for maximum power output. (a) without stiffness compensation; (b) with stiffness compensation.

Figure 20. Power output with optimal resistant load. (a) power output; (b) direct comparison. — with stiffness compensation; — without stiffness compensation.