Modeling and compensation of dynamic effects in camera-based position measurement

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Abstract

Cameras are used as position sensors for camera-based control or metrology, the goal being to estimate the position of some markers or features. When one of these markers has significant displacement over the exposure time, the measurement delivered by image processing cannot be considered as a sample of the actual trajectory as dynamic effects become effective. In this paper, models are proposed that allow to reproduce accurately the measurements provided by a camera. Various cases are considered: with full or partial exposure time; with global shutter mode or rolling shutter mode. An experimental evaluation in the global shutter case shows the accuracies of the models.

When the continuous-time trajectory of a device needs to be reconstructed, these accurate models of the camera-based position measurement can be used in order to improve the trajectory provided by the camera. A methodology based on a hybrid Kalman filter is proposed to solve this general issue. An experimental evaluation is provided for the global shutter case, based on real images inspired from the problem of heart motion reconstruction.

Keywords: Camera-based position measurement, visual servoing, Kalman filter, camera model, dynamic effects

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1. Introduction

Vision systems with real-time image processing have been developed during the last decades and, in association with efficient image processing algorithms, are now able to provide measurements of the position of an object at relatively high frequency. Based on this measurement, specific control schemes have been developed for controlling robotic manipulators [1]. For instance, Andersson developed a robot playing ping-pong based on a 60 Hz stereo vision system [2]. In 2000, Nakabo *et al.* reached a frame rate of 1 000 Hz in a vision servo loop [3].

In vision-based systems, the camera and the associated image-processing algorithm can be considered as a sensor that provides discrete-time (DT) measurements of the continuous-time (CT) image position of a marker or an object. When considering the imperfections of this sensor, one generally focuses on the geometrical aspects, i.e. the projection model whose parameters can be estimated from a calibration procedure [4]. This model is static and deals with the distortion in the projection on the camera plane. Concerning the dynamic effects, one generally considers that the measurement behaves as a sampler that delivers the position at given sampling instants.

When the object or the camera is moving and the displacements over an integration period of the image are significant, the image is altered and the object or the marker appears as a blur or a blob. Several authors have introduced methods that allow to take profit of this effect for the estimation of some features of the movement. Klein and Drummond use the blur effects to estimate the camera rotation [5] whereas Borrachi estimates its translation [6]. Some works also consider the rolling shutter effect: Liang *et al.* compensate the effects due to a translational motion [7]; Ait-Aider *et al.* rely on the deformation of lines to estimate the velocity [8].

Modeling the alteration of the measurement of the position of a marker, in the case of fast displacements, is a different subject. Due to the movement, the markers do not appear any more as simple disks but rather as more complex shapes, depending on the trajectory. Then, the measurement provided by the image processing differs from what would be provided by a pure sampler. Very few papers consider more complex models. Among them, Ranftl *et al.* used a simple discrete-time (DT) model of the camera for performing visual servoing for a fast ultrasonic actuator [9]. Following Luna *et al.*, fast dynamics can be considered as a perturbation and one could try to attenuate them [10].

For reducing the blur effect, common sense recommends to reduce the exposure time. But this leads to images with less amplitude in the pixel values and makes the detection less accurate, particularly when sufficient luminance is not available. Therefore, we consider that there is a need for models reproducing this phenomenon.

The few attention devoted to the current subject can be attributed to the fact that in most of the applications, the DT nature of the measurement is the final information. For instance, in visual servoing, a DT model of the system with a speed reference as input and the DT measurement of the position provided by the camera can be identified and used for control, globally accounting for all the dynamics of the robotic system, including the camera. However, in some applications, the CT signal is of interest. For instance, Cuvillon *et al.* used a high-speed camera in order to reconstruct the CT trajectory of the heart [11]. Neglecting the camera dynamics, it was considered that a high-speed camera was necessary, with a frame rate higher than the heart-beating harmonics with some margin. Instead of selecting a high-speed camera, one can wonder if a more simple one could have been used in combination with an accurate model, therefore reducing the cost of the system. Notice that the reconstruction of the CT trajectory from the DT measurements provided by the camera requires a model. When such a model is available, the Kalman filter is an usual tool allowing to combine both model and measurement data in order to reconstruct the CT trajectory.

The Kalman filter has become a very common tool and is used in many fields including signal processing and robotics. Initially introduced for state and parameter estimation, it appeared to be a very flexible mean for the fusion of heterogeneous data. In its theoretical background, the noises affecting the system are assumed to be the realization of stochastic processes. However, in practice, the covariance matrices of these noises can be used as tuning parameters in order to obtain the proper behavior of the filter.

In robotics and metrology, the goal is often to reconstruct the 3D position of points or objects from a collection of 2D positions in images. Naturally, the evaluation of the effects of the camera on the 3D positions is of interest. However, the current paper focuses on the errors in the image. The models developed herein will be of interest for evaluating the impact on the 3D positions but this issue will not be considered in the current paper.

The scope of this paper is twofold. First, models of the camera-based position measurement (CBPM) are developed for different kinds of cameras.

These models can be used to improve the simulation or to derive a more accurate model dedicated to control design. Second, a methodology is proposed for the estimation of the CT trajectory of a system where the measurement is made by a camera. This methodology is based on a Kalman filter that relies on a CBPM model, such as one of those presented in the first part of the work. The paper is organized as following. Section 2 presents the models of CBPM. An experimental comparison of several models is made in Section 3. The Kalman filter and the adaptations needed for the reconstruction of the CT trajectory from the DT measurement are presented in Section 4. An application for the reconstruction of a periodic trajectory is presented in Section 5. Section 6 concludes the paper.

This paper summarizes several conference papers. The model of a full-shutter mode camera has been initially considered in [12]. In [13], a model of the rolling-shutter mode has been proposed. In addition, this paper presented the evaluation setup and included experimental results for the global-shutter mode camera. In [14], the methodology for the estimation of the CT trajectory was proposed in addition to its use in the reconstruction of a periodic signal.

2. Models of camera-based position measurement

2.1. Measurement of the position by camera

A scene being equipped with visual markers, it is possible to measure their positions in the image by some image processing algorithm. This technique is classically used for processing visual-servoing of robots [15]. The measurements can be used directly in a 2D visual servoing scheme or can be used in order to reconstruct a 3D pose, allowing to derive 3D control. Markers are generally spots of high luminous intensity that can be detected easily. In the visible spectrum, they can be obtained by LEDs or projection of laser beams [16]. It is also possible to use a dark marker over a bright surface [17, 18]. Invisible electromagnetic radiation are also commonly used, as in the Polaris system by NDI that estimates the 3D pose from the 2D position of 3 spheres with high reflexion property in the infrared.

Usually, the elements of interest are simple circular markers and the goal of image processing is to compute the coordinates of their centers of mass in the image. In order to limit the computation cost for real-time implementation, the image processing often makes use of simple operations (binarization,

threshold and computation of the center of mass in several sub-images centered around the current position).

Let us consider one marker and let $q(t) = [x(t) \ y(t)]^T$ denote the vector of the coordinates of its center in the camera image at time t . Images are acquired at the camera frame rate f_c . For Image I_k obtained at time $t_k = kT_c$ where $T_c = 1/f_c$, the vector of the coordinates of the marker center-of-mass is denoted $q_m(t) = [x_m(k) \ y_m(k)]^T$. Let us neglect the noise due to the spatial discretization of the image. In a static case or if displacements are small over the exposure time, one has $q_m(k) = q(t_k)$. In the case of significant displacements over the integration period, the equality is not valid any more. In this paper, we deal with models allowing to compute the DT trajectory $(x_m(k), y_m(k))$ from the CT trajectory $(x(t), y(t))$. Accounting for the dynamic effects induced by the camera, these models are of interest for simulation, identification and control purposes.

For illustration, the following experimental setup has been used: a rigid object containing seven light-emitting diodes (LED) used as markers is moved by a pan-tilt platform, as shown in Fig. 1. The images given by a grey-level camera (see Fig. 2) allow to distinguish the seven markers easily. In the right-hand image in Fig. 2, one can observe the effect of a significant motion over exposure time. The object has been used previously in order to estimate the movements of a beating heart [11].

Based on the analysis of the shape of the blur, more complex image-processing algorithms could be used in order to determine the position of the marker at a given time (for instance at the end of the exposure time). However, in order to limit the complexity of the algorithm for real-time implementation, the simple centre-of-mass computation is generally used in robotics.

$\mathcal{H}_1$	small marker size
$\mathcal{H}_2$	no intersection over the integration period
$\mathcal{H}_3$	high camera resolution
$\mathcal{H}_4$	constant speed amplitude over the integration period

Table 1: List of assumptions for the CBPM model

Figure 1: A pan-tilt robot with seven LEDs mounted on its tip

Figure 2: Two images of the 7-LED platform obtained by a grey-level camera at 25 Hz. On the right-hand side, some significant displacement occur during exposure time.

2.2. Global shutter mode (GSM)

2.2.1. Nonlinear model

In GSM, all the lines of image I_k are acquired over the same time interval $[t_k - T_e, t_k]$ where $t_k = kT_c$ is the instant when the image is available and T_e is the exposure time. Under Assumptions $\mathcal{H}_1 - \mathcal{H}_3$ given in Tab. 1, the image provided by the displacement of an optical marker can be considered as a curve, as represented in Fig. 3. Then the center-of-mass measurement writes $q_m(k) = \frac{1}{l(k)} \int_{q(t_k - T_e)}^{q(t_k)} q \, dl$ where dl is an element of length of the trajectory and $l(k) = \int_{q(t_k - T_e)}^{q(t_k)} dl$ is the length of the trajectory over $[t_k - T_e; t_k]$. Considering that $dl = \|\dot{q}\| \, dt$, the model can be rewritten as a time-integral:

$$q_m(k) = \frac{1}{l(k)} \int_{t_k - T_e}^{t_k} q \, \|\dot{q}\| \, dt \quad (1)$$

with $l(k) = \int_{t_k - T_e}^{t_k} \|\dot{q}\| \, dt$. In the case of full exposure time ($T_c = T_e$), this model results in the block-diagram of Fig. 4 where $\frac{1}{s}$ represents the integration over time. This model, called $\mathcal{M}_1$ in the sequel, is nonlinear and relies on three CT states gathered in $\tilde{X}_b(t)$ and three DT states gathered in $\tilde{X}_c(k)$.

Figure 3: Simplified image obtained from displacement of a marker of small size over integration time

2.2.2. Linear model

Assuming that the speed modulus $\|\dot{q}\|$ remains constant over the integration period ($\mathcal{H}_4$ in Tab. 1), the model can be simplified into:

$$q_m(k) = \frac{1}{T_e} \int_{t_k - T_e}^{t_k} q(t) \, dt \quad (2)$$

This model, called $\mathcal{M}_2$ in the sequel, is linear. It relies on two CT states and two DT states. A corresponding block-diagram is given in Fig. 5 for the full-exposure-time case.

Figure 4: Model $\mathcal{M}_1$: nonlinear model of a camera with global shutter mode and full exposure time (sampling of the CT signal at frequency $f_c = 1/T_c$)

Figure 5: Model $\mathcal{M}_2$: Linear model of a camera with global shutter mode and full exposure time

Model $\mathcal{M}_2$ has the advantage of its simplicity but relies on one more assumption than $\mathcal{M}_1$. Notice that, for constant velocity, they provide the same outputs.

$\mathcal{M}_1$	nonlinear model (Fig. 4)
$\mathcal{M}_2$	linear model (Fig. 5)
$\mathcal{M}_3$	$q_m(k) = q(t_k)$
$\mathcal{M}_4$	$q_m(k) = q(t_k - T_c)$
$\mathcal{M}_5$	$q_m(k) = \frac{1}{2}(q(t_k) + q(t_k - T_e))$
$\mathcal{M}_6$	$q_m(k) = q(t_k - T_e/2)$

Table 2: List of models

2.2.3. Other models

Other more simple models can be used in order to predict the DT measurements from the CT trajectory. These models will be used in the sequel for comparison:

Figure 6: Model of a camera with GSM and partial exposure time (derived from $\mathcal{M}_2$)

- The dynamic effects of the camera can be simply neglected, therefore considering the camera as a simple sampler, leading to $q_m(k) = q(t_k)$ (model $\mathcal{M}_3$) or $q_m(k) = q(t_{k-1})$ (model $\mathcal{M}_4$).
- The measurement can be approximated by the sum of the measurements provided by $\mathcal{M}_3$ and $\mathcal{M}_4$, leading to $q_m(k) = \frac{1}{2}(q(t_k) + q(t_{k-1}))$ (model $\mathcal{M}_5$ proposed in [9]).
- The effects of the camera can be assumed as a pure delay of $T_c/2$, leading to $q_m(k) = q(t_k - T_c/2)$ (model $\mathcal{M}_6$).

The different models are listed in Tab. 2.

2.2.4. Partial exposure time

The previous models can be modified by accounting for partial exposure time, i.e. when the exposure time T_e is less than the camera period T_c . Assume that T_e/T_c is rational and consider a sample time T_s and integers n_c and $n_e \leq n_c$ such that $T_c = n_c T_s$ and $T_e = n_e T_s$. For simplicity, the model derived hereafter from $\mathcal{M}_2$ is developed for coordinate x but is also valid for y . Let us denote:

$$J_x(t) = \int_0^t x(\tau) d\tau \quad (3)$$

One has:

$$x_m(k) = \frac{1}{T_e} (J_x(kn_c T_s) - J_x((kn_c - n_e)T_s)) \quad (4)$$

A block diagram version of the corresponding model is represented in Fig. 6. It includes an integrator $\frac{1}{s}$, a sampler at T_s , a differentiator $1 - z^{-n_e}$ sampled at T_s and a second sampler at T_c . Applying the same modification in the block-diagram of Fig. 4, it is possible to derive a nonlinear model that accounts for partial exposure time for model $\mathcal{M}_1$.

2.3. Rolling shutter mode (RSM)

The paper is mostly devoted to GSM cameras that are more usual than RSM. However, as shown in this section, compact models can be also developed for RSM.

In RSM, the lines of an image are acquired one by one with a delay. Consider that the image is composed of n_l lines L_l numbered from 0 to $n_l - 1$. Lines are acquired in increasing order, so that an image is available as soon as L_{n_l-1} is available. Denoting $t_k = kT_c$ the time at which Image I_k is available and denoting T_d the delay between the acquisitions of the first and the last lines, Line L_l is acquired over $[t_k - T_e - \frac{n_l-l-1}{n_l-1}T_d, t_k - \frac{n_l-l-1}{n_l-1}T_d]$. See Fig. 7 for an illustration.

Neglecting the spatial discretization by considering a camera with infinite resolution, a simplified model can be derived. Consider that y varies over $[0, \bar{y}]$. At time t , Image I_k is affected by a point $(x(t), y(t))$ if the following inequalities are satisfied:

$$\frac{\bar{y}}{T_d}(t + T_d - t_k) \leq y(t) < \frac{\bar{y}}{T_d}(t + T_d + T_e - t_k) \quad (5)$$

Let $C_k(t)$ be a membership function for Image I_k : $C_k(t) = 1$ if (5) is satisfied and $C_k(t) = 0$ otherwise. The residence time $T_r(k)$ for Image I_k is the length of time over which the marker contributes to the image¹ and writes

$$T_r(k) = \int_0^\infty C_k(t) dt \quad (6)$$

Considering the previous assumptions, the measurement obtained from Image I_k is given by:

$$x_m(k) = \frac{1}{T_r(k)} \int_0^\infty C_k(t) x(t) dt \quad (7)$$

$$y_m(k) = \frac{1}{T_r(k)} \int_0^\infty C_k(t) y(t) dt \quad (8)$$

Directly implementing this model is not convenient as it would require the computation of many integrals (as many as the number of images). In the

¹In the GSM, the residence time is constant and equal to the exposure time. In the RSM, this quantity is variable, depending on the displacement of the marker. The maximum residence time is $T_e + T_d$ and can be obtained for $\dot{y} = -\bar{y}/(T_e + T_d)$ (see in Fig. 7 from point $(t_k - T_d - T_e, \bar{y})$ to $(t_k, 0)$). The minimum residence time can reach zero if $T_d \geq T_e$ and would be obtained with a marker starting at the bottom of the image ($y = 0$) at $t = t_k - T_e$ and moving towards the top of the image with an infinite speed $\dot{y}$. For $T_d < T_e$, the minimum residence time is $T_e - T_d$.

Figure 7: Explicative time diagram for the rolling shutter mode

sequel of this subsection, a more convenient formulation relying on a limited number of state variables is given in the remainder of the subsection.

One very specific feature of the RSM, is that several images have to be considered at a given time, as soon as $T_e + T_d > T_c$. In practice, a maximum of two images have to be considered as only one row can be readout at a time in an ordinary camera, yielding $T_d \leq T_c$. However, the theoretical maximum number of images writes $n_i = \lceil \frac{T_e + T_d}{T_c} \rceil$ ($\lceil \cdot \rceil$ being the ceil function) and the model will be presented in the most general form.

Depending on the value of $y(t)$, the marker contributes to Image I_{k+j} , with $j = 0, \dots, n_i - 1$, if $C_{k+j}(t) = 1$. Based on this observation, let I_{xj} and I_{yj} , $j = 0, \dots, n_i - 1$, denote the $2n_i$ state variables corresponding to coordinates x and y . The state equations write:

$$\dot{I}_{xj}(t) = C_{k+j}(t) x(t) \quad (9)$$

$$\dot{I}_{yj}(t) = C_{k+j}(t) y(t) \quad (10)$$

The measurement obtained from Image I_k is finally the sum of the n_i contributions of different vertical strips. For the latest strip, the contribution over $[t_{k-1}, t_k]$ obtained from (7) writes $I_{x0}(t_k) - I_{x0}(t_{k-1})$. The second earlier strip contributes over $[t_{k-2}, t_{k-1}]$ with $I_{x1}(t_{k-1}) - I_{x1}(t_{k-2})$ and so on if more strips were involved. Using equations (7) to (10), the measurements are finally given by:

$$x_m(k) = \frac{1}{T_r(k)} \sum_{j=0}^{n_i-1} I_{xj}(t_{k-j}) - I_{xj}(t_{k-j-1}) \quad (11)$$

$$y_m(k) = \frac{1}{T_r(k)} \sum_{j=0}^{n_i-1} I_{yj}(t_{k-j}) - I_{yj}(t_{k-j-1}) \quad (12)$$

Figure 8: Bode Diagrams of the different models

The residence time $T_r(k)$ can be computed simultaneously with additional state variables. Let τ_0 denote the time length corresponding to the current image, τ_1 the time length corresponding to the next image and more generally τ_j , $j = 0, \dots, n_i - 1$, the time length corresponding to the j^{th} future image. These states are updated according to the active mode:

$$\dot{\tau}_j(t) = C_{k+j}(t) \quad (13)$$

Finally, the residence time is computed by:

$$T_r(k) = \sum_{j=0}^{n_i-1} \tau_j(t_k) - \tau_j(t_{k-1}) \quad (14)$$

The resulting nonlinear model is composed of $3n_i$ states (I_{xj} , I_{yj} and τ_j , $j = 1, \dots, n_i - 1$) and as many state equations ((9), (10) and (13)).

Notice that only the GSM is considered in the next sections of the paper. However, as they rely on the same hypotheses (Tab. 1), one can imagine that the GSM and RSM versions of $\mathcal{M}_2$ have similar accuracies.

2.4. Frequency and time-domain evaluation

Frequency-domain evaluation. Considering the DT-CT equivalence of the advance operator $z = \exp(T_c s)$, it is possible to compute the frequency response

Figure 9: Simulation results of GSM and RSM models for a circular trajectory (plain: original CT trajectory, dash: GSM, dot/dash: RSM)

of the CBPM with full exposure time. For instance, for model $\mathcal{M}_2$, the equivalent CT model writes $\frac{1-\exp(-T_c s)}{T_c s}$. With $s = j\omega$, ω being the angular frequency, the frequency-domain behavior of the different models are given in Fig. 8. One can notice that the models differ either in gain or phase. Models $\mathcal{M}_3$, $\mathcal{M}_4$ and $\mathcal{M}_6$ have the same constant gain. Models $\mathcal{M}_1$, $\mathcal{M}_2$, $\mathcal{M}_5$ and $\mathcal{M}_6$ have the same phase.

Time-domain evaluation. Camera models can be directly implemented for simulation from the models given in the previous subsection². For illustration, the models of the GSM and RSM ($\mathcal{M}_2$ and its adaptation for RSM) were simulated with a circular trajectory at 4 Hz centered in a 200×200 pixel image. The camera period is $T_c = 50$ ms and the exposure time is 20 ms. The GSM is implemented with $T_s = 10$ ms, $n_e = 2$ and $n_c = 5$. For the RSM, a delay $T_d = T_c = 50$ ms is considered. The original CT trajectory and the DT trajectories obtained with the two shutter modes are given in Fig. 9. One can notice that the measurement obtained with GSM has a constant delay whereas the measurement obtained with RSM has a variable delay that in-

²A library with the Simulink models is available on the first author academic web pages (presently <http://eavr.u-strasbg.fr/~laroche/camera/>)

creases with y position. When the CBPM system is implemented on a plant to be simulated, the camera model is simply put at the end of the plant, such as in the block-diagram presented in Fig. 10.a.

2.5. Derivation of a control-oriented model

A DT model of the process, to be used for control or observation, can be obtained as following. Consider a process to be controlled equipped with a camera providing the output to be controlled $Y(k)$ thanks to a DT control signal $u(k)$ given to a CT process through zero-order hold (ZOH) as shown in Fig. 10.a where $v(k)$ and $w(k)$ are noise signals that are neglected at this step.

Considering a linear CT model $G(s)$ of the process in the Laplace domain and choosing the linear model $\mathcal{M}_2$ for CBPM, a simplified model can be obtained such as in Fig. 10.b. The DT model can be obtained using a CT to DT conversion of $G(s)/s$, assuming that $u(t)$ comes from a ZOH, and then multiplying the result by DT differentiator $\frac{z-1}{zT_c}$. It can be also obtained using a direct CT to DT transform based on a causal first-order hold assumption. Indeed, the camera has the same effect as an increase in the order of the hold. The resulting DT model is then obtained by an usual formula [19]:

$$G(z) = \frac{(z-1)^2}{T_c z^2} \mathcal{Z} \circ \mathcal{L}^{-1} \left(\frac{G(s)}{s^2} \right), \quad (15)$$

where $\mathcal{L}^{-1}$ is the inverse Laplace transform and $\mathcal{Z}$ is the Z-transform applied to the signal sampled at period T_c . A minimal realization is finally derived by canceling the poles and zeros at $z = 1$.

Including the noise input w , the MIMO case with n_u inputs u , n_w inputs w and n_q measurements can be formulated in state space, provided that a state-space (SS) realization of $G(s)$ of order n_G is available:

$$\dot{X}_a = A_G X_a + B_{G1} u + B_{G2} w \quad (16)$$

$$q = C_G X_a + D_{G1} u + D_{G2} w \quad (17)$$

The augmented CT model $G_{ab}(s)$ including the CT states $X_b(t)$ of the camera model can be written with the following state-space realization:

$$\begin{bmatrix} \dot{X}_a \\ \dot{X}_b \\ X_b \end{bmatrix} = \left[\begin{array}{cc|cc} A_G & O_{n_G \times n_q} & B_{G1} & B_{G2} \\ C_G & O_{n_q \times n_q} & D_{G1} & D_{G2} \\ \hline O_{n_q \times n_G} & I_{n_q \times n_q} & O_{n_q \times n_u} & O_{n_q \times n_w} \end{array} \right] \begin{bmatrix} X_a \\ X_b \\ u \\ w \end{bmatrix} \quad (18)$$

Figure 10: Block diagram of the hybrid model of the process with CBPM as sensor

The DT model of output $\tilde{q}$ and input $[u^T(k) \ w^T(k)]^T$ writes $G(z) = \frac{1}{T_c}(1-z^{-1}) \text{c2d}(G_{ab}(s), T_c, 'zoh')$ where $\text{c2d}(\bullet, T_c, 'zoh')$ stands for the CT-to-DT transform with the assumption of ZOH on input signals.

Looking more into the details, the resulting model, based on state vector $X(k) = \begin{bmatrix} X_a(t_k) \\ X_b(t_k) \\ X_c(k) \end{bmatrix}$, has a SS representation of the form:

$$X(k+1) = AX(k) + B_1 u(k) + B_2 w(k) \quad (19)$$

$$Y(k) = CX(k) + v(k) \quad (20)$$

Provided that matrices $A_{aa}, A_{ab}, A_{ba}, A_{bb}, B_{au}, B_{aw}, B_{bu}$ and B_{bw} represent a partition of the SS matrices of the DT model from $\begin{bmatrix} u(k) \\ w(k) \end{bmatrix}$ to $X_b(t_k)$ with state

vector $\begin{bmatrix} X_a(t_k) \\ X_b(t_k) \end{bmatrix}$ obtained with $\text{c2d}(G_{ab}(s), T_c, 'zoh')$, the state matrices A, B ,

$$C \text{ write } A = \begin{bmatrix} A_{aa} & A_{ab} & O_{n_G \times n_q} \\ A_{ba} & A_{bb} & O_{n_q \times n_q} \\ O_{n_q \times n_G} & I_{n_q \times n_q} & O_{n_q \times n_q} \end{bmatrix}, B_1 = \begin{bmatrix} B_{au} \\ B_{bu} \\ O_{n_q \times n_u} \end{bmatrix}, B_2 = \begin{bmatrix} B_{aw} \\ B_{bw} \\ O_{n_q \times n_q} \end{bmatrix}$$

and $C = [O_{n_q \times n_G} \ \frac{1}{T_c} I_{n_q \times n_q} \ \frac{-1}{T_c} I_{n_q \times n_q}]$.

A pole-zero cancellation method (for instance `minreal` with Matlab) provides a reduced-order model based on state vector $\bar{X}$ such that $X = U\bar{X}$ and transforms the state matrices into the Kalman decomposition $\bar{A} = U^{-1}AU$,

$\overline{B} = U^{-1}B$, $\overline{C} = CU$. This model $\{\overline{A}, \overline{B}, \overline{C}\}$ can be used for control and observation.

3. Experimental evaluation

3.1. Evaluation method

Experimental validation of the models requires to move a marker at high speed along a given trajectory in a reliable and reproducible way. A convenient setup has been developed for this purpose, based on a high-speed video projector [20] that projects a spot on a screen (see Fig. 11 and 12). A periodic trajectory is chosen and images are acquired by the camera at frame rate $f_c = 1/T_c$. Assuming that the projector rate f_p is high, the displacement of the spot can be considered as continuous and the obtained images are equivalent to those obtained by the displacement of a LED on a moving arm. Assuming that the corresponding CT trajectory in the camera frame is known, the CBPM model will be used to predict the measurements. This prediction will be compared with the measurements based on image processing, thus allowing to evaluate the model.

Notice that the proposed setup has several advantages compared to a more conventional robotic system that would move the visual marker at different velocities. Indeed, the projection setup has no mechanical dynamics. Therefore, it is possible to move the target at arbitrary velocity with a very high accuracy in the marker position. Contrarily, the trajectory produced by a robotic setup would be altered by the dynamics at high frequency.

The steps required for validating the model are: (i) choose a trajectory in the projector frame; (ii) identify the corresponding trajectory in the camera frame; (iii) acquire the images for the corresponding trajectory; (iv) compute the prediction based on the model; (v) synchronize the data (in the case where no synchronization between camera and projector is available as herein); (vi) compare the output data. In the results presented herein, a circular trajectory was chosen.

3.2. Description of the setup

The experimental system, shown in Fig. 11 and 12, consists of a high-speed projector that generates marker patterns and a CCD camera that captures images of the marker. The projector and the camera are set approximately side by side at the distance of about 200 mm from each other. A

Figure 11: View of the setup; on the front-ground: the camera (right) and the videoprojector (left); on the background: the screen (N.B. the circle is obtained by the rotation of the marker over the exposure time of the camera)

Figure 12: Partial view on the camera (left) and the videoprojector (right)

planar screen is set at the distance of about 700 mm in front of the projector-camera system.

The projector is composed of a Texas Instruments Digital Micromirror Device [21] DMD 0.7XGA 12 DDR, its controller board DMD Discovery 1100 and an external memory board ViALUX ALP-1. In this configuration, 1024×768 pixels binary images can be projected at up to 8 kHz. In the presented experiments, the maximum projection frame rate is set at $f_{p\max} = 4$ kHz to ensure stable operations. The external memory board is able to store up to $n_{p\max} = 2730$ frames, and an arbitrary binary image sequence whose length is within $n_{p\max}$ frames can be projected repeatedly. Light from a metal halide lamp Fiber-Lite MH-100 is guided to the DMD by an optical fiber guide, and the light reflected by the DMD is projected through a lens whose focal length is about 27 mm. Since a single mirror size of the DMD is $13.68 \mu\text{m}$, the horizontal view angle is about 29 degrees.

The camera is a Point Grey Research Dragonfly Express IEEE1394b camera, which captures 640×480 -pixel 8-bit monochrome images with the progressive scan in GSM. In the presented experiments, images are acquired at $f_c = 50$ Hz with full-time exposure ($T_e = T_c$). The focal length of the camera lens is about 6 mm and the CCD pixel size is $7.4 \mu\text{m}$, resulting in an horizontal view angle of about 43 degrees.

For illustration, an image obtained with this system is given in Fig. 13.a, in addition with the measurements of the center of mass obtained with (Fig. 13.b) and without (Fig. 13.c) Sobel filter. Notice that the projector and the camera work in asynchronous mode.

3.3. Trajectory design

Consider a given periodic trajectory at frequency $f = 1/T$. It is necessary to choose the corresponding number of projector patterns n_p and projector frame rate f_p that satisfy the constraints $n_p \leq n_{p\max}$ and $f_p \leq f_{p\max}$. Let T_g denote a common multiple of the different periods. Then, there exist integers n_p , n and n_c such that:

$$T_g = n_p T_p = n T = n_c T_c \quad (21)$$

where $T_p = 1/f_p$. In order to be close as possible to a CT trajectory, the projector rate f_p is maximized.

Each projected image is composed of a white point of radius r at a particular position on a circle of center (x_0, y_0) and radius R . The trajectory in the

Figure 13: Images obtained by the camera and image processing in addition with the position measurements. a/ original image and the measurements ($\square$: without Sobel filter; $\diamond$: with Sobel filter); b/ binary image obtained without Sobel filter and its center of mass ($\square$); c/ binary image obtained with Sobel filter and its center of mass ($\diamond$)

projector frame writes $x_p(t) = x_0 + R \cos(\phi(t))$, $y_p(t) = x_0 + R \sin(\phi(t))$ where $\phi(t)$ denotes the instantaneous phase. The center of the trajectory is chosen as the center of the image. The trajectory radius is chosen as $R = 200$ pixels. For most of the tests, we chose $r = 40$ pixels. However, experiments with different values of r were processed. A number of n_p frames are computed and projected at the projector frequency, based on $\phi_k = \phi(kT_p)$, $k = 0, \dots, n_p - 1$.

3.4. Identification of the trajectory in the camera frame

In order to validate the camera models, a projective model is necessary to provide the position in the camera frame for a given position in the projector frame. Taking the advantage of the periodicity of the movement, a simple approach is used consisting in a truncated Fourier series of the trajectory rather than a real calibration.

Let $(x_p(\phi), y_p(\phi))$ denotes the known trajectory in the projector frame and $(x(\phi), y(\phi))$ the unknown trajectory in the image frame, with the same periodicity 2π . The following truncated Fourier series can be used:

$$x(\phi) = a_0 + \sum_{k=1}^m a_k \cos(k\phi) + b_k \sin(k\phi) \quad (22)$$

$$y(\phi) = c_0 + \sum_{k=1}^m c_k \cos(k\phi) + d_k \sin(k\phi) \quad (23)$$

Parameters a_k , b_k , c_k and d_k are evaluated by linear regression in order to fit static measurements obtained for several values of ϕ in $[0; 2\pi]$. A number

λ^*	n	n_c	$\lambda = \frac{n}{n_c}$	T_g (ms)	n_p
0.1	1	10	0.1	200	800
0.192	1	8	0.125	160	640
0.167	1	6	0.167	120	480
0.215	1	5	0.2	100	400
0.278	3	11	0.272	220	880
0.359	1	3	0.333	60	240
0.464	6	13	0.462	260	1040
0.599	3	5	0.6	100	400
0.774	3	4	0.75	80	320
1	1	1	1	20	80

Table 3: Numerical values of the parameters of the constant velocity experiment; T_g is the multiple period and n_p is the number of frames.

of 1024 positions regularly spaced was chosen. Order $m = 2$ was chosen for the truncation, allowing an accuracy around 0.13 pixel, which is close to the residual measurement error.

3.5. Evaluation results

3.5.1. Constant velocity trajectories

The phase of the trajectory writes $\phi(t) = 2\pi ft$. Several experiments were proceeded with various $\lambda = f/f_c = n/n_c$, i.e. various ratio of the trajectory frequency to camera frame rate, from 0.1 to 1. Making use of the explanations of Section 3.2, n and n_c were chosen manually such that n/n_c is close to the desired value λ^* of λ . The maximum projector rate $f_p = 4$ kHz was used for each case. The parameters obtained for 10 values of λ^* logarithmically spaced are given in Table 3.

As an example, the results for $\lambda = 0.333$ are given in Fig. 14. One can notice that model $\mathcal{M}_2$ allows very accurate predictions of the measurements. Remind that $\mathcal{M}_1$ and $\mathcal{M}_2$ provide the same outputs for constant-velocity trajectories. For evaluation of the models, the mean distance error $J = \frac{1}{m} \sum_{k=1}^m \sqrt{(\Delta x_m(k))^2 + (\Delta y_m(k))^2}$ is considered in the sequel, where $\Delta x_m(k)$ and $\Delta y_m(k)$ are respectively the errors on x and y coordinates for measurement $\#k$, $k = 1, \dots, m$. The evaluations obtained for the different values of the ratio f/f_c are given in Fig. 15 considering 5 different models $\mathcal{M}_2$ to $\mathcal{M}_6$ (see Tab. 2 for the list of the models). One can notice that model

Figure 14: Measurements (+) and predictions with model $\mathcal{M}_2$ (o) for $\lambda = 0.359$

$\mathcal{M}_2$ developed in this paper is by far the more accurate. This is notably the case when the signal frequency is close to the camera rate. Whereas the mean distance error J remains lower than 2 pixels with the proposed model, it reaches or exceeds 9 pixels with the other models. One can also notice that models $\mathcal{M}_5$ and $\mathcal{M}_6$ are efficient for low ratios of trajectory to camera frequencies, but the prediction error increases for frequencies close to the camera rate.

3.5.2. Trajectories with varying velocity

Models $\mathcal{M}_1$ and $\mathcal{M}_2$ coincide for constant-speed trajectories ($\mathcal{H}_4$ is the only assumption that is assumed by $\mathcal{M}_2$ and not by $\mathcal{M}_1$), which was the case for constant phase velocity $\dot{\phi}$ in the previous experimental setup. In order to emphasize the better accuracy of model $\mathcal{M}_1$ for variable speed, an experiment was done where the angular speed $\dot{\phi}$ of the trajectory varies from approximately 100 rad/s to 300 rad/s with a periodicity of approximately 50 ms. The numerical values were chosen such that all the frequencies have a common periodicity. A portion of the chosen trajectory is given in Fig. 16. One can notice that after 11 camera periods and 7 turns, the marker comes back to the initial position. Over a camera period, the shift in angle ϕ varies from 2.78 to 5.29 rad. This extreme situation (compared to usual situations where the displacement of the marker over a camera period is small) is intended to discriminate the different models.

Figure 15: Variations with respect to $\lambda = f/f_c$ of the errors in the prediction obtained with the different models

Figure 16: Variable speed circular trajectory for model evaluation (the circles represent the values sampled at the camera rate)

Figure 17: Comparison between measurement ($\times$) and results provided by model $\mathcal{M}_1$ (o)

The values of the criterion J (i.e. the mean distance error) for each model are given in Table 4. One can notice that models $\mathcal{M}_1$ and $\mathcal{M}_2$ provide more accurate results than the other models. Model $\mathcal{M}_1$ is sensibly superior to $\mathcal{M}_2$: the mean error is reduced by 25%. For illustration, the prediction of the measurements with model $\mathcal{M}_1$ are given in Fig. 17 and compared with the values obtained experimentally. One can notice that the model reproduces the measurements with rather good precision.

model	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$
J (pixel)	12.6	16.9	94.3	100.7	66.3	86.1

Table 4: Mean distance error between measurements and the models

3.5.3. Varying the spot radius and the detection algorithm

Due to Assumption $\mathcal{H}_1$, the radius of the spot is assumed to be small compared to the radius of the trajectory. It is then interesting to check the effect of the marker size on the accuracy of the model. The experiments were made at constant velocity with markers of different radii r . The results obtained with $k_1 = 3$ and $k_2 = 5$ are given with two processing protocols for image processing:

- with threshold and center of mass, Fig. 18,

Figure 18: Influence of the marker radius without Sobel filter

- with Sobel filter, threshold and center of mass, Fig. 19.

One can notice that the estimation error varies with respect to r/R . Let focus on Model $\mathcal{M}_2$. As expected, the error globally increases with r/R . In the case without Sobel filter, the mean distance error J is higher for the very low r/R ($r/R \leq 0.1$). Indeed, in this situation, images receive few light and the detection of the center of mass is hazardous. Assuming that the marker is sufficiently luminous, model $\mathcal{M}_2$ is sensibly more accurate without using the Sobel filter.

4. Compensation of the dynamic effects using a Kalman filter

4.1. Presentation of the issue

It is not possible to invert the camera model, i.e. to determine the CT trajectory $q(t)$ only from the DT measurement $q_m(k)$. Therefore, additional information must be used. When a model of the process generating $q(t)$ is available, the Kalman filter is the most common tool allowing to combine the model and the output measurement in order to estimate the trajectory of the internal states. In this section, the use of the Kalman filter is explained for the particular issue of determining the CT trajectory of a marker position measured by camera. Whereas the methodology is rather general, the specific case of a camera with GSM and full exposure time is considered herein.

Figure 19: Influence of the marker radius with Sobel filter

The global model has the form shown in the diagram of Fig. 10. Input vector $u(k)$ corresponds to the known input signals (such as the control signals) and $w(k)$ corresponds to the noise that affects the CT process (unknown input signals are considered as noise). Without any other information, it is sensible to consider that the continuous signals $u(t)$ and $w(t)$ are constant over a sampling interval, i.e. they are provided through ZOH. The process is CT and could be any kind of CT system. The measurement given by the camera is altered by some additive measurement noise $v(k)$. The dashed line with $\dot{q}(t)$ corresponds to the fact that $\dot{q}(t)$ would be necessary for model $\mathcal{M}_1$ but not for the other models. The state of the CT process is denoted by $X_a(t)$. In this paper, we consider that the CT process is linear and the linear model $\mathcal{M}_2$ of the CBPM is used. Extension to nonlinear systems is possible. Capital letters are used for the states (X) and outputs (Y) in order to distinguish them from the x and y coordinates.

4.2. Kalman filter for discrete-time systems

The Kalman filter allows to determine the estimate of the state with minimal variance [22]. Details on the theory and application of the Kalman filter can be found in many publications [23, 24]. Herein, a simplified version is given, corresponding to the case where the gain matrix K has converged,

resulting in a linear filter. At time t_k , the estimation is computed in two steps:

1. compute the prediction $\hat{\bar{X}}(k+1|k)$ of the state at time t_{k+1} from data available at t_k :

$$\hat{\bar{X}}(k+1|k) = \bar{A} \hat{\bar{X}}(k|k) + \bar{B}_1 u(k) \quad (24)$$

2. compute the correction $\hat{\bar{X}}(k+1|k+1)$ based on measurements at t_{k+1} :

$$\begin{aligned} \hat{\bar{X}}(k+1|k+1) = & \hat{\bar{X}}(k+1|k) + K(Y(k+1) \\ & - \bar{C} \hat{\bar{X}}(k+1|k)) \end{aligned} \quad (25)$$

The gain matrix K is computed using the observable model $\{\bar{A}, \bar{B}, \bar{C}\}$ developed in Subsection 2.5.

4.3. Adaptation to the hybrid model

The previous DT Kalman filter can be adapted for estimation of the CT trajectory $q(t)$ and the CT state of the process $X_a(t)$. First, the estimation scheme based on the reduced state $\bar{X}(k)$ can be transposed to $X(k)$, provided that the system is detectable. The corresponding estimation scheme is simply obtained from (24-25) by removing all the bars (i.e. using the state matrices of the model before pole-zero cancellation) and replacing the correction gain K by UK .

Second, the CT nature of the signals to observe must be recovered. This is solved by replacing the computation of the prediction step (24) by:

1. simulation of the CT part of the model (states $X_a(t)$ and $X_b(t)$) on $[t_k; t_{k+1}]$ by any numerical solver dedicated to ordinary differential equations,
2. computation of the DT part ($X_c(k+1) = X_b(t_k)$).

The correction step, based on the DT measurements available at each camera period, is unchanged.

The resulting observer can be considered as a hybrid system in two ways: 1/ it contains both CT and DT states; 2/ the CT states make a jump at each sampling time, due to the correction step. Practically, the simulation of the CT part is done numerically at an arbitrarily small time-step.

5. Trajectory reconstruction

We propose to show the interest of the methodology presented in Section IV by a specific application: the reconstruction of a periodic displacement of one marker.

5.1. Experiment

In heart-beating surgery, the portion of interest of the heart is immobilized by a mechanical stabilizer. However, some residual motion remains that act as a disturbance for the surgeon [11]. Several solutions have been proposed to improve the stability of the region of interest [18, 25, 26]. In order to obtain the best results, there is a need to predict the movement of the heart [27]. This can be done using LEDs attached on the tip of the stabilizer and measured by a camera. The estimation can be done by Kalman filter [28].

The experimental setup shown in Fig. 1 has been used to reproduce the displacements of a beating heart. It consists of a pan-tilt arm equipped with seven LEDs at its tip. A fixed camera (DALSA CAD6) allows acquisition of images (256×256 pixels) at variable frequencies, up to 400 Hz. The pan-tilt arm is controlled in position at 500 Hz. The chosen position reference corresponds to a periodic movement corresponding to one cardiac period of the heart movement recorded by Ginhoux *et al.* [29]. The corresponding trajectory in the image is given in plain line in Fig. 20.

The goal of the study is to show that the use of the camera model allows to reconstruct the trajectory of a complex periodic movement with a low frequency camera whereas without the use of the model, a high-speed camera is required. Therefore, the movement of the platform is first acquired at a relatively high frequency to get an accurate knowledge of the real trajectory. A camera rate $f_{c1} = 250$ Hz was considered as sufficient. The position of a LED is obtained as following: an area of interest centered on the previous position allows to discriminate one LED from the others; a Sobel filter detects the contour; the computation of the center of mass of the pixels of the contour is considered as the marker position. Trajectories at lower frequencies should be acquired for comparison. However, in order to compare synchronized trajectories, it was decided to reconstruct the images at lower frequencies. For instance, the trajectory at $f_{c2} = 25$ Hz is obtained by superimposing $f_{c1}/f_{c2} = 10$ consecutive images. As an illustration, the image computed from 10 successive images acquired at 250 Hz is shown in Fig. 21. One can

Figure 20: CT trajectory obtained in different conditions: $\mathcal{C}_1$ (measured at 250 Hz, plain line), $\mathcal{C}_2$ (Fourier serial of order 9 estimated from measurements at 250 Hz without camera model, dots), $\mathcal{C}_3$ (Fourier serial of order 9 estimated with the linear model of the camera from images acquired at 25 Hz, dashed line), $\mathcal{C}_4$ (Fourier serial of order 9 estimated at 25 Hz without camera model, dashed-dotted line).

see the resemblance with the image given in the right-hand side of Fig. 2 that was acquired at 25 Hz.

5.2. Signal model

Let us explain how to model a periodic trajectory into the scheme of Fig. 10.a in order to use the Kalman filter with the methodology introduced in the previous section.

A periodic signal $x(t)$ of period T and of finite bandwidth can be decomposed into a finite sum of harmonics $x_i(t)$, $i = 1, \dots, n_h$, at multiple frequencies if where $f = 1/T$. Each harmonic can be produced by a second order resonant filter of equation $\ddot{x}_i(t) + (i\omega_0)^2 x_i(t) = 0$ of eigen-frequency $i\omega_0$ with $\omega_0 = 2\pi f$. To allow the Kalman filter to converge to the right phase and amplitude, a noise $w_i^x(t)$ is considered on the acceleration, leading to $\ddot{x}_i(t) + (i\omega_0)^2 x_i(t) = w_i^x(t)$. For the y coordinate, the same approach is used; the corresponding second order filter has independent states ($y_i(t)$ and $\dot{y}_i(t)$) and inputs $w_i^y(t)$.

Figure 21: Image computed from 10 images acquired at 250 Hz for simulation of the 25 Hz acquisition.

By stacking all the 2nd order blocks, a model of the process can be obtained³. Its state X_a is of order $4n_h$. The input vector w is of order $2n_h$. The outputs $x(t)$, $y(t)$, $\dot{x}(t)$ and $\dot{y}(t)$ are respectively computed as the sum of the different harmonics $x_i(t)$, $y_i(t)$, $\dot{x}_i(t)$ and $\dot{y}_i(t)$. This model is linear and has no direct feed-through; it has no control input u .

5.3. Filter tuning

A correct tuning of the covariance matrices is necessary in order to obtain a good behavior of the Kalman filter. The noise $v(k)$ on measurement generally satisfies the assumptions of time-independence and zero mean. Therefore, its covariance matrix R can be tuned according to an estimation of the noise amplitude. In our case, the standard deviation of the measurement is about 0.1 pixel, leading to $R = 0.01 \times \mathbb{I}_2$ where $\mathbb{I}_2$ is the identity matrix of order 2.

The tuning of matrix Q is a more difficult question as input $w(k)$ is an artifact used for allowing the filter to adapt to the real trajectory. It is sensible to restrict it to a diagonal matrix with the same values for the x and y coor-

³For the harmonic of rank $\#i$, the model writes

$$\begin{bmatrix} \dot{x}_i \\ \ddot{x}_i \\ \dot{y}_i \\ \ddot{y}_i \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -i^2\omega_0^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -i^2\omega_0^2 & 0 \end{bmatrix} \begin{bmatrix} x_i \\ \dot{x}_i \\ y_i \\ \dot{y}_i \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} w_i^x \\ w_i^y \end{bmatrix}$$

The state matrices of the complete model of $G(s)$ are obtained by stacking the matrices obtained for $i = 1, \dots, n_h$ in a block-diagonal fashion.

dinates of a given harmonic, yielding $Q = \text{diag}(Q_1, Q_1, Q_2, Q_2, \dots, Q_{n_h}, Q_{n_h})$. There remains the tuning of n_h positive scalars. Notice that a higher value of Q_j corresponds to more noise on w_j^x and w_j^y and will allow the filter to adapt faster. However, a fast adaptation results in high sensitivity to the measurement noise. A trade-off needs to be found. As the amplitude of the harmonics generally decrease with the frequency, we chose Q_j to decrease hyperbolically in j , i.e. $Q_j = Q_1/j$, $j = 2, \dots, n_h$. When the number n_h of considered harmonics vary, there is a need to adapt the tuning. Indeed, a higher n_h corresponds to more components in w and then more possibility for the filter to adapt. Therefore, we decided to choose $Q_1 = Q_0/n_h$ for reducing the weight with respect to the number of harmonics. Finally, the value of $Q_0 = 100$ was chosen so that the filter converges after a few periods of the signal.

5.4. Results

In order to evaluate the interest of including the camera model in the estimation model, the trajectory was estimated at different frequencies with or without camera model and compared. Different curves in the conditions mentioned in Tab. 5 are given in Fig. 20. $\mathcal{C}_1$ is the trajectory obtained at 250 Hz. This trajectory is considered to be the actual trajectory of $(x(t), y(t))$. The other trajectories are reconstructed by Kalman filter with $n_h = 9$ harmonics in different conditions: in $\mathcal{C}_2$, images are acquired at 250 Hz. Notice that at this frequency, the camera does not have much effect, therefore, this trajectory is the reference for the reconstruction of the truncated signal. The other curves ($\mathcal{C}_3$ to $\mathcal{C}_6$) are obtained from images acquired at 25 Hz. $\mathcal{C}_3$ is obtained with the linear camera model $\mathcal{M}_2$, $\mathcal{C}_4$ without any camera model and $\mathcal{C}_5$ with camera model $\mathcal{M}_6$ (it corresponds to trajectory $\mathcal{C}_4$ time-shifted of $T_c/2$).

curve	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_5$
camera rate (Hz)	250	250	25	25	25
camera model	/	/	$\mathcal{M}_2$	/	$\mathcal{M}_6$
order n_h	∞	9	9	9	9

Table 5: The different curves considered for evaluation

Figure 22: Estimation results with the linear model of the camera ($\mathcal{M}_2$). The estimated CT trajectory is given in plain line. The blocked signal in dashed line corresponds to the measurement at 25 Hz. The corrected estimation of the measurement is given in dot. On the left is presented the beginning of the estimation; the estimated curve seems to be attracted by the measurements. On the right is presented the behavior after convergence; the measurements are accurately estimated (the points belong to the dashed curve).

As an illustration of the behavior of the hybrid Kalman filter, the results ($\mathcal{C}_3$) obtained with the linear model ($\mathcal{M}_2$) at 25 Hz are detailed in Fig. 22. During the convergence phase (left-hand side), one can notice the jumps in the CT trajectory at the measurement instants. After convergence (plot on the right-hand side), the estimated measurements (dots) are very close to the effective measurements (dashed line) and the discontinuities due to the correction have vanished.

The different curves being sampled with the same frequency $f_{c1} = 250$ Hz, the mean distance between two curves, after convergence, is given in Tab. 6 for several couples of curves. The comparisons are given with respect to the actual trajectory $\mathcal{C}_1$ and with respect to the estimation from the Kalman filter at high frequency, i.e. $\mathcal{C}_2$. Remind that Curves $\mathcal{C}_1$ to $\mathcal{C}_4$ are shown in Fig. 20.

Let $d(\mathcal{C}_k, \mathcal{C}_l)$ denote the mean distance between $\mathcal{C}_k$ and $\mathcal{C}_l$ reported in Tab. 6. Let us start with comparisons with respect to $\mathcal{C}_1$. The value $d(\mathcal{C}_1, \mathcal{C}_2) = 1.46$ evaluates the approximation due to the Fourier series truncation at order 9. Notice that this error can be reduced by increasing the order n_h of the Fourier series. When estimating the trajectory with a low-frequency camera ($\mathcal{C}_4$), it appears that this error nearly triples if no camera

$d(\mathcal{C}_k, \mathcal{C}_l)$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_5$
$\mathcal{C}_1$	1.46	1.76	4.24	3.13
$\mathcal{C}_2$	0	0.60	3.00	2.33

Table 6: Comparison of the different curves: mean distance d (in pixel)

model is used (compare $d(\mathcal{C}_1, \mathcal{C}_4)$ with $d(\mathcal{C}_1, \mathcal{C}_2)$) but, with the camera model $\mathcal{M}_2$ (curve $\mathcal{C}_3$), the error is brought back to a similar value (see $d(\mathcal{C}_1, \mathcal{C}_3)$).

Now, let us make comparisons with respect to $\mathcal{C}_2$, considered as the reference for the estimation of a truncated signal. The use of the camera model $\mathcal{M}_2$ in $\mathcal{C}_3$ highly improves the results compared to those obtained in $\mathcal{C}_4$ without camera model, the distance being divided by 5 (see $d(\mathcal{C}_4, \mathcal{C}_2)$ compared to $d(\mathcal{C}_3, \mathcal{C}_2)$). As shown with the results obtained with curve $\mathcal{C}_6$, model $\mathcal{M}_5$ is not as efficient as model $\mathcal{M}_2$ (see $d(\mathcal{C}_5, \mathcal{C}_2)$ compared to $d(\mathcal{C}_3, \mathcal{C}_2)$). One can observe that the camera model allows to obtain results with a 25 Hz camera that are very similar to those obtained at 250 Hz without camera model.

6. Conclusion

This paper proposes an insight visit of the dynamic behavior of CBPM. Different models have been developed, depending on the camera technology (GSM or RSM) for full or partial exposure time. A nonlinear model $\mathcal{M}_1$ was first developed. It allows to reproduce the measurement with better accuracy than all the other models. Therefore, it is suitable for refined simulation. For estimation and control, the linear model $\mathcal{M}_2$, assuming constant velocity over the exposure time, is more suitable. An experimental setup and a complete procedure have been proposed for the model validation. The results showed the significant improvements obtained with these models compared to the reference models. Notice that the paper focuses on the 2D movement of points in the image. Investigations of the dynamic effects of other visual features such as lines, and also the effects on a reconstructed 3D trajectory remains to be considered.

In addition, this paper presents a methodology to account for the dynamic effects of the camera when reconstructing the trajectory of a visual marker. Making use of a model of the camera, a Kalman filter is used to estimate the CT trajectory. The methodology was evaluated in the context of the recon-

struction of an unknown periodic signal, corresponding to the displacements of a beating heart. In this context, it was shown that the use of the model allows to obtain results with a low frame-rate camera that resemble those obtained with a high-speed camera. For instance, the trajectory estimated with a Fourier series could be estimated at 25 Hz with an accuracy close to what is given by a camera at 250 Hz without camera model.

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