Domains in multiband superconductors

Y. Tanaka^{a*}, T. Yanagisawa^a, A. Crisan^{b,c}, P. M. Shirage^a, A. Iyo^a, K. Tokiwa^d, T. Nishio^e, A. Sundaresan^f, N. Terada^g

^aNational Institute of Advanced Industrial Science and Technology, Tsukuba, Japan

^bUniversity of Birmingham, Birmingham, United Kingdom

^cNational Institute of Materials Physics, Bucharest, Romania

^dTokyo University of Science, Noda, Japan

^eTokyo University of Science, Kagurazaka, Japan

^fJawaharlal Nehru Centre for Advanced Scientific Research, Bangalore, India

^gKagoshima University, Kagoshima, Japan

Abstract

Multiband superconductors can have several types of domains that are inhibited in conventional single-band superconductors. These domains are phase domains and chiral domains and their domain wall are an interband phase difference soliton. In a superconductor with an odd number of electronic bands (five or more) and with positive interband Josephson interactions, we find other types of domains with different interband phase differences. We call these domains configuration domains because pseudo-order parameters for each band are dispersed in the complex plane and several configurations, which have several local minima. Fractional vortices serve as hubs for

phase difference solitons (configuration domain walls). The divergence of the number of configurations with local minima would pose a serious problem for the stability of superconductivity.

(122 words)

Corresponding author: Yasumoto TANAKA (y.tanaka@aist.go.jp)

Keyword: soliton, multiband superconductor, chiral superconductivity

1. Introduction

Conventional superconductors exhibit one-dimensional unitary ($U(1)$) internal space, which is a superconducting quantum phase θ [1-3]. When the interband Josephson interaction is much smaller than intraband interactions, multiband superconductors can be considered to be multicomponent superconductors that have multiple quantum phases θ_v , where v is the band index [4-8]. These can be called multicomponent superconductors based on the multiband superconductor in which the superconducting order parameters are composed of pseudo-order parameters $\psi_v = |\psi_v| \exp(i\theta_v)$ for each band, where $|\psi_v|^2$ is the pair density. By applying the extended London approximation to this system [7-9], we had presented characteristic phenomena such as an interband phase difference soliton and fractional flux quanta [7-13]. In the extended London approximation, the amplitude of the pseudo-order parameters $|\psi_v|^2$ is held constant. This approximation is based on the fact that the crystal fields (or a band structure) determine the number of carriers for each band.

Sometimes, multicomponent superconductivity based on multiband superconductors is considered as a symmetry-breaking state of a $U(1)^N$ system, where N is the number of bands [14-17]. However, the interband interaction *drags* other gauge fields on higher-dimensional manifolds (such as a manifold having broken $U(N)$ symmetry) [18-21]. For example, in a two-band superconductor, we can identify the electromagnetic $U(1)$ gauge field and three other gauge fields. Because of the extended London approximation and the real current condition $J = 0$, two degrees of freedom are removed; and the Josephson interaction combines three gauges into a single gauge field

[21]. This is the nonmagnetic gauge field for the interband phase difference, and sometimes (depending on the ratio of $m_v^*/|\psi_v|^2$ between two bands, where m_v^* is the mass of the pair) the manifold for this gauge is noncompact. In three-band superconductors, the potential surface determined by the Ginzburg–Landau free energy reduces the number of gauge fields [13]. The omitted gauge fields determine the dynamics of the soliton, such as its vibration and deformation in the quantum phase space, which are ignored in the discussion of the static and semistatic situation (e.g., a slow translational movement of the soliton). In multiband superconductors, we consider that incrementing the number of internal degrees of freedom, the constraints from crystal fields, and the potential surface of the Ginzburg–Landau free energy primarily determines topology of the manifold. The high symmetry and its breakdown do not have complete informations [22-29].

Recently, experimental observations that indicate the existence of these new internal degrees of freedom have been reported [30-35]. For magnetization, phase difference solitons and fractional vortices have been clearly demonstrated by scanning microscopy[30,32,33]. In macroscopic experiments, a curiously strong pinning effect was found in some new superconductors, which raises the suspicion of domain trapping of fractional vortices at the domain wall [12, 36]. There are several mechanisms that lead to chiral superconductivity in which we find chiral domains [12, 22, 24, 25, 27, 29, 37]. The chiral states suggested by three-band superconductivity are prominent [13, 14, 16, 38-41] (we were aware of reports [39, 40] on these when we published our reports on chiral superconductivity based on three-band superconductors [13, 41]). In particular, in a recent report [40], neither the high crystal symmetry nor a multidimensional representation due to this high symmetry is provided. Nondegenerate multiple bands

lead to chiral states by the positive interband Josephson interaction. From this fact, we can confirm that the multiplicity itself is primary rather than the high symmetry and its breakdown. Based on this understanding, we demonstrate a new type of domain, which we refer to hereinafter as the configuration domain of multiband superconductors.

2. Model formula

The extended Ginzburg–Landau formalism is convenient for dealing with multiband superconductors. We also assume the extended London approximation. With this simplification, we can understand the physics on a more intuitive level [9,12,13].

The phenomenological free energy f can be expressed as

$$f = \left(\sum_{v=1}^N \left(\alpha_v |\psi_v|^2 + \frac{\beta_v}{2} |\psi_v|^4 + \frac{1}{2m_v^*} \left| \left(\frac{\hbar}{i} \nabla - \frac{e^*}{c} A \right) \psi_v \right|^2 \right) \right) + \sum_{\mu=1}^N \left(\sum_{v=1, v \neq \mu}^N \frac{\gamma_{\mu v}}{4} (\psi_\mu^+ \psi_v + \psi_v^+ \psi_\mu) \right) + \frac{h^2}{8\pi}, \quad (1)$$

where N is the number of bands, e^* is the mass, A is the vector potential, and c is the speed of light. The sum $\sum_{v=1}^N \alpha_v |\psi_v|^2 + \frac{\beta_v}{2} |\psi_v|^4$ gives the energy due to intraband interactions, and $\frac{h^2}{8\pi}$ is the energy of the magnetic field. The kinetic energy term,

$\sum_{v=1}^N \frac{1}{2m_v^*} \left| \left(\frac{\hbar}{i} \nabla - \frac{e^*}{c} A \right) \psi_v \right|^2$, can be removed to explore the homogeneous state. The

interband Josephson interactions $\frac{\gamma_{\mu v}}{4} (\psi_\mu^+ \psi_v + \psi_v^+ \psi_\mu)$ determine the relative phase in the homogenous ground state, where μ and v are the band indices [12,13].

To discuss the ground state, we can also neglect the magnetic contribution. With the extended London approximation, f can be considerably simplified, as follows [12,13]:

$$f = f_0 + f_\theta, \quad (2)$$

$$f_0 = \sum_{\nu=1}^N \left(\alpha_{\nu} |\psi_{\nu}|^2 + \frac{\beta_{\nu}}{2} |\psi_{\nu}|^4 \right), \quad (3)$$

$$f_{\theta} = \sum_{\mu=1}^N \left(\sum_{\nu=1, \nu \neq \mu}^N \frac{\gamma_{\mu\nu}}{2} |\psi_{\mu}| |\psi_{\nu}| \cos(\theta_{\nu} - \theta_{\mu}) \right) = \sum_{\mu=1}^N \left(\sum_{\nu=1, \nu \neq \mu}^N \frac{\Gamma_{\mu\nu}}{2} \cos(\theta_{\nu} - \theta_{\mu}) \right). \quad (4)$$

When the interband interaction is much smaller than the intraband interaction, we can consider $|\psi_{\nu}|$ is determined by f_0 . Introducing $\Gamma_{\mu\nu} = |\psi_{\mu}| |\psi_{\nu}| \gamma_{\mu\nu}$, we can discuss the quantum phase using f_{θ} . As seen in previous reports, the positive interband Josephson interaction results in the nontrivial wave leading to chiral superconductivity. We also explore the same situation for multiband superconductors having four or more bands. For simplicity, we assume the multiple equivalent band, and let $\Gamma_{\mu\nu} = \Gamma > 0$ for all μ and ν . In this discussion, we carefully inspect the effect of high symmetry due to this simplification and distinguish between the effect due to symmetry and that due to multiplicity.

3. Results

Eq. (4) has local maxima or minima for some values that satisfy $\partial f_{\theta} / \partial \theta_{\nu} = 0$. The condition $\theta_{\nu} - \theta_{\mu} = 0$ or π gives trivial solutions for f_{θ} . When all the interband phase differences are zero, the trivial solution corresponds to the s_{++} wave. In contrast, when some of the phase differences are π , the trivial solution is an $s_{\pm}$ wave. Because we assume a positive interband Josephson interaction, the s_{++} wave gives the highest energy. Among several $s_{\pm}$ waves, the one that corresponds to half the term in the sum of Eq. (4) having $\theta_{\mu} = 0$ and half having $\theta_{\mu} = \pi$ when N is even. When N is three or

more and is odd, this minimum energy is given by $\theta_\mu = 0$ for $(N+1)/2$ bands with the other bands having $\theta_\mu = \pi$. We designate this wave $s_{0\pm}$. When there are an odd number of bands, we have a local minimum at $|\theta_{\mu+1} - \theta_\mu| = \frac{2\pi}{N}$ for every μ , where $\theta_{N+1} = \theta_1$. This wave is designated s_* and is shown as an example in Figs. 1(a) and 1(b) for $N = 3$ and 5, respectively.

We also observe $f_\theta(s_{0\pm}) - f_\theta(s_*) = \frac{\Gamma}{2}$, which means that the wave s_* is more stable than the wave $s_{0\pm}$. When there are an even number of bands, $f_\theta(s_{0\pm}) - f_\theta(s_*) = 0$. We show the wave s_* for $N = 4$ in Fig. 1(c). Moreover, several other waves have the same energy as $f_\theta(s_{0\pm})$. For $N = 4$, $\theta_1 - \theta_3 = \theta_2 - \theta_4 = \pi$ and $\theta_1 - \theta_2$ can be arbitrary, as shown in Fig. 1(d). This can be designated $s_\pm + e^{i\theta} s'_\pm$. Eq. (4) cannot lock $\theta_1 - \theta_2$. Rotation of $\theta_1 - \theta_2$ corresponds to the Nambu–Goldstone mode [42]. To lock in this value, some $\Gamma_{\mu\nu}$ values should differ from other $\Gamma_{\mu\nu}$ values (for example, $\Gamma_{13} = \Gamma_{14} = \Gamma_{23} = \Gamma_{24} > \Gamma_{12} = \Gamma_{34}$ or $\Gamma_{12} = \Gamma_{24} = \Gamma_{13} = \Gamma_{34} > \Gamma_{23} = \Gamma_{14}$). In this case, $\theta_1 - \theta_2$ will be locked at 0 or π and $f_\theta(s_{0\pm})$ becomes stable. In the locked phase, the Nambu–Goldstone mode becomes a phase difference soliton. This instability also occurs for other larger even numbers of bands. To lock in the interband phase difference, some different $\Gamma_{\mu\nu}$ are required (for example, $\Gamma_{\mu \leq \frac{N}{2} \nu > \frac{N}{2}} > \Gamma_{\mu \leq \frac{N}{2} \nu \leq \frac{N}{2}} = \Gamma_{\mu > \frac{N}{2} \nu > \frac{N}{2}}$) for higher even numbers of bands.

4. Discussion

It has been argued that the pure repulsive pair-exchange interaction mediates

superconductivity in multiband superconductors [43-46]. The present model suggests the repulsive channel stabilizes $s_{\pm}$ for an even number of bands without any special symmetry. For some high-symmetry systems, $s_{\pm} + e^{i\theta} s'_{\pm}$ might be possible, and such a system could be interesting if it were considered to be real superconductivity. However, we currently do not have any realistic candidate for such a system, and it may be rather difficult to realize this condition (or it might correspond to the “non-superconducting / incoherent” pseudogap state in the cuprate superconductors.[47, 48]). Thus, this possibility is not interesting for investigations of the present *coherent* superconductivity.

However, an s_* wave for an odd-numbered system should be considered if we are to understand the real phenomenon because the minimum at $|\theta_{\mu+1} - \theta_{\mu}| = \frac{2\pi}{N}$ does not disappear even if some $\Gamma_{\mu\nu}$ values change slightly. The system should retain many local minima of which $(N-1)!$ diverge upon increasing the number of bands. When we have only three bands, these local minima are recognized as different chiral states; but when the number exceeds three, many other configurations that are different from the chiral state become possible. For example, we show varying configurations in Figs. 2(a)–2(d) for $N = 5$. The energy of these four configurations gives the local minimum. Even if all $\Gamma_{\mu\nu}$ values are not equal, the local minimum does not disappear when all $\Gamma_{\mu\nu}$ values are similar. The starfish pattern is deformed but still present at this local minimum. This was shown in the three-band case. The Mercedes Benz emblem deforms with nonequivalent $\Gamma_{\mu\nu}$ but is still at the local minimum. This means many configuration domains can be formed just below the superconducting transition temperature (T_c) in these superconductors. The rapid quenching can freeze these

domains (Kibble-Goryo mechanism) [10, 12, 36, 49, 50].

The single chiral domain can trap fractional vortices on its domain wall. Unlike a chiral domain [10, 12, 13, 24, 25, 27, 36], several configuration domains are necessary to trap fractional vortices, as seen in Fig. 3. In this figure, the name of the domain is same as the name of the order parameter shown in Fig. 2. The boundary of the domain (a) and (b) is the phase difference soliton exchanging ψ_1 and ψ_5 in which $\theta_1 - \theta_5$ varies from $2\pi/5$ through 0 to $-2\pi/5$. The (electro-magnetic) vector potential cannot adjust this mismatch of the configuration. After passing four boundaries, the configuration of the order parameters is restored with a $2\pi/5$ phase shift. Thus, the fractional vortex with $\Phi_0/5$ recovers this $2\pi/5$ phase shift, where Φ_0 is the unit flux quantum. The role of fractional vortices is to serve as hubs for phase difference solitons. (The value of the fractional flux quanta varies for the condition $J = 0$, as seen in ordinary two-band superconductors [7,8].)

The repulsive interband Josephson interaction between many fragmental Fermi surfaces can contribute to opening the superconducting gap. However, it invokes many configuration domains simultaneously. The formation energy of the soliton wall (domain wall) decreases upon incrementing the number of fragments of the Fermi surfaces, which means the configuration domain can easily be invoked in such a system. Even if the superconducting gap opens, the dynamic creation and destruction of the configuration domain tends to destroy the coherence. A movement of the soliton wall also leads to the energy dissipation. These dynamics become problematic when seeking a new high- T_c superconductor based on disconnected Fermi surfaces with the repulsive pair-exchange interaction. However, once these configurations are stabilized by

effective pinning, we can use this topology for new electronics [21].

Taking into account this new topology, the curiously strong pinning found in some superconductors should be surveyed [12, 36].

5. Summary

The positive Josephson interaction in multiband superconductors can create a configuration domain, and the resulting domain wall seriously affects the coherence of superconductivity, resulting in energy dissipation. However, such configuration domains also offer new potential for novel electronics.

Acknowledgments

This work was supported by the Strategic International Cooperative Program of the Japan Science and Technology Agency (JST) and the Department of Science and Technology (India) project “Feasibility study of the application of multiple order parameters in materials to information processing;” a Grant-in-Aid for Specially Promoted Research (20001004) from The Ministry of Education, Culture, Sports, Science, and Technology (MEXT); National Institute of Advanced Industrial Science and Technology (AIST) through a grant for overriding priority research “New Superconducting Revolution,” the Romanian Ministry of Education and Research, and the European Commission through the Marie Curie Excellence Grant “NanoTechPinningHTS.”

References

- [1] M. Tinkham, *Intoduction to superconductivity*, 2nd ed. , McGraw-Hill, New York , 1996, Chap. 1, pp. 9-11.
- [2] D. R. Tilley and J. Tilley, *Superfluidity and superconductivity*, 3rd ed. ,IOP publishing, Bristol, 1990, Chap. 6, pp. 171-174.
- [3] J. A. Annett, N. Goldenfeld, A. J. Leggett, *Physical Properties of High Temperature Superconductors V*, ed. D. M. Ginzberg, World Scientific, Singapore, 1996, Chap. 6, pp. 383– 386.
- [4] H. Suhl, B. T. Matthias, L. R. Walker, Phys. Rev. Lett. **3** (1959) 552.
- [5] A. J. Leggett, Prog. Theor. Phys. **36** (1966) 901.
- [6] X. X. Xi, Rep. Prog. Phys. **71** (2008) 116501, and references therein.
- [7] Y. Tanaka, J. Phys. Soc. Jpn. **70** (2001) 2844.
- [8] Y. Tanaka, Phys. Rev. Lett. **88** (2002) 017002.
- [9] Y. Tanaka, D.D. Shivagan, A. Crisan, A. Iyo, P.M. Shirage, K. Tokiwa, T. Watanabe, N. Terada, J. Phys.: Conf. Ser. **150** (2009) 052267.
- [10] Y. Tanaka, A. Crisan, D. D. Shivagan, A. Iyo, K. Tokiwa, T. Watanabe, Jpn. J. Appl. Phys. **46** (2007) 134.
- [11] Y. Tanaka, A. Crisan, Physica B **404** (2010) 1033.
- [12] Y. Tanaka, P. M. Shirage, A. Iyo, Physica C , in printing.
(doi:10.1016/j.physc.2010.09.006)
- [13] Y. Tanaka, T. Yanagiwasa, J. Phys. Soc. Jpn. **79** (2010) 114706.
- [14] T. Suzuki, Prog. Theor. Phys. **80** (1988) 929.
- [15] E. Babaev, Phys. Rev. Lett. **89** (2002) 067001.
- [16] Y. Koma, M. Koma, D. Ebert, H. Toki, Nucl. Phys. B **648** (2003) 189.

- [17] I. P. Ivanov, Phys. Rev. E **79** (2009) 021116.
- [18] L. F. Chibotaru, V. H. Dao, Phys. Rev. B **81** (2010) 020502.
- [19] R. Geurts, M. V. Milošević, F. M. Peeters, Phys. Rev. B **81** (2010) 214514.
- [20] Y. M. Cho, P. M. Zhang, Eur. Phys. J. B **65** (2008) 155.
- [21] Y. Tanaka, A. Iyo, K. Tokiwa, T. Watanabe, A. Crisan, A. Sundaresan, N. Terada, Physica C **470** (2010) 1010.
- [22] T. D. Lee, Phys. Rev. D **8** (1973) 1226.
- [23] K. Kuboki, M. Sigrist, J. Phys. Soc. Jpn. **65** (1996) 361.
- [24] Yu.A. Izyumov, V.M. Laptev, Phase Transit **20** (1990) 95.
- [25] O. V. Lounasmaa, E. Thuneberg, Proc. Natl. Acad. Sci. U.S.A. **96** (1999) 7760.
- [26] F. Wilczek . A. Zee, Phys. Rev. Lett. **51** (1983) 2250.
- [27] M. Sigrist, K. Ueda, Rev. Mod. Phys. **63** (1991) 239.
- [28] S. Weinberg, Phys. Rev. Lett. **19** (1967) 1264.
- [29] A. J. Leggett, Rev. Mod. Phys. **76** (2004) 999.
- [30] H. Bluhm, N. C. Koshnick, M. E. Huber, K. A. Moler, Phys. Rev. Lett. **97** (2006) 237002. (See also Phys. Rev. Lett. **98** (2007) 209902(E).)
- [31] A. Crisan, Y. Tanaka, D. D. Shivagan, A. Iyo, L. Cosereanu, K. Tokiwa, T. Watanabe, Jpn. J. Appl. Phys. **46** (2007) L451.
- [32] J.W. Guikema, H. Bluhm, D.A. Bonn, R. Liang, W.N. Hardy, K.A. Moler, Phys. Rev. B **77** (2008) 104515.
- [33] L. Luan, O.M. Auslaender, D.A. Bonn, R. Liang, W.H. Hardy, K.A. Moler, Phys. Rev. B **79** (2009) 214530.
- [34] D. D. Shivagan, A. Crisan, P. M. Shirage, A. Sundaresan, Y. Tanaka, A. Iyo, K. Tokiwa, T. Watanabe, N. Terada, J. Phys.: Conf. Ser. **97** (2008) 012212.

- [35] A. Crisan, Y. Tanaka, A. Iyo, J. Supercond. Nov. Magn. **23** (2010) (published on line, doi:10.1007/s10948-010-0892-8).
- [36] Y. Tanaka, P.M. Shirage, A. Iyo, J. Supercond. Nov. Magn. **23** (2010) 253.
- [37] Y. Imry, J. Phys. C: Solid State Phys. **8** (1975) 567.
- [38] H. Suganuma, S. Sasaki, H. Toki, Nucl. Phys. B **435** (1995) 207.
- [39] D. F. Agterberg, V. Barzykin, L. P. Gor'kov, Phys. Rev. B **60** (1999) 14868.
- [40] V. Stanev, Z. Tesanovic, Phys. Rev. B. **81** (2010) 134522.
- [41] Y. Tanaka, T. Yanagisawa, Solid State Commun. **150** (2010) 1980.
- [42] Y. Nambu, Phys. Rev. **117** (1960) 648.
- [43] J. Kondo, Prog. Theor. Phys. **29** (1963) 1.
- [44] C.C. Tsuei, J.R. Kirtley, Rev. Mod. Phys. **72** (2000) 969.
- [45] I. I. Mazin, D. J. Singh, M. D. Johannes, M. H. Du, Phys. Rev. Lett. **101** (2008) 057003.
- [46] K. Kuroki, H. Usui, S. Onari, R. Arita, H. Aoki, Phys. Rev. B **79** (2009) 224511.
- [47] V. J. Emery, S. A. Kivelson, Nature **374** (1995) 434.
- [48] H.-Y. Choi, Y. Bang, D. K. Campbell, Phys. Rev. B **61** (2000) 9748.
- [49] T.W.B. Kibble, J. Phys. A: Math. Gen. **9** (1976) 1387.
- [50] J. Goryo, S. Soma, H. Matsukawa, Europhys. Lett. **80** (2007) 17002.

Figure captions

Figure 1. Some examples of the configuration of order parameters in multiband superconductors. (a) Example of a three-band case. (b) Example of a five-band case. (c) Example of a four-band case. (d) Different configuration that gives the same free energy as shown in (c). ψ_1 and ψ_3 can rotate against ψ_2 and ψ_4 while keeping $\theta_1 - \theta_3 = \theta_2 - \theta_4 = \pi$ and without changing the energy.

Figure 2. Some configurations for $N = 5$ system. The boundary between the domain described by (a) and (b) is composed of the phase difference soliton exchanging ψ_1 and ψ_5 .

Figure 3. Fractional vortex and configuration domains. The order parameters are shown in Fig. 2. The same figures (a)–(e) are used to denote the order parameter. The boundary is the phase difference soliton exchanging the two pseudo-order parameters. Panels (a) and (e) show a phase shift that is recovered by the fractional vortex, which has $\Phi_0/5$.

Figure 1

Figure 2

Figure 3