Identity-based conditional proxy re-encryption with fine grain policy

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Abstract. An identity-based conditional proxy re-encryption scheme (IB-CPRE) allows a semi-trusted proxy to convert a ciphertext satisfying one condition, which is set by the delegator, under one identity to another without the necessity to reveal the underlying message. In ICISC 2012, Liang, Liu, Tan, Wong and Tang proposed an IB-CPRE scheme, and left an open problem on how to construct chosen-ciphertext secure IB-CPRE supporting OR gates on conditions. In this work, we answer this aforementioned problem by constructing an identity-based conditional proxy re-encryption scheme with fine grain policy (IB-CPRE-FG). In an IB-CPRE-FG scheme, each ciphertext is labeled with a set of descriptive conditions and each re-encryption key is associated with an access tree that specifies which type of ciphertexts the proxy can re-encrypt. Furthermore, our scheme can be proved secure against adaptive access tree and adaptive identity chosen-ciphertext attack.

Keywords: Conditional proxy re-encryption, Identity-based proxy re-encryption, Chosen-ciphertext security.

1 Introduction

Identity-based proxy re-encryption (IB-PRE), introduced by Green and Ateniese [1], enables a proxy to convert a ciphertext encrypted under Alice's identity into one encrypted under Bob's identity. To capture the conditional property, Shao, Wei, Ling and Xie [2] introduced the notion of identity-based conditional proxy re-encryption (IB-CPRE), in which only ciphertexts satisfying a special condition can be converted by the proxy. However, their scheme is only proven secure in the random oracle. Further, Liang, Liu, Tan, Wong and Tang [3] proposed an identity-based conditional proxy re-encryption scheme in the standard model.

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Unfortunately, the above scheme [3] does not allow any boolean combinations of conditions. However, boolean combinations of conditions are essential practical property for CPRE. We shall use the on-line medical service systems [4] to illustrate the practicality of our work. In an on-line medical system, a patient prefers to find doctors with the following requirements to remedy his heart disease. The requirements are described by a set of attributes $W = (\text{"heart"}, \text{"protomedicus"}, \text{"specialist"}, \text{"New York"})$. The patient encrypts his medical record with W before uploading to the system. The system then forwards the ciphertext to doctor Alice to in charge of this case. Nevertheless, when the doctor is out for a vacation, he may want to find a trustworthy colleague Bob with policy $P_1 = (\text{"heart"} \wedge (\text{"protomedicus"} \vee \text{"specialist"}) \wedge (\text{Location} : \text{"Downtown of New York"}))$ to check the medical record for him. In a traditional public key encryption system, Alice has to decrypt the ciphertext and recover the data first, and then encrypt the data using Bob's public key. If there are N different patients' medical records that Alice wants to send to N different colleagues, she is required to perform N decryptions and N new encryptions. This is impractical as Alice's workload is linear in N . The situation becomes worse as she should be on-line during each encryption process.

To solve the above problem, many CPRE schemes were proposed to capture a fine-grained control over conditions [5, 6, 7]. However, none of these schemes work in the identity-based setting. In 2011, Fang, Susilo, Ge and Wang proposed an interactive conditional proxy re-encryption with fine grain policy [8] and left an open problem on how to construct a non-interactive conditional proxy re-encryption with fine grain policy.

1.1 Our contribution

This work, for the first time, answers the aforementioned open problems affirmatively by presenting an identity-based proxy re-encryption with fine grain policy. Our contributions are as follows. First, we propose a fine grain policy on conditions. In this work, the access control on conditions is described an access tree. Hence, our work moves one step ahead to solve the open question proposed by [3]. Finally, as our condition proxy re-encryption works in the identity-based setting, and the re-encryption key is generated without any interaction between the delegator and the delegatee, our scheme is also an affirmative answer of the open question proposed by [8]. Moreover, Our scheme is secure against adaptive access tree and adaptive identity chosen-ciphertext attack under the decisional bilinear Diffie-Hellman assumption.

1.2 Related work

Proxy re-encryption. Proxy re-encryption(PRE) was first introduced by Blaze, Bleumer and Strauss [9], which enables a semi-trusted proxy to transform Alice's ciphertext into Bob's ciphertext of the same message. The proxy, however, cannot learn anything about the underlying message. In 2005, the first unidirectional PRE scheme was proposed by Ateniese, Fu, Green and Hohenberger [10]. Follow their work, many PRE schemes [11, 12, 13, 14, 15] with different properties have been proposed. To make flexible control on encrypted ciphertexts, conditional proxy re-encryption(CPRE) schemes [16, 17, 8, 18] were proposed, in which only ciphertexts satisfying a certain condition can be re-encrypted.

Identity-based proxy re-encryption. To extend the notion of proxy re-encryption to the identity-based setting, Green and Ateniese [1] proposed the first identity-based proxy

re-encryption (IB-PRE). In an IB-PRE scheme, the proxy can convert a ciphertext encrypted under Alice's identity into one encrypted under Bob's identity. They proposed two IB-PRE schemes, which are both secure in the random oracle model. Chun and Tzeng [19] introduced two IB-PRE schemes, which are both secure in the standard model based on Waters IBE scheme [20]. However, in previous schemes, the length of re-encrypted ciphertext grows with the time of re-encryption. To solve this problem, Liang et al. [21] proposed a CCA-secure multi-hop identity-based conditional proxy re-encryption scheme with constant-size ciphertexts. Liang, Susilo and Liu [22] proposed an anonymous multi-hop identity-based conditional proxy re-encryption scheme and found it can provide privacy-preserving for big data storage. In ASIACCS 2009, Liang et al. [5] introduced the notion of attribute-based proxy re-encryption (AB-PRE), in which identities are viewed as a set of attribute. To support AB-PRE with any monotonic access structures, ciphertext-policy AB-PRE and key-policy AB-PRE were proposed by Liang et al. [23, 24, 25, 26] and Ge et al. [27]. In 2014, Liang et al. [28] introduced a new notion of DFA-based functional PRE and proposed a concrete DFA-based functional PRE scheme with CCA security.

Lattice-based proxy re-encryption. The vast majority of schemes based on number theoretic problems such as the discrete logarithm are known to be susceptible against quantum computers attack. To improve this problem, Aono et al. [29] proposed an unidirectional proxy re-encryption scheme under LWE. Their scheme is CPA-secure in the standard model and can be transformed into CCA-secure in the random oracle. After their work, Kirshanova [30] proposed a CCA1 secure proxy re-encryption scheme in the selective model based on LWE. In ASIACCS 2015, Nunez, Agudo and Lopez [31] proposed bidirectional and multi-hop proxy re-encryption based on NTRU.

1.3 Roadmap

The rest of the paper is organized as follows. We first provide the basic primitives and the security model for an IB-CPRE-FG scheme in Sec. 2. In Sec. 3, we present our IB-PRE-FG scheme and give the security proofs. We give a comparison and discussion of our scheme in Sec. 4. Finally, we conclude the paper in Sec. 5.

2 Preliminaries

2.1 Negligible function

A function $\varepsilon(n) : N \rightarrow R$ is said to be negligible if for all positive integer $c \in N$ there exists a $n_c \in N$ such that $\varepsilon(n) < n_c$ for all $n > n_c$.

2.2 Bilinear map

Let G and G_T be two multiplicative cyclic groups with the same prime order p , and g be a generator of G . A bilinear pairing is a map $e : G \times G \rightarrow G_T$ with the following properties [32, 33]:

1. $e(g^a, h^b) = e(g, h)^{ab}$ for all $a, b \xleftarrow{R} \mathbb{Z}_p^*$ and $g, h \in G$.
2. $e(g, g) \neq 1$.

3. There is an efficient algorithm to compute $e(g, h)$ for all $g, h \in G$.

2.3 Decisional bilinear Diffie-Hellman(DBDH) assumption

The DBDH assumption [20] is as below: Given a vector of 5 elements

$$(g, g^a, g^b, g^c, T) \in G^4 \times G_T$$

as input, it is hard to distinguish $T = e(g, g)^{abc}$ from a random value in G_T . Formally, for all probability polynomial time adversaries $\mathcal{A}$, the following probability is negligible:

$$\begin{aligned} & | \Pr[a, b, c, r \xleftarrow{R} Z_p^*; \quad T_0 = e(g, g)^{abc}; \quad T_1 = e(g, g)^r; \\ & z \in \{0, 1\}; \quad z' \leftarrow \mathcal{A}(g, g^a, g^b, g^c, T_z) : \quad z = z'] - \frac{1}{2}. \end{aligned}$$

2.4 Strongly unforgeable one-time signature

A one-time signature [34] consists of a triple of algorithms $Sig = (\mathcal{G}, \mathcal{S}, \mathcal{V})$ such that, on input of a security parameter λ , $\mathcal{G}$ generates a one-time key pair (ssk, sk) , while for any message M , $\mathcal{V}(sk, \sigma, M)$ outputs 1 whenever $\sigma = \mathcal{S}(ssk, M)$ and 0 otherwise. As in [11], we need strongly unforgeable one-time signatures, which means that no PPT adversary can create a new signature even for a previously signed message.

$Sig = (\mathcal{G}, \mathcal{S}, \mathcal{V})$ is a strongly unforgeable one-time signature if the probability $Adv^{OTS} = \Pr[(ssk, sk) \leftarrow \mathcal{G}(\lambda); (M, St) \leftarrow F(sk); \sigma \leftarrow \mathcal{S}(ssk, M); (M', \sigma') \leftarrow F(M, \sigma, sk, St) : \mathcal{V}(sk, \sigma', M') = 1 \wedge (M', \sigma') \neq (M, \sigma)]$, where St denotes the state information maintained by F between stages, is negligible for any PPT forger F .

2.5 Pseudorandom function family

We here review the definition of pseudorandom function family [13]. Let $F : \mathcal{K} \times \mathcal{D} \rightarrow \mathcal{R}$ be a function family, where $\mathcal{K}$ is the set of keys of F , $\mathcal{D}$ is the domain, and $\mathcal{R}$ is the range. Let $G : \mathcal{D} \rightarrow \mathcal{R}$ be a true random function family. Let $\mathcal{F}$ be a PPT adversary which outputs a bit. We consider the following two experiments:

$$\begin{aligned} \text{Experiment } Exp_F^{PRF-1}(\mathcal{F}) : & [K \xleftarrow{\$} \mathcal{K}, b \xleftarrow{\$} \mathcal{F}^{F(K, \cdot)}, \text{Return } b]. \\ \text{Experiment } Exp_F^{PRF-0}(\mathcal{F}) : & [g \xleftarrow{\$} G, b \xleftarrow{\$} \mathcal{F}^{g(\cdot)}, \text{Return } b]. \end{aligned}$$

We define $\mathcal{F}$'s advantage $Adv_{F, \mathcal{F}}^P$ in attacking the pseudorandom of the function F as

$$|\Pr[Exp_F^{PRF-1}(\mathcal{F}) = 1] - \Pr[Exp_F^{PRF-0}(\mathcal{F}) = 1]|.$$

If for any PPT adversary $\mathcal{F}$, its advantage in attacking the pseudorandomness of the function family F is negligible, then we say that F is a pseudorandom function family.

2.6 Identity-based proxy re-encryption with fine grain policy

In this subsection, we will provide the definition of identity-based proxy re-encryption with fine grain policy. We use the same approach as [8], where ciphertexts are labeled with a set of descriptive conditions. A tree access structure, in which each interior node of the tree is a threshold gate and the leaves are associated with conditions, is used to identify re-encryption keys. Using this approach, we can present a tree with “AND” and “OR” gate by using n -out-of- n and 1-out-of- n threshold gates, respectively. A user will be able to re-encrypt a ciphertext with a re-encryption key provided if and only if there is an assignment of conditions from the ciphertexts to nodes of the tree such that the tree is satisfied.

Definition 1 (Access tree [35]). Access tree $\mathcal{T}$. We incorporate the definition of access tree as in [35]. Let $\mathcal{T}$ be a tree that represents an access structure. Each non-leaf node of the tree represents a threshold gate, described by its children a threshold value. Let num_x be the number of children of a node x and k_x be its threshold value, then we have $0 < k_x \leq num_x$. When $k_x = num_x$, it is an AND gate, and when $k_x = 1$, the threshold is an OR gate. Each leaf node x of the tree is described by a conditional keyword and a threshold value $k_x = 1$. We fit the root of an access tree to be at depth 0. Let $\overline{LN}_{\mathcal{T}}$ be the set of all the non-leaf nodes, and $LN_{\mathcal{T}}$ be the set of all the leaf nodes. We denote the parent of node x in the tree $p(x)$. The function $keyword(x)$ is defined only if $x \in LN_{\mathcal{T}}$ and it denotes the conditional keyword associated with the leaf node x in the tree. The access tree $\mathcal{T}$ also defines an ordering between the children of every node, that is, the children of a node are numbered from 1 to num . The function $inex(z)$ returns a index associated with the node z , where the index values are uniquely assigned to nodes in the access tree.

Satisfying an access tree. Let $\mathcal{T}$ be an access tree with root r . $\mathcal{T}_x$ is the subtree rooted at the node x . Hence $\mathcal{T}$ is the same as $\mathcal{T}_r$. $\mathcal{T}_x(W) = 1$ denotes a set of conditions W satisfies the access tree $\mathcal{T}_x$. If $x \in LN_{\mathcal{T}}$, then $\mathcal{T}_x(W) = 1$ if and only if $keyword(x) \in W$. If $x \in \overline{LN}_{\mathcal{T}}$, evaluate $\mathcal{T}_z(W)$ for all children z of node x , $\mathcal{T}_x(W)$ return 1 if and only if at least k_x children return 1.

Definition 2 (IB-CPRE-FG). An (single-use) identity-based conditional proxy re-encryption with fine grain policy [3] scheme consists the following algorithms:

1. Setup(λ): The Setup algorithm is run by the private key generator(PKG), on input a security parameter λ , the global public parameters PP and mast secret key msk are outputted.
2. KeyGen(msk, ID): On input the mast secret key msk and an identity ID , output the private key sk_{ID} for identity ID .
3. Enc(ID, m, W): On input an identity ID , a message $m \in M$ and a condition set W , output a second level ciphertext $C_{ID}^{(2)}$. Here M denotes the message space.
4. RKeyGen($sk_{ID_i}, ID_i, \mathcal{T}, ID_j$): On input identities ID_i, ID_j , the secret key sk_{ID_i} and an access tree $\mathcal{T}$, output the re-encryption key $rk_{\mathcal{T}|ID_i \rightarrow ID_j}$.
5. ReEncrypt($C_{ID_i}^{(2)}, rk_{\mathcal{T}|ID_i \rightarrow ID_j}$): On input a re-encryption key $rk_{\mathcal{T}|ID_i \rightarrow ID_j}$ and a second level ciphertext $C_{ID_i}^{(2)}$ associated with W under identity ID_i , output a first level ciphertext $C_{ID_j}^{(1)}$.

6. $\text{Dec2}(ID_i, sk_{ID_i}, C_{ID_i}^{(2)})$: On input an identity ID_i , a private key sk_{ID_i} , and a second level ciphertext $C_{ID_i}^{(2)}$, output the plaintext m or an error symbol $\perp$.
7. $\text{Dec1}(ID_j, sk_{ID_j}, C_{ID_j}^{(1)})$: On input an identity ID_j , a private key sk_{ID_j} , and a first level ciphertext $C_{ID_j}^{(1)}$, output the plaintext m or an error symbol $\perp$.

Note that we omit the global parameters PP as the other algorithms' input for simplicity. The correctness of IB-CPRE-FG means that, for any message $m \in M$, $sk_{ID_i} \leftarrow \text{KeyGen}(msk, ID_i)$, $sk_{ID_j} \leftarrow \text{KeyGen}(msk, ID_j)$, and $rk_{\mathcal{T}|ID_i \rightarrow ID_j} \leftarrow \text{RKeyGen}(sk_{ID_i}, ID_i, \mathcal{T}, ID_j)$ and $C_{ID_i}^{(2)} = (ID_i, m, W)$, where $\mathcal{T}(W) = 1$.

$$\Pr[\text{Decrypt}(C_{ID_i}^{(2)}, sk_{ID_i}) = m] = 1$$

and

$$\Pr[\text{Decrypt}(sk_{ID_j}, \text{ReEncrypt}(C_{ID_i}^{(2)}, rk_{\mathcal{T}|ID_i \rightarrow ID_j})) = m] = 1.$$

2.7 Security notion for IB-CPRE-FG

Next, we will provide the game-based security definition of IB-CPRE-FG. As in [3], our security model considers the indistinguishability of the second level ciphertext and the first level ciphertext in the adaptive identity and adaptive condition set model. The two security game are denoted as IND-level2-CCA and IND-level1-CCA individually.

Definition 3 (IND-level2-CCA). An (single-use) IB-CPRE-FG scheme is IND-level2-CCA secure if no probabilistic polynomial time(PPT) adversary $\mathcal{A}$ can win the game below with non-negligible advantage. Next in the game, assuming $\mathcal{B}$ is the challenger and λ is the security parameter.

1. Setup. $\mathcal{B}$ Runs the $\text{Setup}(\lambda)$ algorithm to get the (PP, msk) , and give PP to $\mathcal{A}$.
2. Query phase 1. $\mathcal{A}$ makes the following queries:
 - (a) $\text{Extract}(ID)$: run the $\text{KeyGen}(msk, ID)$ algorithm to get sk_{ID} , and returns sk_{ID} to $\mathcal{A}$.
 - (b) $\text{RKExtract}(ID_i, \mathcal{T}, ID_j)$: run the $\text{RKeyGen}(sk_{ID_i}, ID_i, \mathcal{T}, ID_j)$ algorithm to get $rk_{\mathcal{T}|ID_i \rightarrow ID_j}$, and returns $rk_{\mathcal{T}|ID_i \rightarrow ID_j}$ to $\mathcal{A}$.
 - (c) $\text{ReEnc}(ID_i, ID_j, (W, C_{ID_i}^{(2)}))$: run the $\text{ReEncrypt}(C_{ID_i}^{(2)}, rk_{\mathcal{T}|ID_i \rightarrow ID_j})$, where $\mathcal{T}(W) = 1$, algorithm to get a first level ciphertext $C_{ID_j}^{(1)}$, and returns $C_{ID_j}^{(1)}$ to $\mathcal{A}$.
 - (d) $\text{Dec2}(ID_i, (W, C_{ID_i}^{(2)}))$: run the $\text{Dec2}(ID_i, sk_{ID_i}, C_{ID_i}^{(2)})$ algorithm to get the underlying message m , and returns m to $\mathcal{A}$.
 - (e) $\text{Dec1}(ID_j, C_{ID_j}^{(1)})$: run the $\text{Dec1}(ID_j, sk_{ID_j}, C_{ID_j}^{(1)})$ algorithm to get the underlying message m , and returns m to $\mathcal{A}$.
3. Challenge. Once $\mathcal{A}$ decides that Phase 1 is over, it outputs two equal length message (m_0, m_1) , a target identity ID^* and a target condition set W^* . The challenger $\mathcal{B}$ chooses a random bit $b \in \{0, 1\}$ and sends the challenge ciphertext $C_{ID^*}^{(2)*} = \text{Enc}(ID^*, m_b, W^*)$ to $\mathcal{A}$, if the following queries are never made:
 - $\text{Extract}(ID^*)$, and
 - $\text{RKExtract}(ID^*, \mathcal{T}, ID_j)$ and $\text{Extract}(ID_j)$, where $\mathcal{T}(W^*) = 1$.

4. Query phase 2. $\mathcal{A}$ continues making queries as in the Query phase 1, except the following queries:
 - (a) $\text{Extract}(ID^*)$;
 - (b) $\text{RKExtract}(ID^*, \mathcal{T}, ID_j)$ and $\text{Extract}(ID_j)$, where $\mathcal{T}(W^*) = 1$;
 - (c) $\text{ReEnc}(ID^*, ID_j, (W^*, C_{ID^*}^{(2)}))$ and $\text{Extract}(ID_j)$;
 - (d) $\text{Dec2}(ID^*, (W^*, C_{ID^*}^{(2)}))$;
 - (e) $\text{Dec1}(ID_j, C_{ID_j}^{(1)})$, if $C_{ID_j}^{(1)}$ is a derivative of $C_{ID^*}^{(2)}$. As defined in [11], derivative of $C_{ID_i}^{(2)} = \text{Enc}(ID_i, m, W)$ is defined as follows:
 - $C_{ID_i}^{(2)}$ is a derivative of itself;
 - If $\mathcal{A}$ has issued $\text{RKExtract}(ID_i, \mathcal{T}, ID_j)$ query to obtain a re-encryption key $rk_{\mathcal{T}|ID_i \rightarrow ID_j}$, where $\mathcal{T}(W) = 1$, and computed $C_{ID_j}^{(1)} \rightarrow \text{ReEnc}(ID_i, ID_j, (W, C_{ID_i}^{(2)}))$, then $C_{ID_j}^{(1)}$ is a derivative of $C_{ID_i}^{(2)}$.
 - If $\mathcal{A}$ has issued $\text{ReEnc}(ID_i, ID_j, (W, C_{ID_i}^{(2)}))$ and obtained $C_{ID_j}^{(1)}$. Then $C_{ID_j}^{(1)}$ is a derivative of $C_{ID_i}^{(2)}$.
5. Guess. $\mathcal{A}$ outputs the guess b' . The adversary wins if $b' = b$.

We say that an IB-CPRE-FG scheme is IND-level2-CCA secure, if the following probability is negligible for all probabilistic polynomial time adversary $\mathcal{A}$:

$$\text{Adv}_{\mathcal{A}}^{\text{IND-level2-CCA}}(\lambda) = |\Pr[b' = b] - 1/2|.$$

Next, we will present the indistinguishability of the first level ciphertext. In this definition, a first level ciphertext is provided to the adversary $\mathcal{A}$ in the challenge phase. As defined in [12], since the first level ciphertext cannot be re-encrypted, $\mathcal{A}$ should be allowed to obtain any re-encryption key including those from the target identity to identities that the adversary have issued Extract query. Furthermore, since $\mathcal{A}$ is allowed to obtain any re-encryption keys, the ReEnc query becomes useless. Moreover, the Dec2 query is also unnecessary.

Definition 4 (IND-level1-CCA). An (single-use) IB-CPRE-FG scheme is IND-level1-CCA secure if no probabilistic polynomial time(PPT) adversary $\mathcal{A}$ can win the game below with non-negligible advantage. Next in the game, assuming $\mathcal{B}$ is the challenger and λ is the security parameter.

1. Setup. $\mathcal{B}$ Runs the $\text{Setup}(\lambda)$ algorithm to get the (PP, msk) , and give PP to $\mathcal{A}$.
2. Query phase 1. $\mathcal{A}$ makes the following queries:
 - (a) $\text{Extract}(ID)$: run the $\text{KeyGen}(\text{msk}, ID)$ algorithm to get sk_{ID} , and returns sk_{ID} to $\mathcal{A}$.
 - (b) $\text{RKExtract}(ID_i, \mathcal{T}, ID_j)$: run the $\text{RKeyGen}(sk_{ID_i}, ID_i, \mathcal{T}, ID_j)$ algorithm to get $rk_{\mathcal{T}|ID_i \rightarrow ID_j}$, and returns $rk_{\mathcal{T}|ID_i \rightarrow ID_j}$ to $\mathcal{A}$.
 - (c) $\text{Dec1}(ID_j, C_{ID_j}^{(1)})$: run the $\text{Dec1}(ID_j, sk_{ID_j}, C_{ID_j}^{(1)})$ algorithm to get the underlying message m , and returns m to $\mathcal{A}$.

3. Challenge. Once $\mathcal{A}$ decides that Phase 1 is over, it outputs two equal length message (m_0, m_1) and a target identity ID^* . The challenger $\mathcal{B}$ chooses a random bit $b \in \{0, 1\}$ and sends the challenge ciphertext $C_{ID^*}^{(1)*} = \text{ReEnc}(\text{Enc2}(ID, m_b, W), rk_{\mathcal{T}|ID \rightarrow ID^*})$ to $\mathcal{A}$, where $ID \neq ID^*$ and $\mathcal{T}(W) = 1$. The restriction is that the adversary has never make the $\text{Extract}(ID^*)$ query.
4. Query phase 2. $\mathcal{A}$ continues making queries as in the Query phase 1, except the following queries:
 - (a) $\text{Extract}(ID^*)$;
 - (b) $\text{Dec1}(C_{ID^*}^{(1)*})$.
5. Guess. $\mathcal{A}$ outputs the guess b' . The adversary wins if $b' = b$.

We say that an IB-CPRE-FG scheme is IND-level1-CCA secure, if the following probability is negligible for all probabilistic polynomial time adversary $\mathcal{A}$:

$$\text{Adv}_{\mathcal{A}}^{\text{IND-level1-CCA}}(\lambda) = |\Pr[b' = b] - 1/2|.$$

The IB-CPRE-FG scheme is said to be IND-CCA secure if both $\text{Adv}_{\mathcal{A}}^{\text{IND-level2-CCA}}(\lambda)$ and $\text{Adv}_{\mathcal{A}}^{\text{IND-level1-CCA}}(\lambda)$ are negligible.

3 Our proposed IB-CPRE-FG

In this section, we first propose our IB-CPRE-FG scheme and then prove its IND-CCA security.

3.1 On achieving IND-CCA secure IB-CPRE-FG

Inspired by [35], in our scheme, the ciphertext is encrypted with a condition set W and the re-encryption key is generated with an access tree $\mathcal{T}$. The proxy is able to re-encrypt the ciphertext if and only if $\mathcal{T}(W) = 1$. One may thought that it is trivial to achieve an IB-CPRE-FG scheme by combining an identity-based encryption and an attribute-based encryption (ABE). As in [8], one may use the the ABE scheme with the mast key x_i , and then subsequently alter the input of the attribute-key generation algorithm form x_i to $x_j - x_i$. The re-encryption key $x_j - x_i$ is then involved in the access tree. Unfortunately, this approach can not be used directly in the identity-based setting. In the former scheme, the secret and public key for an user i is set to be x_i and g^{x_i} , where $x_i \in \mathbb{Z}_p^*$ and $g^{x_i} \in G$. The re-encryption key $sk_j - sk_i = x_j - x_i \in \mathbb{Z}_p^*$ is set to be the exponent for element $g^{x_j - x_i}$. However, in the identity-based setting [20, 33], the secret key for an identity is set be an element in G and the public key is ID . The re-encryption key $sk_j - sk_i$ cannot be used as the exponent anymore. Furthermore, the re-encryption key $sk_j - sk_i$ cannot be involved in the access tree using the threshold method.

To solve the above problem, we use the “token-controlled” approach. In the re-encryption key generation algorithm, we select a random token θ as the secret for the polynomial. And also we let θ involved in ID_j . When performing the re-encryption, the original ciphertext is “semi-decrypted” by the proxy using the re-encryption key. At last, the re-encrypted ciphertext is only relevant to the token θ , which is encrypted under ID_j . Using sk_{ID_j} , he can recover θ and furthermore recover the message.

3.2 Modified Waters IBE scheme

We first review the construction of modified Waters IBE scheme, which is named “Type-III mWIBE” in [3]. The details of the construction is shown as follows [3]:

1. Setup(λ): Let λ be the security parameter, and (p, g, G, G_t, e) be the bilinear map parameters. Randomly choose $\alpha \in Z_p^*$, $g_2, u'_1, u'_2, u_{1,1}, \dots, u_{1,n}, u_{2,0}, u_{2,1}, \dots, u_{2,n} \in G$. Let $PRF : G_T \times G \rightarrow \{0, 1\}^l$ be a pseudorandom function, and $(\mathcal{G}, \mathcal{S}, \mathcal{V})$ be a strongly unforgeable one-time signature. Set $g_1 = g^\alpha$, and define two functions

$$F_1(v) = u'_1 \prod_{i \in \mathcal{V}} u_{1,i}, \quad F_2(\omega) = u'_2 u_{2,0} \prod_{i \in \mathcal{W}} u_{2,i},$$

where v, ω are two n -bit strings and $\mathcal{V}, \mathcal{W}$ are the set of all i for which the i -th bit of v and ω is one, respectively. The public key is $PP = (\lambda, g, g_1, g_2, F_1(\cdot), F_2(\cdot), PRF, (\mathcal{G}, \mathcal{S}, \mathcal{V}))$ and the master secret key is $msk = g_2^\alpha$.

2. KeyGen(msk, ID): Let ID be an n -bit string, then the secret key for ID is computed as

$$sk_{ID} = (sk_{ID1}, sk_{ID2}) = (g_2^\alpha F_1(ID)^r, g^r),$$

where r is randomly chosen in Z_p^* .

3. Enc(ID, m): Perform $\mathcal{G}(\lambda)$ to get a signature pair (ssk, svk) . For a message $m \in \{0, 1\}^{l_1}$, generate the ciphertext

$$\begin{aligned} C_1 &= [PRF(\sigma, C_3)]^{l-l_1} || ([PRF(\sigma, C_3)]_{l_1} \oplus m), \\ C_2 &= \sigma \cdot e(g_1, g_2)^t, \\ C_3 &= g^t, \\ C_4 &= F_1(ID)^t, \\ C_5 &= F_2(svk)^t, \\ S &= \mathcal{S}(ssk, (C_1, C_3, C_4, C_5)), \end{aligned}$$

where $t \xleftarrow{R} Z_p^*$, and $\sigma \xleftarrow{R} G_T$. Return the ciphertext $C = (svk, C_1, C_2, C_3, C_4, C_5, S)$.

4. Dec(sk_{ID}, C): Verify $\mathcal{V}(svk, S, (C_1, C_3, C_4, C_5)) \stackrel{?}{=} 1$, $e(g, C_4) \stackrel{?}{=} e(C_3, F_1(ID))$ and $e(g, C_5) \stackrel{?}{=} e(C_3, F_2(svk))$. If the equations do not hold, output $\perp$. Otherwise compute $\sigma = \frac{C_2 \cdot e(sk_{ID2}, C_4)}{e(sk_{ID1}, C_3)}$. Return $m = [PRF(\sigma, C_3)]_{l_1} \oplus [C_1]_{l_1}$, if $[C_1]^{l-l_1} = [PRF(\sigma, C_3)]^{l_1}$. Otherwise output $\perp$.

We refer to the above modified Waters IBE as mWIBE. As stated in [3], the above mWIBE is secure against CCA. Thus we have the following theorem.

Theorem 1. The mWIBE scheme is CCA secure under the DBDH assumption if PRF is a pseudorandom function and $(\mathcal{G}, \mathcal{S}, \mathcal{V})$ is a strongly unforgeable one-time signature.

3.3 Our construction

We first define the Lagrange coefficient $\Delta_{\omega,S}(x)$ for $\omega \in Z_p$ and a set S , of elements in Z_p :

$$\Delta_{\omega,S}(x) = \prod_{i \in S, i \neq \omega} \frac{x - i}{\omega - i}.$$

Our proposed IB-CPRE-FG consists of the following algorithms:

1. **Setup**(λ): Let λ be the security parameter, and (p, g, G, G_T, e) be the bilinear map parameters. Randomly choose $\alpha, \varepsilon \in Z_p^*, g_2, Z, u'_1, u'_2, u'_3, u_{3,0}, u_{1,1}, \dots, u_{1,n}, u_{2,1}, \dots, u_{2,n}, u_{3,1}, \dots, u_{3,n} \in G$. Let $PRF : G_T \times G \rightarrow \{0, 1\}^l$ be a pseudorandom function, $H_1 : \{0, 1\}^{k_1} \rightarrow Z_p^*$, $H_2 : \{0, 1\}^{k_2} \rightarrow G$ be collision-resistant hash functions. Denote $l_1 = k_1 + k_2$. Let $(\mathcal{G}, \mathcal{S}, \mathcal{V})$ be a strongly unforgeable one-time signature. Set $g_1 = g^\alpha$, and define three functions

$$F_1(v) = u'_1 \prod_{i \in \mathcal{V}} u_{1,i}, \quad F_2(\omega) = u'_2 \prod_{i \in \mathcal{W}} u_{2,i}, \quad F_3(x) = u'_3 u_{3,0} \prod_{i \in \mathcal{X}} u_{3,i},$$

where v, ω, x are three n -bit strings and $\mathcal{V}, \mathcal{W}, \mathcal{X}$ are the set of all i for which the i -th bit of v, ω and x is one, respectively. The public key is

$$PP = (\lambda, l, k_1, k_2, g, g_1, g_2, g^\varepsilon, Z, F_1(\cdot), F_2(\cdot), F_3(\cdot), PRF, H, (\mathcal{G}, \mathcal{S}, \mathcal{V}))$$

and the master secret key is $msk = g_2^\alpha$.

2. **KeyGen**(msk, ID): Let ID be an n -bit string, then the secret key for ID is computed as

$$sk_{ID} = (sk_{ID1}, sk_{ID2}) = (g_2^\alpha F_1(ID)^r, g^r),$$

where r is randomly chosen in Z_p^* .

3. **Enc**(ID, m, W): To encrypt a message $m \in M$, under identity ID and a keyword set W at the second level, the sender proceeds as follows:

- (a) Select a one-time signature key pair as $(ssk, svk) \leftarrow \mathcal{G}(\lambda)$.
- (b) Set $C_0 = W$.
- (c) Select a random value $t \xleftarrow{R} Z_p^*$ and compute

$$\begin{aligned} C_1 &= [PRF(\sigma, C_3)]^{l-l_1} \parallel ([PRF(\sigma, C_3)]_{l_1} \oplus m), & C_2 &= \sigma \cdot e(g_1, g_2)^t, \\ C_3 &= g^t, & C_4 &= F_1(ID)^t, & C_5 &= (C_\omega = F_2(\omega)^t)_{\omega \in W}, \\ C_6 &= F_3(svk)^t, & C_7 &= (g^\varepsilon)^t. \end{aligned}$$

- (d) Run the signing algorithm $\mathcal{S}(ssk, (C_1, C_3, C_4, C_5, C_6))$ and denote the signature as S .
- (e) Output the ciphertext $CT_{ID}^{(2)} = (svk, C_0, C_1, C_2, C_3, C_4, C_5, C_6, C_7, S)$.

4. $\text{RKeyGen}(sk_{ID_i}, ID_i, \mathcal{T}, ID_j)$: On input two identities ID_i, ID_j , an access tree $\mathcal{T}$, and user ID_i 's private key sk_{ID_i} . The ReKeyGen algorithms outputs a re-encryption key $rk_{\mathcal{T}|ID_i \rightarrow ID_j}$ which enables the proxy to re-encrypted a ciphertext under a condition set W , if and only if $\mathcal{T}(W) = 1$. The algorithms proceeds as follows. First chooses random values $\theta_1 \in \{0, 1\}^{k_1}, \theta_2 \in \{0, 1\}^{k_2}$, also chooses a polynomial q_x for each non-leaf node x in the tree $\mathcal{T}$. These polynomials are chosen in the following way in a top down manner, starting from the root node r . It calls the procedure $RKG(\mathcal{T}, H_1(\theta_1))$, where procedure $RKG(\mathcal{T}, H_1(\theta_1))$ is defined as follows:

For each node x in the tree, set the degree d_x of the polynomial q_x to be one less than the threshold value k_x of the note, that is, $d_x = k_x - 1$. Now for the root r , set $q_r(0) = H_1(\theta_1)$ and d_r other points of the polynomial q_r randomly to define it completely. For any other node x , set $q_x(0) = q_{p(x)}(\text{index}(x))$ and choose d_x other points randomly to completely define q_x , once the polynomials have been decided, for each leaf node x , set $\omega = \text{keyword}(x)$. Thus the re-encryption key given to the proxy are calculated as follows:

- (a) Select a random value $s_x \xleftarrow{R} Z_p^*$, computes

$$a_x = Z^{q_x(0)} F_2(\omega)^{s_x} \quad b_x = g^{s_x}$$

Let $a = \{a_x : x \in LN_{\mathcal{T}}\}$ and $b = \{b_x : x \in LN_{\mathcal{T}}\}$.

- (b) Select a one-time signature key pair as $(ssk', svk') \leftarrow \mathcal{G}(\lambda)$.
(c) Select random value $\rho, t' \xleftarrow{R} Z_p^*$ and $\sigma' \xleftarrow{R} G_T$, compute

$$\begin{aligned} rk_0 &= sk_{ID_{i,1}} \cdot (g^\varepsilon)^\rho, & rk_1 &= g^\rho, & rk_2 &= sk_{ID_{i,2}} \cdot H_2(\theta_2), \\ rk_3 &= [PRF(\sigma', rk_5)]^{l-l_1} \parallel ([PRF(\sigma', rk_5)]_{l_1} \oplus (\theta_1 \parallel \theta_2)), & rk_4 &= \sigma' \cdot e(g_1, g_2)^{t'}, \\ rk_5 &= g^{t'}, & rk_6 &= F_1(ID_j)^{t'}, & rk_7 &= F_3(svk')^{t'}. \end{aligned}$$

- (d) Run the signing algorithm $\mathcal{S}(ssk', (rk_3, rk_5, rk_6, rk_7))$ and denote the signature S' .
(e) Return $rk_{\mathcal{T}|ID_i \rightarrow ID_j} = (a, b, \mathcal{T}, svk', rk_0, rk_1, rk_2, rk_3, rk_4, rk_5, rk_6, rk_7, S')$.

5. $\text{ReEnc}(C_{ID_i}^{(2)}, rk_{\mathcal{T}|ID_i \rightarrow ID_j})$: On input a re-encryption key $rk_{\mathcal{T}|ID_i \rightarrow ID_j} = (a, b, \mathcal{T}, svk', rk_0, rk_1, rk_2, rk_3, rk_4, rk_5, rk_6, rk_7, S')$ and a second level ciphertext $C_{ID_i}^{(2)} = (svk, C_0, C_1, C_2, C_3, C_4, C_5, C_6, S)$ under the key word set W . Recover $W = C_0$, for all $\omega \in W$ it check whether the following equalities hold:

$$\mathcal{V}(svk, S, (C_1, C_3, C_4, C_5, C_6)) \stackrel{?}{=} 1, \quad (1)$$

$$e(g, C_4) \stackrel{?}{=} e(C_3, F_1(ID_i)), \quad (2)$$

$$e(g, C_6) \stackrel{?}{=} e(C_3, F_3(svk)), \quad (3)$$

$$e(g, C_\omega)_{\omega \in W} \stackrel{?}{=} e(C_3, F_2(\omega))_{\omega \in W}, \quad (4)$$

$$e(g, C_7) \stackrel{?}{=} e(C_3, g^\varepsilon). \quad (5)$$

If not, output $\perp$. Otherwise, we define a recursive algorithms $\text{NodeReEnc}(C_{ID_i}^{(2)}, rk_{\mathcal{T}|ID_i \rightarrow ID_j}, x)$ that takes as input the ciphertext $C_{ID_i}^{(2)}$, the re-encryption key $rk_{\mathcal{T}|ID_i \rightarrow ID_j}$, and a note x in the tree.

- (a) For leaf node x , let $\omega = \text{keyword}(x)$, if $\omega \in W$, then

$$\begin{aligned}
& \text{NodeReEnc}(C_{ID_i}^{(2)}, rk_{\mathcal{T}|ID_i \rightarrow ID_j}, x) \\
&= \frac{e(C_3, a_x)}{e(b_x, C_\omega)} \\
&= \frac{e(g^t, Z^{q_x(0)} F_2(\omega)^{s_x})}{e(g^{s_x}, F_2(\omega)^t)} \\
&= e(g, Z)^{tq_x(0)}
\end{aligned}$$

Otherwise, outputs $\text{NodeReEnc}(C_{ID_i}^{(2)}, rk_{\mathcal{T}|ID_i \rightarrow ID_j}, x) = \perp$.

- (b) If x is a non-leaf node, consider the recursive case, for all nodes z that are children of x , it calls $\text{NodeReEnc}(C_{ID_i}^{(2)}, rk_{\mathcal{T}|ID_i \rightarrow ID_j}, z)$ and stores the output as F_z . Let S_x be an arbitrary k_x -sized set of child nodes z , such that $F_z \neq \perp$. If no such set exists, then the node was not satisfied and the function $\text{NodeReEnc}(C_{ID_i}^{(2)}, rk_{\mathcal{T}|ID_i \rightarrow ID_j}, x)$ returns $\perp$. Otherwise, let $S'_x = \{\text{index}(z) : z \in S_x\}$, and computes

$$\begin{aligned}
F_x &= \prod_{z \in S_x, i = \text{index}(z)} (F_z)^{\Delta_{i, S'_x}(0)} \\
&= \prod_{z \in S_x, i = \text{index}(z)} (e(g, Z)^{tq_z(0)})^{\Delta_{i, S'_x}(0)} \\
&= \prod_{z \in S_x, i = \text{index}(z)} (e(g, Z)^{tq_{p(z)}(\text{index}(z))})^{\Delta_{i, S'_x}(0)} \\
&= \prod_{z \in S_x, i = \text{index}(z)} (e(g, Z)^{tq_x(i)})^{\Delta_{i, S'_x}(0)} \\
&= e(g, Z)^{tq_x(0)}.
\end{aligned}$$

and return the result. Then we can compute

$$\begin{aligned}
F_r &= \text{NodeReEnc}(C_{ID_i}^{(2)}, rk_{\mathcal{T}|ID_i \rightarrow ID_j}, r) \\
&= e(g, Z)^{tq_r(0)} \\
&= e(g, Z)^{tH_1(\theta_1)}.
\end{aligned}$$

- (c) Compute $C'_2 = \frac{C_2 \cdot F_r \cdot e(rk_2, C_4)}{e(rk_0, C_3) / e(rk_1, C_7)}$.

Finally, output the re-encrypted ciphertext

$$CT_{ID_j}^{(1)} = (svk, C_1, C'_2, C_3, C_4, C_5, C_6, S, svk', rk_3, rk_4, rk_5, rk_6, rk_7, S').$$

6. Dec2($ID_i, sk_{ID_i}, CT_{ID_i}^{(2)}$):

- (a) First check the validity of the ciphertexts as in equations (1)–(5). If the verification fails, output $\perp$ and abort.
- (b) Compute $\sigma = C_2 \cdot \frac{e(sk_{ID_{i,2}}, C_4)}{e(sk_{ID_{i,1}}, C_3)}$. Output $m = \text{PRF}[\sigma, C_3]_{l_1} \oplus [C_1]_{l_1}$ if $\text{PRF}[\sigma, C_3]^{l-l_1} = [C_1]^{l-l_1}$, else output $\perp$ and abort.

7. $\text{Dec1}(ID_j, sk_{ID_j}, CT_{ID_j}^{(1)})$:

(a) Verify

$$\mathcal{V}(svk', S', (rk_3, rk_5, rk_6, rk_7)) \stackrel{?}{=} 1, \quad (6)$$

$$e(g, rk_6) \stackrel{?}{=} e(rk_5, F_1(ID_j)), \quad (7)$$

$$e(g, rk_7) \stackrel{?}{=} e(rk_5, F_3(svk')). \quad (8)$$

If the equations (6)-(8) do not hold, output $\perp$ and abort. Otherwise, proceed.

- (b) Compute $\sigma' = rk_4 \cdot \frac{e(sk_{ID_{j,2}}, rk_6)}{e(sk_{ID_{j,1}}, rk_5)}$. Output $\theta_1 || \theta_2 = PRF[\sigma', rk_5]_{l_1} \oplus [rk_3]_{l_1}$ if $PRF[\sigma', rk_5]^{l-l_1} = [rk_3]^{l-l_1}$, else output $\perp$ and abort.
- (c) Verify equations (1)-(4). If (1)-(4) do not hold output $\perp$ and abort. Otherwise, proceed.
- (d) Compute $\sigma = C'_2 / (e(C_3, Z)^{H_1(\theta_1)} \cdot e(C_4, H_2(\theta_2)))$. Output $m = PRF[\sigma, C_3]_{l_1} \oplus [C_1]_{l_1}$ if $PRF[\sigma, C_3]^{l-l_1} = [C_1]^{l-l_1}$, else output $\perp$ and abort.

Correctness. We now explain the correctness of our scheme:

1. For an original ciphertext $CT_{ID}^{(2)} = (svk, C_0, C_1, C_2, C_3, C_4, C_5, C_6, S)$, we have

$$\begin{aligned} C_2 \cdot \frac{e(sk_{ID_{i,2}}, C_4)}{e(sk_{ID_{i,1}}, C_3)} &= \sigma \cdot e(g_1, g_2)^t \cdot \frac{e(g^r, F_1(ID_i)^t)}{e(g_2^\alpha \cdot F_1(ID_i)^r, g^t)} \\ &= \sigma \cdot e(g_1, g_2)^t \cdot \frac{1}{e(g_2^\alpha, g^t)} \\ &= \sigma. \end{aligned}$$

2. For a re-encrypted ciphertext $CT_{ID_j}^{(1)} = (svk, C_1, C'_2, C_3, C_4, C_6, S, svk', rk_3, rk_4, rk_5, rk_6, rk_7, S')$, we have

$$\begin{aligned} C'_2 &= \frac{C_2 \cdot Fr \cdot e(rk_2, C_4)}{e(rk_0, C_3) / e(rk_1, C_4)} \\ &= \frac{\sigma \cdot e(g_1, g_2)^t \cdot e(g, Z)^{tH_1(\theta_1)} \cdot e(g^r \cdot H_2(\theta_2), F_1(ID_i)^t)}{e(g_2^\alpha F_1(ID_i)^r \cdot (g^\varepsilon)^\rho, g^t) / e(g^\rho, (g^\varepsilon)^t)} \\ &= \sigma \cdot e(C_4, H_2(\theta_2)) \cdot e(g, Z)^{tH_1(\theta_1)}. \end{aligned}$$

Then we have $\sigma = C'_2 / (e(C_3, Z)^{H_1(\theta_1)} \cdot e(C_4, H_2(\theta_2)))$, where θ_1, θ_2 can be correctly computed using the same way as above,

3.4 Security of our key-private IBPRE scheme

Theorem 2. Our IB-CPRE-FG scheme is IND-level2-CCA secure assuming the DBDH assumption holds, PRF is a pseudorandom function, H_1, H_2 are collision-resistant hash functions and $(\mathcal{G}, \mathcal{S}, \mathcal{V})$ is a strongly unforgeable one-time signature.

The proof of Theorem 2 is provided in Appendix A.

Theorem 3. Our IB-CPRE-FG scheme is IND-level1-CCA secure assuming the DBDH assumption holds, PRF is a pseudorandom function, H_1, H_2 are collision-resistant hash functions and $(\mathcal{G}, \mathcal{S}, \mathcal{V})$ is a strongly unforgeable one-time signature.

The proof of Theorem 3 is provided in Appendix B.

4 Comparison and discussion

4.1 Comparison

In this section, we compare our scheme with previous IB-CPRE schemes in terms of private/re-encryption key size, level2/level1 ciphertext size, re-encryption cost, properties and security model in Table 1. Let g, g_t and z be the size of an element in G, G_T and Z_p , respectively. Let m be the size of a message M with m bit in $\{0, 1\}^*$, S be the size of a strongly unforgeable one-time signature, t be the size of an access tree and w be the size of attributes used in the encryption algorithm. Besides, c_e, c_p and v denote the computational cost of an exponentiation, a bilinear pairing and the verification of one-time signature.

Schemes	Private/Re-encryption key size	level2/level1 ciphertext size	Re-encryption cost	without RO? /fine grain policy?
IB-CPRE [2]	$g/3g + m$	$3g + g_t + m$ $/3g + g_t + m$	$2c_p$	$\times/\times$
IB-CPRE [3]	$2g/6g + g_t + S$	$4g + g_t + m + S$ $/7g + 2g_t + m + 2S$	$9c_p + v$	$\sqrt{}/\times$
Our IB-CPRE-FG	$2g$ $/6g + g + t + S + m + t$	$(w + 4)g + g_t + m + w + S$ $/((w + 6)g + 2g_t + 2m + 2S)$	$(9 + 2w + t)c_p + v$	$\sqrt{}/\sqrt{}$

Table 1. Comparison with Shao et al.'s scheme [2] and Liang et al.'s scheme [3]

4.2 Master secret security

Master secret security means that even if a proxy colludes with a set of delegates, it is still impossible for them to expose the delegator's private key [10]. As discussed in [12], the notion of CCA security of level 1 ciphertext implies the master secret security. In our proposed scheme, a collusion of proxy and delegatee can only extract the second part $sk_{ID_i,2}$ of the private key of the delegator. Additionally, the first part $sk_{ID_i,1}$ remains secret to the proxy and delegatee.

4.3 Non-transitivity and non-transferability issue

Non-transitivity means that the proxy alone cannot re-delegate the decryption rights. For example, from $rk_{ID_i \rightarrow ID_j}$ and $rk_{ID_j \rightarrow ID_k}$, he cannot produce $rk_{ID_i \rightarrow ID_k}$. Non-transferability means the proxy and a set of colluding delegates cannot re-delegate decryption rights. For example, from $rk_{ID_i \rightarrow ID_j}$, sk_{ID_j} and ID_j , they cannot produce $rk_{ID_i \rightarrow ID_j}$. Note that, in our scheme, user ID_j can produce the re-encryption key $sk_{ID_j \rightarrow ID_k}$, so non-transitivity is implied by non-transferability. Our scheme is transferable, as the collusion of the proxy with $rk_{ID_i \rightarrow j}$

and a delegatee with sk_{ID_j} can produce a new re-encryption key $rk_{ID_i \rightarrow ID_k}$. As discussed in [10], transferability is “mild-harmful”, as ID_j can always decrypt and forward the plaintext to ID_j . However, this requires that ID_j should be online. Achieving an identity-based conditional proxy re-encryption with fine-grain policy scheme that is non-transferable seems to be an interesting future work.

5 Conclusions

In this work, we give an affirmative answer to an existing open problem by presenting a new IB-CPRE-FG scheme. We formulated the security model of IB-CPRE-FG and proved its IND-CCA security. Many interesting questions still remain to be solved though. In our scheme, the access policy is described by an access structure. First, it is interesting to construct an IB-CPRE scheme supporting AND or OR gates directly. Second, as many proxy re-encryption schemes [36, 37] have been proposed to capture the key-private property, hence, it is also an interesting open problem to construct a key-private IB-CPRE scheme.

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A Proof of Theorem 2

Proof. Suppose there is an adversary $\mathcal{A}$ who can break the IND-level2-CCA security of our IB-CPRE-FG scheme with non-negligible probability ϵ . We can construct a simulator $\mathcal{B}$ to break the mWIBE scheme with non-negligible probability ϵ' using $\mathcal{A}$. Denote $\mathcal{B}_1$ as the challenge of the mWIBE in the CCA simulation. $\mathcal{B}$ first maintains the following tables which are initially empty.

- Key^{List} : records the tuples (β, ID, SK_{ID}) , which are the information of secret keys;
- $ReKey^{List}$: records the tuples $(\gamma, ID_i, \mathcal{T}, ID_j, rk_{\mathcal{T}|ID_i \rightarrow ID_j}, \theta_1, \theta_2, flag_1)$, which are the result of the queries to $RKExtract(ID_i, \mathcal{T}, ID_j)$, where $flag_1 = 1$ denotes the re-encryption key is a valid one, and $flag_1 = 0$ denotes the re-encryption key is a random value.
- $ReEnc^{List}$: records the tuples $(ID_i, ID_j, (W, C_{(ID_j)}^{(2)}), C_{(ID_j)}^{(1)}, flag_2)$, which are the result of the queries to $ReEnc(ID_i, ID_j, (W, C_{ID_i}^{(2)}))$, where $flag_2 = 1$ denotes the re-encrypted ciphertext is generated under a valid re-encryption key, and $flag_2 = 0$ denotes the re-encrypted ciphertext is generated under a random re-encryption key.

1. **Setup:** $\mathcal{B}$ first calls $\mathcal{B}_1$ to get the public parameter $PP = (\lambda, l, k_1, k_2, g, g_1, g_2, F_1(\cdot), F_2(\cdot), F_3(\cdot), PRF, (\mathcal{G}, \mathcal{S}, \mathcal{V}))$, $\mathcal{B}$ also chooses random values $\varepsilon \in Z_p^*$, $Z \in G$ and collision-resistant hash functions $H_1 : \{0, 1\}^{k_1} \rightarrow Z_p^*$, $H_2 : \{0, 1\}^{k_2} \rightarrow G$. Adds $g^\varepsilon, Z, H_1, H_2$ to PP and forwards PP to $\mathcal{A}$.

2. **Query phase 1:** $\mathcal{A}$ issues a series of queries to which $\mathcal{B}$ responds as follows:

(a) *Extract*(ID): $\mathcal{B}$ first searches Key^{List} , if $(1, ID, sk_{ID})$ exists in Key^{List} , returns sk_{ID} as the result. Otherwise, $\mathcal{B}$ generates a biased coin β so that $Pr[\beta = 1] = \delta$ for some δ that will be determined later.

- If $\beta = 0$, $\mathcal{B}$ outputs a random bit and aborts.
- If $\beta = 1$, $\mathcal{B}$ forwards the query to $\mathcal{B}_1$ and obtains the secret key sk_{ID} . Returns sk_{ID} to $\mathcal{A}$ and adds $(1, ID, sk_{ID})$ to Key^{List} .

(b) *RKExtract*($ID_i, \mathcal{T}, ID_j$): $\mathcal{B}$ first searches whether there is a tuple $(*, ID_i, \mathcal{T}, ID_j, rk_{\mathcal{T}|ID_i \rightarrow ID_j}, \theta_1, \theta_2, *)$ in $ReKey^{List}$. If yes, $\mathcal{B}$ returns $rk_{\mathcal{T}|ID_i \rightarrow ID_j}$ as the result, where $*$ is the wildcard. Otherwise, $\mathcal{B}$ proceeds as follows:

- If $(1, ID_i, sk_{ID_i})$ exists in Key^{List} , $\mathcal{B}$ uses sk_{ID_i} to generate the re-encryption key $rk_{\mathcal{T}|ID_i \rightarrow ID_j}$ via algorithm *RKeyGen* as in the real scheme. Returns the re-encryption key to $\mathcal{A}$ and adds $(*, ID_i, ID_j, rk_{\mathcal{T}|ID_i \rightarrow ID_j}, \theta_1, \theta_2, 1)$ to $ReKey^{List}$, where $\theta_1 \in \{0, 1\}^{k_1}$, $\theta_2 \in \{0, 1\}^{k_2}$ are randomly chosen in the *RKeyGen* algorithm.
- Otherwise, $\mathcal{B}$ flips a biased coin β . If $\beta = 1$, $\mathcal{B}$ queries the *Extract*(ID_i) oracle to get sk_{ID_i} , and then generates $rk_{\mathcal{T}|ID_i \rightarrow ID_j}$ via algorithm *RKeyGen* as in the real scheme. Returns the re-encryption key to $\mathcal{A}$ and adds $(1, ID_i, sk_{ID_i})$ and $(*, ID_i, ID_j, rk_{\mathcal{T}|ID_i \rightarrow ID_j}, \theta_1, \theta_2, 1)$ to Key^{List} and $ReKey^{List}$ respectively, where $\theta_1 \in \{0, 1\}^{k_1}$, $\theta_2 \in \{0, 1\}^{k_2}$ are randomly chosen in the *RKeyGen* algorithm. If $\beta = 0$, $\mathcal{B}$ randomly chooses $\theta_1 \in \{0, 1\}^{k_1}$, $\theta_2 \in \{0, 1\}^{k_2}$, $s_x \in Z_p^*$ and computer a, b as in the real scheme. Then $\mathcal{B}$ sets $rk_1 = \phi_1, rk_2 = \phi_2, rk_3 = \phi_3$ for randomly chosen $\phi_1, \phi_2, \phi_3 \in G$. Then $\mathcal{B}$ constructs $rk_3, rk_4, rk_5, rk_6, rk_7, sk', S'$ to encrypted a random $(\theta_1 || \theta_2) \in \{0, 1\}^{l_1}$ as in the real scheme. $\mathcal{B}$ forwards the re-encryption key to $\mathcal{A}$ and adds $(*, ID_i, \mathcal{T}, ID_j, rk_{\mathcal{T}|ID_i \rightarrow ID_j}, \theta_1, \theta_2, 0)$ to $ReKey^{List}$.

(c) *ReEnc*($ID_i, ID_j, (W, C_{ID_i}^{(2)})$): $\mathcal{B}$ first searches whether there is a tuple $(ID_i, ID_j, (W, C_{ID_j}^{(2)}), C_{ID_j}^{(1)}, *)$ in $ReEnc^{List}$. If yes, $\mathcal{B}$ returns $C_{ID_j}^{(1)}$ as the result, where $*$ is the wildcard. Otherwise, $\mathcal{B}$ proceeds as follows:

- If $(*, ID_i, \mathcal{T}, ID_j, rk_{\mathcal{T}|ID_i \rightarrow ID_j}, \theta_1, \theta_2, *)$ exists in $ReKey^{List}$, where $\mathcal{T}(W) = 1$. $\mathcal{B}$ uses $rk_{\mathcal{T}|ID_i \rightarrow ID_j}$ to generate $C_{ID_j}^{(1)}$ via algorithm *ReEnc* as in the real scheme. Returns $C_{ID_j}^{(1)}$ to $\mathcal{A}$ and adds $(ID_i, ID_j, (W, C_{ID_j}^{(2)}), C_{ID_j}^{(1)}, *)$ to $ReEnc^{List}$.

- Otherwise, $\mathcal{B}$ first issues *RKExtract*($ID_i, \mathcal{T}, ID_j$), where $\mathcal{T}(W) = 1$ to obtain re-encryption key. Next, generates $C_{ID_j}^{(1)}$ and adds $(ID_i, ID_j, (W, C_{ID_j}^{(2)}), C_{ID_j}^{(1)}, *)$ to the $ReEnc^{List}$.

(d) *Dec2*($ID_i, (W, C_{ID_i}^{(2)})$): $\mathcal{B}$ first verifies equations (1)-(5). If the equations do not hold, output $\perp$ and abort. Otherwise, $\mathcal{B}$ proceeds:

- If $(1, ID_i, sk_{ID_i})$ exists in Key^{List} , using sk_{ID_i} to recover m .

– Otherwise, $\mathcal{B}$ forwards the query to the decryption oracle of mWIBE to obtain m .

(e) $\text{Dec1}(ID_j, C_{ID_j}^{(1)})$: $\mathcal{B}$ first verifies equations (1)-(4) and (6)-(8). If the equations do not hold, output $\perp$ and abort. Otherwise, $\mathcal{B}$ proceeds:

- If $(ID_i, ID_j, (W, C_{(ID_j)}^{(2)}), C_{(ID_j)}^{(1)}, 1)$ exists in ReEnc^{List} , return $C_{(ID_j)}^{(2)}$ as the result.
- Else, $\mathcal{B}$ searches whether (i, ID_j, sk_{ID_j}) exists in Key^{List} , if yes, $\mathcal{B}$ recovers m using sk_{ID_j} . Otherwise, $\mathcal{B}$ issues $(svk', rk_3, rk_4, rk_5, rk_6, rk_7, S')$ to the decryption oracle of mWIBE and get θ_1, θ_2 . Then using θ_1, θ_2 , $\mathcal{B}$ can recover m as in the real scheme.

3. **Challenge:** Once $\mathcal{A}$ decides that Phase 1 is over, it outputs two equal length message (m_0, m_1) , a target identity ID^* and a target condition set W^* . $\mathcal{B}$ first checks whether the following two conditions hold:

- $(1, ID^*, sk_{ID^*})$ does not exist in Key^{List} ;
- $(*, ID^*, \mathcal{T}, ID_j, rk_{\mathcal{T}|ID^* \rightarrow ID_j}, \theta_1, \theta_2, *)$, where $\mathcal{T}(W^*) = 1$ and $(1, ID_j, sk_{ID_j})$ do not exist in ReKey^{List} and Key^{List} .

If the above two conditions do not hold, $\mathcal{B}$ outputs a random bit and aborts. Else, $\mathcal{B}$ forwards (m_0, m_1, ID^*, W^*) to the challenge oracle of mWIBE, and obtains the challenge ciphertext $C_{ID^*}^{(2)*}$.

4. **Query phase 2:** $\mathcal{A}$ continues making queries as in the query phase 1 with the restrictions described in the IND-level2-CCA game.

5. **Guess:** When $\mathcal{A}$ outputs a guess $b' \in \{0, 1\}$, $\mathcal{B}$ outputs b' .

Probability analysis. If $\mathcal{B}$ does not abort, $\mathcal{A}$'s view is identical to the real scheme. We define *Abort* be the event of $\mathcal{B}$'s aborting during the simulation of *Extract* query. Let q_e denote the total number of *Extract* queries, we have $\Pr[\neg \text{Abort}] \geq \delta^{q_e}(1 - \delta)$, which is maximized at $\delta_{opt} = \frac{q_e}{(1+q_e)}$. Using δ_{opt} , the probability $\Pr[\neg \text{Abort}]$ is at least $\frac{1}{e(1+q_e)}$, where e is the base of the nature logarithm. Therefor, we have

$$\varepsilon' \geq \frac{\varepsilon}{e(1+q_e)} - \text{Adv}_{H_1, \mathcal{A}}^{TCR} - \text{Adv}_{H_2, \mathcal{A}}^{TCR} - \text{Adv}_{F, \mathcal{F}}^{PRF}.$$

This completes the proof of Theorem 2.

B Proof of Theorem 3

Proof. Suppose there is an adversary $\mathcal{A}$ who can break the IND-level1-CCA security of our IB-CPRE-FG scheme with non-negligible probability ε . We can construct a simulator $\mathcal{B}$ to break the mWIBE scheme with non-negligible probability ε' using $\mathcal{A}$. Denote $\mathcal{B}_1$ as the challenge of the mWIBE in the CCA simulation.

1. **Setup:** $\mathcal{B}$ first calls $\mathcal{B}_1$ to get the public parameter $PP = (\lambda, l, k_1, k_2, g, g_1, g_2, F_1(\cdot), F_2(\cdot), F_3(\cdot), \text{PRF}, (\mathcal{G}, \mathcal{S}, \mathcal{V}))$, $\mathcal{B}$ also chooses random values $\varepsilon \in Z_p^*$, $Z \in G$ and collision-resistant hash functions $H_1 : \{0, 1\}^{k_1} \rightarrow Z_p^*$, $H_2 : \{0, 1\}^{k_2} \rightarrow G$. Adds $g^\varepsilon, Z, H_1, H_2$ to PP and forwards PP to $\mathcal{A}$.

2. **Query phase 1:** $\mathcal{A}$ issues a series of queries to which $\mathcal{B}$ responds as follows:
- (a) $Extract(ID)$: $\mathcal{B}$ first searches Key^{List} , if $(1, ID, sk_{ID})$ exists in Key^{List} , returns sk_{ID} as the result. Otherwise, $\mathcal{B}$ generates a biased coin β so that $Pr[\beta = 1] = \delta$ for some δ that will be determined later.
 - If $\beta = 0$, $\mathcal{B}$ outputs a random bit and aborts.
 - If $\beta = 1$, $\mathcal{B}$ forwards the query to $\mathcal{B}_1$ and obtains the secret key sk_{ID} . Returns sk_{ID} to $\mathcal{A}$ and adds $(1, ID, sk_{ID})$ to Key^{List} .

- (b) $RKExtract(ID_i, \mathcal{T}, ID_j)$: $\mathcal{B}$ first searches whether there is a tuple $(*, ID_i, \mathcal{T}, ID_j, rk_{\mathcal{T}|ID_i \rightarrow ID_j}, \theta_1, \theta_2, *)$ in $ReKey^{List}$. If yes, $\mathcal{B}$ returns $rk_{\mathcal{T}|ID_i \rightarrow ID_j}$ as the result, where $*$ is the wildcard. Otherwise, $\mathcal{B}$ proceeds as follows:
 - If $(1, ID_i, sk_{ID_i})$ exists in Key^{List} , $\mathcal{B}$ uses sk_{ID_i} to generate the re-encryption key $rk_{\mathcal{T}|ID_i \rightarrow ID_j}$ via algorithm $RKeyGen$ as in the real scheme. Returns the re-encryption key to $\mathcal{A}$ and adds $(*, ID_i, ID_j, rk_{\mathcal{T}|ID_i \rightarrow ID_j}, \theta_1, \theta_2, 1)$ to $ReKey^{List}$, where $\theta_1 \in \{0, 1\}^{k_1}$, $\theta_2 \in \{0, 1\}^{k_2}$ are randomly chosen in the $RKeyGen$ algorithm.
 - Otherwise, $\mathcal{B}$ flips a biased coin β . If $\beta = 1$, $\mathcal{B}$ queries the $Extract(ID_i)$ oracle to get sk_{ID_i} , and then generates $rk_{\mathcal{T}|ID_i \rightarrow ID_j}$ via algorithm $RKeyGen$ as in the real scheme. Returns the re-encryption key to $\mathcal{A}$ and adds $(1, ID_i, sk_{ID_i})$ and $(*, ID_i, ID_j, rk_{\mathcal{T}|ID_i \rightarrow ID_j}, \theta_1, \theta_2, 1)$ to Key^{List} and $ReKey^{List}$ respectively, where $\theta_1 \in \{0, 1\}^{k_1}$, $\theta_2 \in \{0, 1\}^{k_2}$ are randomly chosen in the $RKeyGen$ algorithm. If $\beta = 0$, $\mathcal{B}$ randomly chooses $\theta_1 \in \{0, 1\}^{k_1}$, $\theta_2 \in \{0, 1\}^{k_2}$, $s_x \in Z_p^*$ and computer a, b as in the real scheme. Then $\mathcal{B}$ sets $rk_1 = \phi_1, rk_2 = \phi_2, rk_3 = \phi_3$ for randomly chosen $\phi_1, \phi_2, \phi_3 \in G$. Then $\mathcal{B}$ constructs $rk_3, rk_4, rk_5, rk_6, rk_7, sk'_1, S'$ to encrypted a random $(\theta_1 || \theta_2) \in \{0, 1\}^{l_1}$ as in the real scheme. $\mathcal{B}$ forwards the re-encryption key to $\mathcal{A}$ and adds $(*, ID_i, \mathcal{T}, ID_j, rk_{\mathcal{T}|ID_i \rightarrow ID_j}, \theta_1, \theta_2, 0)$ to $ReKey^{List}$.

- (c) $Dec1(ID_j, C_{ID_j}^{(1)})$: $\mathcal{B}$ first verifies equations (1)-(4) and (6)-(8). If the equations do not hold, output $\perp$ and abort. Otherwise, $\mathcal{B}$ proceeds:
 - If (i, ID_j, sk_{ID_j}) exists in Key^{List} , $\mathcal{B}$ recovers m using sk_{ID_j} .
 - Otherwise, $\mathcal{B}$ issues $(sk'_1, rk_3, rk_4, rk_5, rk_6, rk_7, S')$ to the decryption oracle of mWIBE and get θ_1, θ_2 . Then using θ_1, θ_2 , $\mathcal{B}$ can recover m as in the real scheme.

3. **Challenge:** Once $\mathcal{A}$ decides that Phase 1 is over, it outputs two equal length message (m_0, m_1) and a target identity ID^* . If $(1, ID^*, sk_{ID^*})$ exists in the Key^{List} , $\mathcal{B}$ outputs random bits and aborts. Else, $\mathcal{B}$ choose an identity $ID_i \neq ID^*$, and forwards (m_0, m_1, ID_i, W^*) to the challenge oracle of mWIBE to obtain the result $C_{ID_i}^{(2)}$. $\mathcal{B}$ then issues $RKExtract(ID_i, \mathcal{T}, ID^*)$, where $\mathcal{T}(W^*) = 1$ to get $rk_{\mathcal{T}|ID_i \rightarrow ID^*}$. $\mathcal{B}$ uses $rk_{\mathcal{T}|ID_i \rightarrow ID^*}$ to re-encrypt $C_{ID_i}^{(2)}$ as the real scheme and get the challenge ciphertext $C_{ID^*}^{(1)}$. Finally, $\mathcal{B}$ sends the challenge ciphertext $C_{ID^*}^{(1)}$ to $\mathcal{A}$.

4. **Query phase 2:** $\mathcal{A}$ continues making queries as in the query phase 1 with the restrictions described in the IND-level1-CCA game.

5. **Guess:** When $\mathcal{A}$ outputs a guess $b' \in \{0, 1\}$, $\mathcal{B}$ outputs b' .

Probability analysis. If $\mathcal{B}$ does not abort, $\mathcal{A}$'s view is identical to the real scheme. We define *Abort* be the event of $\mathcal{B}$'s aborting during the simulation of $Extract$ query. Let q_e denote the

total number of *Extract* queries, we have $Pr[\neg Abort] \geq \delta^{q_e}(1 - \delta)$, which is maximized at $\delta_{opt} = \frac{q_e}{(1+q_e)}$. Using δ_{opt} , the probability $Pr[\neg Abort]$ is at least $\frac{1}{\dot{e}(1+q_e)}$, where $\dot{e}$ is the base of the nature logarithm. Therefor, we have

$$\varepsilon' \geq \frac{\varepsilon}{\dot{e}(1+q_e)} - Adv_{H_1, \mathcal{A}}^{TCR} - Adv_{H_2, \mathcal{A}}^{TCR} - Adv_{F, \mathcal{F}}^{PRF}.$$

This completes the proof of Theorem 3.