A New Correlation to Evaluate the Fracture Permeability Changes as Reservoir is Depleted

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Abstract

The increasing development of unconventional resources plays an important role in filling the gap between demand and conventional oil and gas supply. Due to the nature of low permeability, tight gas and shale reservoirs need fractures, natural and artificial, to produce hydrocarbon at commercial rates. Fracture permeability is a key factor affecting the development of these unconventional reservoirs. It is observed that fracture permeabilities decline as reservoirs are depleted because pore pressure declines lead to the closures of fractures and the permeability reductions. Therefore a correlation to quantify the variations of the fracture permeability with pore pressure is highly needed. In this study we investigate the effects of pore pressures on fracture permeabilities assuming constant in-situ stresses exert on the formations. Starting from the force balance, we derived equations to calculate fracture permeability based on fracture geometry. Our new correlations can also be used to evaluate the changing fracture permeability during the recovery of hydrocarbon. The proposed correlations provide a way to estimate the fracture permeability at initial pressure and the depleted pressure at any production stage. Although some experiments had been conducted to build relationships between fracture permeability and pressure for some types of rocks. It is noted that experiments are time

consuming and cost expensive. Sometimes, the unavailability of shale sample and difficulty in preparing shale core plug also limit the number of experiment. Our research provided a theoretical model. We realize that theoretical results are not necessarily the ground truth because of the many implicit and explicit assumptions made to obtain an analytical formula. Therefore, extensive sensitivity study was conducted to quantify the effects of important parameters on permeability change. The proposed methods considered the laminar flow as well as turbulent flow in the fracture, which is often observed in gas well. Under the conditions of no core sample available to test the impact of pressure on fracture permeability, or special core analyses prohibited by cost and time, the proposed correlations are powerful tools to estimate the change of permeability with producing time.

Introduction

As conventional reservoirs peaked, the increasing development of unconventional resource plays an important role in filling the gap between the ever increasing demand and decreasing conventional oil and gas supply. Due to the nature of low permeability, tight gas and shale reservoirs need fractures, natural and artificial, to produce hydrocarbon at commercial rates. Fracture permeability is a key factor to obtain commercial production rate from these unconventional reservoirs. Field observations indicate that fracture permeabilities decline as reservoirs are depleted due to the closures of fractures and the permeability reductions resulting from depleting pore pressure. To quantify and model the permeability under different pressure, many studies had been conducted both within and outside of petroleum industry.

Nelson (1976) quantified fracture permeability by combining lab and field measurements. A

correlation with three empirical coefficients was proposed to estimate permeability reduction because of the increase in confining pressure. Walsh (1981) investigated the effects of pore pressure and confining pressure on fracture permeability. He presented an empirical correlation that can be applied under low to moderate pressure conditions. Bakhtar et al. (1985) modeled fracture permeability around a well during depletion by a fracture aperture-permeability correlation. The method ignored the friction on top and bottom walls of fracture. Thomas et al. (1987) determined fracture permeability subjected to high confining stress in laboratory. No permeability model was proposed. Saidi (1987) proposed an equation to evaluate fracture permeability at depleted pressure. However, the equation overestimates the permeability decline as pointed out in the following section (Eq. 28). Stearns and Linscott (1994) studied the influence of in-situ stress conditions on fold-related fracture permeability using field data. Darcy equation was used to calculate the permeability in their study. Barton et al. (1995) examined the relationship between in-situ stress, fractures and fluid flow in the Long Valley Caldera exploratory well. No model was developed to estimate permeability under in-situ stress. Matsuki et al. (2001) measured time-dependent closure and permeability of a hydraulic fracture under constant normal stress. Empirical correlations were applied to evaluate aperture and permeability. Cuisiat et al. (2002) tested long-term fracture permeability in shales in lab. The permeability model is same as Bakhtar et al. (1985). Kasap and Bush (2003) developed an empirical relationship between pore pressure and natural fracture permeability for highly stressed reservoirs basing on well test data. Kassis and Sondergeld (2010) utilized Saidi's model to evaluate the effects of roughness, proppant, fracture offset, and confining pressure on fracture permeability. Ulhaq and Salehi (2010) proposed a methodology to estimate fracture permeability in naturally fractured reservoirs. The empirical correlation they used includes a coefficient

controlled by the topography of the fracture surfaces and rock type. Cho et al. (2012) presented an experimental study of pressure-dependent natural-fracture permeability in shale and its effect on shale-gas well production. Their permeability model is same as Bakhtar et al. (1985). Crandall et al. (2013) run the experiment to examine the change of fracture hydraulic with the mechanical changes of a fracture. Their experiment indicated that permeability declines as fracture width decreases. However, no correlation was proposed to relate the permeability to fracture geometry. Phueakphum and Fuenkajorn (2014) measured rock hydraulic aperture and hydraulic conductivity under different normal and shear stresses. They developed empirical correlations to calculate hydraulic conductivity from hydraulic aperture. The limitation of these empirical correlations is that the coefficients are empirical and vary with rock type. Tokan-Lawal et al. (2014) captured the fracture geometry with x-ray microtomography (CT) scan and determined the permeability and tortuosity of the fracture by cubic law equation (Eq. 4 in the following section). Huo et al. (2014) used CT scanning to measure the aperture distribution and numerical simulation to model the change of permeability as a function of effective stress. No equation is shown to calculate permeability in their paper. Raimbay et al. (2014) measured the permeability of rough fracture in limestone, granite, and marble. They observed significant permeability change due to fracture closure. But no permeability correlation was available in the paper. Shaoul et al. (2015) used stress sensitive permeability data derived from core testing to simulate a typical tight gas reservoir performance. Again, the correlation used to calculate the permeability contains empirical coefficients, which vary with rock type. Wang et al. (2015) proposed an approximate analytical model to interpret the pressure transient in the fractured reservoirs. The permeability correlation is similar to the one used by Phueakphum and Fuenkajorn (2014). Farahmand et al. (2015) modeled the effect of fracture dilation angle on

permeability change. Their results showed that a higher dilation angle has a higher contribution to permeability. No correlation related permeability to stress was shown in the article. Chi and Heidari (2015) used nuclear magnetic resonance (NMR) to access fracture permeability. In their study the permeability is calculated by porosity and T_2 relaxation time and the correlations needs constants that are determined by formation type. In summary, literature review indicates that former investigations are mainly experimental study and the corresponding correlations are empirical. Other than Saidi (1987) equation which overestimates the permeability declines, no analytical equation is available to quantify the fracture permeability as the reservoir is depleted. The disadvantage of empirical correlation is that coefficients vary. Coefficients from some studies may not be appropriate for other cases. In this study, we developed a theoretical model including the drag forces on the top and bottom walls of the fracture, which were ignored in the former studies. We also developed a model to estimate pressure drop in fracture under turbulent flow conditions commonly seen in gas flow, which was not taken into account in the previous investigations. Our work indicates that β -factor in Forchheimer equation is not only a function of fracture properties, but also a function of fluid properties.

Derivation of Theoretical Model for Fracture Permeability

The geometry of fracture in the reservoir can vary in many ways. An ideally flat fracture can be represented by the schematic shown in **Fig. 1 (a)**. In case that fracture is in a curvic plane or turns because of the stress field as depicted in **Fig. 1 (b)**, the concept of fracture tortuosity can be introduced to describe the tortuous level. Borrowing from the concept of tortuosity of capillary tube in the porous media, we define the fracture tortuosity as the ratio of length of tortuous fracture to that of flat fracture.

$$\tau_f = \frac{x_{f, tortuous}}{x_{f, flat}} \dots\dots\dots (1)$$

where

$x_{f, flat}$ = length of the flat fracture

$x_{f, tortuous}$ = length of the tortuous fracture

τ_f = fracture tortuosity

It is common to assume a well locates in the center of a fracture. Therefore, **Fig. 2** is usually used to represent one half of a fracture. Considering the fact that the fracture width in reservoir usually is less than 0.3 inches, which is much less than the height of fracture, for either natural or artificial fracture, we represent one half of a fracture with a slot, as shown in Fig. 2. **Fig. 2 (a)** represents the cross-sectional area of fracture with a rectangle; while **Fig. 2 (b)** approximates one half of a fracture with a slot with the dimension of a length of x_f , a width of w , and a height of h . It is not uncommon that the fracture width and height can change at different location even for the same fracture. We can divide the fracture into several sections and assume each section has constant width and height. Then each section can be approximated by a slot.

Theoretical Model for Fracture Permeability under Laminar Flow

The fracture permeability can be derived assuming fluid flows through the slot in Fig. 2 (b). In case of numerous natural fractures or fractures created by multi-stage fracturing stimulation, we consider one portion of rock that contains one fracture, and the whole rock can be represented by an assembly of n fractures distributed uniformly. Before the derivation of fracture permeability, it is necessary to develop equation to calculate pressure drop through the slot using force balance analysis. We consider fluid flows through a slot shown in Fig. 2 (b) at a velocity of $u(y, h)$,

which is not uniform across the whole cross-section area but varies with the location, (y, h) . Because the height of fracture is much longer than the width, the drag forces exerted by the top and bottom of the slot are negligible. It is reasonable to conclude that a velocity of $u(y, h)$ can be approximated by $u(y)$, or in other words, velocity only varies in horizontal direction. Fluid velocity is zero at the both vertical walls of slot and reaches maximum at the center of slot as illustrated in Fig. 2.

To derive the equation to calculate pressure drop along the slot we make the following assumptions:

- 1) Single phase incompressible fluid flows in the slot.
- 2) Steady-state laminar flow in the slot.
- 3) The height of fracture is much longer than the width, $h \gg w$.
- 4) The fluid at the walls of the slot is assumed to be stationary, and the flow rate increases to a maximum at the center of the slot (or $y=0$). No slippage happens during the flow.
- 5) The fluid is homogeneous.
- 6) Temperature is constant throughout the slot.

Now we consider a control element in the slot as shown in **Fig. 3**. Basing on the concept of viscosity, the drag force acting on a vertical fluid layer at location y as it moves ahead at a velocity of $u(y)$ is caused by viscous force. This drag force is balanced by the force resulting from pressure difference. We can get the flow rate through fracture basing on force balance analysis (Croft, 1938) when $h \gg w$.

$$q = \frac{(p_1 - p_2)hw^3}{12\mu x_f} \dots \dots \dots (2)$$

where

h = height

p_1 = inlet pressure

p_2 = outlet pressure

q = volumetric flow rate

w = width of slot, or fracture

x_f = length of the slot, or fracture

μ = fluid viscosity

Recalling Darcy's law

$$q = k_f \frac{hw(p_1 - p_2)}{\mu x_f} \dots \dots \dots (3)$$

and comparing **Eq. (2)** with **(3)** we obtain fracture permeability in terms of fracture width (Tiab and Donaldson, 2004), which is

$$k_f = \frac{w^2}{12} \dots \dots \dots (4)$$

where

k_f = fracture permeability

Applying the concept of capillary tube to represent the connected pores contributing to flow through porous media, Poiseuille's Law (1847) can be used to calculate flow rate and rock permeability in terms of capillary tube radius,

$$q = \frac{(p_1 - p_2)\pi R R^3}{8\mu L} \dots \dots \dots (5a)$$

and

$$k = \frac{R^2}{8} \dots \dots \dots (5b)$$

where

k = permeability

L = length of capillary tube

R = radius of capillary tube

The similarities between **Eqs. (4)** and **(5b)** indicate that the permeabilities of different geometries share similar form. Regarding the fact that circle has the smallest perimeter comparing to other shapes assuming same flow areas, it is easy to conclude that circular pipe has the lowest drag force on the wall, or the highest permeability under same flow rate. This is one of the reasons that circular pipes are used to transport the fluids.

Because one of the assumptions to develop Eq. (4) is the height of fracture is much longer than the width, or $h \gg w$, one can expect that Eq. (4) can only be valid as height to width ratio,

$R_{height-to-width} = \frac{h}{w}$, is larger than a specific value. Considering the fact that

$$\frac{w^2}{12} < \frac{R^2}{8} \dots\dots\dots (6)$$

and

$$hw = R_{height-to-width} ww = \pi R^2 \dots\dots\dots (7)$$

Substituting **Eq. (7)** into **Eq. (6)** yields

$$R_{height-to-width} < \frac{2\pi}{3} \dots\dots\dots (8)$$

where

$R_{height-to-width}$ = height to width ratio

Therefore Eqs. (3) and (4) will introduce error under the conditions of $R_{height-to-width} < \frac{2\pi}{3}$. The

lower the $R_{height-to-width}$, the larger the error is. To reduce the error it is necessary to include the

drag force exerted by the top and bottom sides of fracture. **Fig. 4** illustrates the approximation of drag forces on the four walls of a slot with two semi-cylinders and a cuboid.

The drag forces on two sides of a cuboid and two curvic planes of two semi-cylinders are

$$2F_{d,semi-cylinder} = -\mu 2\pi y x_f \frac{du(y)}{dy} \dots\dots\dots (9a)$$

and

$$F_{d,cuboid} = -\mu(h-w)x_f \frac{du(y)}{dy} \dots\dots\dots (9b)$$

respectively.

where

$F_{d, cuboid}$ = the drag force on the surface of cuboid

$F_{d, semi-cylinder}$ = the drag force on the surface of semi-cylinder

u = velocity

$u(y)$ = fluid velocity at location of y

y = location in y direction, or horizontal direction, measured from center of slot

u_{cuboid} = fluid velocity in cuboid

$u_{semi-cylinder}$ = fluid velocity in semi-cylinder

Force balance gives

$$p_1 \pi y^2 = 2F_{d,semi-cylinder} + p_2 \pi y^2 \dots\dots\dots (10a)$$

and

$$p_1(h-w)y = F_{d,cuboid} + p_2(h-w)y \dots\dots\dots (10b)$$

Substituting **Eqs. (9a)** and **(9b)** into **(10a)** and **(10b)** and integrating give

$$u_{semi-cylinder} = \frac{(p_1 - p_2)}{4\mu x_f} \left(\frac{w^2}{4} - y^2 \right) \dots\dots\dots (11a)$$

and

$$u_{cuboid} = \frac{(p_1 - p_2)}{2\mu x_f} \left(\frac{w^2}{4} - y^2 \right) \dots\dots\dots (11b)$$

Using control element concept we can calculate the volumetric flow rate through two semi-cylinders and cuboid. Integrating the fluid velocity, u , over cross-sectional area gives the total flow rate

$$q = q_{cuboid} + 2q_{semi-cylinder} = \frac{(p_1 - p_2)}{\mu x_f} \left[\frac{(h-w)w^3}{12} + \frac{\pi \left(\frac{w}{2} \right)^4}{8} \right] \dots\dots\dots (12)$$

where

q_{cuboid} = volumetric flow rate through cuboid

$q_{semi-cylinder}$ = volumetric flow rate through semi-cylinder

u_{cuboid} = fluid velocity in cuboid

$u_{semi-cylinder}$ = fluid velocity in semi-cylinder

Darcy's law gives flow rate through fracture as

$$q = k_f \frac{(p_1 - p_2)}{\mu x_f} \left[(h-w)w + \pi \left(\frac{w}{2} \right)^2 \right] \dots\dots\dots (13)$$

Comparing **Eq. (12)** with **(13)** we obtain

$$k_f = \frac{w^2 [32(h-w) + 3\pi w]}{384(h-w) + 96\pi w} \dots\dots\dots (14)$$

Eq. (14) is more general and accurate than Eq. (4). When $w \rightarrow 0$, $k_f \rightarrow \frac{w^2}{12}$, which is Eq. (4).

When $w \rightarrow h$, $k_f \rightarrow \frac{w^2}{32}$, which is Eq. (5b), or the permeability of circle with a radius of $R = \frac{w}{2}$.

It should be noted that if $w \rightarrow 0$, $k_f \rightarrow 0$.

The fluid flow through the tortuous fracture overcomes the viscous force along the length of tortuous path. Applying the concept of fracture tortuosity in Eq. (1), the fracture permeability is expressed as

$$k_f = \frac{w^2 [32(h-w) + 3\pi w]}{384(h-w) + 96\pi w} \frac{1}{\tau_f} \dots\dots\dots (15)$$

Theoretical Model for Fracture Permeability under Turbulent Flow

It is not unusual to observe turbulent flow in fracture in gas wells. Under such circumstance, it is imperative to estimate the permeability or the pressure drop in the fracture. Again, the fracture can be approximated with two semi-cylinders and a cuboid. The drag force on the walls of the slot can be analyzed by employing Reynolds number and friction factor. Fanning friction factor (Darby, 2001), f_f , is the ratio of the shear stress on the slot walls applied by unit volume of fluid to its kinetic energy.

$$f_f = \frac{\text{shear stress on the wall/volume fluid}}{\text{kinetic energy/volume fluid}} = \frac{\tau_{slot}}{\frac{1}{2} \rho_f u^2} \dots\dots\dots (16)$$

where

ρ_f = fluid density

τ_{slot} = shear stress on slot

For turbulent flow where $N_{Re} > 2100$, Fanning friction factor is a function of Reynolds number and relative roughness. It can be calculated from Chen's (1979) correlation, which is

$$f_f = \frac{1}{16 \left\{ \log \left[\frac{\varepsilon}{3.7065} - \frac{5.0452}{N_{Re}} \log \left[\frac{\varepsilon^{1.1098}}{2.8257} + \left(\frac{7.149}{N_{Re}} \right)^{0.8981} \right] \right] \right\}^2} \dots\dots\dots (17)$$

where

ε = relative roughness

N_{Re} is the Reynolds number, which is expressed as

$$N_{Re} = \frac{D_e u \rho_f}{\mu} = \frac{4 r_H u \rho_f}{\mu} \dots\dots\dots (18)$$

The hydraulic radius, r_H , is calculated by

$$r_H = \frac{\pi \frac{w^2}{4} + (h-w)w}{\pi w + 2(h-w)} \dots\dots\dots (19)$$

The roughness of fracture can be evaluated by Brown (1987) study.

Because the shear stress is equilibrated by pressure difference between two ends of slot, we have

$$\tau_{slot} = \frac{(p_1 - p_2) \left[\pi \left(\frac{w}{2} \right)^2 + (h-w)w \right]}{x_f [\pi w + 2(h-w)]} \dots\dots\dots (20)$$

where

τ_{slot} = shear stress on slot

Substituting **Eq. (20)** into **(16)** gives

$$\frac{(p_1 - p_2)}{x_f} = \frac{2\pi w + 4(h-w)}{\pi w^2 + 4(h-w)w} f_f \rho_f u^2 \dots\dots\dots (21)$$

Eq. (21) can be employed to estimate the pressure gradient in fracture for gas flow. The significance of Eq. (21) is that it can be used to replace Forchheimer equation in the situation of unknown β -factor. Forchheimer equation includes laminar flow, which is expressed by Darcy law, and inertial flow, which is the proportional to the fluid flow velocity. Expressed in terms of pressure gradient it is

$$\frac{dp}{dx_f} = \frac{\mu}{k} u + \beta \rho_f u^2 \dots\dots\dots (22)$$

where

β = coefficient of quadratic term in Forchheimer equation, or β -factor

To match Forchheimer equation, we separate right-hand-side of Eq. (21) into laminar and inertial flows. Recalling that under laminar flow, Fanning friction factor is

$$f_f = \frac{16}{N_{Re}} = \frac{16\mu u \rho_f [\pi w + 2(h-w)]}{\pi w^2 + 4(h-w)w} \dots\dots\dots (23)$$

We can recast Eq. (21) into

$$\frac{dp}{dx_f} = \frac{32[\pi w + 2(h-w)]^2}{[\pi w^2 + 4(h-w)w]^2} \mu u + \left[\frac{2\pi w + 4(h-w)}{\pi w^2 + 4(h-w)w} \left(f_f - \frac{16\mu u \rho_f [\pi w + 2(h-w)]}{\pi w^2 + 4(h-w)w} \right) \right] \rho_f u^2 \dots\dots\dots (24)$$

where f_f can be calculated by **Eq. (17)**.

Comparing **Eq. (24)** with **(22)** we find that

$$k_f = \frac{[\pi w^2 + 4(h-w)w]^2}{32[\pi w + 2(h-w)]^2} \dots\dots\dots (25)$$

and

$$\beta = \frac{2\pi w + 4(h-w)}{\pi w^2 + 4(h-w)w} \left(f_f - \frac{16\mu u \rho_f [\pi w + 2(h-w)]}{\pi w^2 + 4(h-w)w} \right) \dots\dots\dots (26)$$

Using **Eq. (25)** we calculate fracture permeabilities under two conditions.

- 1) When $w \rightarrow 0$, $k_f \rightarrow \frac{w^2}{8}$, (Note: Eq. (14) gives $k_f \rightarrow \frac{w^2}{12}$).
- 2) When $w \rightarrow h$, $k_f \rightarrow \frac{w^2}{32}$, (Note: Eq. (14) gives $k_f \rightarrow \frac{w^2}{32}$).

The difference between permeabilities of planar fracture calculated from Eqs. (14) and (25) is caused by non-circular flow channel. When applying Eq. (16) to planar fracture, one implicit assumption is that the friction factor on the curvic walls of two semi-cylinders is same as that on the walls of cuboid. In fact, the friction factors are different. This difference leads to the

difference in fracture permeability calculated from Eqs. (14) and (25).

Fracture Permeability Change as Reservoir is Depleted

As the reservoir is produced, pore and fracture pressures decline, thus lead to the closure of the fracture. To estimate the permeability change as the reservoir is depleted, it is necessary to understand the deformation of fracture when the fluid is drained. It is reasonable to assume fracture length and height are constant and fracture width becomes narrower as the reservoir is depleted. Therefore the width of fracture at fracture pressure, p , can be calculated by

$$hx_f (w_i - w) = c_{fr} (p_i - p) hx_f w_i$$

or

$$\frac{w}{w_i} = [1 - c_{fr} (p_i - p)] \dots \dots \dots (27)$$

where

c_{fr} = fracture compressibility

p = pressure

p_i = initial pressure

w_i = initial width of slot, or fracture

Plugging **Eq. (27)** into (25) we can calculate permeability as reservoir pressure is depleted. Again, plugging Eq. (27) into (24) we can calculate pressure gradient in fracture. **Fig. 5** shows the ratio of permeability at depleted reservoir pressure to permeability at original pressure vs. pressure ratio as reservoir is depleted. **Fig. 6** depicts the ratio of pressure gradient at depleted reservoir pressure to pressure gradient at original pressure vs. pressure ratio under turbulent flow. Figs. 5 and 6 indicate high fracture compressibility has high impact on permeability decline rate and pressure gradient change. Input data for Figs. 5 and 6 are listed in **Table 1**.

Comparisons with Previous Study

Saidi (1987) proposed an equation to evaluate fracture permeability at depleted pressure.

$$\frac{k}{k_i} = [1 - c_{fr}(p_i - p)]^3 \dots\dots\dots (28)$$

where

k_i = initial permeability

The comparisons of permeabilities ratios from Saidi's and our methods for different fracture compressibilities are shown in Fig. 5. It is noted that Saini's method gives higher permeability decline rates. Combining Eqs. (27) with (25) we note that the permeability ratio is proportional to $[1 - c_{fr}(p_i - p)]^2$ in our model, while in Saini's model it is proportional to $[1 - c_{fr}(p_i - p)]^3$. Therefore Saini model gives a higher permeability decline rate. Considering that the fracture length and height are constant and fracture width becomes narrower as the reservoir is depleted, our model gives more accurate results.

Sensitivity Analysis

It should be noted that the proposed models are theoretical; the parameters in these theoretical models can vary significantly in different reservoirs. To better understand the effects of important parameters on the permeability change and pressure gradient as fracture pressure is depleted, it is necessary to conduct sensitivity analysis.

Sensitivity of Fracture Compressibility

Fig. 5 indicates the effect of fracture compressibility on fracture permeability decline rate as reservoir is depleted. The larger the fracture compressibility, the higher the decline rate is. This

agrees with Aybar et al. (2014) study. Their investigation indicated that the decline of fracture permeability of hard shale is faster than that of soft shale as shown in **Fig. 7**. Usually soft formation has a higher fracture compressibility than that of hard formation. Therefore, it is reasonable that two studies have similar results. Fig. 6 depicts the combination effect of fracture closure and gas density reduction on pressure gradient when pressure is reduced. Fracture closure leads to an increase in pressure gradient, while density reduction results in a lower pressure gradient. When the effect of fracture closure exceeds that of density reduction, pressure gradient increases as fracture pressure is depleted. This happens when fracture compressibility is larger, for instance, $c_{fr} = 0.0002$ 1/psi in Fig. 6. When the effect of fracture closure is balanced by that of density reduction, pressure gradient almost keeps constant, for example, $c_{fr} = 0.0001$ 1/psi in Fig. 6. When the effect of density reduction outruns that of fracture closure, pressure gradient decreases, as $c_{fr} = 0.00005$ 1/psi in Fig. 6.

Sensitivity of Initial Fracture Pressure

Fig. 8 illustrates the effect of initial fracture pressure on fracture permeability decline rate as reservoir is depleted. It is noted that the higher the initial pressure, the higher the decline rate is. This is because the large pressure drop in high pressure fracture leads to a narrower fracture. **Fig. 9** shows the comparison of production decline of wells producing high and low pressure reservoirs in Williston Basin in North Dakota. Well A produced a low pressure reservoir while Well B produced a high pressure reservoir. Both wells are horizontal wells with fracturing stimulation. The comparison indicates that Well B has a higher decline rate than Well A. This is mainly due to the high initial pressure of reservoir produced by Well B. **Fig. 10** depicts the effect of fracture closure surpasses that of density reduction at high initial fracture pressure,

while at low initial fracture pressure, the effect of density reduction exceeds that of fracture closure. At moderate pressure, two effects balance.

Sensitivity of Initial Fracture Width

Fig. 11 depicts same permeability decline rates for different initial fracture widths. **Fig. 12** shows that pressure gradient increases quickly in narrow fractures when fractures close due to depletion. It is obvious that wide fractures benefit the production through requiring smaller pressure drawdowns. As a result, production rate declines as fractures become narrower. It should be noted that fracture permeability deterioration is one of the causes leading to production decline. Reservoir pressure decrease is another important contribution to the decline.

Sensitivity of Fracture Height

Fig. 13 illustrates the effect of fracture height on fracture permeability decline rate as reservoir is depleted. The plots show that different fracture heights have same permeability decline rates. Two wells producing Bakken oil in Williston Basin with different reservoir thicknesses are compared in **Fig. 14**. Well C produced a reservoir thickness ranging between 32 and 41 ft. Reservoir thickness in Well D varies from 65 to 86 ft. Hydraulic fracturing has been implemented in both wells and fractures have migrated through reservoir in vertical direction according to microseismic interpretation. Therefore, the fracture height is assumed to be equal to the reservoir thickness. The comparison in **Fig. 14** shows that although Well D has a higher production rate resulting from a thicker reservoir thickness, its production decline rate is similar to that of Well C. This similar decline behavior verifies that the proposed model is reliable. **Fig. 15** shows that different fracture heights have same pressure gradient changes. Therefore fracture

height does not affect permeability decline rate and pressure gradient change assuming same fracture widths.

Sensitivity of Fracture Length

Figs. 16 and 17 shows that fracture length does not affect fracture permeability decline rate and pressure gradient change. Although fracture length does not impact the permeability decline rate, a long fracture length can improve the contact area with reservoir. Therefore, wells with longer fractures have higher production rates and estimated ultimate recoveries (EUR) than those of wells with shorter fractures.

Discussion, Limitation of the Model, and Future Work

In the sensitivity analysis we assume constant total stress. It should be noted that the above analysis applies to fractured reservoirs with low to extremely low matrix porosity. For reservoirs with sufficient matrix porosity, the decrease in pore pressure in the matrix leads to a decrease in total stress (Cheng et al., 1993; Engelder and Fischer, 1994) acting to close the fracture; this decrease in total stress helps to slow down the closure of fracture thus the decrease in fracture permeability due to depletion. Additionally, any asperities along the natural fracture walls will help to reduce the decrease in fracture width with drawdown, and will help to preserve a minimum fracture permeability in natural fractures when the in-situ stress cannot exceed the propped force resulting from the asperity contact of fracture walls. For this type of reservoir, the effect of total stress change on fracture closure is significant and cannot be ignored when modeling fracture permeability. Therefore, the total stress applied to the fracture should be updated when the fracture pore pressure declines. Future work should take the variation in total

stress into account and upgrade the current model to a coupled poroelasticity model. One should note that both induced hydraulic fractures and natural fractures are important contributions to the production. The proposed model not only can be applied to the induced hydraulic fracture but also to natural fractures providing fracture geometry can be described by other technology. In other words, the proposed method can be used for both induced hydraulic fractures and natural fractures if fracture geometry is known. Considering the complex nature of fracture geometry. To apply the model one can divide the fracture into several sections. Each section can be approximated by a planar fracture or a curvic fracture with constant width and height as shown in Fig. 1. Then the new model can be applied. We notice that the approximation may simplify the complicated fracture system and introduce errors. But if we appropriately approximate the fracture system through the combination of several planar and curvic fractures, the introduced errors should be small enough and can be accepted. This approach leads to a limitation of the proposed method: to model very complicated fracture systems one needs to divide the fracture systems into hundreds sections. To simulate a fractured reservoir with hundreds sections of fractures, the intensively computational time and numerical convergence are the main difficulties needed to be addressed. Another limitation of the model is the assumption of smooth fracture wall. In reality, fracture wall is rough. The model should be modified to take the wall roughness into account in the future. Verification of the model by experiment is another critical future work.

Conclusions

Following conclusions can be drawn upon finishing this study:

A more general model that includes drag forces on the top and bottom walls of the fracture, which were ignored in the former studies, was proposed to estimate fracture permeability.

The proposed model indicates that β -factor in Forchheimer equation is not only a function of fracture properties but also a function of fluid properties under turbulent flow conditions. The β -factor, which is often assumed to be a function of rock properties only in other investigations, can be calculated by an explicit expression, or Eq. (26).

When neglecting the frictional forces on the top and bottom walls of the fracture, the new model collapse to the commonly used model, or Eq. (4). In other words, Eq. (4) is a special case of the proposed model.

According to the sensitivity study, high fracture compressibility impacts permeability decline rate and pressure gradient change significantly. High initial fracture pressure has a higher permeability decline rate than low initial fracture pressure case. Fracture height and length do not affect the fracture permeability decline rate materially when reservoirs are depleted.

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Nomenclature

A	= area
c_{fr}	= fracture compressibility
D_e	= equivalent circular diameter
$F_{d, cuboid}$	= the drag force on the surface of cuboid
$F_{d, semi-cylinder}$	= the drag force on the surface of semi-cylinder
f_f	= Fanning friction factor
h	= height
k	= permeability
k_f	= fracture permeability
k_i	= initial permeability
L	= length of capillary tube
N_{Re}	= Reynolds number
p	= pressure
p_1	= inlet pressure
p_2	= outlet pressure
p_i	= initial pressure
q	= volumetric flow rate
q_{cuboid}	= volumetric flow rate through cuboid
$q_{semi-cylinder}$	= volumetric flow rate through semi-cylinder
R	= radius of capillary tube
$R_{height-to-width}$	= height to width ratio

r_H	= hydraulic diameter
u	= velocity
u_{cuboid}	= fluid velocity in cuboid
$u_{semi-cylinder}$	= fluid velocity in semi-cylinder
$u(y)$	= fluid velocity at location of y
$u(y,h)$	= fluid velocity at location of (y, h) , where y is the location in y direction, or horizontal direction, and h is the location in vertical direction
w	= width of slot, or fracture
w_i	= initial width of slot, or fracture
x_f	= length of the slot, or fracture
$x_{f,flat}$	= length of the flat fracture
$x_{f,tortuous}$	= length of the tortuous fracture
y	= location in y direction, or horizontal direction, measured from center of slot
β	= coefficient of quadratic term in Forchheimer equation
ε	= relative roughness
μ	= fluid viscosity
ρ_f	= fluid density
τ_f	= fracture tortuosity
τ_{slot}	= shear stress on slot

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TABLES

Table 1. Input data for fracture permeability and pressure gradient calculations

Original fracture width	0.1	inches
Fracture length	2000	inches
Fracture height	500	inches
Fracture compressibility	0.0002, 0.0001, and 0.00005	1/psi
Original pressure	6000	psia
Fracture relative roughness	0.0005	dimensionless
Gas specific gravity	0.7	air=1.0
Fluid velocity	10	ft/s
Temperature	180	°F

FIGURES

Fig. 1 A schematic of fracture: (a) an ideally flat fracture, (b) a tortuous fracture with curvic plane

Fig. 2 Representing a fracture as a slot, (a) cross-section view of fracture, (b) 3-D view of a fracture

Fig. 3 3-D view of a slot in a rock block

$$u(y, h) \approx u(y)$$

$$u(y) = 0 \text{ at } y = w/2 \text{ and } y = -w/2$$

$$u(y) = \text{maximum at } y = 0$$

Fig. 4 Approximation of drag forces on the four walls of a slot with two semi-cylinders and a cuboid

Fig. 5 Ratio of permeability at depleted reservoir pressure to permeability at initial reservoir pressure vs. pressure ratio

Fig. 6 Ratio of pressure gradient at depleted reservoir pressure to pressure gradient at initial reservoir pressure vs. pressure ratio under turbulent flow conditions

Fig. 7 Natural fracture permeability and conductivity multiplier changes with decreasing pore-pressure for different shale types, (after Aybar et al. (2014))

Fig. 8 Permeability ratio vs. pressure ratio for different initial pressures

Fig. 9 Comparison of production decline trend in two wells producing high and low pressure reservoirs

Fig. 10 Effect of initial pressure on pressure gradient under turbulent flow conditions

Fig. 11 Permeability ratio vs. pressure ratio for different initial fracture widths

Fig. 12 Effect of initial fracture width on pressure gradient under turbulent flow conditions

Fig. 13 Permeability ratio vs. pressure ratio for different fracture heights

Fig. 14 Comparison of production decline trend in two wells with different fracture heights

Fig. 15 Effect of fracture height on pressure gradient under turbulent flow conditions

Fig. 16 Permeability ratio vs. pressure ratio for different fracture lengths

Fig. 17 Effect of fracture length on pressure gradient under turbulent flow conditions