A New Theoretical Method to Calculate Shale Fracture Conductivity Based on the Population Balance Equation

Junjing Zhang^{1*}, Ding Zhu² and A. Daniel Hill²

¹ ConocoPhillips Company, Houston, Texas, U.S.A.

² Texas A&M University, College Station, Texas, U.S.A.

* Corresponding author: Junjing Zhang. Tel: +1 832-486-3861. Fax: +1 832-486-2756

Email: junjing.zhang@conocophillips.com

Abstract

Shale fracture conductivity plays a critical role in determining the long term production of shale wells. High fracture conductivity is achieved by pumping more proppants and minimizing conductivity damage by fracturing fluid. Currently, fracture conductivity is either measured in laboratory following ISO 13503-5 or calculated using correlations derived from extensive laboratory tests by regression analysis. There is a need for a handy and practical tool to calculate shale fracture conductivity reflecting realistic fracturing designs.

This paper presents a new correlation to calculate shale fracture conductivity that considers proppant properties, fracture design variables and formation mechanical properties. This correlation is based on the population balance equation for size reduction. It utilizes the population balance concept to predict the crushed proppant size distribution under increasing closure stress. Numerical solution of the integro-differential equation is validated by comparing the computed results with the measured data. The crushed grain size distribution determines the permeability of the sand pack. Fracture width is calculated separately by considering proppant embedment and proppant grain rearrangement. Fracture conductivity is then calculated and compared with laboratory measured conductivity. Finally, the effect of water damage to shale fracture conductivity is considered by bringing in a new modification term.

Results show that the population balance equation can reasonably predict the crushed proppant size reduction by properly choosing the selection equation and breakage equation. The calculated shale fracture conductivity can match with the measured fracture conductivity using the Barnett Shale. An exponential expression can be used to account for the water damage effect in the Barnett Shale.

Introduction

More and more pilot projects in shale reservoirs have shown that shale well production is positively correlated with the amount of proppants pumped in hydraulic fracturing (LaFollette et al. 2014; Holcomb et al. 2015). This observation indicates that propped fractures with higher conductivity elevate shale well production. The industry uses ISO 13503-2 to standardize the manufacturing of hydraulic fracturing proppants and ISO 13503-5 to measure the long term fracture conductivity. The recommended practices regulate that fracture conductivity should be measured at the concentration of 2 lb/ft². Conductivities of different proppants in compliance with the ISO practices as well as mutated procedures to account for realistic conditions are utilized to select proppants by the operators. A huge amount of time and labors are required to measure the realistic fracture conductivity. To generate a useful correlation from experiments, it needs data from hundreds of laboratory tests. The regression

method was typically utilized to generation a correlation with multiple variables (Wen et al. 2015). So, there is a need for a cost-effective and accurate theoretical model to predict the fracture conductivity.

One of the earliest fracture conductivity calculation equations was derived by Darin et al. (1959) from the Kozeny-Carman relation based on the partial monolayer assumption. In this equation, fracture width is an input calculated by geometric relations between proppant grain diameter and embedment depth (Huitt et al. 1958). The most widely used fracture conductivity calculation toolbox is a Stim-Lab product called Predict-K. This program contains the correlations found by the Stim-Lab consortium between fracture conductivity and proppant embedment, proppant crushing, gel filter cake and non-Darcy flow. These relations are established mainly by statistical method such as regression of extensive experimental datasets. Gao et al. (2012) derived a mathematical model to calculate the proppant embedment depth and fracture conductivity based on the Hertzian elastic contact theory. They matched the predicted results with experimental results by introducing a number of empirical coefficients. A new fracture conductivity correlation was proposed by Awoleke (2013) using factorial design. This correlation links fracture conductivity with proppant concentration, temperature, closure stress and gel loading. Unpropped fracture conductivity calculation has been well studied in acid fracturing where fracture conductivity is created by unevenly etched fracture faces. Conductivity correlations for unpropped rough fractures were discussed and derived by Nierode and Kruk (1973), Gong et al. (1999), Deng et al. (2012) and Zhang et al. (2014a, 2015a).

Water damage is a common mechanism of flow capacity loss in porous media (Bang et al. 2008). Remedial removal of water damage was studied both experimentally and theoretically (Yuan 2012). Shale fracture conductivity damage by water is unique and significant in clay rich shale. Experiments show that over 80% of shale fracture conductivity measured in laboratory conditions can be lost after the samples are exposed to water flow (Zhang et al. 2014b). Thus, the water damage needs to be considered in shale fracture conductivity correlations.

Previous correlations to calculate fracture conductivity either require extensive laboratory tests to calibrate the model or depend on a single parameter that is difficult to measure under laboratory conditions. Besides, there is little published data to show that water damage is considered in the previous conductivity correlations.

This study intends to develop a cost effective and handy correlation that can be used to calculate shale fracture conductivity considering proppant properties, fracturing design variables, formation conditions and water damage effect.

Description of Methodology

Fracture conductivity is calculated through the following steps:

- (1) Review the recognized correlations for sand pack permeability calculation.
- (2) Derive and solve the population balance equation for size reduction that describes proppant crushing in the fracture during elevated stress application.
- (3) Validate the solution of the population balance equation by comparing the calculated results with the sieve analysis data.
- (4) Measure the initial proppant pack width under zero closure stress and calculate the width change due to grain rearrangement and embedment.

- (5) Calculate the fracture conductivity and compare the results with the measured fracture conductivity.
- (6) Modify the correlation that is based on the dry gas measurements to consider water damage.

Fracture Permeability Calculation

Sand Pack Permeability Correlation. It has been recognized that sand pack permeability is a strong function of sand grain size, packing, and sorting. Kozeny (1927) developed a capillary model that accounts for shape factor in actual rocks to calculate the permeability of porous media. Carman (1938) extended Kozeny's model by introducing a factor for grain surface area to represent the irregular surface of pores. One of the most popular and accurate correlations to calculate sand pack permeability was proposed by Berg (1970):

$$k = 5.1 \times 10^{-6} \phi^{5.1} d_{50}^2 e^{-1.385\psi} \dots\dots\dots(1)$$

This model considers porosity of the porous media, grain particle size and sorting. However, one difficulty of applying this model to calculate propped fracture permeability is to determine the median grain size (d_{50}) and phi percentile deviation (ψ) of the crushed proppants, because proppant crushing is a dynamic process during the stress application. It is challenging to collect the low concentration proppants after experiment to do sieve analysis for particle size distribution. Even at higher concentration such as 0.20 lb/ft² where the total mass is 6.40 grams, post experiment sieve analysis brings too much error due to the loss of proppants during the experiment, because some proppants are embedded into the shale matrix and cannot be retrieved. The used proppants are always blended with shale flakes or particles and cannot be separated due to the similar size ranges. More importantly, we are interested in permeability at different stress levels. We cannot measure size distribution at each stress level without disturbing the proppant pack. Therefore, a method to calculate particle size distribution at various stress levels is needed to determine fracture permeability.

Population Balance Equation for Size Reduction. A population balance equation describes the process of population changes of certain species as the net change of population is equal to the rate of concentration change. The net change of population is defined as the difference between species birth and death and they are usually functions of the entire population. So, population balance equations take the form of integro-differential equation. Depending on the form of the birth term and the death term, population balance equations can be solved analytically or numerically. The idea of population balance is widely applied in a variety of engineering areas, such as comminution of minerals, crystallization, polymerization in reactors and monodisperse colloidal suspensions (Ramkrishna 2000).

In petroleum engineering area, the idea of population balance was applied by Guin et al. (1971) and Hill (1978) to study the pore size evolution during matrix acid treatment. Sharma et al. (1986) also used this methodology to study formation permeability damage by fines migration and entrainment. Recently, Maqbool (2011) described the asphaltene precipitation process by applying population balance equations and solved the series of equations numerically. The population balance in size reduction can be illustrated by **Fig. 1**.

Selection Function. In crushing tests, particles usually have a size range and the stress status for the grains is different depending on size, location, number of contacts, and their mechanical properties. Therefore, the chance for each particle to be crushed is not equal. The function to describe the mass fraction of the particles of a specific size to be crushed per unit stress is called the selection function or specific rate of breakage.

Take size interval 3 in Fig. 1 for instance. Particles of this size range are crushed into smaller sizes. Meanwhile, this interval also receives particles crushed from larger sizes. So, a population balance equation for size reduction can be described as,

$$\begin{aligned} &\text{Population density change per unit application of stress} = \\ &\quad \text{Amount of particles crushed from this size interval} + \\ &\quad \text{Amount of particles crushed from larger sizes that this size interval receives.} \end{aligned}$$

For example, the 40/70 mesh sand at 0.1 lb/ft² contains 0.973 gram particles at 50-60 mesh size range at zero closure stress. We apply the closure stress and crush 0.001 gram particles of this size range. Then, the selection function would be 0.001 g/0.973 g.

Assume $P(x, \sigma)$ is the cumulative mass fraction of the proppants smaller than size x at stress σ , the selection function, $S(x)$, can be mathematically expressed as (Gardner and Austin 1962),

$$S(x) = - \left[\frac{\frac{\partial}{\partial \sigma} \left[\frac{\partial P(x, \sigma)}{\partial x} \right]}{\frac{\partial P(x, \sigma)}{\partial x}} \right] \dots \dots \dots (2)$$

Gardner and Austin (1962) experimentally determined the selection function by radioactive tracer techniques. They irradiated the samples for one hour at the maximum thermal neutron flux of the Pennsylvania State University Reactor. The amount of radioactivity in each size fraction was determined by gross counting at every loading cycle with size distributions determined. From the experiments, they plotted the fractional weight of the broken particles versus particle size and determined the selection function (**Fig. 2**).

In our study, we follow the expression of the selection function suggested by Austin (1999),

$$S(x) = a_1 x^{a_2} \dots \dots \dots (3)$$

Where a_1 and a_2 are the empirical coefficients and can be determined by experiment. From Fig. 2, $a_1 = 3\text{E-}4$, $a_2 = 0.565$.

Breakage Function. For particles that are crushed from interval i , some of them fall into the size range of $i+1$, while others might be in interval $i+2$ or $i+5$. So, another function is needed to characterize the amount of particles falling into interval j from a larger size interval i . This function is called the breakage function or distribution function.

Again, let us use 40/70 mesh sand data as an example. Within the 0.001 gram crushed particles, half of them fall into the range of 60-70 mesh. Then the breakage function for this size interval should be 0.0005. The designation B is used for breakage function. Mathematically, B is defined as,

$$B(y, x) = - \frac{\frac{\partial P(x, \sigma)}{\partial \sigma}}{\frac{\partial}{\partial \sigma} \left[\frac{\partial P(y, \sigma)}{\partial y} \right]}, 0 \leq x \leq y \quad (4)$$

The breakage function was experimentally determined by Gardner and Austin (1962) using the same radioactive tracer technology as the selection function (**Fig. 3**). Various models have been proposed to express the breakage function, including the work by Broadbent et al. (1956), Reid (1965), Kapur (1972) and Austin (1999). Here we use the modified Austin model to match the experimental data to generate the breakage function. The model is expressed as,

$$B(y, x) = a_3 \left(\frac{x}{y} \right)^{a_4} + (1 - a_3) \left(\frac{x}{y} \right)^{a_5}, 0 \leq x \leq y \quad (5)$$

Where a_3 , a_4 and a_5 are the empirical coefficients and are determined by experiment. From Fig. 3, $a_3 = 0.65$, $a_4 = 1.25$, $a_5 = 4.5$.

Derivation of the Population Balance Equation. The first rigorous mathematical model for size reduction based on population balance was derived by Bass (1954). But more popular are the mathematical expressions proposed by Gardner and Austin (1962) and Reid (1965). In this study, we follow the procedures recommended by Gardner and Austin (1962) to derive the mathematical model.

It is assumed that: (1) during the stress application, proppants can only be crushed into smaller sizes and there is no feed of particles into the proppant pack; (2) the selection function and the breakage function as stated in the previous sections are valid in the primary breakage of particles at size y to a differential size $y+dy$. The weight percent of proppants smaller than size x at closure stress σ , $P(x, \sigma)$ is equal to the sum of the weight percent of proppants at size x originally and the weight percent of proppants crushed to size x from larger sizes due to the stress increment $d\sigma$.

For a differential size increment from y to $y+dy$, the weight percent of the amount of crushed proppants is,

$$m_p = \left[\frac{\partial P(y, \sigma)}{\partial y} \right] dy, x \leq y \leq x_{\max} \quad (6)$$

According to the definition of the selection function, it can be expressed as,

$$S(y) = \frac{\left[\frac{dm_p}{d\sigma} \right]}{m_p} = \frac{\frac{dm_p}{d\sigma}}{\frac{\partial P(y, \sigma)}{\partial y} dy} \quad (7)$$

Rearrange Eq. 7, we can get,

$$dm_p = S(y) \frac{\partial P(y, \sigma)}{\partial y} dy d\sigma \quad (8)$$

We use the consistent nomenclature for breakage function as $B(y, x)$. The amount of proppants crushed from dm_p into size x can be defined as,

$$m_{p,x} = B(y,x)S(y)\frac{\partial P(y,\sigma)}{\partial y}dyd\sigma \dots\dots\dots(9)$$

Assume the stress interval $[0, \sigma_{max}]$ is equally spaced. The weight percent of proppants at size x in stress interval n can be derived as,

$$P(x, \sigma_n) = P(x, \sigma_{n-1}) + B(y,x)S(y)\frac{\partial P(y,\sigma)}{\partial y}dyd\sigma \dots\dots\dots(10)$$

Eq. 10 can be mathematically mutated as,

$$P(x, \sigma) = P(x, \sigma_0) + \int_0^{\sigma_n \cdot x_{max}} \int_x B(y,x)S(y)\frac{\partial P(y,\sigma)}{\partial y}dyd\sigma \dots\dots\dots(11)$$

Rearrange Eq. 11 and partially differentiate it with respect to σ ,

$$\frac{\partial [P(x, \sigma)]}{\partial \sigma} = \int_x^{x_{max}} B(y,x)S(y)\frac{\partial P(y,\sigma)}{\partial y}dy \dots\dots\dots(12)$$

Differentiate Eq. 12 with respect to size x ,

$$\frac{\partial}{\partial x} \left[\frac{\partial P(x, \sigma)}{\partial \sigma} \right] = -S(x)\frac{\partial P(x, \sigma)}{\partial x} + \int_{y=x}^{x_{max}} \frac{\partial B(y,x)}{\partial x} \frac{\partial P(y, \sigma)}{\partial y} S(y)dy \dots\dots\dots(13)$$

Eq. 13 is the integro-differential population balance equation for size reduction. The population balance equation for proppant crushing satisfies the initial and the boundary conditions below.

$$P(x, \sigma) \Big|_{\sigma=0} = P(x, 0) \dots\dots\dots(14)$$

$$P(x, \sigma) \Big|_{x=x_{max}} = 1 \dots\dots\dots(15)$$

Where $P(x, 0)$ is the initial proppant size distribution which can be acquired from sieve analysis. The boundary condition Eq. 15 means that the cumulative weight fraction of proppant smaller than the maximum size is equal to 1.

Solution of the Population Balance Equation. Solution of the integro-differential population balance equation has been well presented by Ramkrishna (2000). Generally, it can be solved analytically by assuming simple linear selection and breakage functions (Reid 1965; King 1972; Peterson 1986; Campbell et al. 1994). A numerical solution scheme was proposed by Gardner and Austin (1962) and improved by Lucas and Luke (1983) as well as Maqbool (2011). Mishra (1999) solved the equation using the Monte-Carlo method.

This study solves the population balance equation numerically using a finite difference method. The algorithm is illustrated by the flow chart in **Fig. 4**.

Validation of the solution. Solution to the integro-differential equation is validated by matching the computed proppant size distribution with the measured data. Two sets of sieve analysis data are provided to match the solutions.

The first set of data comes from the conductivity experiment with 0.10 lb/ft² proppant concentration at 4,000 psi closure stress. The proppant size was measured before the

experiment as represented by the blue dashed line in **Fig. 5**. After the experiment, the proppant was carefully removed and collected from the fracture and its size distribution was analyzed again. The measured size distribution and modeled size distribution at various closure stresses are plotted in Fig. 5.

We can see that at the smaller size range, the model shows an underpredicted size distribution compared with the measured values. This is probably caused by the shale particles mixed with the proppants after the experiments. The shale particles collected from the fracture surface have similar sizes as the crushed sand grains and cannot be separated. Other than that, the model predicts a reasonable proppant size distribution.

The second dataset is collected from the published literature by Fredd et al. (2001). In their work, they measured the proppant size distribution after the conductivity experiment at 7,000 psi closure stress. The measured data and the modeled results are plotted in **Fig. 6**.

The linear size distribution at higher closure stresses on the log-log plot indicates the fractal distribution of grain sizes (McDowell and Bolton 1998; Lobo-Guerrero and Vallejo 2005). It also honors the assumption of one dimensional loading pattern with no lateral strain (Guimaraes et al. 2007).

Fracture Width Calculation

Width reduction happens due to proppant rearrangement, proppant embedment, proppant crushing, gel filter cake, and so on (Palisch et al. 2007). In this study, proppants are manually placed, so there is no gel damage. We mainly discuss the width reductions caused by proppant grain rearrangement and embedment.

Proppant rearrangement. Loose proppant grains with a wide size range are loaded on the fracture surface. After the stress is applied, grain rearrangement occurs due to the inter-granular frictional slippage and pore collapse, as illustrated in **Fig. 7**.

Define the initial proppant pack void ratio e_0 as the ratio of the pore volume to proppant grain volume at zero closure stress. The fraction of void ratio change is equal to the fraction of width change (**Fig. 8**) assuming:

- (1) The solid particles are incompressible and embedment is negligible during this phase, so that the change in total volume is only related to the change in pore volume.
- (2) The consolidation is one dimensional in the width direction and the strain in the flow direction is negligible, or simply assuming the cross sectional area of the proppant pack in the width direction is constant.

Assume an initial proppant pack width w_0 , width reduction by the grain rearrangement Δw_1 , initial proppant pack void ratio e_0 , and void ratio reduction Δe . Then,

$$\frac{\Delta w_1}{w_0} = \frac{\Delta e}{1 + e_0} \dots\dots\dots(16)$$

Initial porosity is,

$$\phi_0 = \frac{e_0}{1 + e_0} \dots\dots\dots(17)$$

So,

$$\Delta w_1 = w_0(\phi_0 - \phi_1)(1 - \phi_0) \dots\dots\dots(18)$$

Proppant embedment. Proppant embedment depth is determined by laboratory measurements and the width loss due to embedment is shown in Eq. 19 (Palisch et al. 2010). This relation has been used for a variety of sandstone fracture width calculations.

$$\Delta w_E = \frac{2.5d_p}{(1 + \frac{4000 \times E}{\sigma})^{1.5}} \dots\dots\dots(19)$$

Fig. 9 shows the embedment depth Δw_e as functions of the grain diameter d_p , Young's modulus E (MMpsi) and the closure stress σ (psi). The embedment depth has the consistent unit as the proppant grain diameter. At 4,000 psi, the ratio of the embedment depth to the grain diameter is about 0.20 assuming the Young's modulus is 4 MMpsi.

Calculation of Fracture Conductivity

In this section, we propose a new method to predict propped fracture conductivity by modifying the Berg correlation for calculating permeability integrated with the population balance equation for size reduction. In this method, the proppant size, packing, sorting, crushing, grain rearrangement and grain embedment are taken into account. A flowchart describing the conductivity calculation procedure is shown in **Fig. 10**.

We first solve the population balance equation for the proppant size distribution at various stresses. The parameters for the Berg's correlation (1970) to calculate the fracture permeability can be read from the proppant size distribution (**Fig. 11**). The initial porosity and porosity change for clean sands studied by Karner et al. (2005) indicate that, in the Barnett Shale reservoir conditions, the porosity variation in propped fracture is within 3% (**Fig. 12**). So the average porosity is used in the Berg correlation.

The initial fracture widths for various concentrations were measured in lab. We particularly measured the proppant pack thickness at multiple surface valleys indicated by the surface profilometry, because fractures tend to close on the thicker sand packs at the surface valleys. The initial fracture widths are summarized in **Table 1**.

As Kamenov (2014) has pointed out that due to the surface asperity, proppants tend to collect at the valleys of the fracture surface. At the same concentration, the initial propped width of the rough fracture is larger than the fracture width where proppants are evenly distributed due to the thicker sand collections at the surface valleys (**Fig. 13**). The conductivity of the rough fracture would also be higher than tightly packed fracture because of the void spaces above the proppant pack. At lower closure stress, the proppant grains rearrange and accommodate themselves to overcome the stress. During this phase, fracture width decreases and the fracture still closes on the thick sand pack at the surface valleys. At

higher closure stress where proppant embedment happens and point loading effect starts to occur, the sand pack at the valleys experiences severe width reduction. Meanwhile, the rock creeps in response to the higher stress causing the void space above the proppants to close. During this phase, the pressure drop across the fracture at certain flow rate is close to the pressure drop across the fracture where proppants are evenly placed initially.

To account for the extra flow path caused by the open spaces above the sand pack, the Berg model is modified to better predict proppant pack permeability as,

$$k = 283.13d_{50}^2 e^{-1.385\psi} \sigma^{-0.541} \dots\dots\dots(20)$$

Where,

$$\psi = \frac{d_{90} - d_{10}}{2} \dots\dots\dots(21)$$

The unit of the closure stress σ is psi. With this modified model, proppant pack permeability at low concentrations can be predicted considering the uneven proppant distribution.

The exponential relationship expressed by Eq. 22 can be used to calculate the damaged fracture conductivity and reflects the relation between initial and recovered fracture conductivities after the water damage.

$$C_f = 0.0185 \times (k_f w_f)^{1.5168} \dots\dots\dots(22)$$

Matching with Laboratory Data

Undamaged Fracture Conductivity. Fig. 14 and Fig. 15 show the predicted fracture conductivities matching with the measured conductivities of 40/70 mesh and 30/50 mesh sands. In the calculations, the initial porosity is 0.35. Young's Modulus for the Barnett Shale sample is 4 MMpsi.

Damaged Fracture Conductivity. Damaged fracture conductivity, or recovered fracture conductivity, was measured by flowing dry nitrogen through the fracture that was damaged by water. Detailed procedures and the damage mechanisms can be found in the reference (Zhang et al. 2015b).

Initial fracture conductivities and their corresponding recovered fracture conductivities are plotted in Fig. 16. The data points are collected from experiments at the same conditions. In the testing range, there is a power-law trend between the initial and recovered fracture conductivities. The data are more scattered at lower initial fracture conductivity where 40/70 mesh sands at 0.06 lb/ft² and 0.10 lb/ft² are placed. This is related to the existence of unpropped fracture and proppant monolayers at the fracture surface hills and slopes, because these areas are more susceptible to water damage. At elevated stresses, these regions tend to close up easier after water flow than the heavily propped areas. Water damage to shale fracture conductivity mainly happens to the flow areas close to the fracture walls. With the same proppant size, fracture with higher conductivity has more proppant layers and the dominant flow channels are created by the inner pack which is usually much cleaner and more conductive.

The power-law correlation for water damage was developed based on the Barnett shale experiments. Therefore, it is recommended to use the same empirical coefficients for rocks that have comparable properties as the Barnett shale.

Conclusions

A theoretical model of propped fracture conductivity was developed to include the effects of water damage on fracture conductivity. The fracture permeability calculation is based on the population balance equation for size reduction to calculate the crushed proppants size distribution as a function of closure stress. Fracture width correlation considers proppant rearrangement and embedment. Based on the studies, following conclusions are made:

1. The Population Balance Equation based approach to calculate propped fracture conductivity considers the proppant properties, fracture design variables, and formation properties. The predicted results give a reasonable match with the lab data.
2. A correlation to calculate fracture conductivity considering water damage is developed. This correlation expresses the power law relationship between the initial fracture conductivity and the recovered fracture conductivity after water damage.
3. In the testing range, a fracture with higher initial conductivity usually has higher recovered fracture conductivity, because it can better overcome the conductivity damage by water which occurs in the vicinity of the fracture walls. With a thick proppant pack, the internal proppant layers are the dominant flow area, and stay intact in the presence of water damage.

Nomenclature

a_1 to a_6	Coefficients of the selection and breakage functions, dimensionless
B	Breakage function, dimensionless
d_{10}	Proppant diameter at the tenth percentile, L , mm
d_{50}	Median proppant diameter, L , mm
d_{90}	Proppant diameter at the ninetieth percentile, L , mm
e_0	Initial void ratio, fraction
e	Void ratio at certain stress, fraction
E	Young's modulus, $ML^{-1}T^{-2}$, MMpsi [pa]
m_p	Weight percentage of the amount of proppants for size increment of dy
S	Selection function, dimensionless
w_0	Initial fracture width, L , m
w_1	Fracture width at certain stress, L , m
w_f	Fracture width, L , m
x	Grain size, L , mm
y	Dummy variable in the population balance equation
ϕ	Porosity, fraction

ϕ_0	Initial proppant pack porosity, fraction
ϕ_1	Proppant pack porosity at certain stress, fraction
γ_p	Proppant specific gravity, dimensionless
σ	Closure stress, $ML^{-1}T^{-2}$, psi [pa]
ψ	Phi percentile deviation, L , mm
i	Index for the proppant grain size increment
n	Index for the closure stress increment

Acknowledgement

The authors would like to acknowledge the sponsors of Crisman Institute for Petroleum Research in the Petroleum Engineering Department at Texas A&M University as well as the U.S. Department of Energy, and the *Research Partnership to Secure Energy for America* (RPSEA) for their support of this work (RPSEA project 11122-07-TAMU-Zhu).

References

1. Austin, L.G. 1999. A Discussion of Equations for the Analysis of Batch Grinding Data. *Power Technology*, vol. 106: 71-77.
2. Awoleke, O., Romero, J., Zhu, D., and Hill, A.D. 2012. Experimental Investigation of Propped Fracture Conductivity in Tight Gas Reservoirs Using Factorial Design. Paper SPE 151963 presented at the SPE Hydraulic Fracturing Technology Conference, the Woodlands, Texas, U.S.A., 6-8 February.
3. Bang, V., Yuan, C., Pope, G.A., Sharma, M.M., Baran, R. Jr., Skildum, J. and Linnemeyer, H.C. 2008. Improving Productivity of Hydraulically Fractured Gas Condensate Wells by Chemical Treatment. Paper OTC 19599 presented at the SPE Offshore Technology Conference held in Houston, Texas, U.S. A., 5-8 May.
4. Bass, V.L. 1954. Zur Theorie Der Mahlvorgange. *Zeitschrift für angewandte Mathematik und Physik*, 5(4): 283-292.
5. Berg, R.R. 1970. Method for Determining Permeability from Reservoir Rock Properties. *Gulf Coast Association of Geological Societies Transactions*, Vol. 20: 303-317.
6. Broadbent, A.R. and Callcott T.G. 1956. A Matrix Analysis of Processes Involving Particle Assemblies. *Philosophical Transactions of the Royal Society A: Mathematical, Physical & Engineering Sciences*, vol. 149: 99-123.
7. Campbell, Q.P., Everson, R.C., Eyer, D., and Spamer, H.J. 1994. Spline Solution of the Continuous Batch Grinding Equation. *Minerals Engineering*, 8(4/5): 377-387.
8. Carman P.C. 1937. Fluid Flow Through Granular Beds. *Chemical Engineering Research and Design*, vol 75: S32-S48.
9. Darin, S.R. and Huitt, J.L. 1959. Effect of a Partial Monolayer of Proppant Agent on Fracture Flow Capacity. Paper SPE 1291 presented at the Annual Fall Meeting, Dallas, Texas, U.S.A., 4-7 October.
10. Deng, J., Mou, J., Hill, A.D., and Zhu, D. 2012. A New Correlation of Acid-Fracture Conductivity Subject to Closure Stress. *SPE Production & Operations*, 27(2): 158-169.
11. Fredd, C.N., McConnell, S.B., Boney, C.L., and England, K.W. 2001. Experimental

- Study of Fracture Conductivity for Water-Fracturing and Conventional Fracturing Applications. *SPE Journal*, 6(3): 288-298.
12. Gao, Y., Lv, Y., Wang, M., and Li, K. 2012. New Mathematical Models for Calculating the Proppant Embedment and Fracture Conductivity. Paper SPE 155954 presented at the SPE Annual Technical Conference and Exhibition, San Antonio, Texas, U.S.A., 8-10 October.
13. Gardner R.P. and Austin L.G. 1962. A Chemical Engineering Treatment of Batch Grinding. *Symposium on Size Reduction*, April 1962: 217-248.
14. Gong, M., Lacote, S., and Hill, A.D. 1999. New Model of Acid-Fracture Conductivity Based on Deformation of Surface Asperities. *SPE Journal*, 4(3): 206-214.
15. Guimaraes, M.S., Valdes, J.R., Palomino, A.M., and Santamarina, J.C. 2007. Aggregate Production: Fines Generation During Rock Crushing. *International Journal of Mineral Processing*, 81(4): 237-247.
16. Guin, J.A., Schechter, R.S., and Silbergberg, I.H. 1971. Chemically Induced Changes in Porous Media. *Industrial & Engineering Chemistry Fundamentals*, 10(1): 50-54.
17. Hill, A.D. 1978. Flow with Simultaneous Heterogeneous Reactions in Porous Media. PhD dissertation, The University of Texas at Austin, Austin, Texas (May 1978).
18. Holcomb, W.D., LaFollette, R.F. and Zhong, M. 2015. The Third Dimension: Productivity Effects From Spatial Placement and Well Architecture in Eagle Ford Shale Horizontal Wells. Paper SPE 173343 presented at the SPE Hydraulic Fracturing Technology Conference, 3-5 February, The Woodlands, Texas, U.S.A.
19. Huitt, J.L. and Mcglothlin Jr., B.B. 1958. The Propping of Fractures in Formations Susceptible to Propping-sand Embedment. *Drilling and Production Practice*: 115-123.
20. Kamenov, A., Zhang, J., Zhu, D., and Hill, A.D. 2014. Laboratory Measurement of Hydraulic Fracture Conductivities in the Barnett Shale. *SPE Production & Operations*, 29(3): 216-227.
21. Kapur, P.C. 1972. Kinetics of Granulation by Non-Random Coalescence Mechanism. *Chemical Engineering Science*, 27(10): 1863-1869.
22. Karner, S., Chester, J.S., Chester, F.M., Kronenberg, A.K., and Hajash Jr., A. 2005. Laboratory Deformation of Granular Quartz Sand: Implication for the Burial of Clastic Rocks. *AAPG Bulletin*, 89(5): 603-625.
23. King, R.P. 1972. An Analytical Solution to the Batch Comminution Equation. *Journal of the South African Institute of Mining and Metallurgy*, vol. 73: 127-131.
24. Kozeny, J. 1927. Ueber Kapillare Leitung Des Wassers Im Boden. *Sitzungsberichte Wiener Akademie*, 136(2a): 271-306.
25. LaFollette, R.F., Izadi, G. and Zhong, M. 2014. Application of Multivariate Statistical Modeling and Geographic Information Systems Pattern-Recognition Analysis to Production Results in the Eagle Ford Formation of South Texas. Paper SPE 168628 presented at the SPE Hydraulic Fracturing Technology Conference, 4-6 February, The Woodlands, Texas, U.S.A.
26. Lobo-Guerrero, S. and Vallejo, L.E. 2005. Discrete Element Method Evacuation of Granular Crushing under Direct Shear Test Conditions. *Journal of Geotechnical and Geoenvironmental Engineering*, 131(10):1295-1300.
27. Lucas, P.W. and Luke, D.A. 1983. Methods for Analyzing the Breakdown of Food in Human Mastication. *Archives of Oral Biology*, 28(9): 813-819.

28. Maqbool, T. 2011. Understanding the Kinetics of Asphaltene Precipitation from Crude Oils. PhD dissertation, The University of Michigan, Ann Arbor, Michigan (2011).
29. McDowell, G.R. and Bolton, M.D. 1998. On the Mechanics of Crushable Aggregates. *Geotechnique*, 48(5): 667-679.
30. Mishra, B.K. 1999. Monte Carlo Simulation of Particle Breakage Process During Grinding. *Powder Technology*, vol. 110: 246-252.
31. Nierode, D.E. and Kruk, K.F. 1973. An Evaluation of Acid Fluid Loss Additives Retarded Acids, and Acidized Fracture Conductivity. Paper SPE 4549 presented at the Fall Meeting of the Society of Petroleum Engineers of AIME, Las Vegas, Nevada, U.S.A., 30 September-3 October.
32. Palisch, T., Duenckel, R., Bazan, L., Heidt, J.H., and Turk, G.A. 2007. Determining Realistic Fracture Conductivity and Understanding its Impact on Well Performance. Paper SPE 106301 presented at the SPE Hydraulic Fracturing Technology Conference, College Station, Texas, U.S.A., 29-31 January.
33. Palisch, T.T., Vincent, M.C., and Handren, P.J. 2010. Slickwater Fracturing: Food for Thought. *SPE Production & Operations*, 25(3):327-344.
34. Peterson, T.W. 1986. Similarity Solutions for the Population Balance Equation Describing Particle Fragmentation. *Aerosol Science and Technology*, vol. 5: 93-101.
35. Ramkrishna, D. 2000. Population Balances: Theory and Applications to Particular Systems in Engineering. San Diego, California: Academic Press.
36. Reid, K.J. 1965. A Solution to the Batch Grinding Equation. *Chemical Engineering Science*, vol. 20: 953-963.
37. Sharma, M.M. and Yortsos, Y.C. 1986. Permeability Impairment Due to Fines Migration in Sandstones. Paper SPE 14819 presented at the 7th Symposium on Formation Damage Control, Lafayette, Louisiana, U.S.A., 26-27 February.
38. Wen, Q., Zhang, J. and Li, M. 2015. A New Correlation to Predict Fracture Pressure Loss and to Assist Fracture Modeling in Sandy Conglomerate Reservoirs. *Journal of Natural Gas Science and Engineering*, April 2015 (in press). doi:10.1016/j.jngse.2015.04.007
39. Yuan, C. 2012. The Chemical Treatment to Remove Liquid Block in Hydraulic Fractured Well-A Simulation Study with Leak-Off. Paper SPE 158190 presented at the 2012 Energy Conference-Developing Resources for Sustainability at Port-of-Spain, Trinidad, 11-13 June.
40. Zhang, J., Kamenov, A., Zhu, D., and Hill, A.D. 2014a. Laboratory Measurement of Hydraulic Fracture Conductivities in the Barnett Shale. *SPE Production & Operations*, 29(3): 216-227.
41. Zhang, J., Ouyang, L., Zhu, D. and Hill, A.D. 2014b. Experimental and Numerical Studies on Reduced Fracture Conductivity due to Proppant Embedment in the Shale Reservoirs. Paper SPE 170775 presented at the SPE Annual Technical Conference and Exhibition in Amsterdam, The Netherlands, 27-29 October.
42. Zhang, J., Kamenov, A., Zhu, D., and Hill, A.D. 2015a. Measurement of Realistic Fracture Conductivity in the Barnett Shale. *Journal of Unconventional Oil and Gas Resources*, 11(2015): 44-52.
43. Zhang, J., Zhu, D. and Hill, A.D. 2015b. Water-Induced Fracture Conductivity Damage in Shale Formations. Paper SPE 173346 presented at the 2015 Hydraulic

Fracturing Technology Conference in the Woodlands, Texas, U.S.A., 3-5 February.

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65

Fig. 1—Population balance for size reduction.

Fig. 2—Determination of the selection function by Gardner and Austin (1962).

Fig. 3—Determination of the breakage function. Experiment data are from Gardner and Austin (1962).

Fig. 4—Algorithm to solve the population balance equation.

Fig. 5—Measured and modeled proppant size distribution (Dataset 1).

Fig. 6—Measured and modeled proppant size distribution (Dataset 2, Fredd et al. 2001).

Fig. 7—Width reduction by proppant grain rearrangement caused by inter-granular frictional slippage and pore collapse.

Fig. 8—Void volume reduction due to the grain rearrangement.

Fig. 9—The ratio of the proppant embedment depth to grain diameter correlated with closure stress and Young's modulus.

Fig.10—Workflow for the calculation of shale fracture conductivity.

Fig. 11—Median size and phi percentile deviation of proppant grain from the proppant size distribution.

Fig. 12—Porosity of sand packs at various stresses (Karner et al. 2005).

Fig. 13—Proppant distribution on rough shale fracture surface: (a) initial proppant pack with large void space above the proppants; (b) proppant grain rearrangement during the application of closure stress; (c) at high closure stress, proppants are crushed and the fracture surface asperities are deformed.

Fig. 14—Calculated and measured fracture conductivity of 40/70 mesh sands at various concentrations.

Fig. 15—Calculated and measured fracture conductivity of 40/70 mesh sands and 30/50 mesh sands at 0.10 lb/ft².

Fig. 16—Calculated fracture conductivity and measured fracture conductivity considering water damage in the Barnett shale.

Table 1—Measured fracture widths at the fracture surface valleys.

Proppant mesh size	Proppant concentration (lb/ft²)	Initial width (inch)	Initial width (mm)
40/70	0.06	0.024	0.610
40/70	0.10	0.037	0.940
40/70	0.20	0.066	1.676
30/50	0.10	0.035	0.889