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DRAFT: ISOTHERMAL COHERENT STRUCTURES AND TURBULENT FLOW PRODUCED BY A GAS TURBINE COMBUSTOR LEAN PRE-MIXED SWIRL FUEL NOZZLE

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ABSTRACT

The steady and unsteady isothermal fluid dynamics generated by an industrial low emission, lean premixed, fuel swirl nozzle designed by Solar Turbines Incorporated were investigated in this study. The experiments were carried out in a model optical can combustor operating at atmospheric pressures. Non-time resolved, planar Particle Image Velocimetry (PIV) measurements were taken at Reynolds numbers with respect to the nozzle throat diameter of ~50 000, ~100 000, and ~180 000. The time-averaged velocity fields were approximately self-similar, with the highest mass flow exhibiting a central recirculation zone (CRZ) with a slightly larger diameter. The results were analyzed using a methodology based on Proper Orthogonal Decomposition (POD) to extract the periodic structures in the flow and obtain the underlying stochastic turbulence field. This distinction between stochastic and coherent fluctuations is critical to properly model combustor flows. Coherent flow instabilities such as the precessing vortex core (PVC) and the propagation of axial/radial vortices were observed to significantly contribute to the mixing between the nozzle exit flow and the recirculated mass flow. Over 30% of the total fluctuation (difference between instantaneous and time-averaged velocity fields) kinetic energy was attributed to coherent structures throughout the inner shear layer between the swirling jet exiting the nozzle and the CRZ. Stochastic variability was prevalent close the liner wall and throughout the combustor domain after the swirling jet impinged on the wall, with <20% of the total fluctuation attributed to coherent structures. The normalized coherent and stochastic flow fields were also approximately self-similar with Reynolds number.

KEYWORDS:

Gas turbine combustor, swirl flows, proper orthogonal decomposition, combustor turbulence, combustor coherent structures, precessing vortex core

NOMENCLATURE

A	Cross-sectional area of the fuel nozzle [m^2]
a, b	Fourier polynomial coefficients
CRZ	Central Recirculation Zone
D_N	Nozzle diameter
f	Generic field (velocity/turbulence)
$\bar{f}$	Time averaged field
$\tilde{f}$	Coherent/periodic field
f'	Stochastic field
G	Axial flux of momentum [N]
ii	Total number of vectors in the i direction
jj	Total number of vectors in the j direction
K	Stochastic turbulent kinetic energy [$\text{m}^2 \text{s}^{-2}$]
$\dot{m}$	Mass flow [kg/s]
$\dot{m}_{ND}$	Non-dimensionalized mass flow

PIV	Particle Image Velocimetry
POD	Proper Orthogonal Decomposition
PVC	Precessing Vortex Core
P_s	Static inlet pressure to the combustor
R	Nozzle throat radius
S	Swirl number
TKE	Turbulent Kinetic Energy [$\text{m}^2 \text{s}^{-2}$]
T_s	Static inlet temperature to the combustor
u	X-velocity component
V	Velocity [m s^{-1}], bold indicates matrix of all points
V_{ref}	Reference velocity for different mass flows [m s^{-1}]
v	Y-velocity component
w	Z-velocity component
X	Axial coordinate along the combustor axis (direction of the bulk flow) [m]
Y	Vertical coordinate perpendicular to the combustor axis [m]
Z	Horizontal coordinate perpendicular to the combustor axis [m]
χ	Eigenvector , bold indicates matrix of eigenvectors
ϕ	POD mode, bold indicates matrix of POD modes
ν^t	Turbulent (eddy) viscosity
θ	Phase within the period of the coherent fluctuation
τ	Reynolds shear stress
Σ	Covariance matrix
ξ	Generalized coordinate dimension (either X, Y, Z)

Subscripts

k	Eigenvector index
n	Snapshot index (PIV)
R	Radial component
X	Axial component
Y	Vertical component (perpendicular to axial)
Z	Horizontal component (perpendicular to axial)
θ	Tangential component

1. INTRODUCTION

Swirling flows within combustors have been extensively studied for the past 50 years; with current work focusing primarily on understanding the vortex breakdown (VB) and unsteady phenomena that affect combustor operation. Comprehensive reviews on combustion in swirling flows were first written in the 1970s and early 80s by Lilley [1], Beér and Chigier [2], Syred and Beér [3], and Gupta et al. [4]. The vast contributions of the research community led to the current understanding and characterization of the central recirculation zone (CRZ), formed as the swirling jet exiting the fuel nozzle expands into the combustion chamber, recovering axial pressure, and eventually triggering the breakdown of the vortex and subsequent backflow [5]. High intensity flames with high combustion efficiencies are attained with swirling jets without the need of a blunt body for flame stabilization. No longer having to cool a blunt body, swirl fuel nozzles simplify cooling requirements while maintaining flame stability, short combustor length, and high combustion efficiency [6]. Recent work has focused on the unsteady behavior and instabilities that emerge in combustor flows as detailed by the work of Lieuwen [7], Lucca-Negro and O'Doherty [8], Syred [9], and Huang and Yang. [5]. Different types of VB, as well as periodic oscillations in the flow such as the Precessing Vortex Core (PVC) and the presence of axial-radial vortices, have been identified [9,10,11].

The size and characteristics of the CRZ and PVC are heavily dependent on the flow properties and velocity profiles at the exit of the nozzle [12, 13, 14]. Hallet and Toews [15] succinctly demonstrated experimentally and theoretically the impact the inlet velocity radial profiles have on the onset of flow reversal. Larger recirculation zones have been observed when using a central hub or a diverging fuel nozzle [1]. Seminal flow visualizations by Faler and Leibovich [11] described the dependence of the vortex breakdown phenomena on Reynolds numbers up to 10000 with respect to the nozzle diameter. At their highest tested Reynolds numbers only two breakdown modes,

axisymmetric and spiral, were identified. As the Reynolds number was increased, their observations indicated an upstream shift of the recirculation bubble location.

More recently, Terhaar and coauthors [16] have used axial injection to modify the flow profile entering a model combustor and study its effects on the formation and characteristics of the vortex breakdown. They identified a conical type of VB that was triggered at high axial flow injection rates. Carmack et al. [17] have studied radial and axial swirl nozzles used in industrial engines to show the differences in the recirculation bubble and flow characteristics. Strakey and Yip [18] performed isothermal flow field measurements using a representative slot swirler yielding a swirl number of 1.17 and a peak swirl angle of 51° . The authors commented on the unsteadiness and turbulence in the flow, but did not focus on its quantification. Ji and Gore [19] studied the time averaged and instantaneous turbulence and vorticity for an unconfined swirl stabilized flame using a nozzle with a swirl number of 2.4. Their results had limited extent and covered only a portion of the flow field, but provided tremendous insight into the importance of the instantaneous flow and the changes occurring between isothermal and reacting flows.

Notwithstanding, most of the available investigations in the open literature to date have been performed using idealized or representative fuel nozzles that allow for precise control of different geometrical or operating parameters [9,16,18]. The objective of the present investigation is to complement the information on idealized fuel nozzles with steady and unsteady flow field data from a modern industrial fuel injector. The study focuses on the isothermal aerodynamics of the low-emission SoLoNOx® fuel nozzle from Solar Turbines® Inc. Particle image velocimetry (PIV) measurements were taken at three different normalized mass flows, one of which was equivalent to the nozzle design point, matching the corresponding inlet Mach numbers. This information is critical for designers using numerical tools and for the validation and development of computational codes. Reynolds-Averaged Navier Stokes (RANS) calculations are routinely used in industry to simulate the flow within combustors. Strakey and Yip [18] have shown however that unsteady RANS models underpredicted the isothermal flow fluctuations. Modeling efforts by Grinstein [20] have focused on predicting swirling flows of practical importance, for which the author stressed that it is critical to rely on accurate velocity radial profiles at the exit of the nozzle. In view of these requirements for the proper modeling of realistic swirling flows within gas turbine combustors, there is still a need to obtain flow and turbulence data for industrial injectors.

To fully understand the unsteady features observed in the flow, the coherent and stochastic temporal components of the flow field were determined by means of a methodology based on Proper Orthogonal Decomposition (POD). In light of the vast amounts of data generated by modern measurements, it is essential to further develop and foment the use of exploratory data analysis techniques that can summarize the available information. Proper Orthogonal Decomposition (POD), also known in different fields as Principal Component Analysis (PCA) or Empirical Mode Decomposition (EMD), is known to be a suitable tool to study coherent structures in turbulent flows [21-25]. Recent work by Lengani et al. [26] have further improved current vortex identification methodologies [22,23] by incorporating a polynomial fit of the velocity fluctuations to better extract the coherent characteristics of the velocity field. These ideas have been recently applied by Berrino and coauthors to study the effects of confinement in a double swirl nozzle [27], proving the potential of the technique to extract the embedded coherent structures in combustor flows and obtain their phase characteristics. Recent interest in alternative uses of POD within the combustion and gas turbine communities include its use for flow validation [28] and structure identification [16]. The present work follows a methodology based on the work by Lengani and coauthors [26] to separate the flow into its mean, periodic, and stochastic components (triple decomposition), providing further information on the character of the turbulence generated by an actual fuel injector.

This study presents the first of a series of investigations that will cover combustor heat transfer and fluid dynamics at isothermal and reacting conditions. Tests at isothermal conditions provide only a partial view of the fluid dynamics in reacting flows. However, isothermal studies are a necessary prerequisite to fully understand the flow within a combustor, without the additional complications introduced by reaction such as sharp property gradients and multiple heat transfer modes.

2. EXPERIMENTAL SETUP AND METHODOLOGY

The experiments were conducted at the Advanced Propulsion and Power Laboratory (APPL) at Virginia Tech. A schematic of the test section is shown in Figure 1. The mass flow rate was calculated from the readings of a turbine flow meter upstream of the setup, with coincident temperature and pressure measurements. The uncertainty in the mass flow measurement was $\sim 2.03\%$ for all cases. However, the compressor in the laboratory provided a cyclic loading that led to a low frequency oscillation (< 0.005 Hz) on the supplied mass flow to the combustor of $\pm 1.4\%$. Accounting for the compressor oscillation, the overall uncertainty in the mass flow delivered to the test section was $\pm 2.5\%$.

Figure 1. Schematic of the test section.

The test section consisted of a settling chamber where the swirl nozzle was mounted flush with the dome plate, followed by a cylindrical quartz liner with an internal diameter of 203 mm and a length of 216 mm. The primary zone of the combustor was purposely designed to be as simple as possible, with no film or effusion cooling, in order to simplify the validation of numerical models. Pressure and temperature were monitored upstream of the nozzle in the settling chamber. A schematic of the fuel nozzle used, with approximate dimensions, is included in Figure 2. Air enters the nozzle on the left hand side as indicated by the blue arrows, passing through the swirl vanes that turn the flow and introduce a tangential velocity component. Downstream of the swirler vanes, injector ports for fuel are located. The fuel and air mix along the nozzle until it exits and expands into the combustor primary zone. No fuel was injected either through the main or pilot lines for the current experiments (reacting flow measurements are part of ongoing research).

Figure 2. Schematic of the SoLoNOx fuel nozzle. Flow direction is indicated by the blue arrows at the inlet of the nozzle.

The tests were performed at ambient pressures, which prevents any decoupling between Reynolds and Mach numbers for the setup. In the current work, both Mach and Reynolds numbers increase simultaneously for increasing mass flows. However, the Mach number is often a more relevant variable for the study of turbomachinery as it will determine the pressure loss characteristics and overall component performance. This is as long as the corresponding Reynolds number remains away from any critical transition points for the flow or boundary layer [2,29]. For this reason, the design non-dimensional mass flow ($\dot{m}_{ND}$) [30] for the fuel injector was matched in the experiments (Eq. 1).

$$\dot{m}_{ND} = \frac{\dot{m} \sqrt{RT_3}}{A P_3} \quad (1)$$

The design conditions of the fuel injector correspond to $\dot{m}_{ND} = 0.1359$. Additional measurements were taken for normalized mass flows (at ambient conditions) including 0.0377 and 0.0753. The three mass flows tested correspond to Reynolds numbers with respect to the fuel nozzle throat diameter of $\sim 50\,000$, $\sim 100\,000$, and $\sim 180\,000$. The measured static pressure drops across the combustor were 0.52%, 1.37%, and 4.13% respectively with increasing mass flows.

2.1 Velocity field measurements using Particle Image Velocimetry (PIV)

Two dimensional planar particle image velocimetry (PIV) measurements were taken at the planes indicated in Figure 3. Planes A-B are referred to as axial planes (XZ planes) and planes C-G are referred to as cross-sectional, radial-tangential, or azimuthal planes (ZY planes). Planes C-G were located at $X/D_N = \sim 0.01, 0.16, 0.91, 1.54,$ and 2.4 respectively. Plane C captured the swirler exit profiles and plane D the initial jet expansion into the primary combustion zone. Planes E and F are before and after the impingement of the jet on the liner respectively, and Plane G was taken to capture the dynamics close to the outlet of the combustor. The coordinate system origin was defined at the center of the swirl nozzle (the figure shows it displaced for clarity). The measurements were performed using a Nano-L 135-15 dual Nd:YAG laser system (5 mm laser sheet thickness), with a Flowsense 4M MkII camera (2048 $\times$ 2048 pixel resolution), and Dantec Dynamics® acquisition software. Glycol droplets were seeded into the settling chamber upstream of the swirl nozzle to act as tracers for the PIV measurement. The seed particle sizes delivered to the test section were estimated to be $< 2.5\mu\text{m}$ [31] yielding an expected velocity delay of $\sim 20\mu\text{s}$. For the highest mass flow (worst case), this time delay corresponds to a Stokes number of ~ 0.01 , based on the nozzle diameter and the mean axial velocity within the nozzle.

Figure 3. Measurement planes. The origin of the coordinate axis is located at the center of the nozzle (displaced in the figure for clarity).

The image pairs (frame A and B) were analyzed using an adaptive correlation procedure with a final interrogation area of 32×32 pixels and an initial interrogation area of 128×128 pixels. The field of view imaged was approximately $130 \text{ mm} \times 130 \text{ mm}$ for the XZ planes and $215 \text{ mm} \times 215 \text{ mm}$ for the ZY planes, yielding respectively a spatial resolution of $63 \mu\text{m}$ and $106 \mu\text{m}$. Neighborhood (3×3 median) and peak (1.2 maximum to minimum peak correlation ratio) validations were used to identify unphysical vectors. The PIV software substitutes the removed vectors based on neighboring velocities. This however can lead to a velocity bias close to walls or shear layers and hence no substituted vectors (replaced by the peak or neighborhood validation) were used in the analysis. 50% overlapping was used to increase the vector resolution and a total of 800 image pairs were acquired per plane. To acquire the data as accurately as possible, the acquisitions were split into 200 image pair runs, cleaning any fouling of the walls and optical liner between runs. Given the extent of the dataset, an automatic procedure to discriminate invalid velocity fields was implemented. This procedure consisted on removing any vector fields that had more than 80% substituted vectors, indicative of poor seeding, or incorrect background subtraction.

Challenges of PIV in confined spaces involve the presence of light reflections (glare) and background images in the acquisition which may bias or entirely prevent any reliable data retrieval. Variations in the laser intensity and seeding distribution prevent the satisfactory elimination of the background by the subtraction of the time averaged image. Moreover, because of uneven seeding in the flow, local normalization procedures, such as those suggested by Mejia-Alvarez and Christensen [32] for instance, are unsuccessful, as these amplify the noise in regions with few particles. The background removal procedure implemented consisted in 1) subtracting the local minimum for each image (9×9 pixel sliding local minimum), 2) calculating the time averaged corrected image, and 3) normalizing all the corrected images with respect to the averaged image (times a constant factor to increase contrast). An example of the implemented background removal procedure applied to an acquisition of Plane D is shown in Figure 4. Another important challenge to note when measuring complex swirling flows is the large range of speeds that must be resolved. Too large a time delay between laser pulses led to particles exiting either the interrogation area or the laser sheet (due to the strong out of plane motion in swirling flows). Too short a delay on the other hand led to negligible movement of the particles. The higher mass flow measurement represented the more difficult acquisition because of the larger quantities of PIV seed required and the consequent quick fouling of the optical liner. The glycol accumulating on the walls changed the laser glare as the experiment progressed, causing errors in the background removal and limiting the observations close to the liner and dome.

Figure 4. Background removal procedure example for PIV measurements.

The correlation algorithm yields an estimated subpixel accuracy of 0.1 pixels, which results in an error between $0.01V_{\text{ref}}$ and $0.06V_{\text{ref}}$ depending on the plane imaged. The data were also checked for consistency at the intersection between cross-sectional and axial planes, where the radial (Z) velocity was sampled twice independently. The average absolute difference in the radial velocities at the overlap between planes was $0.023V_{\text{ref}}$, consistent with the expected velocity accuracies. Additionally, the time-averaged axial velocity profile at $X/D_N = 0.05$ was used to estimate the airflow delivered to the test section (assuming axisymmetric flow) with an average discrepancy with the metered mass flows of 5.6%.

2.2 Proper Orthogonal Decomposition (POD) to identify coherent structures

The PIV system was capable of acquiring image pairs at a rate of 7.4 Hz, which cannot resolve the high frequency oscillations in the data (such as the PVC) or the convective transport of vortices. Since the measurements taken were not time resolved, separation of the stochastic and coherent structures in the flow was accomplished using a POD methodology as outlined by Lengani and coworkers [26,27]. The instantaneous flow fields (f) were divided into three temporal components (triple decomposition) including a time average ($\bar{f}$), a periodic ($\tilde{f}$) and a stochastic ($\check{f}$) part as described by Eq. 2.

$$f = \bar{f} + \tilde{f} + \check{f} = \bar{f} + f' \quad (2)$$

Where f' is equal to the total fluctuation with respect to the time averaged field.

Other authors have contributed with exhaustive explanations on the mathematics behind POD and its application to the analysis of turbulent flows [21-23]. For this reason the analysis procedure is only briefly described in the present work.

The velocity vector data for each time snapshot was originally arranged in a $i \times j$ matrix, where i and j correspond to the horizontal and vertical locations where a velocity vector was obtained. To perform POD, the data for each time snapshot was first vectorized, that is arranged as a single data vector of length $p = ii * jj$, where each element corresponded to different locations within a single acquisition. The vectorization for snapshot n of the V velocity field is shown by Eq. 3 (V is not restricted to a velocity component, as the analysis can also be applied to any other field such as vorticity or turbulent energy).

$$V_n = [v_{n,1}, v_{n,2} \dots v_{n,p}]^T \quad (3)$$

Which in matrix form, including all the PIV snapshots, becomes Eq. 4. The columns of matrix V contain a single snapshot at a given time, whereas the rows of the matrix contain all the observations at a single acquisition point.

$$V = [V_1, V_2, \dots, V_n] = \begin{bmatrix} v_{1,1} & v_{2,1} & \dots & v_{n,1} \\ v_{1,2} & v_{2,2} & \dots & v_{n,2} \\ \vdots & \ddots & \ddots & \vdots \\ v_{1,p} & v_{2,p} & \dots & v_{n,p} \end{bmatrix} \quad (4)$$

Subtracting the time-average of the dataset at all locations, $\bar{V} = [\bar{v}_1, \bar{v}_2, \dots, \bar{v}_p]^T$, yields the total fluctuation V' .

$$\mathbf{V}' = \mathbf{V} - \bar{\mathbf{V}}\mathbf{1} = \mathbf{V} \left(\mathbf{I} - \frac{\mathbf{1}\mathbf{1}^T}{n} \right) \quad (5)$$

Where $\mathbf{1}$ represents a row vector containing n ones and $\mathbf{I}$ is the $n \times n$ identity matrix. The goal in POD is then to obtain the coefficient column vector χ_1 of length n that yields the maximum variance when the fluctuation dataset is projected onto it. In other words, the objective is to find the linear combination χ_1 of the n flow field snapshots, such that $\mathbf{V}'\chi_1$ has maximum variance. The sample variance of $\mathbf{V}'\chi_1$ is calculated from Equation 6.

$$\text{var}(\mathbf{V}'\chi_1) = \frac{1}{n-1} (\mathbf{V}'\chi_1)^T (\mathbf{V}'\chi_1) = \frac{1}{n-1} (\chi_1^T \mathbf{V}'^T \mathbf{V}' \chi_1) \quad (6)$$

Recognizing that the covariance matrix, $\mathbf{\Sigma}$, of the flow field fluctuations is given by Equation 7, Eq. 6 can be reduced to the form given by Eq. 8.

$$\mathbf{\Sigma} = \frac{1}{n-1} (\mathbf{V}'^T \mathbf{V}') \quad (7)$$

$$\text{var}(\mathbf{V}'\chi_1) = \chi_1^T \mathbf{\Sigma} \chi_1 \quad (8)$$

Equation 8 represents an eigenvalue problem. The eigenvectors of $\mathbf{\Sigma}$ provide a set of p independent (i.e. uncorrelated) linear combinations χ_k of the PIV snapshots. The eigenvector with the highest eigenvalue λ_1 is equal to χ_1 , the eigenvector that yields the maximum projected variance $\text{var}(\mathbf{V}'\chi_1)$.

The covariance is a measure of how observations of a variable change with respect to simultaneous or equivalent observations of a different variable. In this analysis, each velocity location is treated as an observation and each time snapshot as a different variable. Physically, the covariance matrix is a measure of how the fluctuations in the velocity field are related between acquisitions. It calculates how consistent are the features in the different snapshots (either overall positive or negative covariance). Periodic vortical structures convecting with the mean flow will show maximum positive covariance for two snapshots that capture the vortex in the same location (in phase), and decreasing covariance when the vortex signal is shifted (out of phase). The eigenvalues (λ_k) and eigenvectors (χ_k) of the covariance matrix yield the significance of each PIV snapshot to the variance in the data. The eigenvalue represents the relative amount of variance captured by the corresponding eigenvector, while the eigenvectors weight each snapshot according to its significance. A geometric description of this process can be attained by visualizing each flow snapshot representing an axis in an n -dimensional space, where the entire velocity fluctuation data is plotted (e.g. the first point, corresponding to the location $i = j = 1$ in all the snapshots would have coordinates $v'_{1,1}, v'_{2,1} \dots v'_{n,1}$). The eigenvectors of the covariance matrix identify an orthogonal (independent) set of directions along which the plotted data is aligned.

The k^{th} POD mode (ϕ_k) represents the projection of the entire velocity fluctuation dataset ($\mathbf{V}'$) into the k^{th} eigenvector as calculated by Eq. 9.

$$\phi_k = \mathbf{V}'\chi_k \quad (9)$$

The elements of the eigenvector indicate how each PIV acquisition contributes to the corresponding mode. The first mode (formed with the eigenvector of the highest eigenvalue) represents the velocity field that captures the highest amount of time variability in the dataset. Subsequent modes (lower eigenvalues) capture decreasing amounts of time variability. The data can then be decomposed according to the matrix multiplication of the full eigenvector matrix (χ) and the POD mode matrix (ϕ) as shown by Eq. 10.

$$\mathbf{V}' = [\phi_1, \phi_2, \dots \phi_n] \times [\chi_1, \chi_2, \dots \chi_n]^{-1} = \boldsymbol{\phi} \boldsymbol{\chi}^{-1} \quad (10)$$

This formulation can be used to filter a portion of the variation in the data according to selected POD modes, by setting the corresponding unwanted eigenvectors to zero. Other researchers have removed the low energy modes from the dataset to eliminate random errors during the measurement or the contributions of turbulence to the data [23,27]. Since identifying the turbulence is of importance for the current investigation, and POD is unable to

differentiate between random errors in the acquisition and actual low energy flow perturbations, all the modes were retained.

Per the discussion in Legrand [22,23], periodic convective flows yield coupled POD modes, with the two most energetic modes and eigenvectors representing the sine and cosine components of a convecting flow oscillation. Figure 5 shows the first two POD modes for the axial velocity in plane A ($Re = 180\,000$). The patterns in these two modes are consistent with propagating vortices. Reiterating however, the POD modes cannot distinguish between different oscillations in the flow and hence some of the features are only indirectly related to the convecting vortices. It will be shown in Section 3.2 that the high variance identified on the bottom left of the first mode is associated with the inner recirculation zone and the formation of the PVC. The high variability on the corner recirculation side identified on the second mode is associated with a flapping motion of the swirl jets exiting the nozzle as these interact with the convecting vortices. In flows with multiple excitation sources with different phase characteristics, it will be important to accurately select locations where each excitation source dominates the structures when analyzing the data.

Figure 5. The two most energetic POD modes for the axial velocity component (u) observed for the measurements in plane A.

Regions of the velocity field where stochastic turbulence dominated, did not show a strong pattern. Hence, to extract the phase of the periodic oscillation in the flow, it was important to select locations in the flow field where the oscillation signal was highest. As done by Lengani et al. [26], a point in the region where each POD mode showed the highest signal was selected to extract the oscillation (shown by a black dot in Figure 5). Lengani and coauthors [26] showed that in order to obtain the appropriate propagation direction for the extracted vortical features, the location of lowest signal (most negative) in the first POD mode and the location of the highest signal (most positive) in the second POD mode must be selected.

The velocity components (u , v , w) are the actual variables that fluctuate, and hence the data was first projected to the two eigenvectors that carry the oscillation signal (Equations 11 and 12), and then the phase θ_n associated with each PIV snapshot within the oscillation was calculated (Eq. 13) from the assumption that the first eigenvector represents the sine component, and the second the cosine component of a convecting oscillation in the flow [22,23,26].

$$V'_{f,(1)} = [\phi_1, \phi_2, \dots \phi_n] \times [\chi_1, 0_2, \dots 0_n]^{-1} \quad (11)$$

$$V'_{f,(2)} = [\phi_1, \phi_2, \dots \phi_n] \times [0_1, \chi_2, \dots 0_n]^{-1} \quad (12)$$

$$\theta_n = \arctan\left(\frac{V'_{f,(1)}}{V'_{f,(2)}} \frac{\max(V'_{f,(2)})}{\max(V'_{f,(1)})}\right) \quad (13)$$

The POD modes help in two ways, first to identify the location where the oscillation signal was most significant (highest variance in the data), and second to specify the corresponding eigenvector that defines how each PIV snapshot is contributing to the oscillation. After calculating the associated phase for each PIV snapshot, the data was rearranged according to the phase. Figure 6 shows the arranged filtered velocity at the location of highest oscillation signal ($V'_{f,(1)}$ and $V'_{f,(2)}$). Note that the signal reproduces the assumed sinusoidal pattern.

Figure 6. Filtered axial velocity measurements at the location of highest oscillation signal arranged according to the calculated phase.

After identifying the phase of each PIV snapshot, the entire dataset was arranged according to its phase within the oscillation. To perform the phase averaged velocity measurements, Lengani et al. [26] suggest to fit a 5th order polynomial to the velocity history at each pixel location. A limitation of using a 5th order polynomial is that the fit may be heavily biased by outliers in the dataset, particularly at the end/start of the oscillation period. An alternative that was adopted in the current investigation was to instead use a Fourier polynomial at each pixel location as defined by Eq. 14.

$$V'(\theta) = a_0 + a_1 \cos(\theta) + b_1 \sin(\theta) + a_2 \cos(2\theta) + b_2 \sin(2\theta) + a_3 \cos(3\theta) + b_3 \sin(3\theta) \quad (14)$$

The advantages of the Fourier polynomial model is that the results are not heavily biased by the data points close to the end and start of the oscillation period ($\theta = -\pi$ and $\theta = \pi$), and that the fitted model is in better agreement with the underlying physical phenomenon of convecting periodic oscillations. Figure 7 compares the results obtained using a 5th order polynomial fit [26,27] and the Fourier model herein suggested for an arbitrary pixel in plane D that showed a significant oscillation signal. At a phase angle of $-\pi$, the polynomial and Fourier model differ by $\sim 30\%$. The noise introduced by the polynomial fit yielded a less uniform velocity field, compared to that obtained from the Fourier model. The disadvantages of the Fourier model were its complexity and the potential lack of convergence in regions where no oscillation was present.

Figure 7. Comparison between 5th order and Fourier polynomial fits to the measured data arranged according to the calculated phase within the oscillation.

The POD modes for the horizontal and vertical velocities for each plane were calculated, the decision between choosing either the horizontal or vertical velocity modes for phase identification was based on which captured the highest fluctuation energy percentage in the flow (based on the eigenvalues for the first and second eigenvectors).

3. EXPERIMENTAL RESULTS

The analysis of the time-averaged velocity fields is first presented, followed by the analysis of the total velocity fluctuations and unsteady fluid dynamics. Given the extent of the dataset, only measurements that were deemed relevant were discussed. For simplicity, the three operating conditions tested are referred to in terms of the equivalent Reynolds number. The velocity fields were normalized with respect to a reference velocity corresponding

to the ideal average axial velocity exiting the swirler at the target mass flows. The reference velocities were equal to 11.01, 22.02, and 39.64 m s⁻¹ respectively for increasing operating mass flows.

The total fluctuating velocity components (u' , v' , w') were readily available from the instantaneous measurements after the mean field ($\bar{u}$, $\bar{v}$, $\bar{w}$) had been computed. An overall turbulence kinetic energy (TKE), per the traditional Reynolds decomposition, was calculated according to Equation 15.

$$\text{TKE}_{XYZ} = \frac{1}{2} \overline{u'^2} + \frac{1}{2} \overline{v'^2} + \frac{1}{2} \overline{w'^2} \quad (15)$$

The TKE was normalized with respect to the square of the reference velocity (V_{ref}^2) to maintain same order scales between the three operating mass flows tested. When only two components of the velocity field were available, a two-dimensional TKE was calculated (which underestimates the total three dimensional fluctuation energy at any given point). The total TKE as calculated by Eq. 15 is not representative of the actual turbulent field as it includes the contribution of coherent periodic structures in the flow. To obtain the underlying stochastic turbulent field, the overall TKE was separated into its coherent and stochastic components (triple decomposition).

The main non-dimensional parameter to characterize combustor rotating flows is the swirl number. The definition used in this study is given by Eq. 16.

$$S = \frac{G_\theta}{R G_x} \quad (16)$$

Where G_θ is the axial flux of angular momentum, G_x is the axial flux of axial momentum and R is the throat radius of the fuel nozzle. The calculation of the momentum fluxes was simplified as is typically done in the available literature. Fluctuating components were not accounted for, and the axial pressure distribution was not considered for the axial momentum.

3.1 Time averaged results: Effect of increasing mass flow

Figure 8 and Figure 9 show the time averaged results for the maximum and minimum Reynolds numbers measured at planes A and D. The white outline in the velocity field of plane A indicates $\bar{u} = 0$, matching the mean location of the shear layers and the stagnation point at the combustor liner wall.

Figure 8. Time averaged results for Plane A at the minimum and maximum mass flows, flow direction is bottom to top.

Figure 9. Time averaged results for Plane D at the minimum and maximum mass flows.

Other authors have reported that the normalized impingement location and overall flow field is approximately independent of mass flow after a self-similarity threshold [18, 33,34]. The reason for the self-similarity is that the mechanisms that dominate the flow field within the combustor become inertia dominated (viscous effects are negligible). This self-similarity is of practical importance because it implies that the isothermal combustor flow field can be numerically or experimentally modeled at lower Reynolds numbers, incurring in lower costs, without significantly sacrificing the scalability of the results. For the range of mass flows tested, the time averaged results were approximately self-similar, with only small differences related to a slightly larger central recirculation zone with increasing mass flow as observed from the azimuthal cross-sections in Figure 9. The larger recirculation bubble further forced the swirling jets radially outward (barely observable in the time averaged flow of plane A). This larger recirculation for the $Re = 180\,000$ case was not related to a significant upstream shift in the recirculation bubble. An analysis of the maximum and minimum time-averaged axial and radial velocities along the combustor (planes A and B), given by Figure 10, showed that the maximum recirculation velocity occurred at $X/D_N = 0.6 - 0.7$ with a magnitude of $-0.48V_{ref}$ for the three mass flows tested. The maximum axial velocity decayed linearly from the nozzle outlet to $X/D_N \approx 1.4$ and asymptoted to a constant value of $\sim 0.28V_{ref}$. Past $X/D_N \approx 1.4$, the maximum radial velocity also decayed to approximately $0.03V_{ref}$, indicative of the swirling jet reaching an equilibrium in radial expansion as it was constrained by the liner wall.

Figure 10. Maximum and minimum axial and radial velocities along the combustor measured from planes A and B. The minimum axial velocity is approximately equal to the central recirculation velocity at $Z/D_N = 0$.

At individual planes only two dimensional velocity components were available. However, at the intersection between axial and azimuthal planes, the three dimensional profiles were calculated. The time averaged velocity profiles and the total fluctuation kinetic energy (including both stochastic and coherent structures) are shown in Figure 11 and Figure 12 respectively for different axial stations along the combustor. The $X/D_N = 0.01$ location approximately captured the nozzle exit profiles (for the 180 000 Reynolds number case, the closest axial data available was at $X/D_N = 0.05$). The axial and tangential velocity profiles for $Re = 180\,000$ had a shift in the radial

direction ($\sim 0.02D_N$) for $X/D_N = 0.16$. The radial shift resulted from the slightly larger central recirculation zone at the highest mass flow as previously explained. The profiles in general are consistent with the self-similarity assumption. The swirl jet broadens from $X/D_N = 0.16$ to 0.91 due to shear and entrainment with the surrounding flow as it exits the nozzle and expands into the primary combustion zone. Based on the $X/D_N = 0.01$ profiles, the swirl number at the exit of the nozzle (Eq. 16) was on average 0.74 ± 0.04 based on the three mass flows measured. Similarly, the mean flow turning angle $\alpha = \tan^{-1}(V_\theta/V_X)$ was $47.4^\circ \pm 2^\circ$ which compares well with the nozzle vane design angles.

Figure 11. Velocity profiles at several axial locations along the combustor.

Figure 12. Total fluctuation kinetic energy including stochastic and coherent fluctuations at different X/D_N locations along the combustor.

The total fluctuation TKE shows strong anisotropy at all axial locations. It is now well understood that combustor flows feature strong asymmetry due to the presence of coherent structures close to the fuel nozzle, particularly the PVC [8,9]. For the cross-section at $X/D_N = 0.16$, the highest total fluctuation energy was located in the core of the vortex (dominated by the radial component), dropping abruptly across the jet. A second peak in the total TKE was observed at the outer shear layer of the swirl jet, associated with oscillations of the corner recirculation and separation at the edge of the nozzle. Within the jet (shifted slightly to the inner shear layer side), the main component of the fluctuations was in the axial direction. This will be shown in the following section to stem primarily from the coherent structures in the flow. The recirculated mass flow in the recirculation bubble appears to have fairly isotropic total turbulence beyond $X/D_N \geq 0.91$.

After impingement on the liner (estimated at around $X/D_N \approx 1.2$ per plane A measurements), the swirl jet attached to the wall until the flow exited the combustor. Measurements at representative axial locations past the impingement point of $X/D_N = 1.54$ and 2.40 show a distinct decay in the maximum TKE. Prior to the jet interacting with the liner wall, the total maximum TKE decayed due to viscous dissipation and expansion of the coherent vortices from $\sim 0.25V_{ref}^2$ to $\sim 0.16V_{ref}^2$ over a distance of $0.9D_N$. Interaction with the wall led to further attenuation of flow oscillations (dissipation of vortices), decreasing the maximum turbulent energy to 0.05 by $X/D_N = 1.54$.

3.2 Coherent structures in the flow: The precessing vortex core

The total turbulence calculated from the time averaged velocity fields only provides a partial view of the system dynamics as it fails to distinguish between the flow variations introduced by coherent structures and the underlying stochastic turbulence. The distinction is relevant for the modeling of combustor flows. Coherent structures are

periodic and arise due to instabilities in the flow that can be partially predicted by three dimensional unsteady RANS solvers [35]. Large Eddy Simulations (LES) predict both the stochastic and coherent structures, as well as their interaction, at a higher computational cost. Computational models can hence be optimized with information on the scale of the stochastic and coherent structures and the location where either plays a primary role. This information is also relevant to understand how the coherent structures impact the mixing within the burner and the asymmetry of the flow.

As described in the methodology section, the periodic oscillations were extracted from the velocity field using a POD methodology [22,23,26,27]. Swirling flow studies [7-10] have shown that the center of the swirl generated vortex is not located at the geometric center of the swirl nozzle, as would be suggested by the time averaged fields, and actually precesses around the axis of the combustor. This is a well-known oscillation in swirling flows known as the PVC. The PVC constitutes a periodic, non-random oscillation that is enhanced in burners operating with premixed fuel [9].

Figure 13 shows the result of the POD analysis for the measurements at Plane D for the highest mass flow. The progression of the vortex center along the inner shear layer was identified as it precessed around the geometric center of the swirl nozzle (with a precession radius of $R_p \approx 0.17 - 0.2D_N$). As it rotated, it convected a portion of the swirling jet mass towards the axis of the combustor. This led to a depleted swirl jet immediately downstream of the PVC, and to a degradation of the tangential velocity fields by 19 – 25% with respect to the time averaged velocities. Similarly, upstream of the vortex core the PVC strengthened the swirling jets by the same amount by convecting flow from the axis of the combustor to the swirl jet. This enhanced entrainment generated by the PVC in and out of the central region of the combustor is critical to properly predict the mixing between the recirculated gases and the fresh air/fuel mixture carried by the nozzle exit swirl jet. A similar tangential velocity field around the swirling jet was reported by Syred [9] using LDA measurements, but the mixing between the vortex core and the swirling jet was not clear.

Figure 13. Phase progression showing the PVC at plane D for Reynolds number 180 000.

A convecting radial/axial vortex was also observed on plane A, as depicted by the phase-time progression shown in Figure 14 for the design mass flow case. The red outline shows the location of zero axial velocity for the

instantaneous time-phase displayed, while the white outline shows that obtained from the time average (as given by Figure 8). The red outline traces the path of the shed axial/radial vortex along the shear layer. From Fig. 11, the PVC location can be identified in an axial/radial plane (XZ) by the change in the radial velocities within the recirculation bubble that occur as the PVC crosses the plane. Negative radial velocities indicate the start of the PVC, and positive radial velocities its end. The radial velocity was therefore used to synchronize the phase angle between the axial-radial and azimuthal-radial planes (Figure 13 and Figure 14). Figure 15 shows the phase-synchronized average radial velocity within the central recirculation ($0 < Z/D_N < \sim 0.3$) from plane A at $X/D_N = 0.16$, depicted as V_Z , and from plane D at $Y/D_N = 0$, depicted as V_R . Figure 15 shows the normalized coherent velocities from all Reynolds numbers, suggesting that the coherent velocity field is also self-similar. In fact, no significant differences were observed with Reynolds number in the structure, normalized velocity magnitude, or phase characteristics of the coherent flow features. The consistency between Reynolds numbers validated the robustness and repeatability of the analysis methodology. Furthermore, this observation supports that the mechanisms that drive the oscillations are independent of Reynolds number; hence pointing to inviscid instability [7].

Figure 14. Phase progression at plane A showing the downstream convection of axial/radial vortices.

Figure 15. Average radial velocity within the central recirculation ($0 < Z/D_N < \sim 0.3$) for phase synchronization between axial-radial and azimuthal-radial planes (A and D).

Going back to the axial/radial eddies, it is then clear that these form as the PVC crosses the axial-radial plane, with the center of the PVC matching the center of the axial eddies. These measurements however provide no evidence that the vortices are actually following an axial path (constrained only to the XZ plane) as suggested by Figure 14. The three dimensional nature of the flow cannot be neglected. The flow features observed in the instantaneous PIV measurements are more likely explained by an axial/radial vortex that rotates with the PVC along the inner shear layer while propagating downstream. The vortex would then follow a spiral pattern as has been previously reported in literature [5,7].

The identified vortex propagates axially following a sinusoidal pattern around the time-averaged shear layer location. The shedding spatial wavelength, defined as the distance between two consecutive vortices along the inner shear layer (distance between two peaks in the red outline of Fig. 12), was $\sim 1.14D_N$. The axial vortex dissipated fully at approximately $X/D_N = 1.2$, as the vortex interacted with the liner wall. Between successive vortices there was a region of strong shear separating the swirl jet and the recirculation zone, similar to Kelvin-Helmholtz instability patterns [7].

The flow continues to be asymmetric until the jet collides and attaches to the wall. A strong coherent asymmetry was observed until $X/D_N = 0.91$, but no periodic flow oscillations could be identified for $X/D_N = 1.54$, as shown in Figure 16. This result suggests that the large coherent structures associated with the PVC are only relevant prior to interaction with the liner wall.

Figure 16. Extracted oscillations from plane E and F (left and right respectively) measurements. Plane E (left, $X/D_N = 0.91$) showed a significant propagating oscillation, Plane F (right, $X/D_N = 1.54$) showed no oscillation patterns. Note the different color scales.

An additional consequence of the PVC was an oscillation of the axial and radial velocities (and momentum) at the outlet of the fuel nozzle. For the phase-time $2 < \theta < 3$ radians in Figure 14, the mean axial velocity exiting the swirl nozzle was maximum ($1.06 - 1.14 V_{ref}$), decreasing on average by 44% for all mass flows tested at a phase-time of $-1 < \theta < 0$ radians. This decrease in axial flow speeds coincided with a simultaneous increase in radial speeds at the outlet of the nozzle as the trailing portion of the PVC pushed the jet radially outward.

3.3 The stochastic turbulent field

The total fluctuation kinetic energy was separated according to its coherent and stochastic components. Figure 17 shows the areas dominated by coherent structures for planes A and D respectively. The coherent vortices play a significant role in the inner shear layer and recirculation zone close to the nozzle ($>40\%$ of the total TKE in both the XZ and YZ planes). Close to the liner wall and beyond $X/D_N \approx 1.5$, the stochastic turbulence dominated ($<20\%$ contribution to the total fluctuation kinetic energy was attributed to coherent structures). Profiles of the stochastic turbulent kinetic energy at the different cross-sections are shown in Figure 18.

Figure 17. Coherence structure contribution to the total fluctuation kinetic energy (including coherent and stochastic components) for planes A (left) and D (right). Two dimensional approximation.

Figure 18. Stochastic turbulence properties at downstream locations along the combustor. The stochastic TKE was calculated according to triple decomposition.

Close to the combustor outlet, at $X/D_N = 2.4$, the stochastic turbulent kinetic energy is ~ 7 times smaller than the average observed at the nozzle outlet, but the turbulence intensity was on average 47% due to the low flow velocities (the recirculated intensity was 67%). The exiting flow along the combustor walls at $X/D_N = 2.4$ carries on average ~ 1.3 times higher stochastic turbulent energy ($0.011V_{ref}^2$) than does the flow recirculating back to the combustor ($0.0084V_{ref}^2$). The difference between the total and stochastic turbulence at $X/D_N = 2.4$ was $\sim 10\%$, indicating that the coherent structures only play a small role in the fluctuations at the combustor outlet.

For axisymmetric flows, it is generally assumed that the tangential and radial components of the turbulent energy are equal, which is not entirely satisfied in the combustor flow shown, even when the mean velocity profiles appear axisymmetric. The coherent structures further increase the anisotropy as shown in Figure 12. This anisotropy appears to be confined to the swirling jet, with fairly isotropic turbulence in the inner recirculation (except near the nozzle). The results corroborate that the implicit assumption of steady RANS eddy viscosity models ($k - \omega$, $k - \epsilon$ for instance) of isotropic turbulence is invalid for strongly swirling flows with internal recirculation as discussed by Sloan and coworkers [13].

The components of the stochastic TKE given in Figure 18 (TKE_x , TKE_z , TKE_r , TKE_θ) are equivalent to half the normal Reynolds stresses in the respective directions. These are analogous to turbulent pressures in the development of fluid momentum equations and typically contribute little to mean flow momentum transport (gradients are small). An often more important component to understand the interaction between mean and turbulent motion are the Reynolds shear stresses. These are shown in Figure 19 for planes A and D at the design Reynolds number, according to the formulation given by Equation 17.

$$\tau'_{xz} = -\overline{\tilde{v}_x \tilde{v}_z} \quad \tau'_{yz} = -\overline{\tilde{v}_y \tilde{v}_z} \quad (17)$$

Figure 19. Turbulent Reynolds shear stress distribution for plane A (left) and D (right).

The Reynolds shear stresses are highest along the inner and outer shear layers of the annular jet. In these regions the stochastic fluctuations in the respective planes are correlated and play an important role in enhancing transport phenomena. The Reynolds stresses are modeled numerically according to different approximations, of which the Boussinesq approximation is commonly used [13]. The Boussinesq hypothesis relates the turbulent Reynolds stresses to the mean flow velocity gradients using a turbulent or eddy viscosity (ν^t) according to Equation 18 (Einstein notation was used, i.e. repeated indices in a term indicate summation over all indices).

$$-\overline{\tilde{V}_i \tilde{V}_j} = \nu^t \left(\frac{\partial \overline{V}_i}{\partial \xi_j} + \frac{\partial \overline{V}_j}{\partial \xi_i} - \frac{2}{3} \frac{\partial \overline{V}_k}{\partial \xi_k} \delta_{ij} \right) - \frac{2}{3} K \delta_{ij} \quad (18)$$

Where K is the stochastic TKE (not the total fluctuation kinetic energy as given by Eq.15), ξ is a generalized coordinate dimension (X , Y , or Z for the i , j , k indices), and δ_{ij} is the Kronecker delta. For an incompressible flow, conservation of mass yields $\partial \overline{V}_k / \partial \xi_k = 0$. The last term in Equation 18 is included to satisfy the definition of K , and requires the eddy viscosity to be constant for the normal Reynolds stresses ($i = j$). The assumption of constant eddy viscosity is traditionally applied to all components of the Reynolds stresses, which often deviates from observations in complex flows [7]. Experimentally investigating the validity of the Boussinesq hypothesis to the isothermal combustor flow measured is constrained by the accuracy and precision of the data, particularly when evaluating gradients from discrete measurement points. The $\partial \overline{V}_X / \partial X$, $\partial \overline{V}_Y / \partial Y$, and $\partial \overline{V}_Z / \partial Z$ velocity gradients normalized with respect to V_{REF} / D_N at $X / D_N = 0.16$ are shown in Figure 20. The velocity gradients were anti-correlated, but only approximately satisfied continuity assuming the flow was incompressible due to the limited accuracy of the data (the addition of the three normalized velocity gradients was equal to 0.27 ± 0.8). In fact, estimations of the turbulent viscosity from the normal Reynolds stresses yielded large variability (partially because it becomes undefined for zero velocity gradients), and did not satisfy the requirement of isotropy. With this in mind, the relevant normalized mean flow velocity gradients and the Reynolds shear stresses are shown in Figure 21 and Figure 22 respectively at selected X / D_N locations (upstream and downstream of the jet impingement on the liner).

Figure 20. Normalized velocity gradients at $X / D_N = 0.16$. Normalization was done with respect to V_{REF} / D_N . All Reynolds numbers are included following line style conventions from previous figures.

Figure 21. Normalized Reynolds shear stresses (stochastic fluctuation only) at select X/D_N locations.

Figure 22. Normalized mean flow velocity gradients at select X/D_N locations. Normalization was done with respect to V_{REF}/D_N . All Reynolds numbers are included following line style conventions from previous figures.

The Boussinesq hypothesis presented in Equation 18, proposes that the Reynolds shear stress $\overline{\tilde{v}_i \tilde{v}_j}$ is proportional to $\partial \overline{\tilde{v}_i} / \partial \xi_j + \partial \overline{\tilde{v}_j} / \partial \xi_i$, with the proportionality equal to the eddy viscosity. This is qualitatively satisfied for τ'_{xz} at the two cross-sections, with a spatially varying eddy viscosity. A vague anti-correlation to the velocity gradients can be suggested for the τ'_{zy} Reynolds stress component, but this is not clear due to the uncertainties in both the shear stresses and velocity gradients in the radial-azimuthal planes. A detailed analysis of the applicability of the Boussinesq approximation and alternative potential modeling strategies for the turbulent Reynolds stresses is beyond the scope of the current work.

4. CONCLUSIONS

The isothermal steady and unsteady fluid dynamics generated by an industrial low emission fuel nozzle at atmospheric pressures were investigated. A triple decomposition of the velocity field was performed based on a POD methodology, resulting in the separation of the instantaneous velocity into its time-averaged, coherent (periodic), and stochastic components. The methodology allowed for the study of the PVC and its downstream propagation.

It was observed that the coherent structures account for over 30% of the total fluctuation kinetic energy along the inner shear layer and within the recirculation zone close to the nozzle exit plane. No dominant coherent fluctuations were identified beyond the point at which the swirl jet impinged on the combustor wall. Strong asymmetry was also observed in the flow velocities, with the PVC causing oscillations compared to the time-averaged results in the axial and tangential velocities close to the nozzle exit of 44% and over 19% respectively. These results imply that to properly capture mixing between the swirling jet and the recirculated gases, coherent structures must be accounted for in numerical models.

The stochastic velocity fluctuations extracted after identifying the coherent velocity field showed that there was anisotropy in the normal Reynolds stresses. This anisotropy was most significant within the jet (particularly the axial component of the fluctuations) while the inner recirculation had relatively isotropic normal Reynolds stresses. The shear Reynolds stresses were calculated at selected planes and a qualitative comparison with the calculated mean flow velocity gradients suggested that the two are related (Boussinesq approximation). The coherent structures further magnified the anisotropy in the total fluctuation.

Contrary to the common conception that vortex breakdown in combustor flows is sensitive to the operating Reynolds number, for the results in the present investigation, the normalized mean velocities, coherent structures, and stochastic flow were relatively independent of Reynolds number. This self-similarity with mass flow was attributed to the decreasing relevance of viscous effects at high Reynolds numbers, and the inviscid instability mechanisms that drive the oscillations in swirling flows [7]. The data presented is critical to further understand turbulent mixing and the dynamics within combustors, and to develop accurate models for these flows.

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