Gas entrainment behaviors in the formation and collapse of a ventilated supercavity

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Abstract

The present work reports some interesting gas entrainment behaviors in the formation and collapse of a ventilated supercavity under steady and unsteady flow conditions. Our experiments show that the gas entrainment required to establish a supercavity are much greater than the minimum gas entrainment required to sustain it, and these gas entrainment values depend on Froude (Fr) number, cavitator size and the flow unsteadiness. Specifically, the measurements of the formation gas entrainment coefficients under different Fr numbers indicate that it does not monotonically increase with Fr but displays increasing and decreasing trends in different regimes of Fr . On the other hand, the collapse air entrainment coefficient initially decreases with Fr and then approaches to a constant. Similar trends of formation and collapse gas entrainment coefficient are observed for different cavitator sizes. Moreover, the introduction of unsteady gusts causes a slight monotonic increase in the formation and collapse gas entrainment

requirements. Our study points out the crucial factors to be considered in the estimation of gas storage requirements for a ventilated supercavitating vehicle.

Keywords: Supercavitation, ventilation, supercavity formation, supercavity collapse, ventilation hysteresis.

1. Introduction

High speed underwater vehicles experience a tremendous amount of flow resistance when moving underwater due to skin friction drag [1-3]. Supercavitation is a special case of cavitation which can be employed to create a bubble of gas/vapor inside water that is large enough to encompass an object (or vehicle) travelling through the water. Supercavitating vehicles are a revolutionary step in the direction of underwater locomotion because it can lead to drag reduction of as high as 90%, facilitating a substantial increase in speed [4]. The phenomenon of supercavitation is generally characterized by non-dimensional parameters such as cavitation number, $\sigma = 2(P_\infty - P_c)/\rho U^2$ and Froude number ($Fr = U / \sqrt{gd_c}$) where P_∞ , P_c , ρ , U , g and d_c denote the ambient pressure, the internal cavity pressure, water density, flow velocity, gravitational acceleration and cavitator size, respectively. A supercavity is attained at small σ ($\sigma < 0.1$) and this can be accomplished naturally or artificially. A natural supercavity refers to a large attached vapor cavity which is generated by a body travelling at very high speeds (> 90 knots at a depth of 1 m in open waters) or at low ambient pressure [5]. On the other hand, an artificial or ventilated supercavity is generated by blowing non-condensable gas into the low pressure region near the nose of the vehicle. The non-condensable gas increases the cavity pressure, allowing for low σ to be attained at much lower speeds. Ventilating supercavitation has numerous advantages over natural supercavitation viz. greater adaptability

for vehicle maneuvering and control. Also, some of the typical negative effects encountered in natural cavitation viz. surface damage, buffeting and vibrations etc. are absent in ventilated supercavitation [6]. A vehicle can be optimally ventilated in case of maneuvering etc. to circumvent the possibility of cavity collapse at different stages of vehicle operation.

A typical design strategy of a supercavitating vehicle entails accelerating the vehicle to a high speed at which a natural supercavity can be sustained. The drastic drag reduction required to attain high speeds at the initial launch of the vehicle is achieved by ventilated supercavitation. Further, the operation of a ventilated supercavitating vehicle depends on its ability to supply sufficient gas to fill the cavity at different flow conditions and at different stages of vehicle motion. The ventilation requirements for a ventilated supercavity is characterized by the gas entrainment coefficient **at the standard conditions**, $C_Q = \dot{Q}/Ud_c^2$, where $\dot{Q}$ denotes the gas ventilation rate **at standard conditions (i.e. at a temperature of 273 K and a pressure of 1 bar. Experimentally $\dot{Q}$ is a direct reading from a mass flow controller)**. The determination of the gas storage requirements for a ventilated supercavitating vehicle requires information on gas supply rate to form and sustain a steady supercavity at different flow conditions. Thus, it is important to understand the gas entrainment behaviors relating to supercavity formation and collapse.

A number of prior studies have investigated the gas entrainment behavior of ventilated supercavity under a wide range of conditions [5-12]. Ventilation hysteresis is intricately related to the formation and collapse gas entrainments and previous studies on ventilation hysteresis have been reported [12]. Ventilation hysteresis refers to a phenomenon whereby the supercavity can be sustained at much lower values of gas entrainment than required to form it, as shown in Fig. 1.

Fig. 1: A typical ventilation hysteresis curve observed in our experiments for a cavitator of 30 mm in diameter

As the above figure shows, a foamy cavity shows a reduction in σ (or an increase in length) when C_Q is increased. This process continues until C_Q equals C_{Qf} (formation gas entrainment coefficient) and a supercavity is established, after which no further reduction in σ is possible upon change in C_Q . Moreover, the supercavity is maintained even as C_Q drops down to very low values. Eventually, when C_Q drops below C_{Qc} (collapse gas entrainment coefficient), the supercavity transitions back into a foamy cavity. Recently, Karn et al. [13] discussed the phenomenon of ventilation hysteresis, particularly focusing on the transition of closure modes of a supercavity and relating ventilation hysteresis to the internal flows in a supercavity. However, the gas entrainment rate at the formation and collapse of a supercavity has not yet been examined in detail.

In this study, we focus our attention on gas entrainment during formation and collapse under steady and unsteady flow conditions, and study the effect of cavitator size on these behaviors. The current paper is structured as follows: § 2 provides the details of experimental methods. In § 3, experimental observations on formation and collapse gas entrainments are reported for steady states along with the effect of cavitator size on these behaviors. Subsequently, we present the behavior of gas entrainments for unsteady states which is followed by a summary and discussion in § 4.

2. Experimental Setup and Procedures

Experiments are conducted to measure ventilation flow rates and formation and collapse processes of a ventilated supercavity under different flow conditions. The experiments are carried out in the high-speed water tunnel at the Saint Anthony Falls Laboratory. This water tunnel is a closed recirculating facility with a horizontal test-section of 1.20 m (Length) $\times$ 0.19 m (Width) $\times$ 0.19 m (Height). This tunnel is specifically designed for cavitation and gas ventilation studies and is capable of operating at a maximum velocity of 20 m/s. Flow unsteadiness can be introduced in the test-section by means of a gust generator as reported in previous studies [14-16]. Fig.2a below shows a schematic of the experimental setup. The gust generator consists of two oscillating slender hydrofoils, which are placed 180 mm upstream of the cavitator. These hydrofoils oscillate in phase to generate uniform gusts. The motion is activated by a pivot arm that is linked to a flywheel through a connecting arm, which extracts the periodic motion from the motor. The eccentric flywheel allows the generation of gusts of varying amplitudes, represented by angle α as shown in the figure. Time-varying velocity amplitudes of the periodic gust flows are measured with Laser Doppler Velocimetry as

described by Lee et al. [16] and it shows that the frequency of the periodic gust flow in the test-section is equal to the oscillation frequency of the gust generator at each flow condition.

A disk cavitator with its back surface facing the incoming flow and mounted downstream of a hydrofoil-shaped strut (referred to as ‘backward facing model’ in [5], [13] and [14]), is employed in the current experiments. Fig. 2b illustrates the general placement of backward-facing model within the test section. As it shows, to avoid the interaction between the formed cavity and the strut body, a hydrofoil strut is placed upstream of the cavitator leading to a free closure as reported by Logvinovich [17]. The hydrofoil strut is just thick enough to envelope the ventilation pipe running to the cavitator. The maximum thickness of the hydrofoil, and the external diameter of the ventilation pipe equal 5mm. High speed videos of the supercavity show that the slender streamlined hydrofoil upstream does not produce significant disturbances on the cavity and only a faint wake signature can be observed. The model has special ventilation ports at the rear of the cavitator to create a supercavity. The gas flow rate is controlled by a mass flow controller and is kept steady during the experiments.

A hypodermic tube used for pressure measurement runs axially inside the ventilation line as shown in Fig. 2c. Pressures in the test section and the supercavity are measured using a Rosemount absolute pressure sensor and a Validyne DP 15-38 differential pressure transducer, respectively. The Validyne pressure transducer is calibrated before each experiment and the pressure lines are purged with compressed gas once before the pressure measurements are recorded at each condition. Uncertainty in the measurements of test-section pressure and supercavity pressure are less than 1% and 3%, respectively.

During the experiments, the ventilation flow rates for the process of supercavity formation and collapse are measured, and the corresponding high speed videos of this process are recorded

to obtain physical insights into the phenomena. The measurements of C_{Qf} and C_{Qc} are repeated for different cavitator sizes and flow conditions. A cavitator of 30 mm in diameter is used as the reference (also referred to as the ‘reference cavitator’ hereafter), and later experiments are repeated with 20 and 40 mm cavitators to study the effect of change in cavitator size. The tunnel blockages caused by the 20, 30 and 40 mm cavitators are 9%, 14% and 19%, respectively. For steady-state experiments, C_{Qf} and C_{Qc} are measured at a fixed Fr , while for the unsteady state experiments, wave-height and gust frequency are fixed as well. The experiments are carried out at gust frequencies (f) of 1, 3 and 5 Hz for each of the gust amplitudes (α) of 2, 4 and 6 degrees.

Fig. 2: (a) The experimental setup for studying unsteady ventilated supercavitation. (b) The schematic of the backward-facing cavitator model. (c) The close-up side view of the cavitator. All dimensions are in mm. Adapted from [14].

3. Results and Discussion

3.1. Steady state experiments

Fig. 3: Dependence of C_{Qf} upon Fr for the reference cavitator

The gas entrainment behaviors at steady state are studied first for the reference cavitator by varying Fr in the range of 5 – 25 (Reynolds number based on the cavitator size, $Re_d = \rho U d_c / \mu$ ranges from 8.18×10^4 to 4.09×10^5). Fig. 3 shows a typical variation of C_{Qf} with Fr . As the figure shows, with increase in Fr , C_{Qf} initially increases, attains a maximum and subsequently decreases. Two separate regimes can be identified around the maximum obtained in this curve, related to the concentration of individual bubbles as will be discussed later. Low Fr ($Fr < 9$) corresponds to a low bubble concentration regime, whereas high Fr ($Fr > 9$) characterizes a high bubble concentration zone. This trend can be explained based on the assertion that the supercavity formation process is driven by bubble coalescence, as suggested by [13]. In general, for a fixed cavitator size, an increase in Fr implies an increase in flow velocity or turbulence. This increased turbulence generally breaks up the individual bubbles into

smaller sizes as reported by previous wake studies. Our previous observations have shown that there are lesser number of bubbles at low Fr and an increase in Fr results in smaller individual bubbles, with a consequent increase in the number of bubbles [18-20]. With increasing Fr , the formation of a supercavity in this regime essentially requires the coalescence of these resulting smaller individual bubbles. Thus, there is a greater ventilation requirement to increase the size and number of bubbles to form a coalesced supercavity. Previous studies have reported that an increase in C_Q results in an increment in both number and size of bubbles [18-20]. However, in the high concentration regime, this effect of increased number of bubbles offsets the reduction in the size of individual bubbles. In this regime, the bubble concentration becomes so high that bubbles are closely packed together in the bubble cloud. Due to the restricted free space for bubble movement and the concomitant increase in bubble collision frequency, the coalescence process is favored, resulting in decreased gas entrainment requirement for supercavity formation with increasing Fr .

The dependence of C_{Qc} upon Fr is presented in Fig. 4a. As the figure shows, C_{Qc} initially decreases sharply with Fr and then becomes constant. This trend of supercavity collapse can be explained using the postulated framework of supercavity closure presented in [13]. According to [13], the supercavity closure is determined by a balance of pressure inside (P_{in}) and outside (P_{out}) at the supercavity rear portion. The non-dimensional pressure difference defined as, $\Delta\tilde{P} = 2(P_{out} - P_{in})/\rho U^2$, characterizes different supercavity closure modes (e.g. twin-vortex, re-entrant jet) as well as the foamy state of a cavity (the supercavity is yet to form). Their data suggested that $\Delta\tilde{P}$ for a foamy cavity is significantly larger than for a supercavity. In other words, when P_{out} far exceeds P_{in} , the high pressure difference forces water jet to gush inside the supercavity, causing its collapse. Thus, at a high $\Delta\tilde{P}$, larger gas entrainment is required to inhibit

the growth of re-entrant jet and maintain a stable supercavity. Further, based on their results, $\Delta\tilde{P}$ can be estimated from the momentum balance for the supercavity as (neglecting the pressure drop due to viscosity) $\Delta\tilde{P}_{\text{est}} = \sigma - C_D(d/D_T)^2$, where D_T represents the diameter of the water tunnel. As shown in Fig. 4b, $\Delta\tilde{P}_{\text{est}}$ exhibits a similar trend with respect to Fr as that of C_{Qc} . Thus, larger the $\Delta\tilde{P}_{\text{est}}$, the more the supercavity is susceptible to collapse and consequently a greater gas entrainment is required to prevent such a collapse. It is noteworthy that similar to formation gas entrainment curve, the collapse gas entrainment curve can also be divided into two separate regimes. Our experiments have shown that the regime in which $\Delta\tilde{P}_{\text{est}}$ does not change significantly with Fr corresponds to supercavities with stable twin-vortex closure at C_{Qc} . On the other hand, the region in which $\Delta\tilde{P}$ decreases sharply with Fr are characterized by other closure modes at C_{Qc} , including re-entrant jet, quad-vortex or unstable closure modes as reported by [13].

Fig. 4: (a) Dependence of C_{Qc} upon Fr and (b) Dependence of $\Delta \tilde{P}_{est}$ upon Fr for the reference cavitator. Fr of 4 -16 in the above plots corresponds to an Re_d range of 6.54×10^4 - 2.62×10^5 .

Next, the effect of cavitator size on C_{Qf} and C_{Qc} are examined. Figs. 5 and 6 show the comparison of formation and collapse gas entrainment behavior for three different cavitators, respectively. The figures clearly indicate that a similar trend is observed at a cavitator size of 20 mm and 40 mm. Further, the curves shift upwards and towards left with an increase in the cavitator size. As pointed out before, for the formation gas entrainment curve, the maximum in this curve represents a transition between low and high bubble concentration regimes. The leftward displacement of the maxima in this curve suggests that the critical Fr for such transition decreases with increasing cavitator size. This observation can be ascribed to an increasing wake turbulence (characterized by Re_d) which promotes bubble breakup and consequently results in increased concentration of individual bubbles [20]. The upward movement of the curve can then be attributed to the larger C_{Qf} required to drive the bubble

coalescence process into a wider and longer supercavity with increasing cavitator size. On the other hand, for the collapse gas entrainment behavior, the shift in the curves can be attributed to a similar variation of $\Delta\tilde{P}$ with Fr as shown in Figs. 4a and 4b. The upward movement of these curves with the increasing cavitator size is related to the greater gas entrainment values required to maintain a larger $\Delta\tilde{P}$ with increasing cavitator size. This observation is related to the fact that a larger size cavitator results in a supercavity with increase both in cross-sectional area and length, which the gas entrainment coefficient only accounts for the increase in cross-sectional area. As pointed out in Fig. 4b, the flat portion of the $\Delta\tilde{P}$ vs Fr curve corresponds to a stable twin vortex cavity. Thus, the leftward shift in the collapse gas entrainment curve indicates that for a larger cavitator the transition from other supercavity closure modes to a twin vortex cavity mode is established at a low Fr which persists upon further increase in Fr . This indicates that twin vortex is predominantly observed over a larger range of Fr for a larger cavitator, which has been previously reported and explained by [13].

Fig. 5: Comparison of C_{Qf} vs Fr curve for three cavitator sizes

Fig. 6: Comparison of C_{Qc} vs Fr curve for three cavitator sizes

3.2. Unsteady state experiments

Fig. 7: Variation of C_{Qf} with St at different Fr for the reference cavitator. Re_a corresponding to the Fr numbers in the plot are 1.82×10^5 , 2.11×10^5 , 2.41×10^5 and 2.72×10^5 , respectively.

The gas entrainment behaviors under unsteady conditions are studied by first holding Fr and gust amplitude (α) constant and then varying the St ($=fd/U$) by changing the gust

frequencies(f). Fig. 7 shows the formation gas entrainment behavior under different St values for the reference cavitator at four different Fr numbers. As Fig. 7 shows, C_{Qf} increases monotonically with increase in St at a fixed Fr . This trend in C_{Qf} with St indicates that the introduction of unsteady gusts in the flow impacts the bubble coalescence process by imposing a lateral velocity on the bubbles. This imposed lateral motion on bubbles makes it more difficult for the coalescence of individual bubbles into a supercavity, thus increasing the C_{Qf} requirement with an increase in St . It is noteworthy that unlike coalescence events in the flow, breakup events is much less affected by the introduction of unsteadiness since the timescales associated with bubbles breakup are substantially smaller than those of the unsteady gusts.

Fig. 8 shows the effect of the variation of gust amplitudes upon C_{Qf} at a fixed Fr and St . As the figure shows, C_{Qf} increases monotonically with increase in α . High speed videos of the supercavity indicates that an increase in gust amplitude results in the bubbles being spread across a larger lateral cross-section (reducing the local void fraction), which decreases the collision frequency of bubbles and consequently reduces coalescence efficiency. Thus, a greater gas entrainment is required to drive the coalescence of individual bubbles into a supercavity.

Fig. 8: Variation of C_{Qf} with α at different Fr for the reference cavitator

Next, experiments are conducted to study the behavior of collapse gas entrainment under unsteady states for the reference cavitator. Fig. 9 shows the variation of C_{Qc} with St at three different Fr numbers. As the figure shows, at a fixed Fr , C_{Qc} increases monotonically with increase in St . This monotonic increase in C_{Qc} with St can be explained as follows. As pointed out before, the process of supercavity collapse is preceded by a transition to a re-entrant jet closure, which makes the supercavity very unstable, particularly in an unsteady flow. A previous study [17] has shown that the introduction of unsteady gusts in the flow lead to pressure fluctuations at the closure, intermittently causing a re-entrant jet closure. As St increases, a larger C_{Qc} is required to counteract the rapid pressure fluctuations occurring at re-entrant jet closure and sustain the supercavity. A similar explanation can also be given for a larger C_{Qc} obtained at a higher Fr , but nearly same St . Previous studies (For instance, [21]) have shown that, the re-entrant jet velocity is approximately equal to the incoming liquid velocity. Thus, an increase in Fr for the same cavitator size implies an increase in re-entrant jet velocity

and thus a larger C_{Qc} is needed to inhibit the re-entrant jet from reaching the cavity upstream and causing cavity collapse.

Fig. 9: Variation of C_{Qc} with St at different Fr numbers for the reference cavitator. Re_a corresponding to the Fr numbers in the plot are 2.11×10^5 , 2.41×10^5 and 2.72×10^5 , respectively.

The effect of the variation of gust amplitudes upon C_{Qc} is found to be similar to C_{Qf} . Fig.10 shows the effect of the variation of gust amplitudes upon C_{Qc} at a fixed Fr and St . From Fig. 10, it can be seen that C_{Qc} increases monotonically with increase in α . Again, this observation can be related to the increase in the extent of pressure fluctuations at the supercavity rear with the increase in α . Thus, a larger C_{Qc} is required to counteract the rapid pressure fluctuations occurring at the supercavity rear portion and sustain the supercavity.

Fig. 10: Variation of C_{Qc} with α at different Fr for the reference cavitator

4. Summary and Conclusions

In the current study, some interesting insights into the gas entrainment behaviors in the formation and collapse of a ventilated supercavity are presented for steady and unsteady flow conditions. Our experiments have shown that the gas entrainment required to establish a supercavity is much greater than the minimum gas entrainment required to sustain it. Further, these gas entrainment values depend on Fr , cavitator size and the flow unsteadiness. Specifically, the measurements of the formation gas entrainment coefficients under different Fr numbers indicate that it does not monotonically increase with Fr but displays increasing and decreasing trends in different regimes of Fr . These trends are possibly caused by the variation in coalescence efficiency with bubble concentration at different flow conditions. On the other hand, the collapse air entrainment coefficient initially decreases with Fr and then approaches to a constant. This trend has been shown to be related to the change in the non-dimensional

pressure inside and outside the cavity at the supercavity rear portion. Similar trends of formation and collapse gas entrainment coefficient have been observed for different cavitator sizes.

Our unsteady state experiments have shown that both the introduction of unsteadiness and the increase in gust amplitudes cause a slight monotonic increase in the formation and collapse gas entrainment requirements. For the supercavity formation, the observed trends are related to lateral motion and expansion of the bubbles induced by the unsteadiness, which reduces the coalescent efficiency of bubbles. For the collapse case, the trends are determined by the pressure fluctuations occurring at the supercavity closure. These findings have far-reaching implications in the operation of underwater supercavitating vehicles. Specifically, knowledge of the occurrence of ventilation hysteresis and its dependence upon flow conditions can drastically reduce the ventilation demand for extended underwater missions. Our study points out the crucial factors to be considered in the estimation of gas storage requirements for such a ventilated supercavitating vehicle.

Nomenclature

P_{∞}	Test-section pressure	$\dot{Q}$	Gas ventilation rate	C_{Qc}	Collapse coefficient
P_C	Internal cavity pressure	σ	Cavitation number	f	Gust frequency (Hz)
ρ	Density of water	Fr	Froude number	α	Gust amplitude (deg)
U	Incoming liquid velocity	C_Q	Gas-entrainment	St	Strouhal number
d	Cavitator diameter	C_{Qf}	Formation	D_T	Water tunnel diameter

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