Investigation into the turbulent bubbly wake of a ventilated hydrofoil: moving towards improved turbine aeration techniques.

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Abstract

The use of aerating hydroturbines to mitigate the problem of low dissolved oxygen in the discharge of hydroelectric power plants has recently attracted a lot of attention. The design of a ventilated hydroturbine requires a precise understanding of the dependence of the operating conditions (viz. liquid velocity, air ventilation rate, hydrofoil configuration etc.) on the bubble size distribution generated in the bubbly wake and the consequent rise in dissolved oxygen in the downstream water. In the current research, experiments are conducted in the wake of a ventilated NACA0015 hydrofoil by systematic variation of hydrodynamic conditions allowing for quantitative analysis of aeration statistics and capabilities for turbine blade hydrofoil designs. The data concerning bubble velocity distributions, bubble locations and size distribution, void fraction etc. are reported for a chosen reference case. In addition, trends in the variation of bubbly wake are explored particularly in the light of wake physics. It is found out that an increase in Reynolds number (Re) led to greater breakup, while an increase in normalized air ventilation rate (C_Q) favored greater coalescence events in the wake. Further, the PDF(φ) of the normalized bubble size $\varphi = \varphi/\varphi_{32}$, where φ_{32} represents Sauter mean size distribution, is found to have a universally similar shape independent of either Re , C_Q or hydrofoil angle of attack. Finally, a numerical formulation is proposed for the bubble sizes in the hydrofoil wake. This rich dataset will also contribute to the development of a numerical turbulence model, to investigate

turbulence effects on bubble size distribution and predict the rate of air entrainment and the oxygen transfer occurring in the wake at different hydrodynamic conditions.

Keywords: Hydroturbine aeration, Turbulent bubbly wake, Ventilated hydrofoil wake, Auto venting turbines, Particle Shadow Velocimetry.

1. Introduction

Over the past few decades, environmental concerns have broadly affected the electric power industry, including the depletion of fossil fuel supplies, the potential of global climatic changes and challenges involved in long-term nuclear waste management (**March et al., 1992**). In response to such concerns, the electric power industry has increasingly focused attention on renewable non-polluting energy technologies such as hydroelectric power. Hydroelectric power generation is clean, inexpensive and reliable and does not significantly pollute the land, air or water. However, environmental problems resulting from the discharge of water with low dissolved oxygen (DO) levels are a concern at many hydroelectric facilities. The problem of low DO is well documented in the literature. (**Ruane and Hauser, 1991**) and a variety of methods have been suggested to provide dissolved oxygen (**Bohac and Ruane, 1990**). The low DO water when released to the river downstream can adversely affect the aquatic habitat and contribute to other water quality problems viz. dissolution of trace metals, formation of hydrogen sulfide and depression of pH (**Wilhelms et al. 1987**).

One of the most attractive techniques currently being investigated to mitigate low DO while maintaining operation efficiency, is the use of an auto-venting turbine (AVT). AVT is a self-aspirating hydroturbine designed to aerate the turbine discharge through ports located at low pressure regions which are open to the atmosphere. When air is allowed to flow through these ports, it breaks down into small bubbles by the high velocity and turbulence of the water flow through the turbine. DO transfer is augmented by the high interfacial area of these bubbles and the level of turbulence in the water (**Thompson and Gulliver, 1997**).

To maximize the DO transfer in the flow through AVT, the knowledge of the bubble sizes of the entrained air at different locations in the bubbly wake is extremely important. The size of bubbles that are generated by the entrainment and are a result of different bubble breakup and coalescence processes, affects the gas transfer at different locations in the wake. Ultimately, it is desired to have a numerical turbulence model to investigate turbulence effects on bubble size distribution and predict the rate of air entrainment and the oxygen transfer occurring in the wake at different hydrodynamic conditions. The accurate prediction of bubble sizes in the wake at a given flow field is imperative for the prediction of oxygen transfer occurring in the wake. However, both for the operation and validation of such a computational model, there is a desperate need for a high quality experimental dataset that can provide information on bubble sizes and void fraction in the wake, pressure at the air entrainment location and the rate of oxygen transfer. To the best of our knowledge such relevant experimental reports on the bubbly wakes are scarce.

Thus, in the current study, a series of ventilation experiments are conducted at several hydrodynamic conditions at the Saint Anthony Falls Laboratory (SAFL) water tunnel, allowing for quantitative analysis of aeration statistics and capabilities for turbine blade hydrofoil designs. SAFL has pioneered the study of bubble wakes (**Kjeldsen et al. 2000; Arndt et al. 2007**) and has several hydrofoil designs specifically modified for ventilation studies. In the present study, a systematic investigation into a bubbly wake is being carried out at multiple hydraulic operating conditions such as different air ventilation rates, liquid velocities, hydrofoil configuration etc. to serve as a test-bed for computational turbine aeration programs. The data concerning bubble velocity distributions, bubble locations and size distribution, void fraction etc. are obtained. In

addition, some simple numerical formulations are proposed for the bubble sizes in the hydrofoil wake.

This paper is structured as follows: Section 2 provides the details of the experimental facility, and the experimental methodology employed. The obtained results are presented in Section 3 along with a brief discussion which is followed by a final conclusion in Section 4.

2. Description of experimental setup

2.1. Experimental apparatus

The experiments are conducted in the SAFL high-speed water tunnel at University of Minnesota. The tunnel has a horizontal test section of 1 m (Length) $\times$ 0.19 m (Width) $\times$ 0.19 m (Height) with three sides having plexiglass walls for optical access. The tunnel is designed for cavitation and air ventilation studies and is capable of operating at a maximum velocity of 20 m/s. A gas-collector dome in the tunnel provides for fast removal of large quantities of air bubbles generated during cavitation and ventilation experiments, enabling bubbly flow experiments for extended periods with little effect on test section conditions.

2.2. Experimental conditions

The NACA 0015 hydrofoil used in our experiments has a span of 190 mm and a chord length of 81 mm. During the experiments, it is installed in the test section at three different angles of attack (α): 0, 4 and 8 degrees. As shown in Figure 1, a narrow spanwise slot allows air to be injected into the flow near the leading edge of the hydrofoil and the full width of the injection slot is used for measurements of oxygen uptake. This setup results in a dense spanwise bubbly wake and thus, in order to make bubble measurements, ventilation is limited to a narrow 9.6 mm

wide slot (5% of span) at the center of the span. This configuration ensures that bubbles remain mostly within a narrow distance away from the centerline. The findings of **Silberman (1957)** indicated that the bubble diameter is independent of the orifice characteristics for a gas jet coming out of an orifice into a cross flow. Also, the bottom views of the bubbly wake show a minimal lateral drift of the bubbles. Based on our observation of the bottom views of bubbly wake, the study of Silberman and the fact that slot width remained unchanged in our experiments, we posit that the bubbles obtained are a reasonable representative sample of the bubble population that exists when the full span is ventilated. Under this scheme, 45 different experiments are conducted at three different water speeds (5, 7.5 and 10 m/s), ventilation gas flow rates equivalent to 10, 20 and 30 LPM per span length and angles of attack of hydrofoil (0, 4 and 8 degrees). Bubbly wake images are obtained at three different axial locations (109, 243 and 377 mm from the hydrofoil center corresponding to $x/c = 1.34, 3$ and 4.65 respectively, where c is the chord length). In total, 90,000 bubble images are captured using a shadow imaging technique.

Figure 1: Details of the NACA0015 ventilated foil (adapted in part from [24])

2.3. Particle Shadow Velocimetry

Particle Shadow Velocimetry technique (PSV) employs direct in-line volume illumination using low power sources such as LED and an optical setup to produce a narrow depth-of-field for 2D plane imaging (Goss et al., 2007; Broder and Sommerfeld, 2007). Figure 2 shows a schematic of the experimental setup as has been reported previously by Karn et al. (2014). A $1K \times 1K$ pixel Photron APX-RS camera (capable of 3000 frames/s at full sensor size) with a 60 mm lens is used to acquire images. A custom-made pulsed LED light source from Innovative Scientific Solutions Inc. is used to illuminate the flow, with a pulsed LED array having flash rates up to 10 kHz with a $5 \mu s$ pulse width and rise and fall times around 200 ns. To ensure uniform back-

lighting and to eliminate noise in the images, a light shaping diffuser is placed between the light source and the flow.

In the PSV technique, two LED light pulses separated by a short duration are synchronized with camera exposure in order to obtain two consecutive (or, double) images. In our experiments, the time duration between two pulses varied between 100-230 μs depending on free stream velocity. Using the image pairs, the instantaneous velocity field of the bubbles is obtained using commercially available Particle Image Velocimetry (PIV) software (DaVis 7.2 from LaVision). The image pairs are captured at a frame rate of 25 image-pairs/s and the exposure time for individual images is reduced to 15 μs to prevent any blurring in the images. A data set consists of 1000 image-pairs taken over a duration of 40 s. The field of view of the captured images is approximately 60 mm $\times$ 60 mm. The imaging system is calibrated prior to the beginning of the experiments using a 2.5 mm $\times$ 2.5 mm grid located at the water filled test section centerline, providing pixel dimension of 0.059 mm in all the calibrated images. The image depth of field is determined to be approximately 15 mm at the location of image capture. The lateral spread in the bubble plume is measured at the measurement location and the calculated uncertainty in spanwise bubble location translates to a length scale (and calculated velocity) uncertainty of 1.6% near the foil and 3.2%. The bubble sizes are in the range of 2 to 68 pixels, which corresponds to a bubble radius of 0.06 to 2 mm.

Figure 2: Schematic of the experimental setup for shadow imaging of bubbly flows in an aerated hydrofoil wake adapted from Karn et al. (2014)

2.4. Image processing and analysis

The image analysis approach employed to process the bubble images has been described in **Karn et al. (2014)**. It assumes bubbles are generic ellipsoids and extracts the bubble properties such as centroid location, size and shape etc. from the projected area of a bubble. To summarize, the image processing consists of three major steps: (1) binarization of grayscale images; (2) labelling and characterizing bubble regions by a series of metrics including area, centroid, major and minor radius (assuming bubble projected areas as ellipses), circularity factor etc. Based on the area of each region, the bubbles are divided into tiny spherical bubbles, intermediate-sized bubbles and large bubbles or clusters. (3) Separation of bubbles in a cluster and bubble information extraction: A multilevel segmentation approach is used to extract the maximum information from the images. A customized approach named ‘Cluster Processing’ was developed using advanced “morphological operations” and a “watershed transform” (**Meyer 1994**) to

extract individual bubbles from bubble clusters. Finally, the information from all the levels is combined, and the location, as well as the shape of all the individual bubbles characterized by semi-minor and semi-major axes of an ellipse are measured (Karn et al., 2014).

3. Results and Discussion

3.1. Results of a reference case

3.1.1. Validation of image processing of the bubbly wake images

Figure 3 below presents a sample of a bubbly wake image from our experiments at liquid velocity of 5 m/s, 0.5 LPM ventilation rate and at a location $x/c = 4.65$ from the hydrofoil center (referred to as ‘reference case’ from now on). It shows a number of characteristics of the bubbly wake. First, there is a wide range of size distribution of bubbles. Measured bubble sizes ranged from 60 μm to a few millimeters and larger bubble clusters were observed as well.

Figure 3: A sample of original bubbly wake image from our experiments

Second, these bubbles are distributed vertically in the wake in a particular fashion. There is a meandering zig-zag pattern clearly visible. It is observed that the bubbly wake in the hydrofoil

has a high-amplitude oscillation about the wake centerline, which varies depending upon the flow conditions.

Figure 4: The comparison of the actual ventilation flow rate (in blue) and the measured ventilation flow rate at twelve different streamwise locations for the reference case.

However, such a velocity field does not give any information about the size of the bubbles. Consequently, image analysis is done for all the 1000 images to obtain bubble size information. Now, the size and velocity information of all the bubbles in the wake are coupled to calculate the air flow rate in the wake. It is expected that a comparison of this calculated value with the measured volume flow rate with a mass flow controller would give an indication about the accuracy of the image analysis technique. Thus, the volume flow rate of air is estimated based on the volume and velocity of the bubbles at twelve different locations in the wake and the comparison has been shown in Figure 4. As it can be seen in the figure, the measured and calculated values compared fairly well at all the locations within a 10% error. It can also be seen

that the errors decreased to less than 3% at $x/c > 4.5$. The greater amount of bubble clustering close to the hydrofoil can be attributed to the deviations observed at $x/c < 4$.

3.1.2 Bubble size and shape distribution in the wake.

Figure 5: (a) Bubble size distribution for the reference case (b) Bubble shape distribution: fraction of bubbles as a function of eccentricity and size for the reference case.

Figure 5 above shows typical bubble size and shape distributions observed in the wake for a reference case. Bubble size (or bubble diameter) is taken to be the geometric mean of minor-axis (b) and major axis length (a) i.e. $\bar{d} = 2\sqrt{ab}$. Similarly, eccentricity of bubble projected areas (called ‘bubble eccentricity’ from now on) is defined as: $e = 1 - (b/a)^2$, suggesting that a perfectly circular bubble has an eccentricity value of zero. Figure 5a shows that the obtained bubble size distribution is a bimodal one, similar to the reports of **Tse et al. (2002)**, with two modes at 0.25 mm and 0.6 mm respectively. This suggests that the bubbly wake contains an excess of small-sized bubbles (about 30%) which are significantly smaller than the Sauter mean diameter. The fraction of 0.6 mm bubbles reach up to 10-12% and the number fraction of larger bubbles decreases with increase in size. There are very few bubbles with bubble diameter greater than 1.5 mm. Figure 5b presents a three-dimensional histogram of fraction of bubbles with bubble diameter and eccentricity. As observed from the figure, about 25% of the bubbles are very small and have an eccentricity value of zero. Higher values of e can be observed only for bubbles larger than 0.5 mm. For instance, 10% of the bubbles are of size 0.6 mm with $e = 0.6$. A variety of bubble shapes are found in the bubbly wake viz. spherical, ellipsoidal, and wobbling, highly elongated bubbles etc. To determine bubble shape transitions, **Clift et al. (1978)** introduced the Eotvos number defined as $Eo = \frac{g(\rho_l - \rho_g)d^2}{\sigma}$. At the transition between spherical and elongated bubbles, the value of Eotvos number depends on the Morton number, defined as $Mo = \frac{g(\rho_l - \rho_g)d^3}{\nu \sigma}$. For the fluid-pair used in our study, the value proposed by **Clift et al. (1978)** for this transition is $Eo = 0.4$, i.e. for an Mo greater than this value, gas bubbles transition to the wobbling/ ellipsoidal regime. This gives a critical bubble radius of 0.86 mm, which is larger than the critical radius at which ellipsoidal bubbles are observed in our

experiments. Conversely, assuming $d_{crit} = 0.6$ mm gives a critical Re of 0.012. However, it must also be noted that in our experiments, there is an additional effect due to bubble shearing at the injection slot. Such discrepancy between the experimental value of the critical Re and the results obtained by **Clift et al. (1978)** were also noted, both for bubbly flows in horizontal (**P. Andreussi et al. 1999**) and vertical pipes (**Barnea et al. 1982**). On the other hand, **Brodkey (1967)** provided a criteria for bubbles to behave like solid spheres as: $Re_{crit} = [0.4(Re - 1.5)]^{1.5}$. For the air-water fluid pair used in our experiments, this gives a critical bubble size of 5 μ m. **Levich (1962)** postulates that such small bubble sizes are reached when local flow around the bubble is such that turbulence is no longer able to deform the bubble. **Hesketh (1982)** suggested that such small bubble sizes are not encountered under most processing situations. Moreover, our experimental approach is limited in measuring such extremely small microbubbles.

It is well known that the breakup processes in turbulent dispersions are influenced by continuous phase hydrodynamics (**Liao and Lucas 2009**) whereas the contact and collision of bubbles determine the bubble coalescence process (**Liao and Lucas 2010**). The collision between bubbles is caused by their relative velocity and will be affected strongly by the bubble density in the flow. Thus, the experimental parameters viz. liquid velocity, ventilation flow rate, system configuration viz. angle of attack of hydrofoil etc. govern the occurrence of these two processes occurring in the wake and thus determine bubble size and shape distributions. To systematically investigate the physical processes occurring in the bubbly wake, an air-entrainment coefficient is defined similar to other ventilated cavitation studies (**Laali and Michel 1984**) as $Q = \frac{Q}{U L}$, where, Q , U , L denote ventilation air flow-rate, liquid velocity, hydrofoil span and chord, respectively. Reynolds number is defined using hydrofoil chord as length scale: $Re = \frac{U L}{\nu}$. To

study the effect of the experimental parameters on the resulting flow physics in the bubbly wake, two different sets of experiments are performed. In the first set of experiments, ΩQ and $\Omega \Omega$ ($\Omega Q = 1.6 \times 10^{-4}$, $Re = 4.1 \times 10^5$) are fixed and experiments are repeated at three different angles of attack (AoA = 0, 4 and 8 degrees). Subsequently, in the second set of experiments, the experiments are done at a fixed zero angle of attack while ΩQ and $\Omega \Omega$ are varied systematically, keeping either one constant and varying the other.

3.1.3. Void fraction measurements.

Figure 6: Local Void fraction measurements in the bubbly wake for the reference case. Origin (0,0) of the above plot is horizontally displaced $4.65c$ from the hydrofoil center of rotation.

Figure 6 above shows the contour plot of the local void fraction in the bubbly wake. For the calculation of local void fraction, entire field of view is divided into 16×16 pixel grids. First, the projected areas of all the bubbles within each grid are traced out and the volume of all the bubbles is calculated assuming a third radius equal to the semi-minor axis of the projected

ellipse. The ratio of the total volume of all the bubbles and volume of each grid (the depth of each grid is determined by the depth of field) gives the local volume void fraction in the wake. As the figure above shows, the maximum local void fraction values are attained below the centerline. The local void fraction below the centerline can reach as high as 0.15. It should also be noted that the slot is located at 6 mm below the centerline and the void fraction distribution along transverse direction is almost symmetrical about the location of injection slot. The figure shows that the void fraction decreases gradually upon moving away from the injection slot location on either side in the vertical direction. The greater void fraction at the location of the injection slot can be attributed to both larger number of bubbles and greater size of bubbles at these locations.

3.1.4. Bubble Velocity Measurements in the wake.

The turbulence in the wake flow downstream of a hydrofoil affects both bubble size and velocity in the wake. The bubble velocity fluctuation, in turn affects the local liquid velocities. The bubble velocity profiles measurements are made at three downstream locations, $x/c = 1.34, 3$ and 4.65 at all the three angles of attack. First, an average velocity field is obtained for the entire field of view. The bubble average velocity field is obtained in a $0.74c \times 0.74c$ window at $x/c = 4.65$. Each window is further divided into smaller grids of $0.94 \text{ mm} \times 0.94 \text{ mm}$ size and a velocity vector corresponding to a bubble in each grid is determined using double pulse images. Finally, this plot is generated by averaging all the 1000 bubble velocity fields obtained from 1000 image pairs. The SIV analysis of the bubbly wake images show that the bubble velocity remains almost uniform in the wake, varying within 5% of about 5 m/s bubbles, with the lowest velocities at the center of the wake (**Karn et al. 2014**). Later, velocities at the same vertical location are averaged to get an averaged bubble velocity profile at the three streamwise

locations. Figure 7 presents normalized velocity deficit profiles of bubbles at a zero hydrofoil angle of attack. It shows that the maximum velocity deficit occurs in the center of the wake for all the three locations. The velocity deficit peaks close to the hydrofoil, and decreases gradually as the wake continues to spread. Specifically, at $x/c = 1.34$, the maximum velocity deficit is 6.7%, which reduces to 3.8% and 3.4% at $x/c = 3$ and 4.65 is, respectively. Also, the bubble velocity profile changes significantly between $x/c = 1.34$ and $x/c = 3$ but there is a minimal evolution of the velocity profile between $x/c = 3$ and 4.65. Further, the maximum velocity deficit is present at the centerline; yet, the wake is not symmetrical about the centerline. This can be attributed to the preferential location of large size bubbles below the centerline. It is noteworthy that a non-symmetric velocity field is observed with higher velocity above the centerline since the ventilated air is injected from the pressure side of the hydrofoil located below the centerline. In general, it is observed that the larger bubbles had slightly lower velocities compared to the tiny bubbles, which followed the streamlines more closely.

Figure 7: Averaged bubble velocity profile for the reference case at three different stream-wise locations. U_{ref} is the velocity at the edge of the wake at the measurement location.

3.2. Effect of variation in angle of attack of hydrofoil

The variation in the angle of attack of hydrofoil is expected to cause a drift in the bubbly wake downstream. However, it can also be conjectured that bubbles of different sizes may respond to this variation differently. To investigate the effect of variation in AoA upon bubble of a particular size, all the bubbles measured at a particular AoA are classified into six equal size bins spanning over 0 to 3 mm. It is found that the movement of the bubbles in the vertical direction is dependent upon the bubble size. Figure 8 shows how the bubbles are distributed across vertical direction in the wake. For all the bubbles in a particular size bin, Sauter mean diameter (referred to as SMD or d_{32} from now on) is calculated and the extent of its drift in the vertical direction is measured. For a number of bubbles, SMD can be defined as: $d_{32} = \frac{\sum_{i=1}^N d_i^3}{\sum_{i=1}^N d_i^2} = \frac{\sum_{i=1}^N d_i^3}{\sum_{i=1}^N d_i^2}$ (Hesketh et al., 1987 ; Razzaque et al., 2003) where N is the number of bubbles at a particular

diameter d_{32} . It is observed that the vertical drift of the bubbles reduced as the bubble sizes increased. For the $AoA=0$ case, it is evident that smallest sized bubbles drifted considerably from the plane of injection slot from $-0.32c$ to $0.36c$. However, the large sized bubbles remained below the injection slot within much a narrower region. This can be attributed to the greater inertia of the large-sized bubbles. The mean location of the bubbles in the size range of $2.5 - 3$ mm bubbles is located approximately at a distance of $0.08c$ from the plane of injection slot. Further, as the AoA increased from 0 to 8 degrees, the entire bubbly wake drifted upwards.

Figure 8: Spread of bubbles in the bubbly wake at different angles of attack. The red circles denote Sauter mean diameter and mean vertical location of all the bubbles in a size bin. The red, green and blue bars represent the y -locations over which bubbles in each size bin are observed at different AoA .

Apart from causing an upward drift in the bubbly wake, an increase in angle of attack also had an impact upon the total number of bubbles and bubble sizes produced. As the air is introduced through the ventilation slot, the air becomes exposed to the destabilizing effect of the turbulent velocity fluctuations in the liquid which causes the continuous air-jet to break into small bubbles. Upon increasing AoA from 0 to 8 degrees, there are greater levels of turbulence in the wake

downstream, and thus the air bubbles are subjected to increasing values of turbulent stresses on their surface, resulting in their breakup into smaller sizes. Consequently, the value of critical bubble diameter, or the diameter of the largest stable bubble in a turbulent flow, (as defined by **Kolmogorov 1949** as) $d_c \propto \sigma^{3/5} \epsilon^{-2/5}$, decreases (σ , ρ and ϵ denote interfacial surface tension, density of liquid and the dissipation rate, respectively). Figure 9a below presents a PDF of the bubbles at different AoA. As the figure shows, a greater angle of attack leads to a greater fraction of bubbles in the intermediate size range (0.5 – 1 mm). The fraction of bubbles at 0.5 mm increased from 11% at an AoA of 0 degree to 17% at AoA of 8 degrees. Corresponding, there is a decrement in fraction of small size bubbles. It is worth noting that in spite of the increased turbulence and greater fraction of the small sized bubbles, the shape of the PDF largely remains the same, and does not shift towards smaller sized bubbles as reported in a previous study by **Martínez-Bazán et al. (2000)**. To represent the bubble size PDF in non-dimensional variables, a normalized bubble diameter is defined as $d^* = d/d_{32}$. The probability density function of d^* , $PDF(d^*)$ for different angles of attack are shown in figure 9b. It is worth noting that all the $PDFs(d^*)$ collapse on a single curve, which is an indicative of the fact that d_{32} is a effective single parameter to represent the entire bubble size distribution.

Figure 9 : (a) Bubble size PDFs at different AoA. (b) Bubble size PDF of d/d_{32} for different AoA.

Figure 10: (a) Variation of SMD and (b) NOB (for entire dataset) with AoA.

Figure 10a presents the number of bubbles (NOB) generated and *SMD* of all the bubbles at each angle of attack, respectively. It shows that with increasing angle of attack from 0 to 8 degrees, *NOB* increased by 33% while the *SMD* decreased from 0.93 mm to 0.85 mm. Calculations based on Figure 9a and 10b reveal that the number of bubbles in the small size range does not change much since the decrease in fraction and increase in number of bubbles offset each other. Similarly, the decrement in d_{32} upon increase in AoA as shown in Figure 10b, can be attributed to the significant rise in the number of bubbles in intermediate size range and a consequent decrease in number of large size bubbles.

3.3. Effect of variation in Reynolds number

Figure 11a presents a PDF of bubble sizes as *Re* is varied from 2.4×10^5 to 8.1×10^5 at $C_Q = 1.1 \times 10^{-4}$. In general, it is observed that upon increase in *Re*, there is a greater fraction of small size

bubbles while the fraction of large size bubbles decreased. The fraction of small size bubbles at 0.25 mm increased from 18% at the smallest Re to approximately 35% at the greatest value of Re tested. Similarly, in the intermediate size range (viz. 1.1 mm) the fraction of bubbles decreased from 5% to below 1% for the same variation of Re . However, the overall shape of the PDF and the slope of the tails of the PDFs does not change with the change in Re . Again, as shown in figure 11b, the normalized PDFs() for this range of variation in Re collapse onto a single, self-similar curve suggesting that d_{32} is the parameter needed to describe the final state of bubble size PDF.

Figure 11 : (a) Bubble size PDFs and (b) Bubble size PDF of $\bar{d} = \bar{d}/\bar{d}_{32}$ for different Re at $C_Q = 1.1 \times 10^{-4}$

Figure 12: (a) Variation of normalized wake-width ($\bar{w} = w/w_0$) in the wake (b) Number of bubbles (NOB) as a function of Re . (c) Sauter mean diameter of all the bubbles as a function of Re . Circles and squares show the data point while the dashed line shows the power law curve fit. C_Q is kept fixed at 1.1×10^{-4} .

Figure 12a shows the variation of normalized wake-width ($\bar{w} = w/w_0$) with Re . Wake-width is defined as the distance between the two y -locations on the either side of the centerline, which had at least a minimum of 500 bubbles in the entire dataset consisting of 2000 bubble images. Figure 12a indicates that $\bar{w}$ increased from 0.45 to 0.65 as Re is varied from smallest to the largest value. As Re increases, the increase in wake-width implies a spread in the bubbly wake in the transverse direction, thus exposing a number of bubbles to the free-stream turbulence which would further break down bubbles into smaller daughter bubbles. Figure 12b shows the consequent increase in number of bubbles from 0.6×10^6 to 5.6×10^6 over this range of Re for the entire dataset. Based on the results from all the conditions, it is found that NOB has a power law dependence with Re at a fixed C_Q as: $NOB \propto Re^{1.84}$. Since a larger Re also leads to a larger fraction of small-size bubbles, the total number of small size bubbles is significantly higher at a greater Re . This would lead to a decrement in Sauter mean diameter at a greater Re . Figure 12c shows that SMD decreases from 1 mm to 0.6 mm as Re is increased over the given range of Re . For all the conditions at a fixed C_Q , SMD is found to have a power law dependence with Re , but with a negative exponent: $SMD \propto Re^{-0.39}$. A further insight into the variation of SMD across all the y -locations reveals that in spite of the large variation in wake-width, the trend of variation of SMD with vertical location remains similar for all the Re . Figure 13 shows that the curve of size versus y/c shifts to the left as Re is increased. As explained before, the preferential presence of the large size bubbles below the centerline leads to an increase in SMD upon moving downwards and this trend remained unchanged with increase in Re . It can be seen that the SMD of all the

bubbles reported before is approximately equal to the mean value of the *SMD* obtained over all the *y*-locations as shown below.

Figure 13: Sauter mean diameters at each vertical location for the range of *Re* tested at $C_Q = 1.1 \times 10^{-4}$. Note that the data points are shown only for the bubbly wake.

3.4. Effect of variation in air entrainment coefficient

Figure 14a below shows the effect of air entrainment on the bubble size distribution at a fixed $Re = 4.1 \times 10^5$. It suggests that at a lower C_Q , there is a greater fraction of small size bubbles while the opposite is true for the intermediate and larger size bubbles. In our experiments, C_Q is varied by a factor of three and the fraction of 0.25 mm bubbles increased by 7%. In general, it would be expected that larger value of C_Q would lead to a larger number of bubbles since the net amount of air ventilated per unit time has increased. Again, when the bubble size PDF is normalized with their corresponding d_{32} , all the PDFs collapse onto the same universal curve as shown in Figure 14b.

Figure 14: (a) Bubble size PDF with variation in C_Q and, (b) Bubble size PDF of d/d_{32} for different C_Q at $Re = 4.1 \times 10^5$

Normalized wake width, $\bar{w}$ is found to increase from 0.4 to 0.63 as C_Q is increased by three times as shown in Figure 15a. Similar to the previous case, it suggests that there will be greater breakup events due to the spreading of the wake. However, it should also be noted that the larger amount of ventilation flow rate leads to formation of larger bubbles and a greater number of bubbles. Since the increase in $\bar{w}$ is just 57% while C_Q is increased by three times, it implies that bubbles are more closely spaced at a higher C_Q . Thus, bubbles lying in the vicinity of each other will readily undergo coalescence in the bubbly wake increasing the number of bubbles in intermediate size range as well (Tse et al. 2002). Thus, there will be a net rise in total number of bubbles. Figure 15b shows that NOB increases with increase in C_Q . However, unlike the previous case, NOB is observed to have a linear dependence upon C_Q , with number of bubbles increasing from 1.3×10^6 at $C_Q = 1.1 \times 10^{-4}$ to 1.7×10^6 at $C_Q = 3.3 \times 10^{-4}$. This information when coupled with the PDF of bubbles reveals that upon an increase in C_Q , the number of bubbles increase over the entire size range of bubbles. However, the change in the number of small size bubbles is minimal while there is a significant increment in number of large size bubbles upon an incremental change in C_Q . Thus, a net increment is seen in d_{32} as C_Q increased. Figure 15c shows that d_{32} increased from 0.73 to 0.85 as C_Q is varied.

Figure 15: (a) Variation of W/c in the wake (b) NOB (cd_{32} of all the bubbles, as a function of C_Q . at $Re = 4.1 \times 10^5$ Circles and squares show the data point while the dashed line shows the power law curve fit.

The variation of SMD in transverse direction followed similar trend at different values of C_Q . It can be conjectured that the higher void fraction in the flow leads to modification of the turbulent kinetic energy spectra, which leads to a lower value of W/c , increasing critical bubble diameter d_{32} (as defined by Kolmogorov 1949) consequently causing an increment in d_{32} . Moreover, increased coalescence of bubbles at higher C_Q also adds to an increment in d_{32} . As Figure 16 presents, with the increment in C_Q , the curve shifts towards the right. This suggests that at larger C_Q , there is an increase in average size of bubbles across all vertical locations. However, not much difference is seen in the profiles at large values of C_Q ($C_Q > 2.5 \times 10^4$). This is because at higher values of C_Q , there is excessive clustering of bubbles leading to the formation of large bubble clusters/cloud, a situation where the image analysis approach is limited in resolving the big clusters into individual bubbles.

Figure 16: Sauter mean diameters at each vertical location for the range of C_Q tested at a fixed $Re = 4.1 \times 10^5$. Note that data points are shown only for the bubbly wake.

3.5. Bubble size prediction

The increase of the bubble size in the wake may be caused by: bubble generation at the injection point or the numerous coalescence events in the bubbly wake. **Silberman (1957)** investigated the production of bubbles by the disintegration of gas jets coming out of an orifice in liquids. His study indicated that the bubble diameter is independent of orifice size as long as the gas issues as a jet from the orifice into a cross flow. He gave a simple expression for bubble diameter as, $d_b = \sqrt[3]{\frac{6Q}{\pi \rho_b}}$, or $d_b^3 / \rho_b = \sqrt[3]{\frac{6Q}{\pi}}$. While this may be true for the bubbles generated at the injection slot, it does not capture the influence of breakup and coalescence events on the bubble size in the bubbly wake. The bubble sizes measured in our experiments are a function of the generated bubble size at the slot, and breakup and coalescence processes occurring in the wake. **Razzaque et al (2003)**

points out that the breaking of a continuous air jet into bubbles depends upon the turbulent velocity fluctuations in the flow, which is strongly dependent upon U (or Re) as $\sigma \sim U^3$. According to **Sevik and Park (1973)**, surface tension, viscous and inertial forces are three forces involved in the splitting of gas bubbles. Our experiments have proved a strong dependence of bubble size upon both Re and C_Q of the flow. An increase in Re decreased the SMD , while an increase in C_Q is observed to impose an opposite effect. In the literature, many studies have been reported that predict bubble size based on the theories developed by **Kolomogorov (1949)**, **Hinze (1955)** and **Levich(1962)**. These theories are based on the hypothesis that if a Weber number, We (defined based on a maximum bubble diameter, d_{max}) exceeds a critical value We_{crit} , then the bubbles of diameter larger than d_{max} break up due to the effect of turbulent velocity fluctuations. Many studies have suggested correlations based on these theories (**P. Andreussi et al., 1999**) to predict the maximum bubble size flowing in dispersed bubble regime. However, in our study maximum bubble size is not of paramount importance and is not the most suited parameter to characterize the bubble size distribution resulting from the flow physics in the bubbly wake. In the analysis of our experiments, it is necessary to assess the effect of breakup and coalescence on the bubble size, since the Kolomogoroff-Hinze theory is developed for single bubble flow in turbulent liquid flow. According to **Ferriera et al. (2012)**, the SMD is a significant scale for characterizing mass transfer during the interfacial processes, as also advocated previously by **Hesketh et al. (1987)** and **Razzaque et al. (2003)**. Thus, in the present study, we define a Weber number based on the Sauter mean diameter, d_{32} as: $We = (\rho U^2 d_{32} / \sigma)$. In general, We measures the ratio of the hydrodynamic pressure forces to surface tension forces in a bubble. Based on our experimental data across three different $AoAs$ of the hydrofoil, We is observed to depend upon Re and C_Q in the following manner: $We \propto Re^\beta C_Q^\gamma$, where β and γ and

the constant of proportionality are seen to depend upon the flow configuration viz. hydrofoil AoA as shown below in Table 1:

AoA	β	γ	R^2
0	1.706	0.179	0.999
4	1.543	0.140	0.999
8	1.515	0.134	0.999

Table 1: The parameters obtained by the experimental data to the fitting equation.

Table 1 presents the coefficients obtained from fitting our data to the equation given above and the R^2 value which gives a measure of the fit. It can be noted that there is a decreasing trend in the values of the coefficients as AoA is increased. An R^2 value greater than 0.999 for all angles of attack suggests a consistent trend in the dependence of We upon Re and C_Q .

4. Conclusions

In this paper, a series of ventilation experiments is conducted in the SAFL high speed water tunnel, providing aeration statistics and capabilities for the NACA 0015 hydrofoil. First, the results of a standard test-case ($Re = 4.1 \times 10^5$, $C_Q = 1.6 \times 10^{-4}$, AoA = 0) are discussed and then the effects of variation in experimental parameters is investigated. The image-processing approach used is experimentally validated for this reference case and is found to give excellent results. The bubble size distribution is found to be a bimodal one, with maxima at 0.25 mm and 0.6 mm. Broadly speaking, the shape distribution showed that the small size bubbles (< 0.3 mm) are mostly spherical while the large size bubbles are largely ellipsoidal. The spread of different size bubbles is studied and it is observed that because of the injection-slot being located on the pressure side of the foil, the larger bubbles stayed close to the location of the injection slot (i.e. below the centerline) due to greater inertia while the very small size bubbles (< 0.3 mm) are distributed uniformly in the wake. This preferential location of the large sized bubbles had an

impact on both the bubble velocity profiles and local void fractions distribution in the wake and these profiles are symmetrical, not about the centerline, but the vertical location of the injection slot. It is observed that upon moving downstream in the wake, the velocity deficit decreases and the wake tends to spread.

Next, the effect of variation in hydrofoil angle of attack and the flow conditions on the bubbly wake are studied. It is observed that an increase in hydrofoil angle of attack led not only to an upward drift in the wake but also increased liquid turbulence in the wake downstream, which resulted in greater splitting of large size bubbles (> 1 mm) into intermediate size bubbles ($0.5 - 1$ mm). Thus, the NOB increased while the d_{32} decreased.

An increase in Re (at a fixed C_Q) led to an increase in wake width and spread and greater turbulence in the wake. This promoted splitting and thus an increase in the fraction of small sized bubbles (~ 0.25 mm) and a reduced fraction of intermediate and larger size bubbles (> 0.5 mm). The total number of bubbles increased and d_{32} decreased. In contrast, an augmentation in C_Q (at fixed Re) had a contrary effect. The higher the C_Q , the smaller the fraction of small sized bubbles and the greater the fraction of intermediate to large size bubbles. An increase in C_Q resulted not only in increased number of bubbles, but also in increased bubble size due to increased coalescence events in the wake. Interestingly, the PDF (φ) of the normalized bubble size $\varphi = \varphi/d_{32}^3$, is found to have a universally similar shape independent of either Re , C_Q or hydrofoil AoA. Finally, it is proposed that We has a power law dependence upon Re and C_Q and the exact expression is derived by fitting with the experimental data. However, it should be noted that the current experiments are performed only for one hydrofoil. Further research is needed to ascertain the effect of variation in hydrofoil chord, span or the ventilation slot dimensions on bubble characteristics. In sync with the gas transfer measurements at the same conditions, these bubble

size measurements can serve as a test-bed for computational fluid dynamics (CFD) simulations of an auto-venting hydroturbine through the use of a ventilated hydrofoil. These results provide a rich experimental dataset that can be used by researchers to verify their CFD model against known bubble size distributions in a flow that simulates the wake of a vented hydroturbine.

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References

- Andreussi, P., Paglianti, A., & Silva, F. S. (1999). Dispersed bubble flow in horizontal pipes. *Chemical Engineering Science*, 54(8), 1101-1107. doi:10.1016/S0009-2509(98)00289-9
- Arndt, R.E.A., Kawakami, D. & Wosnik, M. (2007). Measurements in Cavitating Flows: Handbook of Fluid Mechanics Measurements. *Springer Verlag, Berlin*.
- Barnea, D., Shoham, O., & Taitel, Y. (1982). Flow pattern transition for vertical downward two phase flow. *Chemical Engineering Science*, 37(5), 741-744. doi:10.1016/0165-1684(94)90060-4
- Bohac, C. E., & Ruane, R. J. (1990). Solving the dissolved oxygen problem. *Hydro Review*, 9(1), 62-73.
- Bröder, D., & Sommerfeld, M. (2002). An advanced LIF-PLV system for analysing the hydrodynamics in a laboratory bubble column at higher void fractions. *Experiments in Fluids*, 33(6), 826-837.
- Brodkey, R. S. (1995). *The phenomena of fluid motions* Courier Dover Publications.
- Clift, R., Grace, J. R., & Weber, M. E. (2005). *Bubbles, drops, and particles*. Courier Dover Publications.

- Ferreira, A., Pereira, G., Teixeira, J. A., & Rocha, F. (2012). Statistical tool combined with image analysis to characterize hydrodynamics and mass transfer in a bubble column. *Chemical Engineering Journal*, 180, 216-228. doi:10.1016/j.cej.2011.09.117.
- Gonzalez, R.C., Woods, R.E. & Eddins, S.L. (2004) Digital image processing using MATLAB. : *Pearson Education India*.
- Goss, L. P., Estevadeordal, J., & Crafton, J. W. (2007). Velocity measurements near walls , cavities , and model surfaces using particle shadow velocimetry. Paper presented at the *22nd International Congress on Instrumentation in Aerospace Simulation Facilities*, Pacific Grove, CA. 1-8. doi:10.1109/ICIASF.2007.4380874
- Hesketh, R., Fraser Russell, T., & Etchells, A. (1987). Bubble size in horizontal pipelines. *AIChE Journal*, 33(4), 663-667. doi:10.1002/aic.690330414
- Hinze, J. (1955). Fundamentals of the hydrodynamic mechanism of splitting in dispersion processes. *AIChE Journal*, 1(3), 289-295.
- Karn, A., Ellis, C., Arndt, R.E.A. & Hong, J. (2014). An integrative measurement technique for dense bubbly flows with a wide size distribution. *Chemical Engineering Science* (In press).
- Kjeldsen, M., Arndt, R. E., & Effertz, M. (2000). Spectral characteristics of sheet/cloud cavitation. *Journal of Fluids Engineering*, 122(3), 481-487. doi:10.1115/1.1287854
- Kolmogorov, A. N. (1949). On the breakage of drops in a turbulent flow. *Dokl. Akad. Nauk. SSSR* 66, 825-828.

Levich, V. G. (1962). *Physicochemical hydrodynamics* Prentice-Hall Englewood Cliffs, NJ.

Liao, Y., & Lucas, D. (2009). A literature review of theoretical models for drop and bubble breakup in turbulent dispersions. *Chemical Engineering Science*, 64(15), 3389-3406.
doi:10.1016/j.ces.2009.04.026

Liao, Y., & Lucas, D. (2010). A literature review on mechanisms and models for the coalescence process of fluid particles. *Chemical Engineering Science*, 65(10), 2851-2864.
doi:10.1016/j.ces.2010.02.020

March, P. A., Brice, T. A., Mobley, M. H., & Cybularz, J. M. (1992). Turbines for solving the DO dilemma. *Hydro Review*, 11(1), 30-36.

Martínez Bazán, C., Montañés García, J. L., & Lasheras, J. C. (2000). Bubble size distribution resulting from the breakup of an air cavity injected into a turbulent water jet. *Physics of Fluids*, 12(1), 145-148.

Meyer, F. (1994). Topographic distance and watershed lines. *Signal Processing*, 38(1), 113-125.
doi:10.1016/0165-1684(94)90060-4

Razzaque, M. M., Afacan, A., Liu, S., Nandakumar, K., Masliyah, J. H., & Sanders, R. S. (2003). Bubble size in coalescence dominant regime of turbulent air–water flow through horizontal pipes. *International Journal of Multiphase Flow*, 29(9), 1451-1471.
doi:10.1016/S0301-9322(03)00123-X

Ruane, J.R. & Hauser, G.E. (1993). Factors Affecting Dissolved Oxygen in Hydropower Reservoirs, *Proceedings, WATERPOWER '93, ASCE*.

Sevik, M., & Park, S. (1973). The splitting of drops and bubbles by turbulent fluid flow. *Journal of Fluids Engineering*, 95(1), 53-60. doi:10.1115/1.3446958

Silberman, E. (1957) Production of bubbles by the disintegration of gas jets in liquid, *Proc. 5th Midwestern Conference on Fluid Mechanics*, U of Mich, Michigan, USA.

Thompson, E. J., & Gulliver, J. S. (1997). Oxygen transfer similitude for vented hydroturbine. *Journal of Hydraulic Engineering*, 123(6), 529-538.
doi:[http://dx.doi.org/10.1061/\(ASCE\)0733-9429\(1997\)123:6\(529\)](http://dx.doi.org/10.1061/(ASCE)0733-9429(1997)123:6(529))

Tse, K., Martin, T., McFarlane, C., & Nienow, A. (2003). Small bubble formation via a coalescence dependent break-up mechanism. *Chemical Engineering Science*, 58(2), 275-286. doi:10.1016/j.ces.2009.04.026

Wilhelms, S. C., Schneider, M. L., & Howington, S. E. (1987). Improvement of hydropower release dissolved oxygen with turbine venting. *Available from the National Technical Information Service, Springfield Virginia. 22161. Technical Report E-87-3.*