Modeling vortex-shedding effects for the stochastic response of tall buildings in non-synoptic winds

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Abstract

This study derives a model for the vortex-induced vibration and the stochastic response of a tall building in strong non-synoptic wind regimes. The vortex-induced stochastic dynamics is obtained by combining turbulent-induced buffeting force, aeroelastic force and vortex-induced force. The governing equations of motion in non-synoptic winds account for the coupled motion with nonlinear aerodynamic damping and non-stationary wind loading. An engineering model, replicating the features of thunderstorm downbursts, is employed to simulate strong non-synoptic winds and non-stationary wind loading. This study also aims to examine the effectiveness of the wavelet-Galerkin (WG) approximation method to numerically solve the vortex-induced stochastic dynamics of a tall building with complex wind loading and coupled equations of motions. In the WG approximation method, the compactly supported Daubechies wavelets are used as orthonormal basis functions for the Galerkin projection, which transforms the time-dependent coupled, nonlinear, non-stationary stochastic dynamic equations into random algebraic equations in the wavelet space. An equivalent single-degree-of-freedom building model and a multi-degree-of-freedom model of the benchmark Commonwealth Advisory Aeronautical Research Council (CAARC) tall building are employed for the formulation and numerical analyses. Preliminary parametric investigations on the vortex-shedding effects and the stochastic dynamics of the two building models in non-synoptic downburst winds are

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discussed. The proposed WG approximation method proves to be very powerful and promising to approximately solve various cases of stochastic dynamics and the associated equations of motion accounting for vortex shedding effects, complex wind loads, coupling, nonlinearity and non-stationarity.

Keywords: Tall buildings; non-synoptic winds; vortex shedding; nonlinear vortex-induced coupled aerodynamics; stochastic response; thunderstorm downburst; wavelet-Galerkin method; Daubechies wavelets.

1. Introduction

1.1. General context and motivation

Tall buildings and slender line-like structures (e.g., tall masts, wind turbines, flexible long-span bridges) are sensitive to wind-induced vibration and complex stochastic response due to the effect of nonlinear, coupled and transient/non-stationary aerodynamics and fluid-structure interaction (e.g., Kareem, 2010; Kareem and Wu, 2013). In the relatively low mean wind speed range, crosswind vortex-shedding effects cannot be neglected as they can produce large vibrations in the cross-wind direction. At larger medium-range mean wind velocities, turbulence-induced vibration often results in complex alongwind and crosswind stochastic response due to the coupling between aeroelastic self-excited forces and buffeting forces. The lock-in regime of the vortex shedding is plausible at high speeds for very tall buildings (Chen, 2013), in which nonlinear self-limiting structural vibration is possible due to the combination between nonlinear aerodynamic self-excited load and harmonic vortex shedding load (e.g., Dyrbye and Hansen, 1997). The combination of the random turbulence-induced load and the deterministic vortex-induced load may also possibly trigger stochastic resonance phenomena on slender vertical structures (e.g., Gammaitoni et al., 1998). Moreover, nonlinear effects of the vortex-shedding force could significantly affect the stochastic dynamics of tall buildings either inside or near the lock-in range at higher wind velocities. For example, it is known that a nonlinear damping effect (Van-der-Pol type) can influence the stochastic dynamic stability of bluff bodies. The quasi-periodic beating phenomenon is also possible with a limit cycle

vibration (e.g., Náprstek and Fischer, 2014); the same phenomenon is therefore plausible in the case of vortex shedding in the proximity of lock-in regime due to the nonlinear terms embedded in the Van-der-Pol equation.

The vortex-induced stochastic dynamics of a tall building requires the simulation of aerodynamic terms, such as the turbulent-induced buffeting loads, vortex-shedding force and self-excited force. In addition, coupling, nonlinear and non-stationary aerodynamics can potentially influence the stochastic dynamics of a tall building subjected to strong wind regimes. These particular loading conditions are seldom investigated even though they could be particularly dangerous for tall buildings, especially in the case of strong wind events such as thunderstorm downbursts, which do not satisfy the ordinary hypotheses of synoptic-wind boundary layer and stationary wind loading. An efficient simulation method for the solution of non-stationary stochastic vibration of tall buildings subjected to vortex shedding effects, nonlinear, coupled and transient aerodynamic loading in strong non-synoptic thunderstorm wind regimes is not fully available and still a challenging task.

1.2. Brief overview of vortex-shedding models for vertical structures in synoptic winds

Numerous studies on the vortex-induced vibration of long and flexible structural systems have been carried out in the case of circular and prismatic non-circular cylinder sections (e.g., Landl, 1975; Vickery and Basu, 1983a-b; Goswami et al., 1992, 1993; Matsumoto 1999). Traditionally, semi-empirical mathematical models have been proposed to replicate the main features of the vortex-induced vibration of line-like structures (e.g., Landl, 1975; Vickery and Basu, 1983a, 1983b, 1983c; Williamson and Govardhan, 2004), which are usually classified as either outside or inside the lock-in range depending on the mean wind speed. In the case of vibration outside the lock-in range, which is common to a large class of vertical structures, the vortex-shedding effects are often modeled as a combination of an aerodynamic self-excited force, either in-phase or out-of-phase with the relative velocity, and fluid-related (aerodynamic) harmonic vortex shedding force. If the wind speed meets certain conditions and the frequency of vortex shedding is close to the structural frequency, self-sustained lock-in vibration is possible, in which aerodynamic vortex

shedding force is negligible and the self-excited nonlinear negative-damping loading effects are predominant. Scanlan (1981) proposed and examined an empirical model to comprehensively describe, in a nonlinear form, the vortex-induced loading inside and outside the lock-in regime; the model is based on a set of physical parameters, which can be obtained from experiments (Ehsan and Scanlan, 1990). In many cases, the nonlinear aerodynamic damping term of the vortex-induced loading outside the lock-in has been neglected for the sake of simplification (e.g., Wu and Kareem 2013). Several semi-empirical models have been employed to simulate the effects of vortex shedding on slender structures, which preserve the relevant features of the loading. For time-domain simulations in wind engineering, models by Scanlan (1981), Ehsan and Scanlan (1990) for long-span bridges and by Goswami et al. (1992, 1993) for tall slender chimneys have been proven to be valid and applicable to a wide range of cases. Recent studies on the dynamic response of slender tall buildings (Chen 2013, 2014a) have also suggested the need for carefully re-examining the effects of vortex shedding, by demonstrating the relevance of “lock-in” and nonlinear vortex-induced-vibration for the next generation of super tall structures. Alternative models for vortex shedding response of slender bridges (Larsen 1995; Wu and Kareem, 2013) and line-like structures (Sun et al., 2014) have been recently examined. It must be noted that the loading parameters of these semi-empirical models are usually determined from the shedding frequency of the von-Kármán vortices outside the lock-in range, while the fundamental structural frequency is applied to estimate the model parameters in the lock-in range. Furthermore, spatial correlation and coherence of the loads is enhanced in the lock-in region. Recent mathematical models for the vortex-induced vibration of line-like structures have usually been derived in the frequency domain, making these models adequate in conventional synoptic winds, but they hardly capture nonlinear, unsteady and non-stationary features of the loading in non-synoptic winds.

1.3. Adaptation of current vortex-shedding models to non-synoptic winds

Currently, analysis of the wind-induced stochastic response of slender vertical structures is preferably carried out under the assumptions of linear structural response, simplified modeling of fluid-structure interaction and multivariate stationary wind loading by Fourier analysis (e.g., Kareem 1985, Piccardo and

Solari, 2000, Caracoglia, 2013). The Fourier transformation allows the coupled and nonlinear motion equations to be reduced to an algebraic form. Nevertheless, the solution of the coupled and nonlinear motion equations in the time domain, necessary in the case of transient wind loads, is still a major challenge for these structures; it is seldom pursued since it may require computationally demanding procedures of analysis (e.g., Kareem and Wu, 2013).

Aerodynamic nonlinearity and coupling of the dynamics is, on occasion, employed to formulate the wind-induced stochastic dynamic problems in a more general form for many types of vertical structures (e.g., Kareem, 2010; Ehsan and Scanlan, 2000; Chen 2013, Chen 2014a, Chen 2014b). Particularly, nonlinearity in super-tall buildings (Irwin, 2009) has been recently pointed out as important for structural design, and investigated due to potential interaction of the vibrating structure with nonlinear vortex-shedding effects (e.g., Chen 2014a). The coupling of aerodynamic loads is often analyzed in the study of the stochastic response of tall and slender buildings during extreme wind events, causing fluid-structure interaction. This approach involves the solution of coupled motion equations, combining turbulence-induced buffeting forces and motion-induced forces.

The hypothesis on multivariate stationary wind loads is no longer acceptable in the case of thunderstorms or downburst storms (e.g., Twisdale and Vickery, 1992). In this case accurate treatment of transient wind loads is needed to examine the stochastic dynamic response of the slender structure (Letchford et al., 2001; Wu and Chen, 2004; Sengupta et al., 2008; Chen and Letchford, 2004a). In recent years, the modeling and simulation of non-stationary winds and “time-frequency-dependent” response of tall buildings, slender vertical structures (Chen and Letchford 2004a, 2004b; Zhang et al., 2014) and long-span bridges (Chen, 2012; Xu, 2013; Cao and Sarkar, 2015) has emerged. Along the same line, the response spectrum technique (Solari et al., 2013; [Solari and De Gaetano, 2015](#)) has been proposed and examined to reproduce the features of the transient wind response of structures. Despite these technological advances, the numerical solution of the dynamic equations with transient/non-stationary loads, including vortex shedding loads, is still a complex task in structural engineering when nonlinearity is included (Huang and Iwan, 2006).

The non-synoptic and non-stationary characteristics of the downburst wind, simulated in this study, are: (i) time-varying mean velocity (magnitude and direction); (ii) non-synoptic vertical profile of the horizontal wind velocity, (iii) transient/non-stationary fluctuating wind velocity. At the present time very limited information on non-stationary wind fields and consequent pressure load distributions are available for the thunderstorm downbursts (e.g., full-scale measurements and experimental data). Since the local non-synoptic extreme winds in a thunderstorm downburst are often characterized by time-space intensification, due to translation velocity and the evolution of the downburst energy source, a hybrid “local-global” wind model can still be employed to investigate local non-synoptic wind events with sufficient accuracy for the purpose of examining the response of a slender vertical structure. Therefore, properties of the local downburst wind field and the global wind field are combined in this study to simulate the downburst loading and the dynamic response of a tall building. Another challenge in modeling the downburst loading is the sudden shift of the principal wind direction; this shift usually coincides with the occurrence of a second peak in the absolute value of the field velocity and subsequent decay of the storms.

For example, the influence of the downburst center touchdown point, relative to the position of the structure, on the mean wind velocity and the principal wind direction has been investigated in Le and Caracoglia (2015b). The results indicate that the touchdown longitudinal coordinate (x_0) is more important than the lateral coordinate (y_0 , downburst offset) of the downburst center. The downburst intensification decays faster with larger lateral coordinate offsets (y_0). The smaller the offsets (y_0) are, the shorter the duration of the wind direction shift is (closer to a 180-degree variation in the principal wind direction). For instance, small offsets (y_0) are preferable in order to simulate high intensification of the downburst wind loading: $y_0=150\text{m}$ is therefore employed in this study following the works by Holmes and Oliver (2000) and Chen and Letchford (2004a); $y_0=50\text{m}$ is recommended by Chay et al. (2006). With the application of a small offset y_0 , the plane of the downburst loading is primarily observed and intensified along the principal wind direction (x direction coordinate for the building, as later defined) whereas the participation of the “transverse” mean wind velocity component in the y direction can be neglected. To some extent, the assumption of constant wind direction, which is considered in this study, can be accepted for the simulation

of the downburst loading, owing to: (i) the shift of the wind direction is very brief with small offsets y_0 ; (ii) opposite wind direction occurs around the “secondary” peak of the velocity; (iii) this simplification usually produces the largest intensification of the downburst loading and the worst-case effect on the building.

1.4. Applicability of vortex shedding load models, developed for stationary winds, to downburst winds

In stationary synoptic winds, the vortex shedding is a physical phenomenon that requires an “activation time” (or memory effect in the fluid), similar to the concept of indicial functions in aeronautics or bridge aeroelasticity (e.g., Scanlan, 2000). The same phenomenon should also delay the formation of periodic fully-developed aerodynamic loading due to vortex shedding. This observation is related to the fact that the development of unstable shear layers around the surface of a tall building is not immediate. The hypothesis, used in this study, is that the time delay in the case of vortex-shedding loads should be somehow proportional or similar to the time delay needed by the static force components to become fully developed if a sudden change in the flow field or boundary conditions is observed (i.e., by similarity with the indicial function approach). Typical examples are the variation of the lift component on a flat plate due to a sudden change of angle of attack (Wagner function - Scanlan, 2000; Scanlan et al. 1974) or the corresponding unsteady aerodynamic loads on bluff bodies using the concept of indicial functions (e.g, Caracoglia and Jones, 2003). The simulation of the time delay and the memory effect in the vortex-shedding forces would be important if the duration of the transitory regime were of the same order of magnitude as the temporal variations in the approaching flow field. The slowly-varying flow field in a thunderstorm downburst is approximately stationary within a short duration (roughly 2 minutes). This time period corresponds to approximately between 200 and 250 dimensionless time units in a typical high wind downburst; the dimensionless time may be defined as $s=Ut/D$ with t time, U reference wind speed (approximately equal to 70 m/s or above in the case of a strong downburst, as later discussed in this study) and D a reference dimension of the bluff body. This range of dimensionless time values is at least one order of magnitude larger than the typical time needed for reaching a fully developed stationary load on a bluff body when the load is suddenly applied (indicial load). Therefore, there is sufficient time for the flow field to “adjust” and

to develop periodic shear layers, which are a prerogative of vortex shedding; the temporal duration is also long in comparison with the typical vibration period of tall structures (5 seconds or more).

It is plausible, therefore, to assume the existence of a fully-developed vortex-shedding regime during the strong regime of a thunderstorm. Also, it is possible to approximately neglect the transitory regime and to use a simplified approach, i.e., the vortex-shedding model developed for stationary winds (section 1.2). Nevertheless, more experimental investigation would be needed to examine the non-steady effects in the loading and the “activation time” of vortex-shedding effect in the non-synoptic downburst winds.

1.5. Objectives of the study

This paper examines, for the first time, the influence of vortex-shedding effects on the stochastic dynamics of a tall building in the time domain due to non-synoptic wind loads, by taking into account aerodynamic vortex-shedding loads, turbulent-induced stochastic loading and self-excited forces. Spatial correlation of these aerodynamic loads is simulated by introducing appropriate correlation lengths in the governing equations. The typical case of a translating thunderstorm downburst is employed in simulating the non-synoptic strong winds and the non-stationary wind loading on the structure. The study also examines the use of the Wavelet-Galerkin (WG) numerical algorithm to approximate the vortex-induced stochastic dynamics of tall buildings in the wavelet domain. The WG analysis method combines the features of the Galerkin approach, which converts a continuous operator such as a differential equation (Amaratunga et al. 1994; Amaratunga and Williams, 1997) into a discrete algebraic system, with orthogonal compactly-supported discrete Daubechies wavelets (Daubechies, 1988), used as orthonormal basis functions by the Galerkin projection. This combination of approaches decomposes and converts time-dependent nonlinear stochastic dynamic equations to random algebraic equations.

The use of the WG analysis method has been inspired by recent advances in the field of wavelet transform and wavelet analysis, used as examination tools for engineering and scientific computations. In structural dynamics, the WG method has been first introduced to study vibrations of continuous single-degree-of-freedom and two-degree-of-freedom systems with linear and time-dependent parameters

(Ghanem and Romeo 2000, 2001), and later used for the non-stationary seismic response of single-degree-of-freedom systems (Basu and Gupta, 1998) and for the analysis and modeling of continuous mechanical systems (Gopalakrishnan and Mitra, 2010). Even though wavelets in its various forms have been extensively employed for signal analysis, limited applications of the WG method are available in wind engineering for the simulation of wind loads and stochastic response of civil engineering structures. An initial investigation on the WG methods, numerical challenges, feasibility and applicability to wind engineering problems, in the presence of quasi-steady wind forces only, has been recently reported by Le and Caracoglia (2015).

Two simulated models of the tall buildings are investigated: (1) single-degree-of-freedom (*sdof*) lumped-mass model and (2) multi-degree-of-freedom (*mdof*) full-scale model. Both structural models are derived from the 183-m CAARC benchmark tall building (Commonwealth Advisory Aeronautical Research Council; Melbourne, 1980). In the former case, the vortex-induced stochastic dynamics of the *sdof* building model under the turbulent-induced buffeting loading and the vortex-induced loading outside the lock-in is investigated; this examination includes Van-der-Pol-type damping nonlinearity. In the latter case, the WG method is applied to study the vortex-induced stochastic dynamics of a full building model under vortex-induced loading and turbulence-induced loading with coupling between aerodynamic, buffeting and self-excited forces. Investigation also simulates the effect of a non-stationary/transient thunderstorm wind. The transient downburst wind loading is based on the following assumptions: translation effect of the thunderstorm simulated by constant horizontal thunderstorm velocity, time-independent downburst wind velocity profile, constant wind direction, non-stationary turbulent velocity fluctuations coupled with the translation effect. The time series of stationary wind speed fluctuations, from which the non-stationary field is derived, are digitally generated at different elevations along the building height by accounting for multivariate correlation.

2. Vortex-induced stochastic dynamics of the tall buildings: formulation

2.1. Sdof building model

The governing equation of the vortex-induced stochastic vibration of an equivalent *sdof* building model in turbulent wind flows in the y crosswind direction (Fig. 1) can be described in general form outside the lock-in regime as (e.g., Simiu and Scanlan, 1986; Ehsan and Scanlan, 1990):

$$m_y^* \ddot{y}(t) + c_y^* \dot{y}(t) + k_y^* y(t) = [f_b(t)]_{\{\ell_b^*\}} + [f_{se}(\dot{y}, y, t) + f_{vs}(\omega, t)]_{\{\ell_s^*\}}, \quad (1)$$

where y is the crosswind coordinate; m_y^* , c_y^* , k_y^* respectively are equivalent mass, damping and stiffness of the tall building in the y crosswind coordinate; $f_b(t)$ is the turbulent-induced buffeting force per unit length; $f_{se}(\dot{y}, y, t)$, $f_{vs}(\omega, t)$ are self-excited force and vortex-shedding force per unit length; $\{\ell_b^*\}$, $\{\ell_s^*\}$ denote equivalent correlation lengths of the turbulent-induced force and the vortex-induced forces on the building height, corresponding to the equivalent *sdof* building model. Correlation lengths also imply that the turbulent-induced and vortex-induced loads are either partially or fully correlated on the entire building height in the non-synoptic wind regime. Equivalent dynamic properties of the tall building can be estimated as:

$$m_y^* = c^* H m, \quad (2a)$$

$$k_y^* = 4\pi^2 n_y^2 m_y^*, \quad (2b)$$

$$c_y^* = 4\pi \zeta_y n_y m_y^*. \quad (2c)$$

In previous equation, H denotes total building height; m is equivalent uniform mass per unit length; c^* denotes the equivalent mass coefficient, $c^* = \ell^*/H < 1$ ($c^* \approx 0.333$ in a case of a normalized linear mode shape); n_y , ζ_y are, respectively, natural frequency and damping ratio of the building in the y -crosswind coordinate. The turbulent-induced force and the vortex-induced forces per unit length are expressed as (Ehsan and Scanlan, 1990; Piccardo and Solari, 2000):

$$f_b(t) = \left(\frac{1}{2}\right) \rho \bar{U} D [2C_L u(t) + (C_L' - C_D) v(t)], \quad (3a)$$

$$f_{se}(\dot{y}, y, t) = \left(\frac{1}{2}\right) \rho \bar{U}^2 (2D) \left[Y_1(K_v) (1 - \epsilon \frac{y(t)^2}{D^2}) \frac{\dot{y}(t)}{\bar{U}} + Y_2(K_v) \frac{y(t)}{D} \right], \quad (3b)$$

$$f_{vs}(\omega, t) = \left(\frac{1}{4}\right) \rho \bar{U}^2 (2D) C_{L,v}(K_v) \sin(\omega_v t). \quad (3c)$$

In the previous equations ρ is the air density; $\bar{U}$ is mean wind speed at the elevation of the lumped mass; D is a reference crosswind dimension of the building section; K_v is a reduced frequency ($K_v = \omega_v D / \bar{U} = 2\pi S_t$). In the absence of more experimental evidence, the frequency of the vortex shedding effect is evaluated independently of time in a “frozen” downburst state corresponding to the maximum intensification and maximum $\bar{U}$. Moreover, C_D and C_L are drag and lift force coefficients (the prime symbol designates first derivative with respect to the angle of attack); $u(t)$, $v(t)$ are time series of fluctuating wind velocities at the lumped mass in u -alongwind and v -crosswind directions; ω_v is the circular frequency of the vortex shedding determined from the Strouhal relationship; S_t is the Strouhal number of the building cross section; $Y_1(K_v)$, $Y_2(K_v)$, ϵ are model parameters of the vortex-induced force related to the aerodynamic damping, the aerodynamic stiffness and the nonlinear term, which are generally determined by empirical formulae or experiments; $C_{L,v}(K_v)$ is lift coefficient of the vortex shedding load.

The circular frequency ω_v of vortex shedding is distant from the pulsation of the generalized system $\omega_y = \sqrt{k_y^*/m_y^*}$; this assumption implies that the vortex-induced vibration is established in the Eqs.(1-3) outside the lock-in regime. Thus, the generalized dynamic response can be described as:

$$m_y^* \ddot{y}(t) + c_y^* \dot{y}(t) + k_y^* y(t) = \left(\frac{1}{2}\right) \rho \bar{U}^2 D \left[\begin{aligned} &2C_L \ell_b^* \frac{u(t)}{\bar{U}} + (C_L' - C_D) \ell_b^* \frac{v(t)}{\bar{U}} + \\ &2Y_1(K_v) (1 - \epsilon \frac{y(t)^2}{D^2}) \ell_s^* \frac{\dot{y}(t)}{U} + 2Y_2(K_v) \ell_s^* \frac{y(t)}{D} \\ &+ C_{L,v}(K_v) \ell_s^* \sin(\omega_v t) \end{aligned} \right], \quad (4a)$$

$$\begin{aligned} \ddot{y}(t) + \left(2\omega_y \zeta_y - \frac{1}{m_y^*} \rho \bar{U} D Y_1(K) (1 - \epsilon \frac{y(t)^2}{D^2}) \ell_s^*\right) \dot{y}(t) + \left(\omega_y^2 - \frac{1}{m_y^*} \rho \bar{U}^2 Y_2(K) \ell_s^*\right) y(t) = \\ = \left(\frac{1}{2m_y^*}\right) \rho \bar{U}^2 D \left[\begin{aligned} &2C_L \ell_b^* \frac{u(t)}{\bar{U}} + (C_L' - C_D) \ell_b^* \frac{v(t)}{\bar{U}} \\ &+ C_{L,v}(K_v) \ell_s^* \sin(\omega_v t) \end{aligned} \right]. \end{aligned} \quad (4b)$$

Eq. (4b) is the governing dynamic equation of the motion of the *sdof* building model in the turbulent wind flows, subjected to the vortex-induced loading. The governing equation can be converted to normalized y crosswind coordinate $\eta_y(t) = y(t)/D$ as:

$$\begin{aligned}
& \ddot{\eta}_y(t) + 2\omega_y \left(\zeta_y - \frac{\rho D^2}{m_y^*} \frac{1}{2K_y} Y_1(K_v) (1 - \epsilon \eta_y^2(t)) \ell_s^* \right) \dot{\eta}_y(t) + \omega_y^2 \left(1 - \frac{\rho D^2}{m_y^*} \frac{1}{K_y^2} Y_2(K_v) \ell_s^* \right) \eta_y(t) = \\
& = \left(\frac{\rho \bar{U}^2}{2m_y^*} \right) \left[\begin{aligned} & 2C_L \ell_b^* \frac{u(t)}{\bar{U}} + (C_L' - C_D) \ell_b^* \frac{v(t)}{\bar{U}} \\ & + C_{L,v}(K_v) \ell_s^* \sin(\omega_v t) \end{aligned} \right], \quad (5a)
\end{aligned}$$

or, equivalently:

$$\ddot{\eta}_y(t) + 2\omega_y \left(\zeta_y - \frac{F_d' \ell_s^*}{c^* H} \right) \dot{\eta}_y(t) + \omega_y^2 \left(1 - \frac{F_s' \ell_s^*}{c^* H} \right) \eta_y(t) = \left(\frac{\rho \bar{U}^2}{2m_y^*} \right) \left[\begin{aligned} & 2C_L \ell_b^* \frac{u(t)}{\bar{U}} + (C_L' - C_D) \ell_b^* \frac{v(t)}{\bar{U}} \\ & + C_{L,v}(K_v) \ell_s^* \sin(\omega_v t) \end{aligned} \right]. \quad (5b)$$

In the previous equation, K_y is reduced natural frequency; $K_y = \omega_y D / \bar{U}$; F_d' denotes an equivalent aerodynamic damping ratio, defined as $F_d' = \zeta_a = \frac{\rho D^2}{m} \frac{1}{2K_y} Y_1(1 - \epsilon \eta_y^2(t))$; F_s' is an equivalent aerodynamic stiffness term, as $F_s' = \frac{\rho D^2}{m} \frac{1}{K_y^2} Y_2$. The equivalent aerodynamic damping and stiffness terms can also be expressed by using two quasi-linear (i.e., frequency and time dependent) relationships during vibration; these are $F_d = \frac{m}{\rho D^2} F_d'$, $F_s = \frac{m}{\rho D^2} F_s'$, in which F_d , F_s are empirical aerodynamic damping and stiffness functions, later discussed. It will be described in a subsequent section how $Y_1(K_v)$ and $Y_2(K_v)$ can be derived from known empirical aerodynamic damping and stiffness properties, when the nonlinear term is neglected ($\epsilon = 0$), as:

$$Y_1(K_v) = F_d / \left(\frac{1}{2K_y} \right), \quad (6a)$$

$$Y_2(K_v) = F_s / \left(\frac{1}{K_y^2} \right). \quad (6b)$$

Therefore, the governing equation of the stochastic dynamics of the equivalent *sdof* model can be explicitly solved.

2.2. *Mdof building model*

The governing dynamic equation of the motion of the full-scale building model, subjected to turbulent-induced buffeting force, self-excited force and vortex-induced force (vortex-shedding effects) in non-

synoptic wind regimes can be expressed as a function of x -alongwind and y -crosswind generalized reduced-order coordinates of the building as (e.g., Caracoglia, 2013, Le and Caracoglia 2015):

$$m_x \ddot{x}(t) + c_x \dot{x}(t) + k_x x(t) = [q_{b,x}(t) + q_{se,x}(t, x, \dot{x})]_{\{\ell_b\}}, \quad (7a)$$

$$m_y \ddot{y}(t) + c_y \dot{y}(t) + k_y y(t) = [q_{b,y}(t) + q_{se,y}(t, y, \dot{y})]_{\{\ell_b\}} + [q_{v,y}(t, y, \dot{y})]_{\{\ell_s\}}. \quad (7b)$$

In the previous equations m_p , c_p and k_p respectively are the generalized mass, damping and stiffness of the p -th coordinate with $p = \{x, y\}$. The quantities $q_{b,p}(t)$, $q_{se,p}(t, p, \dot{p})$ and $q_{v,p}(t, p, \dot{p})$ are the generalized turbulent-induced buffeting force, the generalized self-excited force and the vortex-induced force, respectively. Subscript indexes $\{\ell_b\}$ and $\{\ell_s\}$ respectively denote the correlation lengths of the turbulent-induced forces and of the vortex-induced forces. These correlation lengths are used in the notation of the previous equations to imply that: (i) the turbulent-induced forces and the vortex-induced forces can be partially or fully correlated on the entire building height in the simulated wind regime, (ii) the parameters of the models describing turbulent-induced forces and vortex-induced forces have the same values along the correlation length and one can take them out of the integrals. In the unfavorable loading scenario, both the turbulent-induced and vortex-induced forces are fully correlated along the entire building height H ($\ell_b = \ell_s = H$) and one can integrate to estimate the turbulent-induced forces and the vortex-induced forces on the $[0, H]$ domain with constant model parameters.

The rotational motion is neglected since the main objective of this study is to examine the general phenomenon of vortex shedding in non-synoptic winds; if the primary wind direction is not skewed (i.e., orthogonal to one of the vertical faces of the benchmark building, Fig. 1b, later described) the effect on the building response is fundamentally observable in the transverse direction (y coordinate in this study). Even though lateral-torsional motion is possible, for example close to the building corners (e.g., Kareem, 1985) or when mode shapes are non-planar (e.g., Tse et al., 2007), the effects of torsional rotation are not considered in this paper. Regardless of this assumption, this effect could readily be included in future studies since the formulation is general.

The generalized structural dynamic properties pertinent to the generalized coordinates $p=\{x, y\}$ in Eq. (7) are:

$$m_p = \int_0^H \phi_p^2(z) m(z) dz, \quad (8a)$$

$$k_p = 4\pi^2 n_p^2 m_p, \quad (8b)$$

$$c_p = 4\pi \zeta_p n_p m_p, \quad (8c)$$

where z is the vertical coordinate along the building height; $\phi_p(z)$ is a continuous mode shape function; $m(z)$ is the distributed mass of the building per unit height; n_p, ζ_p are the fundamental natural frequencies and damping ratios. Generalized loading quantities are determined in the p -th generalized coordinates $p=\{x, y\}$, from the corresponding loading the per unit height, by integration as:

$$q_{b,p}(t) = (\ell_b/H) \int_0^H \phi_p(z) f_{b,p}(z, t) dz, \quad (9a)$$

$$q_{se,p}(t, p, \dot{p}) = (\ell_b/H) \int_0^H \phi_p^2(z) f_{se,p}(z, t, p, \dot{p}) dz, \quad (9b)$$

$$q_{v,y}(t, y, \dot{y}) = (\ell_s/H) \int_0^H \phi_y(z) f_{v,y}(z, t, y, \dot{y}) dz. \quad (9c)$$

In the previous equation, $f_{b,p}(z, t), f_{se,p}(z, t, p, \dot{p})$ are, respectively, the distributed buffeting forces and the self-excited forces per unit height in the p -th generalized coordinates with $p=\{x, y\}$; $f_{v,y}(z, t, y, \dot{y})$ is the vortex-induced force per unit height of the building in the y crosswind coordinate only. Global responses in the structural coordinates (P) can be estimated from the generalized responses in the generalized coordinates (p) as:

$$P = \phi_p p. \quad (10)$$

The distributed wind forces $f_{b,p}(z, t), f_{se,p}(z, t, p, \dot{p})$ per unit height z in the generalized coordinates $p=\{x, y\}$ are derived as a first-order approximation by quasi-steady aerodynamic theory of a rectangular cross section under the turbulent winds (e.g., Piccardo and Solari, 2000; Caracoglia, 2013; Le and Caracoglia, 2015). The force $f_{v,y}(z, t, y, \dot{y})$ is simulated by following Scanlan (1981) and Ehsan and Scanlan (1980). These forces are:

$$f_{b,x}(z, t) = \left(\frac{1}{2}\right) \rho \bar{U}(z, t) D [2C_D u(z, t) + (C'_D - C_L) v(z, t)], \quad (11a)$$

$$f_{b,y}(z, t) = \left(\frac{1}{2}\right) \rho \bar{U}(z, t) D [2C_L u(z, t) + (C'_L - C_D) v(z, t)], \quad (11b)$$

$$f_{se,x}(z, t, \dot{x}, \dot{y}) = \left(\frac{1}{2}\right) \rho \bar{U}(z, t) D \left[-2C_D \dot{x}(z, t) - \left(C'_D - C_L\right) \dot{y}(z, t) \right], \quad (11c)$$

$$f_{se,y}(z, t, \dot{x}, \dot{y}) = \left(\frac{1}{2}\right) \rho \bar{U}(z, t) D \left[-2C_L \dot{x}(z, t) - \left(C'_L - C_D\right) \dot{y}(z, t) \right], \quad (11d)$$

$$f_{v,y}(z, t, y, \dot{y}) = \left(\frac{1}{2}\right) \rho \bar{U}^2(z, t) (2D) \left[Y_1(K_v) (1 - \epsilon \phi_y^2(z) \frac{y(t)^2}{D^2}) \phi_y(z) \frac{\dot{y}(t)}{\bar{U}(z, t)} + Y_2(K_v) \phi_y(z) \frac{y(t)}{D} \right. \\ \left. + \frac{1}{2} C_{L,v}(K_v) \sin(\omega_v t) \right]. \quad (11e)$$

In the previous equations, $\bar{U}(z, t)$ is a time-dependent mean wind velocity of non-synoptic winds (e.g., the slowly varying wind speed in a thunderstorm downburst), which becomes $\bar{U}(z, t) = \bar{U}(z)$ in a synoptic winds; B, D are the alongwind and crosswind dimensions of the building section (floor plan); C_D, C_L are, respectively, the static force coefficients of the building sections in the x alongwind and the y crosswind coordinates (normalized by D); C'_D, C'_L are first-order derivatives with respect to the angle of attack; $u(z, t), v(z, t)$ are the zero-mean stationary Gaussian velocity fluctuations in the alongwind and crosswind coordinates; $\dot{x}$ and $\dot{y}$ designate the physical velocities of the building motion at z in the alongwind and crosswind coordinates (noting for example that $\dot{y}(z, t) \approx \phi_y(z) \dot{y}(t)$ in generalized form). It is evident from the Eq. (11a-e) that the dynamic equations of motion in the non-synoptic winds are coupled, nonlinear and non-stationary. It is also noted that traditional analysis methods for the solution by numerical integration can be extremely complex.

The coupled motion equations of the full-scale building model as a function of generalized coordinates $p=\{x, y\}$ in the non-synoptic winds can be written as:

$$m_x \ddot{x}(t) + [c_x - q_{x\dot{x}}] \dot{x}(t) - q_{x\dot{y}} \dot{y}(t) + k_x x(t) = q_{b,x}(t), \quad (12a)$$

$$m_y \ddot{y}(t) + [c_y - q_{y\dot{y}} - q_{y\dot{v}i}] \dot{y}(t) - q_{y\dot{x}} \dot{x}(t) + (k_y - q_{y\dot{v}i}) y(t) = q_{b,y}(t) + q_{vs,y}(t), \quad (12b)$$

where m_x, m_y, c_x, c_y, k_x and k_y are derived from Eq. (8a-c) for the x and y coordinates; $q_{x\dot{x}}$ and $q_{x\dot{y}}$ are generalized (quasi-steady) self-excited loading terms associated with the x coordinate, linearly depending on the velocities $\dot{x}$ and $\dot{y}$; $q_{y\dot{y}}$ and $q_{y\dot{x}}$ are generalized (quasi-steady) self-excited loading terms related to

the y coordinate, linearly depending on the velocities $\dot{x}$ and $\dot{y}$; $q_{\dot{y},vi}$ and $q_{y,vi}$ are generalized unsteady vortex-induced self-excited loading terms in the y coordinate; $q_{b,x}(t)$ and $q_{b,y}(t)$ are generalized buffeting forces in the x coordinate and the y coordinate; $q_{vs,y}(t)$ is generalized vortex shedding force in the y coordinate.

The quantities in Eq. (12a-b) are determined as follows:

$$q_{x\dot{x}} = -(\ell_b/H) \int_0^H \rho \bar{U}(z,t) D \phi_x^2(z) C_D dz, \quad (13a)$$

$$q_{x\dot{y}} = -(\ell_b/H) \int_0^H \left(\frac{1}{2}\right) \rho \bar{U}(z,t) D \phi_x^2(z) (C'_D - C_L) dz, \quad (13b)$$

$$q_{y\dot{y}} = -(\ell_b/H) \int_0^H \left(\frac{1}{2}\right) \rho \bar{U}(z,t) D \phi_y^2(z) (C'_L - C_D) dz, \quad (13c)$$

$$q_{y\dot{x}} = -(\ell_b/H) \int_0^H \rho \bar{U}(z,t) D \phi_y^2(z) C_L dz, \quad (13d)$$

$$q_{\dot{y},vi} = (\ell_s/H) \int_0^H \left(\frac{1}{2}\right) \rho \bar{U}(z,t) (2D) \phi_y^2(z) Y_1(K_v) (1 - \epsilon \phi_y^2(z) \frac{y(t)^2}{D^2}) dz, \quad (13e)$$

$$q_{y,vi} = (\ell_s/H) \int_0^H \rho \bar{U}^2(z,t) \phi_y^2(z) Y_2(K_v) dz, \quad (13f)$$

$$q_{b,x}(t) = (\ell_b/H) \int_0^H \left(\frac{1}{2}\right) \rho \bar{U}(z,t) D \phi_x [2C_D u(z,t) + (C'_D - C_L) v(z,t)] dz, \quad (13g)$$

$$q_{b,y}(t) = (\ell_b/H) \int_0^H \left(\frac{1}{2}\right) \rho \bar{U}(z,t) D \phi_y [2C_L u(z,t) + (C'_L - C_D) v(z,t)] dz, \quad (13h)$$

$$q_{vs,y}(t) = (\ell_s/H) \int_0^H \left(\frac{1}{4}\right) \rho \bar{U}^2(z,t) (2D) \phi_y(z) C_{L,v}(K_v) \sin(\omega_v t) dz. \quad (13i)$$

As a result, the Eqs. (12a-b) with Eqs. (13a-i) represent the governing dynamic equation of the motions of the tall buildings, simultaneously subjected to the buffeting force, the self-excited force and the vortex-induced force in the non-synoptic winds. The time-varying mean wind velocity $\bar{U}(z,t)$ of the non-synoptic winds is presented in the next section 3. In Eq. (13i) the wind velocity value used to calculate the shedding frequency is height-dependent, i.e., $\omega_s = 2\pi S_t \bar{U}(z)/D$, in which $\bar{U}(z) = \max_t \{\bar{U}(z,t)\}$ is the maximum value of the horizontal velocity in non-synoptic profile (frozen downburst state, as discussed in Section 2.1) by similarity with the case of vertical tapered structure in synoptic boundary layer shear flow (Vickery and Clark, 1972); the magnitude of the vortex shedding force varies with time and depends on the coordinate z since it is proportional to the square value of $\bar{U}(z,t)$.

The two principal coordinates of the *mdof* full-scale building model can also be normalized by the building depth (D) in the same way as the formulation of the *sdof* building model, $x(t) = D\eta_x(t)$ and $y(t) = D\eta_y(t)$. The model parameters of the vortex-induced forces can be estimated as the same manner as presented in section 2.1.

3. Non-synoptic “strong” wind model: the thunderstorm downburst

3.1. Downburst wind field

This section presents an analytical model of the downbursts for simulating the non-synoptic winds and the non-stationary stochastic wind loads on the tall buildings. Downburst was defined (Fujita, 1985) “as a strong downdraft, which induces an outburst of damaging winds on or near the ground”, which is often associated with thunderstorms. Thunderstorm downbursts are often non-synoptic, short-duration, strong wind events. They can produce large-amplitude transient response to tall buildings and flexible vertical structures in the thunderstorm-prone regions (e.g., Holmes and Oliver, 2000; Letchford et al., 2001). A hybrid “deterministic-stochastic” model is often employed to simulate effects of thunderstorm downbursts on structures. The downburst wind can be simulated as a combination of two concurring phenomena (e.g., Chen and Letchford, 2004a; Chay et al., 2006): a deterministic slowly-varying mean wind velocity (time scales of the order of few minutes), also known as the non-turbulent component, and a stochastic multivariate fluctuating wind velocity field, known as the turbulent component (time scales of the order of seconds). The deterministic mean wind velocity is often described in terms of time-space intensification of the horizontal component of the wind velocity; the concept of intensification results from the combination of the radial outflow velocity in a downburst and the translation velocity of the moving thunderstorm downburst. The stochastic fluctuating wind velocity is also non-stationary, as a result of the various stages in the life cycle of a downburst (Hjelmfelt, 1988); it is generally simulated by evolutionary spectral representation using amplitude modulation functions (e.g., Chen and Letchford, 2004b; Le and Caracoglia, 2015a).

Some assumptions have been used in this study to simplify the downburst wind model: (i) downburst translates along a straight line along the thunderstorm track; (ii) the downburst translation velocity is constant and height independent; (iii) the thunderstorm track is parallel to one principal coordinate of the building (x alongwind); (iv) the “average” horizontal wind direction of the total wind velocity vector is constant during the storm evolution, see Fig. 2. The non-stationary downburst wind along the p principal coordinates of the tall building with $p=\{x, y\}$ is determined as (e.g., Chen and Letchford, 2004a):

$$U(z, t) = \bar{U}(z, t) + u'(z, t) \text{ and } v'(z, t) \quad (14)$$

in which $U(z, t)$ is the total downburst wind velocity; $\bar{U}(z, t)$ is the deterministic time-dependent mean wind velocity (or “mean” velocity for brevity); $u'(z, t)$ and $v'(z, t)$ are the stochastic transient/non-stationary fluctuating wind velocities in the two principal directions, alongwind and crosswind. The time-dependent mean wind velocity of the non-synoptic downburst winds can be decomposed as:

$$\bar{U}(z, t) = \bar{U}(z)f(t) \quad (15)$$

In previous equation, $\bar{U}(z)$ denotes height-dependent horizontal wind velocity (known as the vertical wind velocity profile); $f(t)$ is a time-dependent weighting function. **The previous equation can be used not only to describe the magnitude of the velocity but also the variation in the approaching wind direction as the downburst center approaches the building. In this study (Section 1.3) the wind directionality effect is not considered since the duration of the shift in the mean wind direction (180 degrees) is short and observable only in the proximity of the secondary peak of the downburst. In any case, the directionality effect could be readily included in Eq. (15) by modifying the sign of the time-dependent weighting function $f(t)$.**

3.2. Deterministic time-dependent mean wind velocity

Analytical models for deterministic time-dependent mean wind velocity (the non-turbulent component) have been derived from past downburst observations and measurements, inspired by the pioneering work of the NIMROD and JAWS projects (Fujita, 1985). In these models, the horizontal velocity of the

downburst is found, at any time and position along the height z of the structure, from the vector sum of the downburst radial velocity and the downburst translation velocity. The radial velocity is determined from the maximum value of the horizontal wind velocity in the downburst vertical wind profile; it depends on the relative distance between the building and the time-varying position of the downburst. Two empirical models for the deterministic time-dependent mean wind velocity have been introduced and employed by researchers, which differ in the criterion used to estimate the time-space intensification function (e.g.; Holmes and Oliver, 2000; Chay et al., 2006). The downburst vertical wind velocity profile $\bar{U}(z)$ can be estimated using empirical formulae (Oseguera and Bowles, 1989; Vicroy, 1992; Wood and Kwok, 1998).

This study employs the Holmes-and-Oliver's model with modification by adding a time-dependent intensification function to reflect the various stages of the downburst life cycle. The downburst wind velocity components are first expressed in vector form as (Chen and Letchford, 2004a, Chay et al., 2006), see Fig. 2:

$$\vec{U}(z, t) = \vec{U}_r(z, t) + \vec{U}_{tran}, \quad (16a)$$

$$\bar{U}_r(z, t) = \Pi(t) \bar{U}(z) I(r). \quad (16b)$$

In the previous equations, $\vec{U}_r(z, t)$ and $\vec{U}_{tran}$ are, respectively, the downburst radial velocity and the downburst translation velocity vectors; $\bar{U}_r(z, t)$ is the modulus of $\vec{U}_r(z, t)$; $\bar{U}(z)$ is the downburst wind velocity profile, $\Pi(t)$ is the time-dependent intensification function of the downburst radial velocity; $I(r)$ is the space-dependent intensification function of the downburst radial velocity; r is the variable describing the relative horizontal radial distance between the center of the downburst and the geometric center of the building model (a function of time, downburst location and translation velocity).

Time-dependent and space-dependent intensification functions can be empirically determined as (Holmes and Oliver, 2000; Chay et al., 2006):

$$\Pi(t) = \begin{cases} t/t_0 & t \leq t_0, \\ \exp(-(t - t_0)/T) & t > t_0 \end{cases} \quad (17a)$$

$$I(r) = \begin{cases} r/r_{max} & r \leq r_{max} \\ \exp(-(r - r_{max})/r^*)^2) & r > r_{max} \end{cases} \quad (17b)$$

In previous equation, t_0 is the time at which the mean velocity reaches the maximum intensification and T is the total time (duration of the downburst); in the $I(r)$ function, which is illustrated in Figure 3, r_{max} is the radial distance at which the downburst wind velocity reaches the maximum value; r^* is the radial length scale, which is roughly taken as half the length of r_{max} .

The downburst vertical wind velocity profile at the building height is based on Vicroy's model (Vicroy, 1992; Chen and Letchford, 2004a):

$$\bar{U}(z) = 1.22[e^{-0.15z/z_{max}} - e^{-3.2175z/z_{max}}]\bar{U}_{max}, \quad (18)$$

where $\bar{U}_{max}$ is the maximum mean velocity in the downburst wind profile, z_{max} is the elevation at which the maximum velocity occurs.

The relative radius, magnitude and direction of the total horizontal velocity are (Fig. 2):

$$r(t)^2 = (x_0 - \bar{U}_{tran}t)^2 + y_0^2 \quad (19a)$$

$$\bar{U}(z, t)^2 = \bar{U}_r(z, t)^2 + \bar{U}_{tran}^2 + 2\bar{U}_r(z, t)\bar{U}_{tran}\cos\beta, \quad (19b)$$

$$\cos\theta = [\bar{U}(z, t)^2 + \bar{U}_{tran}^2 - \bar{U}_r(z, t)^2]/[2\bar{U}(z, t)\bar{U}_{tran}]. \quad (19c)$$

In the previous equation, x_0, y_0 are the coordinates of the downburst initial touchdown point; $\bar{U}_r(z, t)$, $\bar{U}_{tran}$ are, respectively, magnitudes of the radial velocity vector and the translation velocity vector; β is the angle between radial and translation velocity vectors, θ is the angle between horizontal and translation velocity vectors.

Figure 2b illustrates an example of downburst vertical wind velocity profile along the building height, employing Vicroy's model with the parameters $\bar{U}_{max}=67.0$ m/s and $z_{max}=80$ m. This elevation z_{max} is adjacent to the building nodes 18 and 19, while the maximum horizontal wind velocity at the node 41 (rooftop) is 57.95 m/s.

3.3 Stochastic fluctuating wind velocity

The transient/non-stationary stochastic wind fluctuations (rapidly-evolving turbulent components), i.e., u -alongwind and v -crosswind velocity components referenced to x and y coordinates, are also required. The

evolutionary spectral representation method using time-dependent amplitude modulation functions can be considered to simulate the transient turbulent components of the downburst winds on the different building nodes (Le and Caracoglia, 2015). Furthermore, the extended “frozen” downburst wind profile model at the initial touchdown state of the downburst is used. Numerical generation of the downburst rapidly-evolving velocity fluctuations is implemented through the following steps: (i) multivariate zero-mean Gaussian wind speed fluctuations are digitally simulated at the building nodes using the spectral representation method (e.g., Deodatis, 1998; Di Paola, 1998; Carassale and Solari, 2006), (ii) amplitude modulation functions are used to convert the stationary fluctuations to transient/non-stationary fluctuations at the building nodes. The cosine modulation function is used in this study (Le and Caracoglia, 2015).

In spite of the advances in the simulation of transient downburst wind fields, the following issues still need careful attention: (i) the use of either “frozen” downburst vertical wind velocity profile at the downburst touchdown point (initial intensity state) or the maximum downburst intensification (maximum intensity state), (ii) adequacy of amplitude modulation approximation to simulate non-stationary rapidly-evolving wind fluctuations from the theory of synoptic winds, (iii) applicability of the target spectral function, coherence function with corresponding parameters of synoptic winds to a non-synoptic downburst wind. At the present time very limited validation, based on comprehensive downburst wind measurements, is available. It is therefore arguable that a refined simulation model of the wind fluctuations should be considered in order to reproduce the basic characteristics of a downburst, such as a variable downburst wind velocity profile and direction, thunderstorm translation, turbulence intensity and length scales, power spectrum, time-dependent coherence of wind turbulence in a downburst. Promising refined approaches for simulating downburst wind fluctuations and dynamic response could be based either on the concept of evolutionary power spectrum of a stochastic process (Liang et al., 2007; Failla et al., 2011) or the response spectrum of the thunderstorm downbursts (Solari et al., 2013; Solari and De Gaetano, 2015). These are, however, beyond the scope of this study but could readily be incorporated in future investigations.

4. Wavelet-Galerkin approximation method and solution to stochastic dynamics of tall buildings

The principle of the wavelet-Galerkin approximation method is briefly introduced in this section for the sake of completeness. For the details about the WG method, the interested readers may refer to, for example, Gopalakrishnan and Mitra (2010) and Le and Caracoglia (2015).

4.1. Theoretical background

This section briefly introduces the WG approximation method. Further reading on the methodology can be found in Amaratunga and Williams (1997), Romine and Peyton (1998) and Le and Caracoglia (2015).

Wavelets $\psi_{a,b}(t)$, in which t is a generic variable, are defined as a family of piecewise functions, generated from a “mother” wavelet by scaling (a) and translation (b) parameters as $\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}} \psi(\frac{t-b}{a})$ (e.g., Daubechies, 1988). Wavelets are dilated and translated on the time-scale plane. The Daubechies wavelet family is applicable to a wide class of problems in time-scale plane computations, thanks to the properties of orthogonality, compactness and multi-resolution signal decomposition. The Daubechies wavelet of order or genus N consists of the twin functions of a “father” scaling function $\varphi(t)$ and the mother wavelet function $\psi(t)$. The values of the functions at various t can be recursively found from $\varphi(t) = \sum_{k=0}^{N-1} a_k \varphi(2t - k)$ and $\psi(t) = \sum_{k=-1}^{N-2} (-1)^k a_{k+1} \varphi(2t + k)$, where k denotes time translation and a_k are scaling coefficients. The dilated and translated father scaling function, at a resolution level j , is defined as $\varphi_{j,k}(t) = 2^{j/2} \varphi(2^j t - k)$. The main features of the scaling function and the wavelet function of the Daubechies wavelet of the order N are: (i) support over the finite interval $[0, N - 1]$; (ii) scaling coefficients satisfy $\sum_{k=0}^{N-1} a_k = 2$; (iii) orthogonality $\sum_{k=0}^{N-1} a_k a_{k+l} = \delta_{0,l}$, $\int_{-\infty}^{+\infty} \varphi_l(t) \varphi_k(t) dt = \delta_{0,l-k}$ and $\int_{-\infty}^{+\infty} \psi_l(t) \psi_k(t) dt = \delta_{0,l-k}$ ($\delta_{0,l}$, $\delta_{0,l-k}$ are Kronecker delta and l is translation); (iv) $\int_{-\infty}^{+\infty} \varphi(t) dt = 1$, $\int_{-\infty}^{+\infty} \psi(t) t^k dt = 0$; (v) $\varphi_l(t)$ is a continuous and differentiable function.

Figure 4 illustrates, as an example, the Daubechies wavelet of order 6 ($D6$), used in the further computation. The pair of the mother wavelet function and the father scaling function of the $D6$ is indicated

in Fig. 4a, while Fig. 4b depicts digital generation of the $D6$ wavelets at selected values of dilation and translation parameters on a 100-second duration interval. Obviously, the $D6$ scaling function expands with the increment of the dilation a (i.e., the time resolution reduces while the frequency resolution increases) and it contracts with the decrease of the dilation a (i.e., the time resolution increases while the frequency resolution reduces). The location of the dilated wavelets is determined by the translation parameter b ; concretely the $D6$ wavelets are located respectively at 10, 30 and 60 seconds on the time axis; these times correspond to the values of the translation b , see Fig. 4b. The Daubechies wavelet $D6$ has been selected in this study due to its optimality in computing time efficiency and numerical accuracy (Le and Caracoglia, 2015).

A generic motion variable, denoted by $x(t)$ in the following treatment, can be decomposed at a pre-selected resolution level j_0 by using the concept of multi-resolution analysis (Newland 1993; Le and Caracoglia, 2015):

$$x(t) = \sum_{k=1}^{N_x} c_{j_0,k} \varphi_{j_0,k}(t) + \sum_{j=0}^{j_0} \sum_{k=1}^{N_x} c_{j,k} \psi_{j,k}(t). \quad (20)$$

In the previous equation, $c_{j_0,k} = \langle x(t), \varphi_{j_0,k}(t) \rangle$ are “approximation” coefficients at the j_0 -th resolution, with the symbol $\langle . \rangle$ denoting inner product; $c_{j,k}$ are “detailed” coefficients at smaller scales $p < j$; $c_{j,k} = \langle x(t), \psi_{j,k}(t) \rangle$; k is a translation parameter (time index); N_x is the computational domain (support interval). The generic motion variable $x(t)$ can be approximated at a selected resolution level $j_0=j$ by neglecting smaller-scale resolution:

$$x(t) \approx \sum_{k=1}^{N_x} x_k \varphi_{j,k}(t), \quad (21)$$

where k is time index, determined in a finite time interval $[1, 2, \dots, N_x]$; x_k are approximation coefficients [similar to term $c_{j,k}$ in Eq. (20)], derived from $x_k = \langle x(t), \varphi_{j,k}(t) \rangle = \int_0^{N_x} x(t) \varphi_{j,k}(t) dt$. The advantage of Eq. (21) is the fact that it only requires definition of the $\varphi_{j,k}$ scaling function.

First and second derivatives of the motion variable $\dot{x}(t), \ddot{x}(t)$ are (Amaratunga et al., 1994):

$$\dot{x}(t) = \sum_{k=1}^{N_x} x_k \dot{\varphi}_{j,k}(t), \quad (22a)$$

$$\ddot{x}(t) = \sum_{k=1}^{N_x} x_k \ddot{\varphi}_{j,k}(t). \quad (22b)$$

In the previous equations $\dot{\varphi}_{j,k}(t)$, $\ddot{\varphi}_{j,k}(t)$ are the first-order and second-order derivatives of the scaling function, respectively. It is noted that the wavelet scaling function and its first, second derivatives are only supported on the interval $[0, N-1]$, often very short in comparison with the entire computational domain $[0, N_x-1]$, i.e., the relevant duration of the signal.

To utilize benefits of orthonormality and compactness of the Daubechies wavelets, the inner product operation between the approximating solutions $x(t)$, $\dot{x}(t)$, $\ddot{x}(t)$ and the scaling functions has been employed (e.g., Latto et al., 1991; Amaratunga and Williams, 1997; Le and Caracoglia, 2015):

$$\langle \varphi_{j,l}(t), x(t) \rangle = \langle \varphi_{j,l}(t), \sum_{k=1}^{N_x} x_k \varphi_{j,k}(t) \rangle = \sum_{k=1}^{N_x} x_k \Omega_{j,k-l}^{0,0}, \quad (23a)$$

$$\langle \varphi_{j,l}(t), \dot{x}(t) \rangle = \langle \varphi_{j,l}(t), \sum_{k=1}^{N_x} x_k \dot{\varphi}_{j,k}(t) \rangle = \sum_{k=1}^{N_x} x_k \Omega_{j,k-l}^{0,1}, \quad (23b)$$

$$\langle \varphi_{j,l}(t), \ddot{x}(t) \rangle = \langle \varphi_{j,l}(t), \sum_{k=1}^{N_x} x_k \ddot{\varphi}_{j,k}(t) \rangle = \sum_{k=1}^{N_x} x_k \Omega_{j,k-l}^{0,2}, \quad (23c)$$

where $\dot{x}(t)$, $\ddot{x}(t)$ are resultant velocity and acceleration; $\dot{\varphi}_{j,k}(t)$, $\ddot{\varphi}_{j,k}(t)$ are defined above; $\Omega_{j,k-l}^{0,0}$, $\Omega_{j,k-l}^{0,1}$, $\Omega_{j,k-l}^{0,2}$ are so-called 2-term connection coefficients in the wavelet space (Latto et al., 1991, Romine and Peyton, 1997). They are computed as:

$$\Omega_{j,k-l}^{0,0} = \int_0^{N_x} \varphi_{j,l}(t) \varphi_{j,k}(t) dt = \langle \varphi_{j,l}(t), \varphi_{j,k}(t) \rangle = \delta_{0,k-l}, \quad (24a)$$

$$\Omega_{j,k-l}^{0,1} = \int_0^{N_x} \varphi_{j,l}(t) \dot{\varphi}_{j,k}(t) dt = \langle \varphi_{j,l}(t), \dot{\varphi}_{j,k}(t) \rangle, \quad (24b)$$

$$\Omega_{j,k-l}^{0,2} = \int_0^{N_x} \varphi_{j,l}(t) \ddot{\varphi}_{j,k}(t) dt = \langle \varphi_{j,l}(t), \ddot{\varphi}_{j,k}(t) \rangle. \quad (24c)$$

The 2-term connection coefficients at any order of derivation are $\Omega_{j,k-l}^{d_1, d_2}$, in which d_1, d_2 are derivative orders ($d_1 \geq 0, d_2 \geq 0$); d_1 and d_2 with $\{0,1,2\}$ are usually required for linear and nonlinear second-order dynamical motion equations. The index $(k-l)$ denotes the “order of appearance” of the connection coefficients in the support interval. If the Daubechies wavelet of the order 6 is used in the approximation, a total of 9 ($2N-3$) connection coefficients are computed at the central translation point of the scaling function in relative terms $(k-l)$; the indices are usually designated in relative terms starting from this central

location as $(k-l)=\{-4,-3,-2,-1,0,1,2,3,4\}$. Furthermore, the connection coefficients are usually assembled into matrix form (Le and Caracoglia, 2015). It must be noted that the connection coefficients exclusively depend on the resolution level j and the order of the scaling functions.

Estimation of the wavelet connection coefficients, treatment of the initial condition treatment, determination of the resolution level, establishment of the computational domain, selection of the Daubechies wavelet and assemblage of the connection coefficients for the WG approximation solution of the stochastic dynamics of tall buildings have been presented in Le and Caracoglia (2015).

4.2. Solution of vortex-induced stochastic dynamics of *sdo*f building model in wavelet space

The WG approximation can be applied to transform the dynamics of the *sdo*f building model in section 2.1 by following the steps: (i) time-varying motion variables $\eta = \frac{y(t)}{D}$, $\dot{\eta} = \frac{\dot{y}(t)}{D}$, $\ddot{\eta} = \ddot{y}(t)/D$ in the crosswind direction and the wind force in Eq. (5b) are approximated in the wavelet space using Eqs. (21, 22a-b); (ii) the inner product operations are employed using Eqs. (23a-c, 24a-c); (iii) the nonlinear damping effect in the Van-der-Pol model is linearized and simulated using equivalent parameters F_d' and F_s' ; (iv) WG approximation solution is found from the approximating functions of the responses. Concretely, Eq. (1) is converted to the wavelet space as:

$$\mathbf{\Omega}^{0,2} \mathbf{\eta}_{\{k\}} + 2\omega_y \left(\zeta_y - \frac{F_d' \ell_s}{c^* H} \right) \mathbf{\Omega}^{0,1} \mathbf{\eta}_{\{k\}} + \omega_y^2 \left(1 - \frac{F_s' \ell_s}{c^* H} \right) \mathbf{\Omega}^{0,0} \mathbf{\eta}_{\{k\}} = [\mathbf{f}_{\{b,k\}} + \mathbf{f}_{\{vs,k\}}]. \quad (25)$$

In the previous equation $\mathbf{\Omega}^{0,2}$, $\mathbf{\Omega}^{0,1}$, $\mathbf{\Omega}^{0,0}$ are N_y -by- N_y connection coefficient matrices with $\mathbf{\Omega}^{0,2} = [\Omega_{k,l}^{0,2}]_{N_y \text{--by--} N_y}$, $\mathbf{\Omega}^{0,1} = [\Omega_{k,l}^{0,1}]_{N_y \text{--by--} N_y}$, $\mathbf{\Omega}^{0,0} = \mathbf{I} = [\delta_{k,l}]_{N_y \text{--by--} N_y}$ (Le and Caracoglia, 2015); N_y is the computational domain (equivalent to the generic index N_x used for the $x(t)$ variable in subsection 4.1); $\mathbf{\eta}_{\{k\}}$ is an unknown N_y -element vector of the unknown wavelet coefficients expressed in terms of the dimensionless motion variable ($\eta=y/D$) $\eta_k=y_k/D$. The subscript within braces $\{k\}$ is used to describe the fact that the elements of this vector span the whole domain of investigation. Similarly, $\mathbf{f}_{\{b,k\}}$ and $\mathbf{f}_{\{vs,k\}}$ are, respectively, known N_y -element independent vectors, derived from the WG approximation of the stochastic

buffeting force f_b and harmonic vortex shedding force f_{vs} in Eq. (5a); the scalar terms of each vector can be approximately found as $\mathbf{f}_{\{s,k\}} = \langle \varphi_{j,l}(t), f_s(t) \rangle = \langle \varphi_{j,l}(t), \sum_{k=1}^{N_x} f_{s,k} \varphi_{j,k}(t) \rangle = \sum_{k=1}^{N_x} f_{s,k} \delta_{j,k-l}$, where $s = \{b, vs\}$, $f_b = \frac{\rho \bar{U} \ell_b}{2mc^*H} \left[2C_L u(t) + (C_L' - C_D) v(t) \right]$, $f_{vs} = \frac{\rho \bar{U}^2 \ell_s}{2mc^*H} C_{L,v}(K_v) \sin(\omega_v t)$; k and l are integer indices with $(1-N) \leq k, l \leq (N_y - N + 1)$. It is also noted that the resolution level j is pre-determined and therefore its index is omitted in the expression used to designate the connection coefficient terms.

Next, Eq. (25) can be re-written in compact matrix form as:

$$\mathbf{A} \boldsymbol{\eta}_{\{k\}} = \mathbf{f}_{\{b,k\}} + \mathbf{f}_{\{vs,k\}}, \quad (26)$$

where the N_y -by- N_y system coefficient matrix is $\mathbf{A} = \boldsymbol{\Omega}^{0,2} + 2\omega_y \left(\zeta_y - \frac{F_d' \ell_s}{c^*H} \right) \boldsymbol{\Omega}^{0,1} + \omega_y^2 \left(1 - \frac{F_s' \ell_s}{c^*H} \right) \boldsymbol{\Omega}^{0,0}$; $\mathbf{f}_{\{b,k\}}$, $\mathbf{f}_{\{vs,k\}}$ are the buffeting and vortex-shedding force vectors.

The initial conditions in dimensionless form, $\eta(0) = y(0)/D = 0$ and $\dot{\eta}(0) = \dot{y}(0)/D = 0$, are also needed. Equation (26) is a linear algebraic equation that can be solved for the resultant displacement of the building model in wavelet space. After the vector $\boldsymbol{\eta}_{\{k\}} = \mathbf{y}_{\{k\}}/D$ is found, the resultant velocities and accelerations are determined in dimensional form as (Gopalakrishnan and Mitra, 2010, Le and Caracoglia, 2015):

$$\dot{\mathbf{y}}_{\{k\}} = \boldsymbol{\Omega}^{0,1} \mathbf{y}_{\{k\}}, \quad \ddot{\mathbf{y}}_{\{k\}} = \boldsymbol{\Omega}^{0,2} \mathbf{y}_{\{k\}}, \quad (27)$$

in which $\dot{\mathbf{y}}_{\{k\}}$ and $\ddot{\mathbf{y}}_{\{k\}}$ are resultant velocity and acceleration vectors in terms of wavelet coefficients.

4.3. Solution of vortex-induced stochastic dynamics of full-scale building model in wavelet space

One can employ similar procedure, presented in section 4.2, for the WG approximation analysis to the vortex-induced dynamics of the full-scale building model, derived from section 2.2. The coupled dynamic equations of motion in the generalized coordinates (the x -alongwind and the y -crosswind directions of the building model, Fig. 1) are converted to the wavelet space, as:

$$\{m_x \boldsymbol{\Omega}^{0,2} + (c_x - q_{x\ddot{x}}) \boldsymbol{\Omega}^{0,1} + k_x \boldsymbol{\Omega}^{0,0}\} \mathbf{x}_{\{k\}} - \{q_{x\dot{y}} \boldsymbol{\Omega}^{0,1}\} \mathbf{y}_{\{k\}} = \mathbf{q}_{\{bx,k\}}, \quad (28a)$$

$$\{m_y \boldsymbol{\Omega}^{0,2} + (c_y - q_{y\dot{y}} - q_{\dot{y}}) \boldsymbol{\Omega}^{0,1} + (k_y - q_y) \boldsymbol{\Omega}^{0,0}\} \mathbf{y}_{\{k\}} - \{q_{y\ddot{x}} \boldsymbol{\Omega}^{0,1}\} \mathbf{x}_{\{k\}} = \mathbf{q}_{\{by,k\}} + \mathbf{q}_{\{vs,k\}}. \quad (28b)$$

Similarly, in the previous equations $\mathbf{\Omega}^{0,2}, \mathbf{\Omega}^{0,1}, \mathbf{\Omega}^{0,0}$ are N_y -by- N_y connection coefficient matrices with $\mathbf{\Omega}^{0,2} = [\Omega_{k,l}^{0,2}]_{N_x-by-N_x}, \mathbf{\Omega}^{0,1} = [\Omega_{k,l}^{0,1}]_{N_x-by-N_x}, \mathbf{\Omega}^{0,0} = \mathbf{I} = [\delta_{k,l}]_{N_x-by-N_x}$ with indices k, l satisfying $(1-N) \leq k, l \leq (N_x - N + 1)$; N_x is the computational domain (similar to N_y); $\mathbf{x}_{\{k\}}, \mathbf{y}_{\{k\}}$ are the vectors of the WG approximation coefficients of the generalized displacements (in dimensional form) in the x and y coordinates; the index $\{k\}$ again describes the elements of this vector ranging in the whole computational domain; $\mathbf{q}_{\{bx,k\}}$ and $\mathbf{q}_{\{by,k\}}$ are vectors of the WG approximation coefficients of generalized turbulent-induced buffeting forces in the x and y coordinates. The quantity $\mathbf{q}_{\{vs,k\}}$ is the vector of the generalized vortex shedding force. Equations (28a-b) are extended to all $l, k = \{1, \dots, N_x\}$ to establish two coupled systems of algebraic matrix equations with random coefficients due to the presence of turbulent-induced buffeting forces and vortex-induced ones:

$$\mathbf{A}_{11}\mathbf{x}_{\{k\}} + \mathbf{A}_{12}\mathbf{y}_{\{k\}} = \mathbf{B}_1, \quad (29a)$$

$$\mathbf{A}_{21}\mathbf{x}_{\{k\}} + \mathbf{A}_{22}\mathbf{y}_{\{k\}} = \mathbf{B}_2. \quad (29b)$$

In the previous equations the quantities $\mathbf{B}_1 = \mathbf{q}_{\{bx,k\}}$ and $\mathbf{B}_2 = \mathbf{q}_{\{by,k\}} + \mathbf{q}_{\{vs,k\}}$ are column vectors of the approximation coefficients of the known generalized buffeting forces in x, y coordinates; $\mathbf{A}_{11}, \mathbf{A}_{12}$ and $\mathbf{A}_{21}, \mathbf{A}_{22}$ are equivalent coefficient matrices, determined as:

$$\mathbf{A}_{11} = m_x \mathbf{\Omega}^{0,2} + (c_x - q_{x\ddot{x}}) \mathbf{\Omega}^{0,1} + k_x \mathbf{\Omega}^{0,0}, \quad (30a)$$

$$\mathbf{A}_{12} = -q_{x\dot{y}} \mathbf{\Omega}^{0,1}, \quad (30b)$$

$$\mathbf{A}_{21} = -q_{y\ddot{x}} \mathbf{\Omega}^{0,1}, \quad (30c)$$

$$\mathbf{A}_{22} = m_y \mathbf{\Omega}^{0,2} + (c_y - q_{y\ddot{y}} - q_{y\dot{v}i}) \mathbf{\Omega}^{0,1} + (k_y - q_{y,vi}) \mathbf{\Omega}^{0,0}. \quad (30d)$$

Finally, resultant velocity and acceleration of the motion in generalized coordinates can be found in a similar way as in Eq. (27).

5. Numerical examples

5.1. Building model

The numerical example is the benchmark CAARC tall building (Melbourne, 1980), with dimensions $B=30.5$ m (width), $D=45.7$ m (depth) and $H=183$ m (height), see Fig. 1. Sectional aspect ratio of the building is $D/B \approx 1.5$. Distributed mass per unit height is constant, with $m(z)=220800$ kg/m independent of z . Natural frequencies of two fundamental lateral modes in the x alongwind and the y crosswind directions are $n_x=0.20$ Hz and $n_y=0.22$ Hz. Fundamental mode shapes are linearly related to the building height $\phi_x(z) = \phi_y(z) = (z/H)^\gamma$, $\gamma=1$; z is the nodal height. Structural modal damping ratios are equal to $\zeta_x=\zeta_y=0.01$ (Melbourne, 1980). The aerodynamic static coefficients and their first-order derivatives per unit height are roughly constant along the height and estimated as described in Smith and Caracoglia (2011) or Wei and Caracoglia (2015); these are approximately: $C_D = 1.1$, $C'_D = -1.1$ (alongwind) and $C_L = -0.1$, $C'_L = -2.2$ (crosswind). The building model is discretized into 41 nodes along the height, equally spaced at a distance of 4.575m. For the vortex shedding parameters, the Strouhal number of the reference cross section $B/D=1.5$ is $S_f=0.116$ (ESDU, 1998). Reduced frequency at vortex shedding is $K_v=2\pi S_f=0.728$. The lift force coefficient, which simulates the harmonic vortex shedding force, is $C_{L,v}(K_v)=0.278$ as a first approximation (ESDU, 1998; Wei and Caracoglia, 2015).

Two building models of the CAARC tall building have been employed in the study: (i) equivalent *s dof* building model and (ii) *mdof* full-scale building model. In the former model, a single concentrated mass is lumped at the rooftop node (Node 41), $m_y \approx 0.333M$, where M denotes the total building mass (Dyrbye and Hansen, 1997). Full-scale discrete lumped mass model in two generalized coordinates of the fundamental lateral modes in the x alongwind and the y crosswind directions are taken into consideration in the latter.

5.2. Non-synoptic downburst winds

The empirical model of a translating thunderstorm downburst is employed to simulate the non-synoptic winds and the non-stationary wind loading. The translation velocity of the downburst is $\bar{U}_{tran}=12$ m/s (Holmes and Oliver, 2000; Chen and Letchford, 2004a). Vicroy's model is applied to generate the non-synoptic vertical wind velocity profile, in which the maximum horizontal mean wind velocity is $\bar{U}_{max}=67$

m/s at the height $z_{max}=80$ m (Fujita, 1985). The downburst departs from its initial touchdown point of coordinates $x_0=\{2500 \text{ m}, 1500 \text{ m}\}$, $y_0=\{300 \text{ m}, 150 \text{ m}, 0 \text{ m}\}$, relative to the location of the building; various combinations of x_0 and y_0 are considered in the parametric investigations to study the influence of the time-varying mean velocity and the variation of the wind direction at the formation of the downburst. The space-dependent intensification function Eq. (17b) is determined with $r_{max}=1000$ m and $r^*=700$ m (Holmes and Oliver, 2000), while the values $t_0=200$ s and $T=400$ s are assigned to the time-dependent intensification function in Eq. (17a). Furthermore, the following properties of the synoptic turbulent winds are employed in simulating the downburst non-synoptic winds: (i) aerodynamic force coefficients are the same as those for synoptic winds, (ii) turbulence intensity is constant during downburst's evolution, (iii) empirical power spectral functions are similar to those of the synoptic winds, (iv) spatial coherence functions are the same as those of the synoptic winds. Concretely, the turbulence intensity is $I_u=I_v=20\%$; Harris' empirical power spectrum is used in the target spectrum (Melbourne, 1980); Davenport-type coherence is employed, with the decay factors $C_u=10$, $C_v=6.77$ (Le and Caracoglia, 2015).

5.3. Empirical aerodynamic damping and stiffness

The model parameters of the vortex-induced forces $Y_1(K_v)$, $Y_2(K_v)$ and μ , are not directly available for the a building section with aspect ratio and characteristics of the CAARC building from Melbourne (1980) or Saunders and Melbourne (1975); nevertheless these can be estimated by the empirical formulae, presented in Watanabe et al. (1997). Aerodynamic stiffness and damping of the vortex-induced loading per unit height can be determined from (Watanabe et al., 1997):

$$F_1 = A_m \frac{-2H_s(V_y/V_v)^2}{[1-(V_y/V_v)^2]^2 + 4H_s^2(V_y/V_v)^2}, \quad (31a)$$

$$F_2 = A_m \frac{(V_y/V_v)[1-(V_y/V_v)^2]^2}{[1-(V_y/V_v)^2]^2 + 4H_s^2(V_y/V_v)^2}, \quad (31b)$$

$$F_s = F_1 \cos \alpha + F_2 \sin \alpha, \quad (31c)$$

$$F_d = -F_1 \sin \alpha + F_2 \cos \alpha + F_p. \quad (31d)$$

Following the treatment in Watanabe et al. (1997), F_s and F_d in Eqs. (31c-d) are, respectively, empirical aerodynamic stiffness and damping functions, which can be found using Eqs. (31a-b); F_p is a mass-damping parameter. Moreover, A_m , H_s and α in the previous equations are parameters, which can be obtained from experimental wind tunnel data using either a reference maximum tip amplitude vibration (displacement of the aeroelastic model), turbulent intensity or sectional aspect ratio or section shape (square, rectangular, etc.); V_y and V_v are, respectively, reduced velocities at the crosswind natural frequency and at vortex shedding, $V_y = \bar{U}/(n_y D)$ and $V_v = \bar{U}/(n_v D)$, which depend on the height; n_y and n_v are frequency of the building in the y crosswind direction and the frequency of the vortex shedding.

The equivalent aerodynamic stiffness and damping can be determined via F_s and F_d , and the model parameters $Y_1(K_v)$ and $Y_2(K_v)$ are subsequently found by Eqs. (6a-b).

5.4. Empirical correlation lengths

Correlation coefficient functions of the vortex-induced forces are estimated using the empirical formula for oscillating squared cross section (Ehsan and Scanlan, 1990), as:

$$R_s(\Delta z) = \exp \left[-f_1(\eta_{max}) \left(\frac{\Delta z}{D} \right)^{f_2(\eta_{max})} \right], \quad (32a)$$

$$f_1(\eta_{max}) = \frac{0.052}{0.298 + \eta^{0.25}}, \quad (32b)$$

$$f_2(\eta_{max}) = \frac{0.065}{0.042 + \eta}. \quad (32c)$$

where η_{max} is the ratio between maximum vortex induced amplitude (y_{max}) and building depth (D), as $\eta_{max} = y_{max}/D$; Δz is spanwise spacing between any two nodal points along the building height. Meanwhile, the correlation coefficient functions of the buffeting forces can be approximated as spanwise coherence functions at the natural frequencies of the tall buildings (Dyrbye and Hansen, 1997), as:

$$R_{b,p,w}(n_p, \Delta z) = \exp \left[-\frac{2C_w n_p \Delta z}{\bar{U}(z_{q1}) + \bar{U}(z_{q2})} \right]. \quad (33)$$

In previous equation, p are the two principal coordinates of the building $p=\{x,y\}$; w are the two principal turbulence components or directions $w=\{u,v\}$; C_w are decay factors in the two wind directions; n_p are the

fundamental frequencies corresponding to the modes whose principal coordinate is $p=\{x,y\}$; z_{q1}, z_{q2} are vertical coordinates of any two building nodes; Δz is the spanwise spacing $\Delta z = |z_{q1} - z_{q2}|$; $\bar{U}(z_{q1})$, $\bar{U}(z_{q2})$ are time-independent mean wind velocities of the non-synoptic downburst wind velocity profile at the two elevations.

The generalized correlation lengths of the vortex-induced force and the turbulent-induced force can be determined from the corresponding correlation coefficient functions or the corresponding coherence function as:

$$\ell_f = \int_0^H R_f(\Delta z) d\Delta z, \quad (34a)$$

$$\ell_f^2 = \int_0^H \int_0^H R_f(\Delta z) dz_{q1} dz_{q2}, \quad (34b)$$

where the symbol $f=\{s,b\}$ denotes the vortex-induced ($R_s(\Delta z)$) and the turbulent-induced ($R_{b,p,w}(n_p, \Delta z)$) terms.

It is noted that the maximum displacement of the building due to the vortex-induced vibration is required to estimate the initial inputs of the model, i.e., to calculate F_s and F_d in Eqs. (31c-d) (consequently, Y_1 and Y_2) and the correlation length of the vortex-induced forces in Eqs. (32a-c). In this study, the initial maximum amplitude of the vortex-induced vibration of the building, which is needed to find F_s and F_d , is approximated by empirical formulae provided in ESDU (1998):

$$\eta_{rms} = \eta_{max}/\sqrt{2} = \left[0.00633 \frac{\rho B D}{m_y} \frac{1}{s_t^2} \right] \frac{C_{L'j}}{\zeta_{s,j}}, \quad (35)$$

In the previous equation, η_{rms} is the narrow-banded maximum amplitude; m_y denotes the mass per the unit length; $\zeta_{s,j}$ is the damping ratio; $C_{L'j} = C_{L'0} f_{ar} f_{Ls} f_{\eta}$ is a mode-dependent generalized coefficient, with the quantities in $C_{L'j} = C_{L'0} f_{ar} f_{Ls} f_{\eta}$ respectively accounting for the crosswind force coefficient, the aspect-ratio correction parameter, and the integral coefficients related to the spanwise correlation length. These parameters and coefficients have been determined using available plots in ESDU (1998).

It is noted that the critical mean wind velocity of lock-in for the building is estimated at $\bar{U}_{cr} = n_y D / S_t = 86$ m/s, which is larger than the maximum mean velocity of the non-synoptic downburst, $\bar{U}_{max} = 67$ m/s

(a velocity ratio $\bar{U}_{max}/\bar{U}_{cr} = 0.78$). Therefore, the vortex-induced aerodynamic loading is formulated outside the lock-in; it consists of both motion-dependent loading, expressed by Y_1 and Y_2 , and harmonic vortex shedding force at the vortex shedding frequency, satisfying the Strouhal relationship. Furthermore, the Van de Pol-type nonlinear aerodynamic damping term, which is defined by the parameter ϵ in Eqs. (5a-b) and is significant in the lock-in range, can be neglected (e.g., Ehsan and Scanlan, 1990). This observation leads to the simplification in Eqs. (6a-b) used for both *s dof* and *mdof* building model for F_d and F_s , i.e., Y_1 and Y_2 .

Basic building properties, which include geometry, dynamics, vortex shedding, downburst model, aerodynamic damping and aerodynamic stiffness of the vortex-induced loading, correlation lengths of the loads are summarized in Table 1 and Table 2.

6. Results and discussion

6.1. Simulated vortex-induced loading in a downburst wind field

Figure 5 illustrates empirical aerodynamic damping (F_d) and empirical aerodynamic stiffness (F_s) of the vortex-induced forces with various maximum amplitude levels $\eta_{max}=\{0.01, 0.015, 0.02, 0.025, 0.05\}$ and reduced velocity ratio range V_y/V_v between 0 and 3, derived from the empirical formulae Eq. (31a-d) by Watanabe et al. (1997). The aerodynamic damping and the aerodynamic stiffness, employed in the study, are calculated at the reduced velocity ratio $V_y/V_v = 0.773$ for the pre-selected maximum amplitude level $\eta_{max} = 0.01$, see Table 2. Accordingly, empirical aerodynamic damping and aerodynamic stiffness of the building are $F_d=1.474$ (equivalent aerodynamic damping ratio is 1.7%) and $F_s=-0.674$, respectively. The model parameters of the vortex shedding of the building (Ehsan and Scanlan, 1990) are determined through the estimated aerodynamic damping and stiffness, as indicated in Table 2.

Figure 6a depicts the correlation coefficient functions of the vortex-induced force with various maximum amplitude limits $\eta_{max}=\{0.001, 0.005, 0.01, 0.015, 0.02, 0.025, 0.05\}$. The correlation length is defined by integration of the correlation coefficient function on the entire building height, as in Eq. (34). It

is noted that the correlation lengths of the vortex-induced force are larger (i.e., this loading is more correlated along the building height) with higher correlation coefficient functions. Apparently, the maximum amplitude levels significantly influence the correlation lengths of the vortex-induced force on the tall building. Pre-selected maximum amplitude level of the vortex-induced is initially estimated by Eq. (35) of ESDU (1998). Pre-selected maximum amplitude level approximately $0.01D$ is employed in this study. The correlation coefficient functions between two principal wind directions $\{u, v\}$ and two principal building coordinates $\{x, y\}$ of the turbulent-induced loading are presented in Fig. 6b. Averaged values of the two correlation lengths for yu and yv combinations are used for computing the correlation length of the turbulent-induced forces. As a result, $\ell_b/H = 0.868$ and $\ell_s/H = 0.821$ are employed in the computations, as shown in Table 2.

6.2. *Sdof equivalent building model: examination of the dynamic response*

Transient wind turbulence components in the u -alongwind and the v -crosswind directions are generated at the lumped mass elevation (rooftop) for the equivalent *sdof* building model. Model parameters $Y_1(K_v)$, $Y_2(K_v)$ and $C_{L,v}(K_v)$ of the vortex-induced loads are computed for the vortex shedding relationship. The mean wind velocity at the lumped mass elevation is $\bar{U} = 57.95$ m/s, according to Vicroy's downburst profile. The values are also given in Tables 1 and 2.

The WG method is employed to numerically solve the governing equation of motion in Eqs. (25-26) to obtain stochastic displacement, velocity and acceleration. Figure 7 shows an example of normalized displacement ($\eta = y/D$) of the equivalent *sdof* building model in the y -crosswind direction, subjected to: (i) combined vortex-induced and turbulent-induced buffeting loads in Fig. 7b, (ii) vortex-induced loads only in Fig. 7c, (iii) turbulent-induced loads only in Fig. 7d. The corresponding realizations of the turbulent-induced buffeting force and the vortex-shedding force at the lumped mass elevation are presented in Fig. 7a. Maximum amplitudes of resultant displacements and the relative contribution of the buffeting and vortex shedding loads on the resultant response are examined. The maximum amplitudes at the rooftop node are respectively $0.0075D$ due to combined vortex-induced and turbulent-induced loads, $0.0016D$ due

to the vortex shedding load only, $0.0064D$ due to the buffeting load only. It is observed that the buffeting load is dominant (85% contribution) on the resultant displacement, while the contribution of the vortex shedding is smaller (21%).

Influence of the downburst initial touchdown point $\{x_0, y_0\}$ and the downburst translation on the time-dependent mean wind velocities and on the principal wind direction at the building position are examined. Figure 8 illustrates the evolution of the downburst mean velocities and the variation of the principal wind direction at various touchdown points $x_0=\{1500 \text{ m}, 2500 \text{ m}\}$, $y_0=\{0 \text{ m}, 150 \text{ m}, 300 \text{ m}\}$, determined at elevation $z=80 \text{ m}$ (at which maximum radial velocity occurs). It can be seen in Fig. 8a that the position of the velocity peaks along the time axis and the shape of the downburst mean wind velocities predominantly depend on the horizontal coordinate (x) rather than the lateral one (y). This observation is clearly influenced by the fact that the downburst translates along the horizontal direction of coordinate (x). The initial touchdown coordinate (x_0) influences the starting point of the downburst along the time axis but it does not alter the shape of the mean velocities. Moreover, a variation in the initial lateral coordinate $y_0=\{0 \text{ m}, 150 \text{ m}, 300 \text{ m}\}$ only affects the background values of the downburst mean velocities.

Figure 8b illustrates the evolution of the principal wind direction at the building location during the downburst evolution and translation. Initial touchdown coordinates $\{x_0, y_0\}$ moderately influence the wind directions with investigated values. However, abrupt variation (shift) of the alongwind direction is observed in a very short time interval (approximately 150 seconds) between two velocity peaks during the downburst evolution. During the evolution between the two velocity peaks, the alongwind directions are opposite, shifted by almost 180 degrees. The abrupt variation in the principal wind direction between two velocity peaks reduces with an increase of the initial coordinate y_0 , but it is independent of the initial coordinate x_0 . This observation can be explained by recalling the structure and the life cycle of a thunderstorm downburst (Hjelmfelt, 1988): two maxima of the space-dependent intensification are possible as the frontward and backward lobes of a symmetrical downburst ring structure pass over the building. When the initial lateral coordinate y_0 is closer to zero, the variation in the direction angles is 180 degrees and the principal wind direction (which corresponds to the x -alongwind direction) is abruptly reversed at the building location.

This observation implies that the downburst time-dependent mean velocity is still dominated by the contribution of the x -alongwind component and the secondary effect of the y -crosswind component can consequently be neglected. This finding also agrees with the initial assumptions on the wind velocity decomposition according to the two principal wind coordinates in Eq. (14) and in the equations of the aerodynamic forces in section 2.

Figure 8c illustrates the maximum mean crosswind velocities ($\bar{V}$) at the reference touchdown point coordinates $x_0=\{2500 \text{ m}\}$, $y_0=\{0 \text{ m}, 50 \text{ m}, 150 \text{ m}, 300 \text{ m}\}$ and at the elevation $z_{max}=80 \text{ m}$ corresponding to maximum downburst intensification. The curves at various y_0 confirm that the effect of the crosswind mean velocity component increases if the touchdown offset (y_0) increases. If the downburst track passes directly through the central axis (x) of the tall building, the mean crosswind velocities are zero. If the offsets y_0 are incremented to 50 m, 150 m and 300 m the maximum mean crosswind velocities are equal to, respectively, 7.6%, 22.7% and 44.4% to the maximum horizontal mean velocities ($\bar{U}_h$). From the analysis of Fig. 8(c) it is suggested that the time-varying mean crosswind velocity components should possibly be taken into consideration to determine the aerodynamic loads, when the downburst touchdown offset is larger than 200 m (with a nearly 30% ratio between crosswind and alongwind downburst reference velocities). In this study, the main numerical results are obtained with a downburst touchdown offset equal to $y_0=150 \text{ m}$ and, therefore, the mean crosswind velocity component of the downburst wind has been neglected for the sake of simplification.

6.3. *Mdof building model: examination of downburst wind field*

Figure 9 shows examples of downburst time-dependent mean wind velocities at the representative building nodes 5, 10, 20, 30 and 41 (rooftop) for two downburst initial touchdown positions $\{x_0=1500 \text{ m}, y_0=150 \text{ m}\}$ and $\{x_0=2500 \text{ m}, y_0=150 \text{ m}\}$. The distance between the thunderstorm track-line and x -alongwind building coordinate is kept constant with $y_0=150 \text{ m}$. The time series represents a 400-second duration record. The location of the nodes along the building height and the non-synoptic vertical wind profile are illustrated in Figs. 1b and 2b. The time-dependent mean velocity at node 20 is the largest one since node

20 is the closest to the maximum of Vicroy's velocity profile with $\bar{U}_{max}=67$ m/s at $z=80$ m. In contrast the mean wind velocity at node 5 is relatively smaller than the typical values of the corresponding synoptic boundary layer winds (Le and Caracoglia, 2015). Two peaks in the time-dependent mean wind velocities can be observed in the 400-second interval. They corresponds to the largest magnitudes in the vector sum of the slowly-varying downburst wind velocity components as the downburst center moves parallel to the x direction of the building (Oliver and Holmes, 2000).

Transient downburst high-frequency wind fluctuations (u -alongwind and v -crosswind components in the horizontal directions x and y with respect to the building floor plan) are digitally simulated at all the building nodes. The extended frozen downburst model with weighted downburst wind velocity profile and cosine modulation function are employed for simulating the transient downburst wind fluctuations, as described in section 3b and explained in Le and Caracoglia (2015). The mean speed value suggested by the Vicroy's model is used for converting the dimensionless frequency of the turbulence spectrum model (Harris' spectrum) to dimensional frequency. Figure 10a shows realizations of u -alongwind transient stochastic wind fluctuations (high-frequency and zero-mean turbulent component) at building nodes 20 and 41 (rooftop), over a 400-second time interval. The simulated time series of the v -crosswind stochastic wind fluctuations (turbulent component) at nodes 20 and 41 are presented in Fig. 10b. The transient turbulent wind fluctuations of the downburst at other building nodes are not shown for the sake of brevity. From the inspection of Figure 10 it can be noticed that the turbulent wind field components are not stationary due to the application of the amplitude modulation function; for instance, the maximum turbulence intensity (20%) is achieved in the proximity of the center of the simulated record (maximum intensification of the downburst effects) while it gradually decreases towards the end of the record. A more detailed study on the "design" of the modulation function for replicating the downburst turbulence features may be found in Appendix C of Le and Caracoglia (2015) and it not replicated here for the sake of brevity.

Figure 11 illustrates 400-second realizations of total alongwind downburst wind velocities $U(z, t)$ at building nodes 41 (rooftop) and 20 (adjacent to the elevation corresponding to the maximum radial wind velocity) for two initial touchdown positions $\{x_0=1500 \text{ m}, y_0=150 \text{ m}\}$ and $\{x_0=2500 \text{ m}, y_0=150 \text{ m}\}$. It is

noted that the total downburst wind velocities are determined as the summation of the time-dependent mean wind velocities (non-turbulent component) and the transient stochastic wind fluctuations (turbulent component) at the building nodes. Downburst wind velocities at other nodes for the u -alongwind field and for the v -crosswind field are also simulated but are not shown.

6.4. *Mdof building model: examination of the dynamic response*

After the two velocity fields (non-turbulent and turbulent fields) of the downburst winds are digitally simulated at all discrete building nodes, the generalized wind loads are computed as in Eq. (13a-i) by coupling buffeting, quasi-steady self-excited forces and vortex-induced loads. The model parameters F_d , F_s , $Y_1(K_v)$ and $Y_2(K_v)$ of the vortex-induced loads in the case of the full-scale *mdof* are inherited from the equivalent *sdof* building model. Moreover, the frequency of the vortex shedding effect is computed as a function of the height (coordinate z) but it is time independent; it is based on the modulus of the mean wind velocity derived from the non-synoptic vertical wind profile at the building nodes in the frozen configuration (section 2.2).

The WG method is subsequently applied to approximate the generalized and global building response of the full-scale building in the x -alongwind and y -crosswind coordinates by solving Eqs. (28-29) in the wavelet domain under the simulated downburst wind fields (single realization). Two types of downburst wind velocity and load models are investigated: (i) downburst wind velocities with time-dependent mean excluded (i.e., loads caused by transient stochastic wind fluctuations only) from Fig. 10, (ii) downburst wind velocities with both time-dependent mean velocity and transient stochastic fluctuations (high-frequency turbulence) from Fig. 11.

Figure 12a illustrates realizations of 400-second time histories of the normalized global displacements ($\eta=x/D$) in the x -alongwind direction at node 41 (rooftop) and node 20 (close to elevation $z_{\max}=80$ m at which the maximum horizontal velocity $U_{\max}=67$ m/s occurs in the downburst wind profile) with initial touchdown position $\{x_0=1500$ m, $y_0=150$ m}. In the figure load contributions from both transient high-frequency turbulence and time-dependent mean velocity fields are considered. The time-varying “mean” of

the resultant total displacement is an evolutionary temporal process, in which a quasi-static displacement of the building due to the “mean” velocity effect slowly evolves over time to construct the response. Therefore, the stochastic response due to high-frequency turbulence fluctuations can be determined by excluding the quasi-static time-evolving response from the total resultant response. Realizations of the stochastic displacements at nodes 41 and 20 in the x -alongwind direction, associated with loads pertaining to the downburst turbulent fluctuations, are indicated in Fig. 12b. These stochastic responses can alternatively be estimated from the fluctuating loads, directly applied to the building nodes. As a result, maximum displacements (in modulus) at the building nodes can be determined from the time histories of the fluctuating displacements. For instance, the maximum amplitudes of the fluctuating downburst displacements in the x -alongwind direction are, respectively, $0.0295D$ at the rooftop node 41 and $0.014D$ at the node 20, as shown in Figure 12b.

Figure 13 shows time series of normalized displacements in the y -crosswind direction ($\eta=y/D$) at building nodes 41 and 20 with initial touchdown position $\{x_0=1500 \text{ m}, y_0=150 \text{ m}\}$ due to: load case (a) associated with both vortex-induced and turbulent-induced forces in Fig. 13a, load case (b) vortex-induced forces only in Fig. 13b, load case (c) derived from turbulence-induced forces and vortex-induced self-excited forces by excluding the harmonic vortex-shedding force in Fig. 13c. The maximum absolute values of the displacements in the y -crosswind direction for the three load cases are, respectively, $0.00295D$ (node 41) and $0.00134D$ (node 20) for case (a) in Fig. 13a, $0.00036D$ (node 41) and $0.00017D$ (node 20) for case (b) in Fig. 13b, $0.00243D$ (node 41) and $0.00115D$ (node 20) for case (c) in Fig. 13c. Apparently, contribution of the vortex shedding loading on the global downburst displacements in the y -crosswind direction is secondary compared to that of the buffeting loading in this particular investigation. Concretely, the vortex shedding loading approximately contributes 12% (node 41) and 13% (node 20) to the y -crosswind displacements in comparison with 88% (node 41) and 87% (node 20), respectively.

Figure 14 illustrates realizations of time histories of normalized global displacements at nodes 40 and 21 in the x -alongwind direction for downburst touchdown position $\{x_0=2500 \text{ m}, y_0=150 \text{ m}\}$, due to

combined non-turbulent and turbulent wind velocities and loads. The quasi-static slowly time-varying mean displacements are included in the total responses.

Figure 15 presents the resultant x -alongwind displacements at nodes 41 and 20 for downburst touchdown position $\{x_0=1500 \text{ m}, y_0=150 \text{ m}\}$; the graphs are obtained by WG method, by either accounting for or neglecting the time-dependent mean velocities of the downburst winds in the estimation of the aerodynamic loads. The slowly-varying time-dependent mean displacements, induced by loads due to time-dependent mean velocity field, are extracted from the total displacements *a posteriori*, by moving average operation with 30-second segmental windows. After removal of the slowly-varying time-dependent displacements, the remaining displacements (due to high-frequency fluctuating velocity field) are also decomposed and compared to the stochastic displacements obtained by exclusively applying the loads associated with the downburst turbulent fluctuations. Adequate agreement can be observed in both node 41 and node 20 between the stochastic displacements, directly and indirectly obtained by WG method with and without the loads pertaining to the deterministic time-dependent mean wind velocity field.

The stochastic nature of the peak dynamic response of the tall building in the simulated downburst winds is further investigated. The generation of several realizations of the turbulent wind fields in the downburst wind has been repeated 50 times by synthetically constructing 50 independent turbulent fields. In each realization the time series of the turbulent velocities at various building heights are random but they all share the same turbulent properties of the downburst wind. From each independent record of the sample the wind loads are calculated and the time series of the x -alongwind and y -crosswind displacements at various building heights (floors) are reconstructed using the afore-mentioned analytical procedure and the proposed WG simulation method. The peak displacements are extracted from each random time series as the maxima during the simulation period, corresponding to the duration of the thunderstorm downburst. Subsequently, the mean and the standard deviations of the estimated peak displacements at the various building floors are empirically obtained by examining the result obtained with the 50 turbulent field and load datasets. Figure 16 depicts mean and standard deviation of the stochastic peak displacements (total response) of the building in the x -alongwind and y -crosswind directions at the reference building nodes

(floors) 20, 30 and 41 due to repeated simulations of a downburst wind by with given deterministic initial touchdown position $\{x_0=1500 \text{ m}, y_0=150 \text{ m}\}$. Concretely, the mean value of the x -alongwind peak displacement at the rooftop node 41 is $0.0184D$, while the standard deviation of the x -alongwind peak displacement is $0.0055D$ (approximate 29%), in Fig. 16a. Figure 16b presents the mean and the standard deviation values of the y -crosswind displacements in two cases with and without the effect of the vortex-induced loads. It is observed that the vortex-induced loading effect on the total response of the building is not predominant in this specific case of downburst field and building configuration. The mean value and the standard deviation of the y -crosswind peak displacements are, respectively, $0.0027D$ and $0.00089D$ by including vortex-induced loads and vortex shedding load; in contrast, they are $0.0026D$ and $0.00093D$, respectively, when the vortex-induced load is excluded.

The sample size (50 realizations) has been selected in order to guarantee sufficient accuracy in the estimation of the mean value of the peak and its standard deviation. For example, the relative error in the estimation of the mean peak response, based on one standard deviation and defined as $\varepsilon = (\pm \sigma[X]/\sqrt{N})/E[X]$ (where X is the random set of simulated peak values, $E[X]$ and $\sigma[X]$ are empirical mean and standard deviation of the sample, N is the number of realizations), is only 4% and 5% in the x -alongwind and y -crosswind directions, respectively, at the rooftop (node 41). This approach, which is based on Monte Carlo sampling of the response to estimate the peak effects on the structure, does not account for the randomness in the downburst properties, such as the touchdown point (x_0 and y_0); more investigation beyond the scope of this study should possibly be devoted to the examination of these and other random features of the downburst.

Table 3 compares the maximum absolute values of the normalized global displacements at node 41 (rooftop) and node 20 (close to the elevation of maximum radial velocity) in both x -alongwind and y -crosswind directions of the *sdof* building model and the *mdof* building model. The results correspond to a simulated downburst with initial touchdown point $\{x_0=1500\text{m}, y_0=150 \text{ m}\}$. For the y -crosswind direction, the displacements are separately computed by considering the effects of: (i) combined vortex-induced and

the turbulent-induced loads, (ii) vortex-induced loads only and (iii) turbulence-induced loads only. It is noted that there is a significant difference in the y-crosswind resultant displacements between the two building models (*sdof* and *mdof*). The maximum amplitudes of the displacement, predicted by the *sdof* building model (e.g., $\eta=0.0075$ at node 41), are larger than those obtained with the *mdof* building model ($\eta=0.0028$ at node 41). This finding can be explained by remarking that the equivalent lumped mass, placed at the rooftop node of the *sdof* model, and the corresponding derivation of the load correlation lengths along the building height employed to simulate the behavior of the full-scale tall building, might produce disproportionate response amplitudes for the investigated simplified structure.

7. Conclusions

This study formulates a model to investigate the stochastic dynamic response of tall buildings in the time domain with loads accounting for vortex-induced effects, generated for a non-synoptic strong wind velocity field associated with a thunderstorm downburst. A novel wavelet-Galerkin approximation method is examined and used to solve the complex governing equations of motion accounting for partial correlation between turbulence-induced forces and vortex-induced forces, and for the coupling between turbulence-induced forces and self-excited forces. An equivalent *sdof* building model and an *mdof* building model of the CAARC benchmark tall building have been employed in the simulations and the numerical analyses.

The main contributions of this study are: (i) formulation of the stochastic vortex-induced dynamics of a tall building in the time domain, subjected to turbulence-induced and vortex-induced loads by accounting for spatial load correlation; (ii) examination of the wavelet-Galerkin method for solving the vortex-induced response of a tall building potentially exposed to lock-in phenomenon during a strong and sustained downburst wind; (iii) derivation of the equations of motion and dynamic response for a generic tall building, subjected to thunderstorm downburst loads, in the wavelet domain.

Apart from the influence of the downburst's center translation on the horizontal plane (e.g., Chen and Letchford, 2004a), the study investigated the effects of non-stationary loading features (intensity), evolving

in both space and time along the building height, touchdown position and time-space intensification on the downburst loading. Future studies will be directed towards the improvement of the downburst loading model by critically reviewing the assumptions used to simulate the stochastic wind fluctuations (for example by examining the adequacy of the wave superposition method with time-varying amplitude modulation function **or the hypothesis on wind directionality**) and by relaxing the hypothesis of quasi-steady forces (for example by comparison with pressure data; Zhang and Sarkar, 2014).

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Figure 5. Empirical aerodynamic damping and stiffness of the vortex-induced forces: (a) aerodynamic stiffness, (b) aerodynamic damping

Figure 6. Correlation functions: (a) vortex-induced force, (b) turbulent-induced force

Figure 7. Time series of normalized displacement ($\eta=y/D$) of the equivalent *sdo*f building model due to: (a) turbulent-induced and vortex-induced forces, (b) vortex-induced and turbulent-induced forces, (c) vortex-induced forces only

Figure 8. Downburst mean wind velocities and principal wind directions as a function of the initial touchdown point $\{x_0, y_0\}$ and height $z=80$ m: (a) time-dependent mean wind velocities, (b) principal wind directions, (c) examination of mean crosswind velocities $\bar{V}$

Figure 9. Simulated time series of time-dependent mean wind velocities (non-turbulent component) at selected building nodes: (a) downburst touchdown position $\{x_0=1500 \text{ m}, y_0=150 \text{ m}\}$, (b) downburst touchdown position $\{x_0=2500 \text{ m}, y_0=150 \text{ m}\}$

Figure 10. Simulated time series of downburst fluctuating wind velocities (turbulent components) at selected building nodes for downburst touchdown position $\{x_0=1500 \text{ m}, y_0=150 \text{ m}\}$: (a) u -alongwind turbulence, (b) v -crosswind turbulence

Figure 11. Simulated time series of total alongwind wind velocities at node 41 (rooftop) for downburst touchdown positions: (a) $\{x_0=1500 \text{ m}, y_0=150 \text{ m}\}$, (b) $\{x_0=2500 \text{ m}, y_0=150 \text{ m}\}$

Figure 12. Time series of x -alongwind displacements at building nodes for downburst touchdown position $\{x_0=1500 \text{ m}, y_0=150 \text{ m}\}$ with loads derived from: (a) combination of non-turbulent and turbulent wind velocity fields, (b) turbulent wind field only (without the effect of time-dependent mean wind velocity)

Figure 13. Time series of y -crosswind displacements at selected building nodes for downburst touchdown position $\{x_0=1500 \text{ m}, y_0=150 \text{ m}\}$ due to: (a) vortex shedding and turbulence-induced forces, (b) vortex-shedding forces only, (c) turbulence-induced forces only ($q_{y,vi}$ and $\dot{q}_{y,vi}$ terms included)

Figure 14. Time series of x -alongwind displacements at selected building nodes for downburst touchdown position $\{x_0=2500 \text{ m}, y_0=150 \text{ m}\}$ with loads derived from the combination of non-turbulent and turbulent wind velocity fields

Figure 15. Comparison of WG solutions describing the x -alongwind displacements due to simulated downburst wind with initial touchdown position $\{x_0=1500 \text{ m}, y_0=150 \text{ m}\}$: (a) Node 41, (b) Node 20

Figure 16. Mean and standard deviation of stochastic peak responses at reference building nodes 20, 30 and 41 due to simulated downburst wind with initial touchdown position $\{x_0=1500 \text{ m}, y_0=150 \text{ m}\}$: (a) x -alongwind displacements, (b) y -crosswind displacements with and without the effects of vortex-induced loads

Table 1.

Notations	Description	Assigned
B	Width (m)	30.5
D	Depth (m)	45.7 m
H	Height (m)	183 m
B/D	Aspect ratio	0.67
$m(z)$	Mass per unit height (kg/m)	220800
ζ_x, ζ_y	Damping ratio	0.01
n_x	Natural frequency (x) (Hz)	0.20
n_y	Natural frequency (y) (Hz)	0.22
ϕ_x, ϕ_y	Mode shape in (x, y), $\gamma=1$	$(z/H)^\gamma$
C_D	Drag coefficient	1.1
C'_D	First derivative of C_D	-1.1
C_L	Lift coefficient	-0.1
C'_L	First derivative of C_L	-2.2
S_t	Strouhal number	0.116
K_v	Reduced frequency (V. S.)	0.728
$\bar{U}_{cr}$	Critical velocity (L.I.) (m/s)	86
$C_{L,v}(K_v)$	Lift coefficient (V. S.)	0.278
$\bar{U}_{max}$	Max horizontal velocity	67
z_{max}	Height of $\bar{U}_{max}$ (m)	80
$\bar{U}_{tran}$	Translation velocity (m/s)	12
$\{x_0, y_0\}$	Touchdown position (m), case 1	{ 1500,150 }
	Touchdown position (m), case 2	{ 2500,150 }

Notes: “V. S.”: Vortex shedding; “L.I.”: Lock-in

Table 2.

Notations	Description	Assigned
V_y/V_v	Reduced velocity ratio	0.773
$m/\rho D^2$	Mass ratio	84.578
F_d	Aerodyn. damping	1.474
F_s	Aerodyn. stiffness	-0.674
ζ_a	Aerodyn. damping ratio	0.0174
$Y_1(K_v), \epsilon=0$	Model parameter (V. S.)	0.51
$Y_2(K_v)$	Model parameter (V. S.)	-0.02
ℓ_b/H	Correlation length (Turb.)	0.868
ℓ_s/H	Correlation length (V. S.)	0.821 (0.61)

Notes: “Turb.”: Turbulence; “Aerodyn.”: Aerodynamic; (.): by ESDU

Table 3.

Node		Vortex- & Turbulence- induced	Vortex- induced only	Turbulence- induced only
Equivalent <i>sdof</i> building model (in y-crosswind)				
Node 41	Disp. (y/D)	0.0075	0.00165	0.0064
	Cont. (%)		21	85
Full-scale <i>mdof</i> building model (in y-crosswind) ^(#)				
Node 41	Disp. (y/D)	0.00276	0.00015	0.00261
	Cont. (%)		5.4	94.6
Node 20	Disp. (y/D)	0.00130	0.00008	0.00122
	Cont. (%)		6.2	93.8
Full-scale <i>mdof</i> building model (in x-alongwind) ^(#)				
Node 41	Disp. (x/D)	-	-	0.0184
Node 20	Disp. (x/D)	-	-	0.0087

Notes: “Disp.”: Normalized displacement (peak values)

“Cont.”: Contribution to the total response (in percentage)

^(#) : Average peak values based on 50 stochastic realizations

(a)

(b)

Figure 1.

(a)

(b)

Figure 2.

Figure 3.

(a)

(b)

Figure 4.

(a)

(b)

Figure 5.

(a)

(b)

Figure 6.

(a)

(b)

(c)

(d)

Figure 7.

(a)

(b)

(c)

Figure 8.

(a)

(b)

Figure 9.

(a)

(b)

Figure 10.

(a)

(b)

Figure 11.

(a)

(b)

Figure 12.

Figure 13.

Figure 14.

(a)

(b)

Figure 15.

(a)

(b)

Figure 16.