Research on variational mode decomposition in rolling bearings fault diagnosis of the multistage centrifugal pump

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Abstract: Rolling bearing faults are among the primary causes of breakdown in multistage centrifugal pump. A novel method of rolling bearings fault diagnosis based on variational mode decomposition is presented in this contribution. The rolling bearing fault signal calculating model of different location defect is established by failure mechanism analysis, and the simulation vibration signal of the proposed fault model is investigated by FFT and envelope analysis. A comparison has gone to evaluate the performance of bearing defect characteristic extraction for rolling bearings simulation signal by using VMD and EMD. The result of comparison verifies the VMD can accurately extract the principal mode of bearing fault signal, and it better than EMD in bearing defect characteristic extraction. The VMD is then applied to detect different location fault features for rolling bearings fault diagnosis via modeling simulation vibration signal and practical vibration signal. The analysis result of simulation and experiment proves that the proposed method can successfully diagnosis rolling bearings fault.

Keywords: Rolling element bearings; Fault diagnosis; Variational mode decomposition; Fault signal modeling and simulation; Vibration signal analysis;

1 Introduction

Multistage centrifugal pump is the key equipment of the process industry. Since its high efficiency, wide performance and stable operation, it is widely used in refineries, power plants, chemical plants, etc [1]. Rolling bearings are frequency encountered in multistage centrifugal pump due to their carrying capacity and low-friction characteristics. As the connection between the rotor and the support, the safety and stability of rolling bearings are the key to ensure the safety and stability of the multistage centrifugal pump. Therefore, it is very important to diagnosis the rolling bearing fault at its incipient stage in order to prevent long-term breakdowns or in some cases possibly catastrophic failures.

Various diagnosis techniques are used to prevent machinery failures caused by the rolling bearings and new methods are being developed. Sugumaran et al. [2] have comprehensively studied of fault diagnostics of rolling bearings using continuous wavelet transform. Ali et al. [3] realize automatic rolling bearings fault diagnosis by using the method of empirical mode decomposition and artificial neural network. Xue et al. [4] have proposed adaptively fast ensemble empirical mode decomposition method and applied to rolling bearings fault diagnosis. Safizadeh and Latifi[5] presents a new method for rolling bearings fault diagnosis by using the fusion of an accelerometer sensor and a load cell sensor. Zhao et al. [6] via discriminative subspace learning algorithm effectively recognize different rolling bearings fault.

Variational mode decomposition is an adaptive signal decomposition method recently proposed by Dragomiretskiy and Zosso [7], which is an entirely non-recursive signal decomposition method. VMD can extract the principal mode of the signal and their respective center frequencies. VMD has been applied in many fields for signal extraction and has achieved a certain effect. For example, Zhao et al. [8] denoise the power transformer partial discharge by using VMD and achieve a better result of signal de-noising. Wang et al. [9] make use of VMD to extract features of rub-impact fault in the rotor system. The result of comparison VMD, EWT, EEMD, EMD

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shows that the rub-impact features can be better extracted with the VMD. Viswanath et al. [10] detect spike of disturbed power signal by using VMD and found that the proposed methodology gives good results in case of single tone signals. Upadhyay and Pachori [11] use VMD to detect instantaneous voiced/non-voiced of speech signals. The experimental results at different signal to noise ratios indicate the effectiveness of the proposed method. At the same time, VMD was compared with some other signal decomposition method such as EMD, EWT, EEMD et al, and it shows superior performance in signal decomposition and feature extraction [12-14].

In this paper, VMD is used to detect the different location defect signal of multistage centrifugal pump rolling bearings. The rolling bearings fault of outer race, inner race and rolling element can be diagnosis by the extracted mode signal. In order to analyze the validity of the proposed method, we studied the failure mechanism of rolling bearings, established the different location defect signal model of rolling bearing, and simulated the fault signal of outer race defect, inner race defect, and rolling element defect in section 2. Then, we analyze and diagnosis the simulation vibration signal of different fault bearing by using VMD and EMD in section 3. The fault vibration signal is given to VMD and it is observed that the defect information of the original signal is available in one of its modes, which can be obtained by energy ratio calculation on all modes. The mode with defective signal has higher energy ratio than other modes and this is observed in the discussion for the simulation and experimental signal. The fault of rolling bearings is simulated by using multistage centrifugal pump test rig, and the proposed method is verified in section 4.

2 Simulation and analysis of vibration signal of rolling bearings with different location defect

The vibration signal of rolling bearings is complex either under normal or with defect, resulting from its structure, tolerance, and surface deterioration. Many researchers predict the vibration response of rolling bearings by modeling analysis. McFadden and Smith [15, 16] developed a mathematical modeling to predict the vibration produced by a single defect on the inner race of a rolling bearing under constant radial load. Tandon [17, 18] proposed an analytical model for predicting the vibration frequencies of rolling bearings and the amplitudes of significant frequency components due to a localized defect on the outer race, inner race or on one of the rolling elements under radial and axial load. Kiral and Karagulle [19, 20] presented a model for defect detection in rolling bearings with single or multiple defects under the action of an unbalanced force based on the finite element vibration analysis. Cong et al. [21] derived the defect signal calculating the equation of the outer race and inner race by dynamic and kinematic analysis. The derivation takes into consideration of the gravity of the rotor-bearing system, the imbalance of the rotor, and the location of the defect on the surface. In these models, the vibration signal of different location defect is established by a series of impulses. When a defect on one surface strikes its matching surface, the impulse will be produced which can excite resonances of bearing and housing structures. As the bearing rotates in constant speed, these impulses are generated periodically and the frequency of impulses can be determined by the position of the defect.

In this section, the single point defect impulse signal model of out race defect and inner race defect was set up based on reference [21], and then the signal model of rolling element defect was derived. Various location defect simulation signal will realize by using the proposed model and the vibration characteristic frequency. Simulation signal of outer race defect, inner race defect, and rolling element defect will be discussed and analyzed by using envelope analysis method and fast Fourier transform (FFT).

2.1 Rolling bearings fault signal modeling.

The simulated signal of rolling bearings defect calculating model can be expressed as [21]:

$$x(t) = \sum_{i=1}^N A_i \cdot s(t - iT_0 - \tau_i) \quad (1)$$

where T_0 is the period of impulses. τ_i is the minor random fluctuation around the average period T_0 . N is the number of the simulated impulses and i is the sequence number of the impulse. A_i is the amplitude modulator, whose periodical feature can be determined by the following equation:

$$A(t) = \rho \cdot (M(t) + T(t)) \quad (2)$$

where $M(t)$ represents the gravity load(G) and $T(t)$ represents the imbalance force load(Fe) indicated in Fig.1. ρ is the factor between amplitude and load.

$s(t)$ in Eq.(2) is an impulse exponential decay oscillation[22], and can be expressed as:

$$s(t) = e^{-Bt} \cos(2\pi f_n t) \quad (3)$$

where f_n is the system natural frequency and B is the decaying coefficient.

(1) Outer race defect signal modeling

In the case of the out race defect, $M(t)$ and $T(t)$ definitions as follows:

$$M(t) = mg \cos(\psi_m) \quad (4)$$

$$T(t) = me\omega^2 \cos(2\pi f_r t + \psi_t) \quad (5)$$

where ψ_m is the angle between gravity direction and outer race defect position and ψ_t is the angle between mass eccentric position and outer race defect position. ψ_m and ψ_t indicated in Fig.1. f_r is the rotational frequency of bearing. ω is the bearing angular velocity. m , g and e is the rotor mass, gravity acceleration and eccentricity, respectively.

In this condition, A_i can be determined by the follow equation:

$$A_i = \rho(mg \cos(\psi_m) + me\omega^2 \cos(2\pi f_r t + \psi_t)) \quad (6)$$

The outer race defect simulation signal calculating model can be concluded as follows:

$$x(t) = \sum_{i=1}^N \rho(mg \cos(\psi_m) + me\omega^2 \cos(2\pi f_r t + \psi_t)) \cdot s(t - iT_0 - \tau_i) \quad (7)$$

(2) Inner race defect signal modeling

In the case of inner race defect, $M(t)$ and $T(t)$ definitions as follows:

$$M(t) = mg \cos(2\pi f_r t + \psi_m) \quad (8)$$

$$T(t) = me\omega^2 \cos(\psi_t) \quad (9)$$

where ψ_m is the angle between gravity direction and inner race defect initial position and ψ_t is the angle

between mass eccentric position and inner race defect position. ψ_m and ψ_i indicated in Fig.1. Other parameters are same as the outer race defect's.

In this condition, A_i can be determined by the follow equation:

$$A_i = \rho(mg \cos(2\pi f_r t + \psi_m) + me\omega^2 \cos(\psi_i)) \quad (10)$$

The outer race defect simulation signal calculating model can be concluded as follows:

$$x(t) = \sum_{i=1}^N \rho(mg \cos(2\pi f_r t + \psi_m) + me\omega^2 \cos(\psi_i)) \cdot s(t - iT_0 - \tau_i) \quad (11)$$

(3) Rolling element defect signal modeling

When the rolling bearing is running in constant speed, there is a speed difference between the cage speed and the inner speed. This means that the angle between the rolling element defect and the gravity direction is real-time changed, and the angle between the rolling element defect and the unbalance force is real-time changed, too.

Therefore, in the case of rolling element defect, $M(t)$ and $T(t)$ definitions as follows:

$$M(t) = mg \cos(2\pi f_{cage} t + \psi_m) \quad (12)$$

$$T(t) = me\omega^2 \cos(2\pi(f_r - f_{cage})t + \psi_i) \quad (13)$$

where ψ_m is the angle between gravity direction and rolling element defect initial position and ψ_i is the angle between mass eccentric position and rolling element defect initial position. ψ_m and ψ_i indicated in Fig.1. f_{cage} is the rotation frequency of rolling bearing cage. Other parameters are same as the outer race defect's.

In this condition, A_i can be determined by the follow equation:

$$A_i = \rho(mg \cos(2\pi f_{cage} t + \psi_m) + me\omega^2 \cos(2\pi(f_r - f_{cage})t + \psi_i)) \quad (14)$$

The outer race defect simulation signal calculating model can be concluded as follows:

$$x(t) = \sum_{i=1}^N \rho(mg \cos(2\pi f_{cage} t + \psi_m) + me\omega^2 \cos(2\pi(f_r - f_{cage})t + \psi_i)) \cdot s(t - iT_0 - \tau_i) \quad (15)$$

2.2 Rolling bearing fundamental characteristic frequency

For the bearing with fixed outer race, there are five fundamental frequencies. These frequencies are defined as follows [17]:

$$\text{Cage frequency: } f_{cage} = \frac{f_r}{2} (1 - \frac{d}{D} \cos \alpha) \quad (16)$$

$$\text{Ball spinning frequency: } f_{ball} = \frac{f_r D}{2d} [1 - \frac{d^2}{D^2} \cos^2 \alpha] \quad (17)$$

$$\text{Outer race defect frequency: } f_{od} = \frac{Zf_r}{2} (1 - \frac{d}{D} \cos \alpha) \quad (18)$$

$$\text{Inner race defect frequency: } f_{id} = \frac{Zf_r}{2} \left(1 + \frac{d}{D} \cos \alpha\right) \quad (19)$$

$$\text{Ball defect Frequency: } f_{bd} = f_r \frac{D}{d} \left(1 - \frac{d^2}{D^2} \cos^2 \alpha\right) \quad (20)$$

where f_r is the shaft rotation frequency in Hz, d is the ball diameter, D is the pitch diameter, Z is the number of rolling elements and α is the contact angle.

2.3 Simulation signal analysis with different location defect

The envelope analysis method is an effective method to extract rolling bearing different location defect features frequency from the vibration signal [23]. The defect vibration signal is dominated by high frequency oscillation waveform which carrying the information about the impulse response of the structure. The envelope signal can be obtained by using the Hilbert transform.

Hilbert transform of the simulated signal $x(t)$ with different location defect is defined as [24, 25]:

$$\tilde{x}(t) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{x(\tau)}{t - \tau} d\tau \quad (21)$$

The original signal $x(t)$ and its Hilbert transform $\tilde{x}(t)$ can generate the complex analytic signal $z(t)$ as follows:

$$z(t) = x(t) + i\tilde{x}(t) \quad (22)$$

where $i = \sqrt{-1}$.

In order to facilitate comparison defect vibration signal in this paper, SKF6312 rolling bearing vibration signal of different location defect was simulated in this section. Geometrical parameters of the SKF6312 rolling bearing are shown in Table 1.

According to Eq. (16) to Eq. (20), SKF6312 rolling bearing fundamental frequencies at 3000 rpm are shown in Table 2.

Assume $m = 50kg$, $g = 9.8m/s^2$, $e = 0.05mm$, $\rho = 0.001$, $\psi_m = 0$, $\psi_i = 0$, $f_n = 8000Hz$ and $B = 800$, the simulation signal of outer race defect, inner race defect and rolling element defect are shown in Fig.2(a), Fig.3(a) and Fig.4(a), respectively. The frequency spectrum of different location defect simulated signal can be calculated by FFT is shown in Fig.2 (b), Fig.3 (b) and Fig.4 (b). The envelope waveform of simulation signal is obtained by using Eq. (21) and Eq. (22) and its frequency spectrum can be calculated by FFT, too. The envelope waveform and its spectrum of outer race defect are shown in Fig.2 (c) and Fig.2 (d), inner race defect are shown in Fig.3 (c) and Fig.3 (d), and rolling element defect are shown in Fig.4 (c) and Fig.4 (d).

Some conclusions can be obtained from Fig.2, Fig.3 and Fig.4 as follows:

- 1) The impulse decaying oscillation of different location defect is repeated with rolling bearing characteristic frequency and modulated by load of different defect.
- 2) The spectrum energy of outer race defect, inner race defect and rolling element defect simulation signal is

concentrated in the natural frequency. This means the characteristic frequency modulation signal is modulated by the natural frequency carrier signal.

3) The characteristic frequency of the outer race defect, inner race defect and rolling element defect is successfully extracted in the envelope spectrum, and different location defect envelope spectrum has the different side frequency band.

3 Vibration signal analysis and diagnosis of rolling bearing based on variational mode decomposition

Variational mode decomposition (VMD) is a novelty, fully intrinsic and adaptive, variational signal decomposition method and it can non-recursively decompose the vibration signal into a number of principal modes which are band-limited intrinsic mode functions. A real valued signal f can be decomposed into a discrete number of modes u_k with specific sparsity properties which are bandwidth in the spectral domain. Each mode is compact around a center pulsation ω_k , and its bandwidth is estimated by the squared L^2 -norm of the gradient. The solution of VMD is treated as a constrained variational problem, it is expressed as follows [7]:

$$\min_{\{u_k\}, \{\omega_k\}} \left\{ \sum_k \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t} \right) * u_k(t) \right] e^{-j\omega_k t} \right\|_2^2 \right\} \quad \text{subject to} \quad \sum_k u_k = f \quad (23)$$

where $\{u_k\} : \{u_1, u_2, \dots, u_K\}$ and $\{\omega_k\} : \{\omega_1, \omega_2, \dots, \omega_K\}$ are shorthand notations for the set of all modes and their center frequencies, respectively.

3.1 Variational mode decomposition algorithm

The original minimization problem (23) can be addressed by introducing a quadratic penalty and Lagrangian multipliers. The augmented Lagrangian is given as follows:

$$L(\{u_k\}, \{\omega_k\}, \lambda) = \alpha \sum_k \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t} \right) * u_k(t) \right] e^{-j\omega_k t} \right\|_2^2 + \left\| f(t) - \sum_k u_k(t) \right\|_2^2 + \left\langle \lambda(t), f(t) - \sum_k u_k(t) \right\rangle \quad (24)$$

Eq.(24) can be solved with the alternate direction method of multipliers(ADMM). The algorithm of VMD summarized as follows:

Initialize $\{u_k^1\}, \{\omega_k^1\}, \lambda^1, n \leftarrow 0$

Repeat

$n \leftarrow n + 1$

For $k = 1 : K$ do

Update u_k :

$$u_k^{n+1} \leftarrow \arg \min_{u_k} L(\{u_{i < k}^{n+1}\}, \{u_{i \geq k}^n\}, \{\omega_i^n\}, \lambda^n) \quad (25)$$

End for

For $k = 1 : K$ do

Updated ω_k :

$$\omega_k^{n+1} \leftarrow \arg \min_{\omega_k} L(\{u_i^{n+1}\}, \{\omega_{i < k}^{n+1}\}, \{\omega_{i \geq k}^n\}, \lambda^n) \quad (26)$$

End for

Dual ascent:

$$\lambda^{n+1} \leftarrow \lambda^n + \tau(f - \sum_k u_k^{n+1}) \quad (27)$$

Until convergence: $\sum_k \|u_k^{n+1} - u_k^n\|_2^2 / \|u_k^n\|_2^2 < \varepsilon$.

The solution of Eq.25 in the spectral domain as follows:

$$\hat{u}_k^{n+1}(\omega) = \frac{f(\omega) - \sum_{i \neq k} \hat{u}_i(\omega) + \frac{\hat{\lambda}(\omega)}{2}}{1 + 2\alpha(\omega - \omega_k)^2} \quad (28)$$

where ω_k is computed at the center of gravity of the corresponding mode's spectrum.

The Eq.26 solved as:

$$\omega_k^{n+1} = \frac{\int_0^\infty \omega |\hat{u}_k(\omega)|^2 d\omega}{\int_0^\infty |\hat{u}_k(\omega)|^2 d\omega} \quad (29)$$

The solution of Eq.27 in the spectral domain as follows:

$$\hat{\lambda}^{n+1}(\omega) \leftarrow \hat{\lambda}^n(\omega) + \tau(f(\omega) - \sum_k \hat{u}_k^{n+1}(\omega)) \quad (30)$$

3.2 Rolling bearing vibration signal analysis

Wold state that any stationary process can be decomposed into a deterministic and a non-deterministic part [26]. Wold's theorem shows that any stationary process can be expressed as follows:

$$X(n) = p(n) + r(n) \quad (31)$$

Where $p(n)$ is a deterministic process, $r(n)$ is a zero-mean stationary process, and $p(n)$ and $r(n)$ are uncorrelated with each other. Here, deterministic is to be understood as the quality of a process of which any value $p(n)$ can be predicted from its recent values $p(n-m)$ ($m > 0$), meanwhile, the non-deterministic process can't be predicted from its recent values. In brief, a vibration signal can consist of a periodic and a non-deterministic signal based on the Wold's theorem [27].

The vibration signal $f(t)$ of the fault bearing is composed of the fault signal $x(t)$ and some additive noise η [28]. It can be expressed as follows:

$$f(t) = x(t) + \eta \quad (32)$$

In section 2, it was indicated that vibration fault signal for outer race defect, inner race defect and rolling element defect is a series of periodic repeated impulse decaying oscillation waveforms, and is modeled as AM-FM (Amplitude modulated – Frequency modulated) signal. The impulse occurs periodically at the fault characteristic frequency, the oscillation frequency is the natural frequency, and the decaying associated with system structure. The formula in section 2 shows that A_i is the amplitude modulator and $s(t - iT_0 - \tau_i)$ is the frequency modulator. The fault signal $x(t)$ is multiplied by A_i and $s(t - iT_0 - \tau_i)$, which makes the fault signal become into a complicated frequency mixing signal. According to the Wold's theorem, the fault signal $x(t)$ is a deterministic process, so the additive noise η is a zero-mean non-deterministic process which uncorrelated with $x(t)$.

The vibration signal of different defect is constructed by simulation fault signal modeled in section 2 and additive noise. In this paper, the sampling frequency of the simulation signal is 25.6KHz, the sampling length is 16384, and the noise obeys gauss distribution with mean value 0 ($\mu = 0$) and variance value 0.15 ($\sigma = 0.15$). The simulated noise signal is shown in Fig.5(a), and the probability density histogram of the noise is shown in Fig.5(b). The simulated vibration signal for outer race fault, inner race fault and rolling element fault is show in Fig.6(a), Fig.6(b) and Fig.6(c), the signal-to-noise ratio(SNR) of the vibration signal reached -2.01db, -1.92db and -3.16db, and the correlation between fault signal and noise is -0.84%, -0.0062% and 0.58%, respectively. In brief, the noise is stronger than the fault signal, and there is no correlation between the noise and the fault signal. The simulated vibration signal with outer race fault, inner race fault or rolling element fault meets the wold' theorem.

3.3 Simulation vibration signal extraction and diagnosis based on VMD

Before the vibration signal is decomposed, we should predefine the number of the modes K , the balancing parameter of the data-fidelity constraint α and time-step of the dual ascent τ . The correlation and the energy ratio are used as a basis for quantitative comparison in this paper. The correlation can be expressed as follows:

$$r_{xy} = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^n (X_i - \bar{X})^2} \sqrt{\sum_{i=1}^n (Y_i - \bar{Y})^2}} \quad (33)$$

where n is the sampling length, $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$ and $\bar{Y} = \frac{1}{n} \sum_{i=1}^n Y_i$.

The energy ratio can be expressed as follows:

$$e_{xy} = \frac{\sum_{i=1}^n (X_i)^2}{\sum_{i=1}^n (Y_i)^2} \quad (34)$$

where n is the sampling length.

Fig.7 shows the correlation and energy ratio between the fault signal and the main mode gained by VMD. The main mode is the maximum energy mode in the modes of VMD decomposition. It is obvious that the correlation and energy ratio are the largest when $K=1$, and the correlation is basically the same and the energy ratio slightly decrease when $K \leq 5$ for different fault, as shown in Fig.7 (a) (b). From Fig.7 (c) (d), we can find that the correlation is stable and almost reaches the maximum, and the energy of the main mode is almost equal the fault signal's energy, when α is between 1000 and 2000. Fig.7 (e) shows that the correlation of different fault reaches maximum when $\tau=0$. The different modes are uncorrelated and sum of modes' energy should close to the composite vibration signal's energy. Fig.8 shows the energy ratio between the sum of modes and simulated vibration signal $f(t)$ with different K and α for outer race fault, inner race fault and rolling element fault.

The sum of decomposed modes' energy is equal to the energy of $f(t)$ when $\alpha=0$, and it becomes smaller and smaller with the increase of α . When the $0 < \alpha < 500$ and $K=4$, the sum of modes' energy reaches maximum. When the value of α is more than 500, the sum of modes' energy increases with the increase of K .

In order to ensure the fidelity of the main mode gained by VMD and preserve the energy of the original signal as far as possible, we decide to choose $K=5$, $\alpha=1500$ and $\tau=0$ in this paper. Fig.9 shows the decomposed modes by VMD and their frequency spectrum for different fault simulated vibration signal. In order to illustrate the effectiveness of VMD, we provide a comparison with EMD based on the same signal in Fig.6. The Imfs obtained by the decomposition of EMD and their frequency spectrum display in Fig.10. To highlight the difference between VMD and EMD, covariance matrix of modes/Imfs can be included. Take modes or Imfs as rows of a matrix (Say A). The covariance matrix can be expressed as follows:

$$Cov = (A \times A') / \sum_{i=1}^n (f_i)^2 \quad (35)$$

where A' is the transpose of A , n is the sampling length and f is the original signal. The covariance matrix calculation result of VMD and EMD is shown in appendix A. The results indicate that different modes and Imfs are uncorrelated. In VMD, the balancing parameter α controls the data preservation. The sum of the element of the VMD resulting matrix for outer race fault, inner race fault and rolling element fault is 0.5676, 0.5706 and 0.5556, respectively. The sum of the element of the EMD resulting matrix for outer race fault, inner race fault and rolling element fault is 0.9846, 0.9811 and 0.9801, respectively. These results illustrate that VMD sacrifices part of the original signal in the process of decomposition, on the contrary, the EMD basically saves the entire original signal.

According to the above, the energy of the fault signal is concentrated in the natural frequency which is 8000Hz. From Table 4 and Fig.9, we can find that the impulses in mode4 are clearer than other modes and the raw signal, the mode4's energy is larger than other mode's energy and its frequency spectrum is concentrated in the vicinity of 8000Hz for different fault. From Table 5 and Fig.10, we can find that the impulses in Imf1 are difficult to be found, but the Imf1's energy is larger than other Imf's energy and its frequency spectrum is concentrated in the vicinity of 8000Hz for different fault. Clearly, the mode4 including bearing fault information is the main mode gained by VMD and the Imf1 including bearing fault information is the main Imf gained by EMD for outer race fault, inner race fault and rolling element fault. Table 4 shows the center frequency, the energy ratio between the main mode and the simulated vibration signal $f(t)$, and the correlation between the main mode and the fault signal $x(t)$ for outer race fault, inner race fault and rolling element fault. Meanwhile, Table 5 shows the energy ratio between the Imfs and $f(t)$ and the correlation between the Imfs and $x(t)$. The energy ratio between $x(t)$ and $f(t)$ for

the outer race fault, inner race fault and rolling element fault is 38.93%, 39.14% and 32.41%, respectively. Clearly, the energy ratio between the main mode and $f(t)$ is close to the energy ratio between $x(t)$ and $f(t)$, while the energy ratio between the Imf1 and $f(t)$ is much larger than the energy ratio between $x(t)$ and $f(t)$. The correlation between the main mode and $x(t)$ is more than the correlation between Imf1 and $x(t)$. By comparison, we can easily find that although VMD sacrifices part of the original signal, the main mode gained by VMD is still more fidelity than the Imf1 gained by EMD for different fault signal. The main mode's center frequency which can't obtain by EMD is close to the preset natural frequency, and it is a very useful feature for fault diagnosis of the rolling bearing. Envelope spectrum of the different fault's main mode display in Fig.11 and fault frequency can be clearly obtained in Fig.11. It is clearly observed that the VMD can extract the fault signal of rolling bearing, and shows better result as compared to EMD.

In this section, we review the VMD algorithm and discuss the natural characteristics of rolling bear vibration signal with different fault. Then, the VMD is used to decompose the simulation vibration signal of rolling bearing with the outer race fault, inner race fault or rolling element fault. Through the quantitative comparison and analysis, it is confirmed that the VMD can extract the fault signal of rolling bearing, and it shows better result as compared to EMD. Finally, we conclude that the envelope spectrum of the main mode decomposed by VMD can be utilized to diagnose the rolling bearing fault accurately.

4 Experiment and discussion

The dominant signal caused by different location defect in the rolling bearing vibration signal can be extracted by VMD from the above study. Therefore, the rolling bearing fault diagnosis method based on the VMD algorithm implementation procedure is shown in Fig.11. The bearing vibration signal is collected through the data acquisition system. Subsequently, the modes are gained by using VMD to decompose the bearing vibration signal under $K = 5$, $\alpha = 1500$ and $\tau = 0$. The main mode is obtained by comparing the energy ratio of different modes. Eventually, the bearing fault can be identified by the analysis of the main mode's envelope spectrum.

4.1 Experimental setup

The multistage centrifugal pump test rig used in this study is illustrated in Fig.12. It consists of a drive motor, pump body, rolling bearing support, and acceleration sensors. Acceleration sensors are horizontally and vertically mounted on each bearing position of the motor and each bearing house of the pump. Some acceleration sensors are vertically mounted on the pump's foundation and pipeline. The SKF6312 rolling bearing near the drive motor is used to simulate different location fault. The sensor1 and sensor2 are horizontal and vertical mounted acceleration sensors of SKF6312 bearing house. To realize the target of this paper's research, practical signal from one acceleration sensor on the SKF6312 bearing house is necessary for fault analysis. The practical vibration signal is acquired by using PCB 608A11 accelerometer (acceleration range of $\pm 50g$, frequency range of 0.5-10KHz, and sensitivity is 100mV/g). The accelerometer's output is sent to the data acquisition system which is composed of a computer and data acquisition card NI USB-6002(analog input resolution is 16 bits). In order to verify the effectiveness of the proposed method, we use the electrical-discharge machining method to make defect on different location of SKF6312 bearing and each location has two defect sizes. The diameter of the defect is about 0.1mm and 0.3mm and the practical vibration signal is measured when the pump is running at 1750 rpm and 2960 rpm for different fault. The sampling frequency of practical signal is 25.6 KHz and the sampling length is 16384. According to the bearing characteristic frequency calculation formula, SKF6312 bearing defect frequencies of the

test rig at different speed are shown in Table 3.

4.2 Results and discussion

The practical vibration signal of outer race fault at different test speed and different defect size are shown in Fig.14 and their corresponding VMD and EMD results are shown in appendix B. From appendix B, we can find that mode4 which spectrum energy concentrated near 8000Hz has very significant impulses while it is difficult to find clearly impulses in each Imfs for the outer race fault at different test speed and different defect size. Table 6 shows the center frequency and energy ratio between modes and the practical signal for outer race fault. The mode4 has the maximum energy and its center frequency is close to 8100Hz. Therefore, the mode4 is the main mode for the outer race fault at different test speed and different defect size, and the main modes are shown in Fig.14. By comparing the practical vibration signal and the main mode, we can find that the impulses of the main modes are clearer than the practical signal's, which means the noise floor has been weakened enough. Fig.15 shows the envelope spectrums of the outer race fault main modes. The outer race fault characteristic frequency and its harmonics are clearly displayed. With the increase of the defect size, the amplitude of the characteristic frequency increases. The test speed directly affects the characteristic frequency. When the test speed is 1750rpm and 2960rpm, the characteristic frequency of the main mode is basically the same as the defect frequency in Table 3. The experimental result shown in Fig.15 obviously verifies that the prediction derived from the proposed outer race fault model in section 2.

Fig.16 shows practical vibration signals of inner race fault and their main modes at different test speed and different defect size. VMD and EMD results related to the inner race fault are shown in appendix C. Similar to outer race fault, mode4 has significant impulses while Imfs haven't in the case of inner race fault. In Table 7, the mode4's energy is the largest, so the mode4 is the main mode whose center frequency near 8300Hz. Fig.16 shows that the impulses of the main mode are more distinctly. Envelope spectrums of the inner race fault main modes are shown in Fig.17. In addition to the distinct characteristic frequency of inner race fault and its harmonics, the rotating frequency is also very obvious, which is agreeable with the theory and simulation result of the proposed inner race fault signal model. The results in Fig.17 indicate that the defect size of inner race fault affects the amplitude of the characteristic frequency and the rotating frequency and the test speed affects the characteristic frequency which is almost equal to the defect frequency in Table 3.

Fig.18 shows practical vibration signals of rolling element fault and their main modes at different test speed and different defect size. VMD and EMD results related to the rolling element fault are shown in appendix D. As for rolling element fault, the impulses are clearly displayed in the mode4 and hardly found in Imfs. This phenomenon is the same as the outer race fault and the inner race fault. The mode4 has the largest energy is the main mode and its center frequency is about 7700Hz. Fig.19 shows that the characteristic frequency of rolling element fault is clearly distinguished at different test speed and different defect size, and the amplitude of the characteristic frequency ascends with the defect size increases which is in accordance with the defect frequency at different speed. This kind of rolling element fault expression is consistent with the presented rolling element fault signal model.

In this experiment, the outer race fault, inner race fault and rolling element fault at different test speed and different defect size is investigated in which the VMD can effectively extract the fault signal of rolling bearing and the rolling bearing fault expression is given to prove the correctness of the fault signal model. According to the experimental results, we can clearly obtain the characteristic frequency of outer race fault, inner race fault and rolling element fault and its harmonics at different test speed and different defect size, and the characteristic frequency and its harmonics is consistent with the theory formula mention in section 2 and the simulation result of proposed fault signal model in Fig.11. In conclusion, the method put forward in this contribution can accurately extract the main mode and successfully diagnosis outer race fault, inner race fault and rolling element fault and

different fault signal model is verified.

5 Conclusions

The rolling bearing fault diagnosis method based on the VMD was presented in this article. The rolling bearing failure mechanism of outer race fault, inner race fault and rolling element fault was investigated by modeling the dynamic response. The gravity and the unbalance force were taken into consideration for dynamic response modeling formula of different location fault. The simulation vibration signal of different location fault was constructed by the derived modeling formula and gauss noise. The result of quantitative comparison in section 3 shows VMD can extract the principal signal of rolling bearing different location fault, and it shows better result as compared to EMD. The analysis result of simulation vibration signal and the practical vibration signal for outer race fault, inner race fault and rolling element fault has proven the validity of the proposed method for diagnosing the rolling bearing fault.

The VMD method needs to predefine some parameters, so we can obtain precise results. In practical applications, choosing parameters will occupy most of the time, and inapposite parameters will affect the accuracy of the results. This is the shortcoming in this article which we are currently working to solve. In the future, it is important to construct criteria for automatically determine the parameters, so that the results can be improved.

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Fig.1. Schematic diagram of force analysis.

Fig.2. Outer race defect simulation signal related waveform and spectrum. (a) The time domain waveform of simulation signal; (b) Frequency spectrum of (a); (c) Envelope waveform of (a); (d) Envelope spectrum of (a).

Fig.3. Inner race defect simulation signal related waveform and spectrum. (a) The time domain waveform of simulation signal; (b) Frequency spectrum of (a); (c) Envelope waveform of (a); (d) Envelope spectrum of (a).

Fig.4. Rolling element defect simulation signal related waveform and spectrum. (a) The time domain waveform of simulation signal; (b) Frequency spectrum of (a); (c) Envelope waveform of (a); (d) Envelope spectrum of (a).

Fig.5. (a) Simulation signal of noise; (b) The probability density histogram of (a).

Fig.6. The simulation vibration signal of different fault. (a) The outer race fault; (b) The inner race fault; (c) The rolling element fault.

(a)

(b)

(c)

(d)

(e)

Fig.7. Comparison of the correlation and energy ratio between main mode gained by VMD with different parameters for different fault. (a),(b) Correlation and energy ratio with different K ; (c),(d) Correlation and energy ratio with different α ; (e) Correlation with different τ .

Fig.8. The energy ratio between the sum of modes and the vibration signal with different K and α for different fault. (a) The outer race fault; (b) The inner race fault; (c) The rolling element fault;

(c)

(d)

(e)

(f)

Fig.9. The modes obtained by using VMD to decompose $f(t)$ and their frequency spectrum for different fault. (a)

The outer race fault modes; (b) The frequency spectrum of (a); (c) The inner race fault modes; (d) The frequency spectrum of (c); (e) The rolling element fault modes; (f) The frequency spectrum of (e).

Fig.10. The Imfs obtained by using EMD to decompose $f(t)$ and their frequency spectrum for different fault. (a)

The outer race fault Imfs; (b) The frequency spectrum of (a); (c) The inner race fault Imfs; (d) The frequency spectrum of (c); (e) The rolling element fault Imfs; (f) The frequency spectrum of (e).

Fig.11. The main mode's envelope spectrum for different fault. (a) The outer race fault; (b) The inner race fault; (c)

The rolling element fault.

Fig.12. The process of proposed bearing fault diagnosis method.

Fig.13. Multistage centrifugal pump test rig.

Fig.14. The practical vibration signal and its main mode for outer race fault. (a),(b) The signal and its main mode when the test speed is 1750rpm and the defect size is 0.1mm; (c),(d) The signal and its main mode when the test speed is 1750rpm and the defect size is 0.3mm; (e),(f) The signal and its main mode when the test speed is 2960rpm and the defect size is 0.1mm; (g),(h) The signal and its main mode when the test speed is 2960rpm and the defect size is 0.3mm.

Fig.15. The main mode's envelope spectrum for outer race fault. (a) when the test speed is 1750rpm and the defect size is 0.1mm; (b) when the test speed is 1750rpm and the defect size is 0.3mm; (c) when the test speed is 2960rpm and the defect size is 0.1mm; (d) when the test speed is 2960rpm and the defect size is 0.3mm.

Fig.16. The practical vibration signal and its main mode for inner race fault. (a),(b) The signal and its main mode when the test speed is 1750rpm and the defect size is 0.1mm; (c),(d) The signal and its main mode when the test

speed is 1750rpm and the defect size is 0.3mm; (e),(f) The signal and its main mode when the test speed is 2960rpm and the defect size is 0.1mm; (g),(h) The signal and its main mode when the test speed is 2960rpm and the defect size is 0.3mm.

Fig.17. The main mode's envelope spectrum for inner race fault. (a) when the test speed is 1750rpm and the defect size is 0.1mm; (b) when the test speed is 1750rpm and the defect size is 0.3mm; (c) when the test speed is 2960rpm and the defect size is 0.1mm; (d) when the test speed is 2960rpm and the defect size is 0.3mm.

Fig.18. The practical vibration signal and its main mode for rolling element fault. (a),(b) The signal and its main mode when the test speed is 1750rpm and the defect size is 0.1mm; (c),(d) The signal and its main mode when the test speed is 1750rpm and the defect size is 0.3mm; (e),(f) The signal and its main mode when the test speed is 2960rpm and the defect size is 0.1mm; (g),(h) The signal and its main mode when the test speed is 2960rpm and the defect size is 0.3mm.

Fig.19. The main mode's envelope spectrum for rolling element fault. (a) when the test speed is 1750rpm and the defect size is 0.1mm; (b) when the test speed is 1750rpm and the defect size is 0.3mm; (c) when the test speed is 2960rpm and the defect size is 0.1mm; (d) when the test speed is 2960rpm and the defect size is 0.3mm.

Table 1 Geometrical parameter of SKF6312.

Ball diameter d(mm)	Pitch diameter D(mm)	Ball number Z	Contact angle α
22.2	95.987	8	0

Table 2 The fundamental frequencies of SKF6312.

$f_r(Hz)$	$f_{cage}(Hz)$	$f_{ball}(Hz)$	$f_{od}(Hz)$	$f_{id}(Hz)$	$f_{bd}(Hz)$
50	19.218	204.626	153.744	246.256	204.626

Table 3 SKF6312 bearing defect frequencies.

Test speed	Outer defect frequency	Inner defect frequency	Ball defect frequency
1750rpm	89.68Hz	143.65Hz	119.36Hz
2960rpm	151.70Hz	242.97Hz	201.89Hz

Table 4 The center frequency, energy ratio between modes gained by VMD and simulated vibration signal $f(t)$

and correlation between modes gained by VMD and simulated fault signal $x(t)$ for different fault.

Modes gained by VMD	Outer race fault			Inner race fault			Rolling element fault		
	Center frequency	Energy ratio	Correlation	Center frequency	Energy ratio	Correlation	Center frequency	Energy ratio	Correlation
Mode1	1167 Hz	4.81%	1.64%	1160Hz	4.68%	1.52%	1173Hz	5.08%	1.12%
Mode2	3935 Hz	4.73%	2.84%	3882Hz	4.69%	3.13%	3860Hz	4.98%	1.99%
Mode3	6299 Hz	5.03%	10.87%	6273Hz	5.16%	11.46%	6233Hz	5.43%	9.12%
Mode4	8006 Hz	32.24%	88.56%	8014Hz	32.71%	88.33%	8000Hz	29.84%	87.12%
Mode5	11019 Hz	4.98%	3.64%	10983Hz	4.89%	3.79%	11029Hz	5.37%	3.19%

Table 5 The energy ratio between Imfs gained by EMD and simulated vibration signal $f(t)$ and correlation

between Imfs gained by EMD and simulated fault signal $x(t)$ for different fault.

IMFs gained by EMD	Outer race fault		Inner race fault		Rolling element fault	
	Energy ratio	Correlation	Energy ratio	Correlation	Energy ratio	Correlation
Imf1	76.92%	67.55%	76.80%	68.34%	74.33%	63.55%
Imf2	15.71%	4.09%	14.83%	4.47%	16.17%	4.60%
Imf3	7.11%	1.11%	7.00%	1.49%	8.46%	1.19%
Imf4	4.93%	1.28%	4.19%	1.38%	5.41%	1.02%

Imf5	2.94%	0.09%	3.18%	0.96%	2.80%	0.52%
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Table 6 The center frequency and the energy ratio between modes gained by VMD and practical outer race fault signal for different test speed and fault size.

Outer race fault								
Test Speed	1750rpm		1750rpm		2960rpm		2960rpm	
Fault Size	0.1mm		0.3mm		0.1mm		0.3mm	
Modes gained by VMD	Center frequency	Energy ratio	Center frequency	Energy ratio	Center frequency	Energy ratio	Center frequency	Energy ratio
Model1	1008Hz	6.33%	1451Hz	5.59%	1172Hz	5.38%	1216Hz	4.63%
Mode2	3542Hz	6.79%	3604Hz	6.58%	3761Hz	5.74%	3616Hz	5.01%
Mode3	6314Hz	6.49%	6565Hz	6.98%	6776Hz	7.36%	6768Hz	6.53%
Mode4	8108Hz	12.98%	8133Hz	16.94%	8150Hz	17.58%	8130Hz	21.35%
Mode5	11414Hz	6.70%	10982Hz	6.05%	10179Hz	6.30%	9917Hz	7.75%

Table 7 The center frequency and the energy ratio between modes gained by VMD and practical inner fault signal for different test speed and fault size.

Inner race fault								
Test Speed	1750rpm		1750rpm		2960rpm		2960rpm	
Fault Size	0.1mm		0.3mm		0.1mm		0.3mm	
Modes gained by VMD	Center frequency	Energy ratio	Center frequency	Energy ratio	Center frequency	Energy ratio	Center frequency	Energy ratio
Model1	855Hz	6.71%	702Hz	6.51%	1237Hz	4.97%	1079Hz	5.18%
Mode2	4276Hz	7.81%	3994Hz	6.93%	4214Hz	5.31%	4705Hz	5.40%
Mode3	6749Hz	9.37%	6822Hz	14.75%	6793Hz	12.27%	6833Hz	12.85%
Mode4	8324Hz	10.94%	8230Hz	15.45%	8272Hz	18.69%	8277Hz	19.83%
Mode5	11147Hz	6.15%	11061Hz	4.74%	11297Hz	5.57%	11233Hz	5.36%

Table 8 The center frequency and the energy ratio between modes gained by VMD and practical rolling fault signal for different test speed and fault size.

Rolling element fault								
Test Speed	1750rpm		1750rpm		2960rpm		2960rpm	
Fault Size	0.1mm		0.3mm		0.1mm		0.3mm	
Modes gained by VMD	Center frequency	Energy ratio	Center frequency	Energy ratio	Center frequency	Energy ratio	Center frequency	Energy ratio
Model1	1135Hz	6.26%	1129Hz	4.92%	1188Hz	4.80%	1033Hz	3.94%
Mode2	3392Hz	5.82%	3525Hz	5.59%	3484Hz	6.41%	3547Hz	5.37%
Mode3	5775Hz	7.29%	5657Hz	12.35%	5836Hz	10.77%	5875Hz	12.21%
Mode4	7755Hz	12.80%	7709Hz	13.02%	7749Hz	13.56%	7754Hz	17.79%
Mode5	10696Hz	6.51%	10820Hz	5.78%	10875Hz	5.86%	10952Hz	5.24%

APPENDIX A

Outer race fault covariance matrix of modes gain by VMD:

$$Cov = \begin{bmatrix} 0.0481 & 0.0029 & 0.0008 & 0.0006 & 0.0002 \\ 0.0029 & 0.0473 & 0.0038 & 0.0017 & 0.0005 \\ 0.0008 & 0.0038 & 0.0503 & 0.0092 & 0.0011 \\ 0.0006 & 0.0017 & 0.0092 & 0.3224 & 0.0039 \\ 0.0002 & 0.0005 & 0.0011 & 0.0039 & 0.0498 \end{bmatrix} \quad (A.1)$$

Inner race fault covariance matrix of modes gain by VMD:

$$Cov = \begin{bmatrix} 0.0468 & 0.0030 & 0.0008 & 0.0006 & 0.0002 \\ 0.0030 & 0.0469 & 0.0037 & 0.0017 & 0.0005 \\ 0.0008 & 0.0037 & 0.0516 & 0.0090 & 0.0011 \\ 0.0006 & 0.0017 & 0.0090 & 0.3271 & 0.0040 \\ 0.0002 & 0.0005 & 0.0011 & 0.0040 & 0.0489 \end{bmatrix} \quad (A.2)$$

Rolling element fault covariance matrix of modes gain by VMD:

$$Cov = \begin{bmatrix} 0.0508 & 0.0033 & 0.0009 & 0.0006 & 0.0003 \\ 0.0033 & 0.0498 & 0.0040 & 0.0016 & 0.0005 \\ 0.0009 & 0.0040 & 0.0544 & 0.0083 & 0.0011 \\ 0.0006 & 0.0016 & 0.0083 & 0.2984 & 0.0038 \\ 0.0003 & 0.0005 & 0.0011 & 0.0038 & 0.0537 \end{bmatrix} \quad (A.3)$$

Outer race fault covariance matrix of Imfs gain by EMD:

$$Cov = \begin{bmatrix} 0.7692 & -0.0191 & -0.0151 & -0.0098 & -0.0035 \\ -0.0191 & 0.1571 & 0.0030 & -0.0022 & -0.0009 \\ -0.0151 & 0.0030 & 0.0711 & 0.0019 & -0.0007 \\ -0.0098 & -0.0022 & 0.0019 & 0.0493 & 0.0007 \\ -0.0035 & -0.0009 & -0.0007 & 0.0007 & 0.0294 \end{bmatrix} \quad (A.4)$$

Inner race fault covariance matrix of Imfs gain by EMD:

$$Cov = \begin{bmatrix} 0.7680 & -0.0147 & -0.0141 & -0.0085 & -0.0041 \\ -0.0147 & 0.1483 & 0.0026 & -0.0010 & -0.0010 \\ -0.0141 & 0.0026 & 0.070 & 0.0022 & -0.0015 \\ -0.0085 & -0.0010 & 0.0022 & 0.0419 & 0.0005 \\ -0.0041 & -0.0010 & -0.0015 & 0.0005 & 0.318 \end{bmatrix} \quad (A.5)$$

Rolling element fault covariance matrix of Imfs gain by EMD:

$$Cov = \begin{bmatrix} 0.7433 & -0.0180 & -0.0165 & -0.0081 & -0.0030 \\ -0.0180 & 0.1617 & 0.0030 & -0.0022 & -0.0006 \\ -0.0165 & 0.0030 & 0.0846 & 0.0007 & -0.0010 \\ -0.0081 & -0.0022 & 0.0007 & 0.0541 & 0.0001 \\ -0.0030 & -0.0006 & -0.0010 & 0.0001 & 0.0280 \end{bmatrix} \quad (A.6)$$

APPENDIX B

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

Fig.B.1. The modes obtained by VMD and their frequency spectrum for outer race fault. (a) when the test speed is 1750 rpm and the defect size is 0.1 mm; (b) The frequency spectrum of (a) ;(c) when the test speed is 1750 rpm and the defect size is 0.3 mm; (d) The frequency spectrum of (c); (e) when the test speed is 2960 rpm and the defect size is 0.1 mm; (f) The frequency spectrum of (e); (g) when the test speed is 2960 rpm and the defect size is 0.3 mm; (h)

The frequency spectrum of (g).

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

Fig.B.2. The Imfs obtained by EMD and their frequency spectrum for outer race fault. (a) when the test speed is

1750 rpm and the defect size is 0.1 mm; (b) The frequency spectrum of (a) ;(c) when the test speed is 1750 rpm and the defect size is 0.3 mm; (d) The frequency spectrum of (c); (e) when the test speed is 2960 rpm and the defect size is 0.1 mm; (f) The frequency spectrum of (e); (g) when the test speed is 2960 rpm and the defect size is 0.3 mm; (h)

The frequency spectrum of (g).

APPENDIX C

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

Fig.C.1. The modes obtained by VMD and their frequency spectrum for inner race fault. (a) when the test speed is 1750 rpm and the defect size is 0.1 mm; (b) The frequency spectrum of (a) ;(c) when the test speed is 1750 rpm and the defect size is 0.3 mm; (d) The frequency spectrum of (c); (e) when the test speed is 2960 rpm and the defect size is 0.1 mm; (f) The frequency spectrum of (e); (g) when the test speed is 2960 rpm and the defect size is 0.3 mm; (h)

The frequency spectrum of (g).

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

Fig.C.2. The Imfs obtained by EMD and their frequency spectrum for inner race fault. (a) when the test speed is 1750 rpm and the defect size is 0.1 mm; (b) The frequency spectrum of (a) ;(c) when the test speed is 1750 rpm and the defect size is 0.3 mm; (d) The frequency spectrum of (c); (e) when the test speed is 2960 rpm and the defect size is 0.1 mm; (f) The frequency spectrum of (e); (g) when the test speed is 2960 rpm and the defect size is 0.3 mm; (h) The frequency spectrum of (g).

APPENDIX D

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

Fig.D.1. The modes obtained by VMD and their frequency spectrum for rolling element fault. (a) when the test

speed is 1750 rpm and the defect size is 0.1 mm; (b) The frequency spectrum of (a) ;(c) when the test speed is 1750 rpm and the defect size is 0.3 mm; (d) The frequency spectrum of (c); (e) when the test speed is 2960 rpm and the defect size is 0.1 mm; (f) The frequency spectrum of (e); (g) when the test speed is 2960 rpm and the defect size is 0.3 mm; (h) The frequency spectrum of (g).

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

Fig.D.2. The Imfs obtained by EMD and their frequency spectrum for rolling element fault. (a) when the test speed is 1750 rpm and the defect size is 0.1 mm; (b) The frequency spectrum of (a) ;(c) when the test speed is 1750 rpm and the defect size is 0.3 mm; (d) The frequency spectrum of (c); (e) when the test speed is 2960 rpm and the defect size is 0.1 mm; (f) The frequency spectrum of (e); (g) when the test speed is 2960 rpm and the defect size is 0.3 mm; (h) The frequency spectrum of (g).