Non-stationary frequency domain system identification using time-frequency representations

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Abstract

System properties of buildings and bridges may vary with time due to temperature changes, aging or extreme loadings. To identify these time-varying system properties, this study proposes a new output-only non-stationary system identification (SI) framework based on instantaneous or marginal spectra derived from the time-frequency representation, e.g., short time Fourier or wavelet transform. Spectra derived from these time-frequency representations are very popular in tracking time-varying frequencies; however, they have seldom been used to identify the time-varying damping ratio because a short window needed to capture the time-varying information amplifies the bandwidth significantly, which may lead to considerably overestimating the damping ratio. To overcome this shortcoming, this study modifies the theoretical frequency response function (FRF) to explicitly account for the windowing effect, and therefore enables SI directly using instantaneous or marginal spectra derived from the wavelet or short time Fourier transform. The response spectrum estimated using the short time window and the modified FRF are both influenced by the same time window, thus the instantaneous or time-localized marginal spectrum of response can be fitted to the modified FRF to identify frequency and damping ratio at each time instant. This spectral-based SI framework can reliably identify damping in time-varying systems under non-stationary excitations. The efficacy of the proposed framework is demonstrated by both numerical and full-scale examples, and also compared to the time-domain SI method, e.g. stochastic subspace identification (SSI),

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since these SI approaches and their extensions are popular in identifying time-varying systems utilizing recursive algorithms or moving windows.

Key words: non-stationary, time-varying, system identification, damping, short time Fourier transform, wavelet transform

1. Introduction

Full-scale monitoring has witnessed that structural dynamic properties may change over time due to the temperature, aging or extreme loadings. These changes may indicate structural damage or deterioration, therefore, identification of these time-varying systems is critical in structural health monitoring practice.

Research efforts in this area have resulted in the development of many time-domain methods. A number of these methods involve recursive algorithms based on subspace [1-3] or autoregressive and moving average (ARMA) model [4]. Some others track time-varying systems by applying moving windows on time histories. Examples include the several subspace-based methods [5-11] and a sensitivity-based method [12]. To further improve the tracking ability, the use of basis functions to express time-varying model parameters [13-18] has been suggested. In this context, the problem of time-varying system identification (SI) becomes the identification of a finite number of time-invariant parameters associated with the basis expansion [19]. Recently, the Hilbert transform has been employed to identify instantaneous model parameters [20-23].

Compared to a wide spectrum of time-domain methods, spectral-based methods are rather limited, despite their dominance and popularity in the time-invariant SI. The frequency domain decomposition (FDD) method based on auto- and cross-spectra of response within a short moving window over time have been utilized in time-varying modal identification [24-26]. Beyond the Fourier spectra based methods, time-varying spectra obtained from time-frequency

representations (e.g. wavelet transform, STFT) have widely demonstrated their ability to track time-varying frequencies and mode shapes [e.g. 27, 28, 29]. Unfortunately, most of these works did not quantitatively identify damping ratios; rather, couple of them qualitatively analyzed the change of damping ratios based on the instantaneous wavelet spectra [30]. This is mainly because short windows necessary for tracking time-varying information in these approaches usually adds to the unreliability of damping estimates. As shown in Figure 1, the short window amplifies the bandwidth of response spectrum significantly, which may cause over-estimation of the damping ratio. The resulting bias error of damping ratio for a lightly damped single degree of freedom (SDOF) system (e.g. 1% damping ratio) could be as high as 100% if a boxcar window with the length of 30 cycles of response is used (Figure 2). In the literature, only a couple of studies by Ceravolo and his co-workers have attempted to identify damping ratio using spectral information of the time-frequency representations [31, 32]; however, the windowing effect introduced here has thus far not been explicitly addressed.

In this paper, a rather novel output-only SI framework for non-stationary response is proposed to enable reliable damping estimation based on time-varying spectra regardless of the conservative nature of the windowing effect. In this framework, frequencies and mode shapes can also be identified concurrently. The efficacy of the proposed framework and the associated uncertainty are investigated.

2. Methodology for a SDOF system

2.1 Spectral-based system identification (SI) of a time-varying SDOF system

A time-varying SDOF system

$$m(t)\ddot{x}(t) + c(t)\dot{x}(t) + k(t)x(t) = f(t) \quad (1)$$

can be considered as an equivalent time-invariant system within a short window of length T moving along time, i.e.

$$m_i \ddot{x}(t) + c_i \dot{x}(t) + k_i x(t) = f(t) \quad (2)$$

where $t \in [t_i - T/2, t_i + T/2]$, so that the identification of the time-varying system is reduced to the identification of the equivalent time-invariant system within each window i . The theoretical power spectral density of the equivalent time-invariant system response under a broadband excitation is equal to a scaled version of the magnitude-squared frequency response function (FRF) of the system, i.e.

$$S_{xx}(f) = S_o |H(f; f_n, \xi)|^2 \quad (3)$$

where S_o is the spectral density of the excitation, $H(f; f_n, \xi)$ is the FRF of the system, for the displacement and acceleration responses, it is given as (with a factor $1/k_i$ omitted)

$$H^{(d)}(f; f_n, \xi) = \frac{1}{(1 - (f/f_n)^2 + 2\xi(f/f_n)j)} \quad (4)$$

and

$$H^{(a)}(f; f_n, \xi) = \frac{-(2\pi f)^2}{(1 - (f/f_n)^2 + 2\xi(f/f_n)j)} \quad (5)$$

respectively, where f_n and ξ are the natural frequency and damping ratio of the system, $j = \sqrt{-1}$. Thus frequency and damping ratio can be identified by fitting the response spectra $S_{xx}(f)$ to a scaled version of $H(f; f_n, \xi)$ [33].

On the other hand, the actual response spectrum estimated within a short time window can be given by the convolution of theoretical response spectrum and the magnitude-squared Fourier transform of the window function, i.e.

$$\hat{S}_{xx}(f;T) = S_o |H(f;f_n,\xi)|^2 * |U(f;T)|^2 \quad (6)$$

where $U(f;T)$ is Fourier transform of an arbitrary window function $u(t;T)$ of length T , $*$ denotes the operation of convolution. The windowing effect will cause amplification of the bandwidth of the response spectrum (Figure 1). If fitting the amplified response spectrum $\hat{S}_{xx}(f;T)$ to the scaled versions of $H(f;f_n,\xi)$, the damping ratio cannot be reliably identified. Unfortunately, it is very difficult, if not impossible, to remove this windowing effect from the response spectrum for an unknown system. But it is possible to add the windowing effect to the FRF of a known system. Noticing this point, our idea is that modifying the FRF by explicitly adding the windowing effect, i.e.

$$|H_w(f;f_n,\xi,T)|^2 = |H(f;f_n,\xi)|^2 * |U(f;T)|^2 \quad (7)$$

where $H_w(f;f_n,\xi,T)$ is the modified FRF, so that the response spectrum estimated within the short window and the modified FRF are both influenced by the same window of length T . Then we have

$$\hat{S}_{xx}(f;T) = S_o |H_w(f;f_n,\xi,T)|^2 \quad (8)$$

As suggested in Figure 3, system parameters can be reliably identified by fitting $\hat{S}_{xx}(f;T)$ to the scaled version of the modified FRF $H_w(f;f_n,\xi,T)$ instead of the original FRF $H(f;f_n,\xi)$. Based on this concept, this study proposes a general framework for spectral-based SI. Within the framework, different types of estimators can be used to estimate the response spectrum within a short window, e.g., instantaneous or marginal spectrum of the wavelet or short time Fourier transform (STFT), or simply traditional Fourier spectra $\hat{S}_{xx}(f;T)$ as discussed here. Accordingly, the FRFs are needed to be modified for each type of spectral estimator. In the next

section, time-frequency transforms used for generating spectral estimators are introduced, followed by the formulation of the corresponding modified FRFs.

2.2 Generalized time-frequency/scale transforms

The continuous wavelet transform of a signal is given by

$$WT(a, t) = \frac{1}{\sqrt{a}} \int x(\tau) \cdot \psi^* \left(\frac{\tau - t}{a} \right) d\tau = \sqrt{a} \int X(f') \cdot \Psi^*(af') e^{j2\pi f't} df' \quad (9)$$

where $x(\tau)$ is the signal, $\psi(\tau)$ is wavelet, $*$ denotes complex conjugate, a is the scale factor,

$X(f') = \int x(\tau) \cdot e^{-j2\pi f'\tau} d\tau$, and $\Psi(f') = \int \psi(\tau) \cdot e^{-j2\pi f'\tau} d\tau$. The right-hand side of Eq. (9)

allows a fast computation of wavelet transform by using the fast Fourier transform (FFT).

Accordingly, a continuous STFT is generalized in a similar manner as the continuous wavelet transform. This generalization permits an arbitrary variable discretization of the frequency, as opposed to the fixed frequency discretization in the discrete STFT. In this way, it is possible to obtain finely discretized spectra to achieve a higher quality of the subsequent curve fitting. The continuous STFT is given by

$$STFT(f, t) = \int x(\tau) \cdot u(\tau - t; T_s) \cdot e^{-j2\pi f(\tau - t)} d\tau = \int x(\tau) \cdot \varphi^*(\tau - t, f; T_s) d\tau \quad (10)$$

where $\varphi(\tau, f; T_s) = u(\tau; T_s) \cdot e^{j2\pi f\tau}$ is just like a “wavelet”, $u(\tau; T_s)$ is the window function of length T_s . Introducing the concept of convolution, we have

$$STFT(f, t) = \int X(f') \cdot \Phi^*(f', f; T_s) \cdot e^{j2\pi f't} df' \quad (11)$$

where

$$\Phi(f', f; T_s) = \mathcal{F} \{ \varphi(\tau, f; T_s) \} = \int (u(\tau; T_s) \cdot e^{j2\pi f\tau}) \cdot e^{-j2\pi f'\tau} d\tau$$

Similarly, Eq. (11) can be calculated by an FFT. A comparison between Eq. (9) and Eq. (11) suggests that the continuous wavelet transform and STFT share the same framework. The only

difference is that the former transforms with respect to scale a , while the latter transforms with respect to frequency f . Accordingly, the squared modulus of the wavelet transform and STFT are called scalogram and spectrogram, respectively.

2.3 Spectral estimators and formulation of modified FRF

The continuous wavelet transform or STFT can generate two types of spectral estimators, namely, instantaneous spectrum and marginal spectrum. The instantaneous spectrum is a slice of the scalogram or spectrogram at a given time (Figure 4), representing frequency density at that instant and given by

$$I_{WT}(a, t_i) = |WT(a, t_i)|^2 \quad (12)$$

$$I_{STFT}(f, t_i) = |STFT(f, t_i)|^2 \quad (13)$$

The marginal spectrum is a frequency density function obtained by integrating scalogram or spectrogram over the time variable (Figure 4), as given by

$$M_{WT}(a) = \int |WT(a, t)|^2 dt \quad (14)$$

$$M_{STFT}(f) = \int |STFT(f, t)|^2 dt \quad (15)$$

It is obvious that the instantaneous spectrum is time-varying. On the other hand, the marginal spectrum does not have time information; however, it can be estimated for data segments within a short boxcar window of length T moving along time, in order to track the time-varying frequency densities.

Next, let us derive the modified FRF for each kind of spectral estimator. The expected instantaneous wavelet or STFT spectrum of a time-varying system at time instant t_i is equivalent to the expected instantaneous spectrum of the corresponding equivalent time-invariant system, given by

$$E[I_{WT}(a, t_i)] = E[|WT(a, t)|^2] = a \int S_{xx}(f') \cdot |\Psi(af')|^2 df' \quad (16)$$

$$E[I_{STFT}(f, t_i)] = E[|STFT(f, t)|^2] = \int S_{xx}(f') \cdot |\Phi(f', f; T_s)|^2 df' \quad (17)$$

where $E[\square]$ denotes the expected value. Substitute Eq. (3) into Eqs. (16) and (17), we have

$$E[I_{WT}(a, t_i)] = S_o \cdot a \int |H(f'; f_n, \xi)|^2 \cdot |\Psi(af')|^2 df' \quad (18)$$

$$E[I_{STFT}(f, t_i)] = S_o \int |H(f'; f_n, \xi)|^2 \cdot |\Phi(f', f; T_s)|^2 df' \quad (19)$$

Then we define the modified FRFs for instantaneous spectra of wavelet transform and STFT as

$$|H_{WT_inst}(a; f_n, \xi)|^2 \square a \int |H(f'; f_n, \xi)|^2 \cdot |\Psi(af')|^2 df' \quad (20)$$

$$|H_{STFT_inst}(f; f_n, \xi)|^2 \square \int |H(f'; f_n, \xi)|^2 \cdot |\Phi(f', f; T_s)|^2 df' \quad (21)$$

Note that the modified FRFs are still functions of system properties f_n and ξ , like the original FRF; while the only difference is that they explicitly incorporate the “wavelet”-like window effects through terms $|\Psi(af')|^2$ and $|\Phi(f', f; T_s)|^2$ in Eqs. (20) and (21).

Similarly, the expected marginal wavelet or STFT spectrum of a time-varying system within a short boxcar window is equivalent to the expected marginal spectrum of the corresponding equivalent time-invariant system, given by

$$E[M_{WT}(a; T)] = a \int \hat{S}_{xx}(f'; T) \cdot |\Psi(af')|^2 df' \quad (22)$$

$$E[M_{STFT}(f; T)] = \int \hat{S}_{xx}(f'; T) \cdot |\Phi(f', f; T_s)|^2 df' \quad (23)$$

Substitute Eq. (8) into Eqs. (22) and (23), we have

$$E[M_{WT}(a; T)] = S_o \cdot a \int |H_w(f'; f_n, \xi, T)|^2 \cdot |\Psi(af')|^2 df' \quad (24)$$

$$E[M_{STFT}(f; T)] = S_o \cdot \int |H_w(f'; f_n, \xi, T)|^2 \cdot |\Phi(f', f; T_s)|^2 df' \quad (25)$$

173 And we define the modified FRFs for marginal spectra of wavelet transform and STFT as

$$174 \quad \left| H_{WT_marg}(a; f_n, \xi, T) \right|^2 \propto a \int \left| H_w(f'; f_n, \xi, T) \right|^2 \cdot \left| \Psi(a f') \right|^2 df' \quad (26)$$

$$175 \quad \left| H_{STFT_marg}(f; f_n, \xi, T) \right|^2 \propto \int \left| H_w(f'; f_n, \xi, T) \right|^2 \cdot \left| \Phi(f', f; T_s) \right|^2 df' \quad (27)$$

176 It is very important to note that not only are the marginal spectra influenced by the “wavelet”-

177 like window through terms $\left| \Psi(a f') \right|^2$ and $\left| \Phi(f', f; T_s) \right|^2$ like instantaneous spectra, but they are

178 also influenced by the length (T) of the boxcar window used to estimate the marginal spectra, as

179 reflected by the term $\left| H_w(f'; f_n, \xi, T) \right|^2$. Based on a theory of spectral estimation given in

180 Reference [34-36], $\left| H_w(f'; f_n, \xi, T) \right|^2$ for the displacement and acceleration responses can be

181 estimated by

$$182 \quad \left| H_w^{(d)}(f; f_n, \xi, T) \right|^2 = \frac{1}{4\xi^2} \text{Re} \left[\left(1 - \frac{j\xi}{\sqrt{1-\xi^2}} \right) \hat{\mathbf{H}}_u \left(\frac{f - f_n \sqrt{1-\xi^2}}{\xi}, \xi T \right) + \left(1 + \frac{j\xi}{\sqrt{1-\xi^2}} \right) \hat{\mathbf{H}}_u \left(\frac{f + f_n \sqrt{1-\xi^2}}{\xi}, \xi T \right) \right] \quad (28)$$

183 and

$$184 \quad \left| H_w^{(a)}(f; f_n, \xi, T) \right|^2 = (2\pi f_n)^4 \left\{ 1 + \frac{1}{4\xi^2} \text{Re} \left[\left(1 - 4\xi^2 + j \frac{\xi(3-4\xi^2)}{\sqrt{1-\xi^2}} \right) \hat{\mathbf{H}}_u \left(\frac{f - f_n \sqrt{1-\xi^2}}{\xi}, \xi T \right) + \left(1 - 4\xi^2 - j \frac{\xi(3-4\xi^2)}{\sqrt{1-\xi^2}} \right) \hat{\mathbf{H}}_u \left(\frac{f + f_n \sqrt{1-\xi^2}}{\xi}, \xi T \right) \right] \right\} \quad (29)$$

185 respectively, where $\hat{\mathbf{H}}_u(f, T)$ is the biased FRF of the first-order system corresponding to the

186 second-order system to be identified here, given as

$$187 \quad \hat{\mathbf{H}}_u(f, T) = \frac{2\pi f_n}{T} \int_0^T [u(t; T) * u(-t; T)] e^{-(2\pi f_n)(1+jf/f_n)t} dt \quad (30)$$

The natural frequency and damping ratio can be reliably identified by fitting instantaneous or marginal spectrum of wavelet transform or STFT (i.e. $E[I_{WT}(a, t_i)]$, $E[I_{STFT}(f, t_i)]$, $E[M_{WT}(a)]$, $E[M_{STFT}(f)]$) to the corresponding modified FRF defined in Eqs. (20), (21), (26) and (27), respectively.

3. Methodology for a MDOF system

By assuming modal damping, an n -DOF system

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F}(t) \quad (31)$$

can be decomposed into a set of SDOF systems by modal analysis, i.e.

$$\ddot{q}_l + 2\xi_l(2\pi f_l)\dot{q}_l + (2\pi f_l)^2 q_l = \Phi_l^T \mathbf{F}(t) \quad (32)$$

where Φ_l is the l th mode shape, f_l and ξ_l are modal frequency and damping, respectively.

The power spectral density matrix of response is expressed as

$$\begin{aligned} \mathbf{S}_{\mathbf{xx}}(f) &= \begin{bmatrix} S_{x_1 x_1} & S_{x_1 x_2} & \cdots & S_{x_1 x_N} \\ S_{x_2 x_1} & S_{x_2 x_2} & \cdots & S_{x_2 x_N} \\ \vdots & \vdots & \ddots & \vdots \\ S_{x_N x_1} & S_{x_N x_2} & \cdots & S_{x_N x_N} \end{bmatrix} \\ &= \mathbf{\Phi} \begin{bmatrix} \Phi_1^T H_1(f; f_1, \xi_1) \\ \vdots \\ \Phi_N^T H_N(f; f_N, \xi_N) \end{bmatrix} \cdot \mathbf{S}_o \cdot \begin{bmatrix} H_1^*(f; f_1, \xi_1) \Phi_1 & \cdots & H_N^*(f; f_N, \xi_N) \Phi_N \end{bmatrix} \mathbf{\Phi}^H \end{aligned} \quad (33)$$

where $S_{x_m x_n}$ is the cross spectra of responses measured at location m and n , $\mathbf{S}_o$ is the spectral density matrix of input, N is the number of DOFs, $H_l(f; f_l, \xi_l)$ is the FRF of the SDOF system corresponding to the l th mode. In the vicinity of the l th mode, Eq. (33) can be approximated by

$$\mathbf{S}_{\mathbf{xx}}(f) \approx \mathbf{\Phi} \text{diag} \begin{bmatrix} 0 & \cdots & c_l |H_l(f; f_l, \xi_l)|^2 & \cdots & 0 \end{bmatrix} \mathbf{\Phi}^H \quad (34)$$

204 where $c_l = \Phi_l^T \mathbf{S}_o \Phi_l$ is a real scale factor. Generalizing Eq. (34), a similar relationship for
 205 instantaneous or marginal spectrum of wavelet transform or STFT can be derived and expressed
 206 as

$$207 \quad \mathbf{G}_{\mathbf{xx}} \approx \mathbf{\Phi} \text{diag} \begin{bmatrix} 0 & \cdots & c_l |H_{l, \text{adapted}}|^2 & \cdots & 0 \end{bmatrix} \mathbf{\Phi}^H \quad (35)$$

208 where $\mathbf{G}_{\mathbf{xx}}$ is the generalized spectra matrix of response; and when wavelet instantaneous
 209 spectral estimator is used,

$$210 \quad \mathbf{G}_{\mathbf{xx}} = E[\mathbf{I}_{WT}(a, t_i)] = E \begin{bmatrix} WT_{x_1 x_1}^c(a, t_i) & WT_{x_1 x_2}^c(a, t_i) & \cdots & WT_{x_1 x_N}^c(a, t_i) \\ WT_{x_2 x_1}^c(a, t_i) & WT_{x_2 x_2}^c(a, t_i) & \cdots & WT_{x_2 x_N}^c(a, t_i) \\ \vdots & \vdots & \ddots & \vdots \\ WT_{x_N x_1}^c(a, t_i) & WT_{x_N x_2}^c(a, t_i) & \cdots & WT_{x_N x_N}^c(a, t_i) \end{bmatrix} \quad (36)$$

211 where $WT_{x_m x_n}^c(a, t_i) = WT_{x_m}(a, t_i) \cdot WT_{x_n}^*(a, t_i)$; when STFT instantaneous spectral estimator is
 212 used,

$$213 \quad \mathbf{G}_{\mathbf{xx}} = E[\mathbf{I}_{STFT}(f, t_i)] = E \begin{bmatrix} STFT_{x_1 x_1}^c(f, t_i) & STFT_{x_1 x_2}^c(f, t_i) & \cdots & STFT_{x_1 x_N}^c(f, t_i) \\ STFT_{x_2 x_1}^c(f, t_i) & STFT_{x_2 x_2}^c(f, t_i) & \cdots & STFT_{x_2 x_N}^c(f, t_i) \\ \vdots & \vdots & \ddots & \vdots \\ STFT_{x_N x_1}^c(f, t_i) & STFT_{x_N x_2}^c(f, t_i) & \cdots & STFT_{x_N x_N}^c(f, t_i) \end{bmatrix} \quad (37)$$

214 where $STFT_{x_m x_n}^c(f, t_i) = STFT_{x_m}(f, t_i) \cdot STFT_{x_n}^*(f, t_i)$; when wavelet marginal spectral estimator
 215 is used,

$$216 \quad \mathbf{G}_{\mathbf{xx}} = E[\mathbf{M}_{WT}(a)] = E \left[\int \begin{bmatrix} WT_{x_1 x_1}^c(a, t) & WT_{x_1 x_2}^c(a, t) & \cdots & WT_{x_1 x_N}^c(a, t) \\ WT_{x_2 x_1}^c(a, t) & WT_{x_2 x_2}^c(a, t) & \cdots & WT_{x_2 x_N}^c(a, t) \\ \vdots & \vdots & \ddots & \vdots \\ WT_{x_N x_1}^c(a, t) & WT_{x_N x_2}^c(a, t) & \cdots & WT_{x_N x_N}^c(a, t) \end{bmatrix} dt \right] \quad (38)$$

217 when STFT marginal spectral estimator is used,

$$\mathbf{G}_{\mathbf{xx}} = E[\mathbf{M}_{STFT}(f)] = E \left[\int \begin{bmatrix} STFT_{x_1 x_1}^c(f, t) & STFT_{x_1 x_2}^c(f, t) & \cdots & STFT_{x_1 x_N}^c(f, t) \\ STFT_{x_2 x_1}^c(f, t) & STFT_{x_2 x_2}^c(f, t) & \cdots & STFT_{x_2 x_N}^c(f, t) \\ \vdots & \vdots & \ddots & \vdots \\ STFT_{x_N x_1}^c(f, t) & STFT_{x_N x_2}^c(f, t) & \cdots & STFT_{x_N x_N}^c(f, t) \end{bmatrix} dt \right] \quad (39)$$

$|H_{l,adapted}|^2$ in Eq. (35) is the corresponding modified FRF of the l th mode and needs to be chosen

as $|H_{l,WT_inst}(a; f_l, \xi_l)|^2$, $|H_{l,STFT_inst}(f; f_l, \xi_l)|^2$, $|H_{l,WT_marg}(a; f_l, \xi_l, T)|^2$ or

$|H_{l,STFT_marg}(f; f_l, \xi_l, T)|^2$ correspondingly.

Applying SVD to $\mathbf{G}_{\mathbf{xx}}$, we have

$$\mathbf{G}_{\mathbf{xx}} = \mathbf{U} \mathbf{S} \mathbf{V}^H \quad (40)$$

where $\mathbf{U}$ and $\mathbf{V}$ are left and right singular vectors; $\mathbf{S}$ is a diagonal matrix of singular values.

The analogy between Eq. (35) and Eq. (40) suggests that the mode shape of the l th mode can be

estimated by the first singular vector at the vicinity of the l th mode. After identifying mode

shapes, the frequencies and damping ratios of p modes in a limited frequency band of the

response spectra can also be identified as follows. We first separate the contributions of different

modes in the response spectra. By assuming that the mode shapes are orthogonal and based on

$$\mathbf{G}_{\mathbf{xx}} = \mathbf{\Phi} \text{diag} \left[c_1 |H_{1,adapted}|^2, \dots, c_p |H_{p,adapted}|^2, 0, \dots, 0 \right] \mathbf{\Phi}^H \quad (41)$$

the contributions of different modes to spectra can be separated by

$$\mathbf{s}_b = \mathbf{\Phi}_b^{-1} \mathbf{G}_{\mathbf{xx}} (\mathbf{\Phi}_b^H)^{-1} \quad (42)$$

where $\mathbf{\Phi}_b = [\Phi_1, \Phi_2, \dots, \Phi_p]$ are the identified p mode shapes in the limited frequency band,

diagonal elements of $\mathbf{s}_b$, i.e., $\text{diag}(\mathbf{s}_b) = [s_1(f), s_2(f), \dots, s_p(f)]$ are the scaled version of

modified FRFs of the p modes, respectively. Therefore, the frequency and damping ratio of each

mode in the frequency band can be identified by fitting $s_i(f)$ to the corresponding modified FRF defined in Section 2.

4. Numerical examples of non-stationary system identification (SI)

4.1 Identification of a time-varying SDOF system

The proposed framework was used to identify a time-varying SDOF system under non-stationary excitation. The non-stationary excitation was simulated by modulating white noise signal with a window function given in Appendix. The system properties are also provided in Appendix. The time-varying frequency and damping ratio are presented in Figure 5. A total of 100 responses were simulated by using 100 realizations of white noise. To compare the performance of different types of spectral estimators in the framework, SI was conducted based on four types of spectra, i.e. instantaneous and marginal spectra of wavelet transform and STFT, respectively. For the instantaneous spectrum, the continuous Morlet wavelet transform and STFT were performed on the full length of data. Ideally, the instantaneous spectrum at t_i obtained from different realizations of response should be averaged to estimate the expected instantaneous spectrum; however, in practice, only one realization is available. In this case, by assuming local stationarity and ergodicity, one can estimate the expected instantaneous spectrum by averaging the instantaneous spectra within a short window. In this study, the instantaneous spectra within a moving short window of 30 s with a shift of 10 s were averaged to estimate the expected spectrum. To obtain the marginal spectrum, the continuous Morlet wavelet transform and STFT were performed on data within a short analysis window of 30 s, respectively. Then the window was translated along the time with a step of 10 s, to track the time-varying system properties. Note that ideally the expected marginal spectrum is desirable for SI. But only one realization of marginal spectrum is available for non-stationary response in practice, thus it is used for SI

instead. The frequency and damping ratio were identified through fitting the marginal and averaged instantaneous spectra to the corresponding modified FRFs. The SI results of 100 simulations by using four types of spectra are compared, with both mean values and coefficient of variation (c.o.v) presented in Figure 5. A quite comparable accuracy is observed for the four types of spectra. Note that instantaneous spectra generally tend to result in lower uncertainties (lower c.o.v) in damping estimates. In practice, the choice between instantaneous and marginal spectra also depends on the application needs. The instantaneous spectrum might be more suitable for an offline analysis, while the marginal spectrum can be readily used for an online analysis as it can be obtained from each streaming short data segment. In the second half of the time histories, an increased uncertainty (c.o.v) of frequency estimates and a decreased uncertainty (c.o.v) of damping estimates are also observed. This is mainly owing to the increase of damping ratio of the system over time, confirmed by a comparison study between the cases where the frequency is constant and the damping varies and where the damping stays constant and the frequency changes (Guo, 2015). This result suggests that the proposed framework achieves more reliable damping estimates for systems with higher damping ratios. One should also note that using longer analysis window will lead to less uncertainty in SI; however, a compromise has to be made between the ability to track time-varying information and the uncertainty of estimates.

It is well known that the central frequency and window length are the governing parameters of wavelet transform and STFT, respectively. Therefore, a study was conducted to investigate the influence of these parameters on the SI results. Due to the space limitation, only the study for the Morlet wavelet marginal spectra based method are presented in detail, while the analysis based on other spectra estimators can be found in Guo [37]. In this study, responses of

different segment lengths (represented as the approximate number of cycles) from a time-invariant system (system properties are given in Appendix) under white noise excitation were used for SI. The frequency and damping ratio of the system are 2.0 Hz and 2 %, respectively. For each data length, different central frequencies were used in wavelet transform. A total of 100 simulations were conducted and the results are shown in Figure 6. It is seen that the accuracy of the SI results is not sensitive to the central frequency. This is because the FRF is modified based on the wavelet with the same central frequency as the wavelet used for estimating the response spectrum. Similarly, the wavelet instantaneous spectrum based SI is not sensitive to the central frequency either. And the accuracy of SI based on marginal and instantaneous spectra of STFT is not sensitive to the window length. However, one should avoid using a very large central frequency for the wavelet transform or window length for the STFT because the resulting spectra will not be sufficiently smooth to guarantee a good curve fitting for the subsequent SI.

4.2 *Identification of a time-varying MDOF system with closely spaced modes*

The performance of the proposed framework is evaluated by identifying a time-varying three-DOF (3DOF) system with closely spaced modes under non-stationary excitation. Similar to the SDOF example, the non-stationary excitation was simulated by multiplying the white noise signal with a time varying window function (given in Appendix). The time-varying system properties are also provided in the Appendix. As Section 4.1 has shown that the SI based on four different types of spectral estimators leads to comparable results, here only the Morlet wavelet based marginal spectrum was used for SI. The window length for calculating the wavelet marginal spectrum is 300 s. The results are presented in Figure 7. The three time-varying frequencies were identified with a very high accuracy (the maximum error and c.o.v are 0.33% and 0.51%). The modal assurance criterion (MAC) values of Modes 2 and 3 are lower than that

of Mode 1, indicating that the accuracy of mode shape identification decreases when modes become closely spaced. As the damping increases, the MAC values of Modes 2 and 3 decrease. This is because higher damping ratios increase the bandwidth of the spectra and result in a more severe energy overlap between the two modes, rendering modal separation more difficult. The time-varying damping ratios were also successfully identified. The mean of the maximum errors of damping estimates of the three modes is 26%. In the first half of the time histories where the damping ratios of the system are lower, the uncertainties of damping estimates are higher than those of the second half. This trend is the same as that observed in the SDOF example. The c.o.v for the damping estimates for Modes 2 and 3 are lower than that for Mode 1, as higher frequency components have more oscillations in the analysis windows, leading to less uncertainty in spectral estimates. The existence of closely spaced modes does not seem to influence the damping estimation.

As the moving window concept is used in the non-stationary SI, the ability of the proposed framework to track the time-varying information is essentially the ability to identify systems using short data. Next, the performance of the proposed framework using short data is compared to the traditional frequency domain method, FDD, and time-domain method, reference based covariance driven SSI (Figure 8), which have been popular in the identification of time-varying systems using the moving windows [5-8, 24-26, 38]. Note that within the proposed framework, Morlet wavelet marginal spectra were used for SI. A 3DOF time-invariant system subjected to white noise excitation was used to simulate the short response data. The system properties are given in Appendix. The modal parameters of the system are listed in Table 1. It is noted that all the three methods can identify natural frequencies very well. For damping estimation, the proposed method was either comparable with or slightly better than the SSI;

while due to the significant windowing effect, the FDD performed much worse than the other two with much higher bias error and higher uncertainty (c.o.v) when data was very short. For the mode shape identification, the three methods were comparable for the first mode; while for the closely spaced second and third modes, the proposed method and SSI outperformed the FDD.

The effect of measurement noise on the proposed framework was also studied using the aforementioned 3DOF time-invariant system. A window of 300 s was used in the Morlet wavelet marginal spectrum based SI. The white noise with different noise to signal ratios (NSRs) was added to the response to simulate the measured response. The NSR is defined as the ratio between standard deviations of white noise and response (i.e., $NSR = std(white\ noise) / std(response)$). For each NSR, a total of 100 simulations were conducted. The SI results are shown in Figure 9. It is observed that both the error and c.o.v of the identified system properties are not sensitive to the noise, suggesting that the proposed method is robust in a noisy environment.

5. Application to full-scale data of Burj Khalifa

A long-term SHM system was implemented in the world's tallest building, Burj Khalifa, based on an advanced SmartSync monitoring technology (Kijewski-Correa et al. 2013b). Three triggered acceleration responses (1-hour each) were collected from Burj Khalifa on April 6, 2013 during a windstorm. The translational responses (both x- and y-directions) measured from four different levels (generically referred to as L1-4 from lower level to higher level) of the structure were analyzed. The responses show non-stationary characteristics through amplitude modulations (e.g. Figure 10). Note that in light of the confidentiality of building information, the amplitude of response, frequencies and damping ratios are normalized. The first two translational modes (Mode 1(Y) and Mode 1 (X)) are closely spaced. A strong beating phenomenon with

considerable amplitude modulations is observed in the x-direction response (e.g. Figure 10). The proposed framework using wavelet marginal spectra was tested on the three non-stationary data sets. As the fundamental frequency is very low, a relatively larger window (30-min) was used to identify system properties. The SI results for the first two modes are compared to those by the SSI in Figure 11 and Figure 12. The natural frequencies identified by the two methods are very close, while the damping ratios identified by the two methods show a consistent trend. The mode shapes by the two methods also show a good agreement, with the minimum MAC of 94%. It is seen that the frequencies and mode shapes of the structure remain constant during the windstorm. The variation in damping ratios might be caused by the amplitude dependence and the inherent uncertainties in damping estimates.

6. Concluding remarks

This study presents a rather novel output-only non-stationary SI framework based on spectra derived from the wavelet and short time Fourier transforms. To address the problem in damping identification caused by the short windows necessary for analyzing non-stationary data, this framework modifies the theoretical frequency response function (FRF) to explicitly account for the windowing effect, and therefore enables SI by directly using instantaneous or marginal spectra derived from the wavelet or short time Fourier transform. In this manner, the instantaneous or time-localized marginal spectrum of the response and the corresponding modified FRF are both influenced by the same window, therefore the response spectrum can be directly fitted to a scaled version of the modified FRF to identify frequency and damping ratio at each time instant.

The performance of the proposed framework has been evaluated using both numerical and full-scale data. The proposed framework proves to be able to track time-varying system

properties for both SDOF and MDOF systems with even closely spaced modes under non-stationary excitations. This framework successfully eliminates the generally recognized limitations of spectral-based SI methods in handling short data and achieves comparable accuracy and uncertainty as SSI, a popular time domain SI scheme. Through its inherent capability to filter time-varying frequency contents, the proposed frequency domain framework is more expedient in tracking time-varying systems than the time domain methods, which usually need additional advanced filtering techniques (e.g. wavelet filtering) to pre-process the data when the system is time-varying [4]. Also, the implementation of the proposed framework is very straightforward compared with the time-domain methods, which often requires more sophisticated techniques [39] to select the model order and/or size of the Hankle matrix when the noise effects and/or modes with low vibration energy are present.

The proposed framework may be extended to more general non-stationary cases where the excitation is not sufficiently white at the vicinity of structural modes. By virtue of spectrum employed in this framework, the non-white spectrum of the non-stationary excitation may be directly modeled in frequency domain and taken into account in SI. For example, one can model non-white wind excitation as a linear function of frequency and introduce the slope as an extra parameter to be identified in SI [40].

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Appendix

The white noise signal used in the numerical examples is simulated by generating normal distributed random sequence. The time domain window function used to modulate the white noise and simulate the non-stationary excitation is expressed as

$$E(t) = \begin{cases} \exp(-(t-t_o) \cdot a), & t \geq t_o \\ \exp((t-t_o) \cdot a), & t < t_o \end{cases} \quad (\text{A.1})$$

For the example in Section 4.1, $t_o = 150$ s, and $a = 0.01$; while for the example in Section 4.2, $t_o = 1500$ s, and $a = 0.001$.

The system properties of the time-varying SDOF system in Section 4.1 are given as follows.

$$m(t) = m^c, \quad 0 \leq t < 300 \text{ s} \quad (\text{A.2})$$

where $m^c = 1000$ kg.

$$k(t) = \begin{cases} 18 \cdot k^c, & t < 120 \text{ s} \\ (18 - (t-120)/15) \cdot k^c, & 120 \leq t < 180 \text{ s} \\ 14 \cdot k^c, & 180 \leq t < 300 \text{ s} \end{cases} \quad (\text{A.3})$$

where $k^c = 10000$ N/m.

$$c(t) = 2 \cdot \xi(t) \cdot \omega(t) \cdot m(t) \quad (\text{A.4})$$

where $\omega(t) = \sqrt{k(t)/m(t)}$, and

$$\xi(t) = \begin{cases} 0.02, & t < 120 \text{ s} \\ 0.02 + (t-120)/2000, & 120 \leq t < 180 \text{ s} \\ 0.05, & 180 \leq t < 300 \text{ s} \end{cases} \quad (\text{A.5})$$

The system properties of the time-invariant SDOF system in Section 4.1 are provided below.

$$m = m^c, \quad k = 15.8 \cdot k^c, \quad \xi = 0.02 \quad (\text{A.6})$$

$$c = 2\xi\omega m \quad (\text{A.7})$$

where $\omega = \sqrt{k/m}$.

The system properties of the time-varying 3DOF system in Section 4.2 are given as follows.

$$\mathbf{M}(t) = m^c \cdot \mathbf{I}, \quad 0 \leq t < 3000 \text{ s} \quad (\text{A.8})$$

where $\mathbf{I}$ is a 3-by-3 identity matrix.

$$\mathbf{K}(t) = k^c \cdot \begin{bmatrix} 4 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 4 \end{bmatrix} \cdot r(t) \quad (\text{A.9})$$

$$r(t) = \begin{cases} 1.2, & t < 1200 \text{ s} \\ 1.2 - (t - 1200) / 3000, & 1200 \leq t < 1800 \text{ s} \\ 1.0, & 1800 \leq t < 3000 \text{ s} \end{cases} \quad (\text{A.10})$$

$$\mathbf{C}(t) = \mathbf{\Phi}^{-1}(t) \begin{bmatrix} 2\omega_1(t)\xi_1(t) & 0 & 0 \\ 0 & 2\omega_2(t)\xi_2(t) & 0 \\ 0 & 0 & 2\omega_3(t)\xi_3(t) \end{bmatrix} \mathbf{\Phi}(t) \quad (\text{A.11})$$

where $\mathbf{\Phi}(t) = [\Phi_1 \quad \Phi_2 \quad \Phi_3]$ and $\omega_l(t)$ ($l = 1, 2, 3$) can be found by eigenvalue problem, i.e.

$$\mathbf{M}^{-1}(t)\mathbf{K}(t)\Phi_l(t) = \omega_l^2\Phi_l(t) \quad (\text{A.12})$$

$$\xi_{1,2,3}(t) = \begin{cases} 0.005, & t < 1200 \text{ s} \\ 0.005 + (t - 1200) \cdot 7 / 60000, & 1200 \leq t < 1800 \text{ s} \\ 0.012, & 1800 \leq t < 3000 \text{ s} \end{cases} \quad (\text{A.13})$$

The system properties of the time-invariant 3DOF system in Section 4.2 are given below.

$$\mathbf{M}^c = m^c \cdot \mathbf{I} \quad (\text{A.14})$$

$$\mathbf{K}^c = k^c \cdot \begin{bmatrix} 4 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 4 \end{bmatrix} \quad (\text{A.15})$$

$$\mathbf{C}^c = (\mathbf{\Phi}^c)^{-1} \begin{bmatrix} 2\omega_1^c \xi_1^c & 0 & 0 \\ 0 & 2\omega_2^c \xi_2^c & 0 \\ 0 & 0 & 2\omega_3^c \xi_3^c \end{bmatrix} \mathbf{\Phi}^c \quad (\text{A.16})$$

where $\xi_1^c = \xi_2^c = \xi_3^c = 0.01$, $\mathbf{\Phi}^c = [\Phi_1^c \quad \Phi_2^c \quad \Phi_3^c]$ and ω_l^c ($l=1,2,3$) can be found by eigenvalue problem

$$(\mathbf{M}^c)^{-1} \mathbf{K}^c \mathbf{\Phi}_l^c = (\omega_l^c)^2 \mathbf{\Phi}_l^c \quad (\text{A.17})$$

Table 1. Modal parameters of 3DOF time-invariant system

Mode	Frequency (Hz)	Damping ratio (%)	Mode shape
1	0.5667	1	$[0.0103 \ 0.0281 \ 0.0103]^T$
2	1.0066	1	$[-0.0224 \ 0.0000 \ 0.0224]^T$
3	1.0948	1	$[-0.0199 \ 0.0145 \ -0.0199]^T$

Figure 1. Concept of windowing effect on bandwidth of spectrum

Figure 2. Effect of a boxcar window on damping estimation for a SDOF system: bias error of damping estimates versus window lengths (represented as number of cycles in response)

Figure 3. Comparison between response spectrum estimated within a short window with the scaled version of (a) original FRF, and (b) modified FRF

Figure 4. Instantaneous and marginal spectra derived from wavelet transform or STFT

Figure 5. SI results for the time-varying SDOF system

Figure 6. Study of influence of central frequency on SI accuracy

Figure 7. SI results for the time-varying 3DOF system

Figure 8. Comparison of proposed method, SSI and FDD

Figure 9. Noise effect on SI accuracy

Figure 10. Acceleration response measured at L2

Figure 11. Identified frequencies and damping ratios of **Burj** Khalifa

Figure 12. Identified mode shapes of **Burj** Khalifa

Figure Captions

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